RD Sharma Solutions Class 11 Maths Chapter 20 Ex 20.2

Geometric Progressions Ex 20.2 Q 1

Let the three number in G.P be $\frac{a}{r}, a, ar$

Sum of these numbers $= \frac{a}{r} + a + ar = 65$

$$3375 = \text{Product of these numbers}$$

$$3375 = \left(\frac{a}{r}\right)(a)(ar) = a^3$$

$$a^3 = (5)^3 \times (3)^3 = (15)^3$$

$$\Rightarrow \quad a = 15$$

$$a\left(\frac{1}{r} + 1 + r\right) = 65$$

$$15\left(\frac{1}{r} + 1 + r\right) = \frac{65}{15} = \frac{13}{3}$$

$$\frac{1 + r + r^2}{r} = \frac{13}{3}$$

$$3 + 3r + 3r^2 = 13r$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$r = 3, \ \frac{1}{3} \qquad r = \frac{1}{3} \text{ or } r = 3$$

$\therefore$ G.P. is a, ar, ar^2

$\therefore$ G.P. is 45, 15, 5 or 5, 15, 45

Geometric Progressions Ex 20.2 Q 2

Let the three numbers be a, ar, ar^2 in G.P., where a is first teror and r is the common ratio.

Then,

$$a + ar + ar^2 = 38$$

$$a\left(1 + r + r^2\right) = 38 \qquad \text{---(i)}$$

and

$$(a)(ar)(ar)^2 = 1728$$

$$a^3r^3 = 1728 = 4^3 3^3 = (12)^3$$

$$a^3 = \frac{12^3}{r^3} \Rightarrow \frac{12}{r} = a$$

Putting $a = \frac{12}{r}$ in (i)

$$\frac{12}{r}\left(1 + r + r^2\right) = 38$$

$$12 + 12r + 12r^2 = 38r$$

$$12r^2 - 26r + 12 = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(3r - 3) - 2(3r - 3) = 0$$

$$r = \frac{3}{2}, \frac{2}{3}$$

$$a = \frac{12}{\frac{3}{2}} = 8 \text{ or } \frac{12}{\frac{2}{3}} = 18$$

$\therefore$ G.P. is 8, 12, 18.

Geometric Progressions Ex 20.2 Q 3

Let the first three terms of G.P. are $\frac{a}{r}$, a, ar

Here,

$$\frac{a}{r}+a+ar=\frac{13}{12} \qquad \text{---(i)}$$

and $\frac{a}{r}\times a\times ar=-1$

$\Rightarrow$ $a^3=-1$

$\Rightarrow$ $a=-1$

Put $a=-1$ in equation (i),

$$\frac{-1}{r}+(-1)-r=\frac{13}{12}$$

$\Rightarrow$ $-1-r-r^2=\frac{13}{12}r$

$\Rightarrow$ $-12-12r-12r^2=13r$

$\Rightarrow$ $12r^2+12r+13r+12=0$

$\Rightarrow$ $12r^2+25r+12=0$

$\Rightarrow$ $12r^2+16r+9r+12=0$

$\Rightarrow$ $4r(3r+4)+3(3r+4)=0$

$\Rightarrow$ $(4r+3)(3r+4)=0$

$r=\frac{-3}{4},\ \frac{-4}{3}$

So,

Required G.P. is, $\frac{4}{3},\ -1,\ \frac{3}{4},\ \dots$

or $\frac{3}{4},\ -1,\ \frac{4}{3},\ \dots$

Geometric Progressions Ex 20.2 Q4

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$ then product of these numbers $\left(\frac{a}{r}\right)(a)(ar)$

$$\Rightarrow \quad a^3 = 125 = 5^3$$

$$a = 5$$

Also, sum of these products in pair

$$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar) = 87\frac{1}{2} = \frac{195}{2}$$

$$\frac{a^2}{r} + a^2 r + a^2 = a^2\left(\frac{1}{r} + r + 1\right)$$

$$= (5)^2\left(\frac{1 + r^2 + r}{r}\right) = \frac{195}{2}$$

$$1 + r^2 + r = \left(\frac{195}{2 \times 25}\right)^r$$

$$2\left(1 + r^2 + r\right) = \frac{39}{5}r$$

$$10 + 10r^2 + 10r = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r - 5) - 2(2r - 5) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

$\therefore$ G.P. is $\frac{a}{r}, a, ar$

$$10, 5, \frac{5}{2}, \ldots \text{ or } \frac{5}{2}, 5, 10, \ldots$$

Geometric Progressions Ex 20.2 Q5

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$

then product of them is $\left(\frac{a}{r}\right)(a)(ar) = 21$ ---(i)

$$= \frac{a}{r}\left(1 + r + r^2\right) = 21$$

and sum of their squares

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = a^2\frac{\left(1 + r^2 + r^4\right)}{r^2} = 189 \quad \text{---(ii)}$$

