RD Sharma Solutions Class 11 Maths Chapter 20 Ex 20.5

Geometric Progressions Ex 20.5 Q 1

Here,

$a,\ b,\ c$ are in G.P.

$b^2 = ac$ ---(i)

Now,

$2\log b = \log b^2$

$= \log ac$

$2\log b = \log a + \log c$

$\log b - \log a = \log c - \log b$

$\Rightarrow$ $\log a,\ \log b,\ \log c$ are in A.P.

Geometric Progressions Ex 20.5 Q 2

Here,

$a,\ b,\ c$ are in G.P., so

$b^2 = ac$

$\frac{2}{\log_b m} = 2\log_m b$

$= \log_m b^2$

$= \log_m ac$

$= \log_m a + \log_m c$

$\frac{2}{\log_b m} = \frac{1}{\log_a m} + \frac{1}{\log_c m}$

$\Rightarrow$ $\frac{1}{\log_b m} - \frac{1}{\log_a m} = \frac{1}{\log_c m} - \frac{1}{\log_b m}$

$\Rightarrow$ $\frac{1}{\log_a m},\ \frac{1}{\log_b m},\ \frac{1}{\log_c m}$ are in A.P.

Geometric Progressions Ex 20.5 Q 3

Here,

a, b, c are in A.P.

$2b = a + c$ ---(i)

and a, b, d are in G.P., so

$b^2 = ad$ ---(ii)

Now,

$$(a-b)^2 = a^2 + b^2 - 2ab$$
$$= a^2 + ad - a(a+c)$$

Using equation (i) and (ii)

$$= a^2 + ad - a^2 - ac$$
$$= ad - ac$$
$$(a-b)^2 = a(d-c)$$
$$\frac{(a-b)}{a} = \frac{(d-c)}{(a-b)}$$

$\Rightarrow$ $a, (a-b), (d-c)$ are in G.P.

Geometric Progressions Ex 20.5 Q 4

Here, Let R be common ratio,

a_p, a_q, a_r, a_s of AP are in GP

$$R = \frac{a_q}{a_p} = \frac{a_r}{a_q}$$
$$= \frac{a_q - a_r}{a_p - a_q} \quad \text{(Ratio property)}$$
$$= \frac{[a+(q-1)d] - [a+(r-1)d]}{[a+(p-1)d] - [a+(q-1)d]}$$

$$= \frac{(q-r)d}{(p-q)d}$$

$$R = \frac{q-r}{p-q} \text{ ------ } (1)$$

Now,

$$R = \frac{a_r}{a_q} = \frac{a_s}{a_r}$$

$$= \frac{a_r - a_s}{a_q - a_r} \quad \text{(Ratio property)}$$

$$= \frac{[a+(r-1)d]-[a+(s-1)d]}{[a+(q-1)d]-[a+(r-1)d]}$$

$$= \frac{(r-s)d}{(q-r)d}$$

$$R = \frac{r-s}{q-r} \text{ ------ } (2)$$

From equation as (1) and (2)

$$\frac{q-r}{p-q} = \frac{r-s}{p-r}$$

$\Rightarrow$ $(p-q), (q-r), (r-s)$ are in GP

Geometric Progressions Ex 20.5 Q 5

$$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c} \text{ are in A.P.}$$

$$\frac{2}{2b} = \frac{1}{(a+b)} + \frac{1}{(b+c)}$$

$$\frac{1}{b} = \frac{b+c+a+b}{(a+b)(b+c)}$$

$$\frac{1}{b} = \frac{2b+c+a}{ab+ac+b^2+bc}$$

$$ab+ac+b^2+bc = 2b^2+bc+ba$$

$$b^2+ac = 2b^2$$

$$b^2 = ac$$

So,

$a,\ b,\ c$ are in G.P.

Geometric Progressions Ex 20.5 Q 6

$$x^a = x^{\frac{b}{2}} z^{\frac{b}{2}} = z^c = \lambda \text{ (say)}$$

$$x = \lambda^{\frac{1}{a}},\ z = \lambda^{\frac{1}{c}}$$

$$x^{\frac{b}{2}} \times z^{\frac{b}{2}} = \lambda$$

$$\lambda^{\frac{1}{a}\left(\frac{b}{2}\right)} \times \lambda^{\frac{b}{2}\times\frac{1}{c}} = \lambda$$

$$\lambda^{\frac{b}{2a}+\frac{b}{2c}} = \lambda^1$$

$$\frac{b}{2a} + \frac{b}{2c} = 1$$

$$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

$\Rightarrow$ $\frac{1}{a},\ \frac{1}{b},\ \frac{1}{c}$ are in A.P.

Geometric Progressions Ex 20.5 Q 7

$k + 9,\ k - 6,\ 4$ are in G.P.

$(k - 6)^2 = (k + 9)4$

$k^2 + 36 - 12k = 4k + 36$

$k^2 - 16k = 0$

$k(k - 16) = 0$

$k = 0,\ k = 16$

Geometric Progressions Ex 20.5 Q 8

Let $a-d,\ a,\ a+d$ be numbers in A.P.

Here,

$$a-d+a+a+d=15$$
$$3a=15$$
$$a=5$$

Find

$$[(5-d)+1],\ (5+3),\ [(5+d)+9] \text{ are in G.P.}$$

$\Rightarrow$ $(6-d),\ 8,\ (14+d)$ are in G.P.

$$(8)^2=(6-d)(14+d)$$
$$64=84+6d-14d-d^2$$
$$d^2+8d-20=0$$
$$(d+10)(d-2)=0$$
$$d=2,\ -10$$

So,

Numbers are $3,5,7$ or $15,5,-5$

Geometric Progressions Ex 20.5 Q 9

Let three numbers in A.P. be $a-d,\ a+d$

Here,

$$a-d+a+a+d=21$$
$$3a=21$$
$$a=7$$

And,

$(7-d),