Now,

$$a\left(1 + r + r^2\right) = 21r \quad \text{---(iii)}$$

Then, $a^2\left(1 + r + r^2\right)^2 = 441r^2$ [suqaring]

$$a^2\left(1 + r^2 + r^4\right) + 2a^2r\left(1 + r + r^2\right) = 441r$$

$$189r^2 + 2ar \times 21r = 441r^2$$

Dividing both sides by $21r^2$

$$9 + 2a = 21$$

$$2a = 21 - 9 = 12$$

$$a = 6 \Rightarrow a = 6$$

Putting in(iii)

$$6\left(1 + r + r^2\right) = 21r$$

$$6 + 6r + 6r^2 - 21r = 0$$

$$6r^2 - 15r + 6 = 0$$

$$6r^2 - 12r - 3r + 6 = 0$$

$\Rightarrow$ $6r(r - 2) - 3(r - 2) = 0$

$$r = 2, \ \frac{1}{2}$$

$\therefore$ G.P. is 3, 6, 12 or 12, 6, 3.

Geometric Progressions Ex 20.2 Q6

Let the numbers are: $\frac{a}{r}, a$ and ar.

Then

$$\frac{a}{r}+a+ar=14$$

Again the numbers $a+1, ar+1$ and ar^2-1 are in A.P, therefore

$$2(a+1)=(ar-1)+\left(\frac{a}{r}+1\right)$$

$$2(a+1)=ar+\frac{a}{r}$$

$$2(a+1)=14-a$$

$$3a=12$$

$$a=4$$

Now we have

$$\frac{4}{r}+4+4r=14$$

$$2-5r+2r^2=0$$

$$2r^2-4r-r+2=0$$

$$2r(r-2)-1(r-2)=0$$

$$(r-2)(2r-1)=0$$

$$r=2, \frac{1}{2}$$

Thus the numbers are: 2, 4, 8 or 8, 4, 2.

Geometric Progressions Ex 20.2 Q7

Let the number in G.P. are $\frac{a}{r}$, a, ar

So,

$$\frac{a}{r} \times a \times ar = 216$$

$\Rightarrow \quad a^3 = 216$

$\Rightarrow \quad a = 6$

And also given,

$$\frac{a}{r} + 2,\ a + 8,\ ar + 6 \text{ are in A.P.}$$

$$2(a+8) = \left(\frac{a}{r} + 2\right) + (ar + 6)$$

$\Rightarrow \quad 2(6+8) = \left(\frac{6+2r}{r}\right) + 6r + 6$

$\Rightarrow \quad 28r = 6 + 2r + 6r^2 + 6r$

$\Rightarrow \quad 6r^2 - 20r + 6 = 0$

$\Rightarrow \quad 6r^2 - 18r - 2r + 6 = 0$

$\Rightarrow \quad 6r(r-3) - 2(r-3) = 0$

$\Rightarrow \quad (r-3)(6r-2) = 0$

$$r = 3,\ r = \frac{1}{3}$$

So,

Required G.P. is 18, 6, 2, ...

or, 2, 6, 18, ...

Geometric Progressions Ex 20.2 Q8

Let three numbers in G.P. are $\frac{a}{r}$, a, ar

Here,

$$\frac{a}{r} \times a \times ar = 729$$

$\Rightarrow \quad a^3 = 729$

$\Rightarrow \quad a = 9$

And,

$$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(\frac{a}{r} \times ar\right) = 819$$

$\Rightarrow \quad \frac{81}{r} + 81r + 81 = 819$

$\Rightarrow \quad \frac{9}{r} + 9r + 9 = 91$

$\Rightarrow \quad 9 + 9r^2 + 9r = 91r$

$\Rightarrow \quad 9r^2 - 82r + 9 = 0$

$\Rightarrow \quad 9r^2 - 81r - r + 9 = 0$

$\Rightarrow \quad 9r(r-9) - 1(r-9) = 0$

$$r = 9,\ \frac{1}{9}$$

So, required G.P. are

81, 9, 1, ...

or, 1, 9, 81, ...

Geometric Progressions Ex 20.2 Q9

Let the numbers are $\frac{a}{r}, a$ and ar. Then we have

$$\frac{a}{r}+a+ar=\frac{39}{10}$$

And

$$\frac{a}{r}\cdot a\cdot ar=1$$
$$a^3=1$$
$$a=1$$

Now we have

$$\frac{1}{r}+1+r=\frac{39}{10}$$
$$1+r+r^2=\frac{39}{10}r$$
$$r^2-\frac{29}{10}r+1=0$$
$$10r^2-29r+10=0$$
$$(2r-5)(5r-2)=0$$
$$r=\frac{5}{2},\frac{2}{5}$$

Thus the numbers are: either $\frac{2}{5},1,\frac{5}{2}$ or $\frac{5}{2},1,\frac{2}{5}$.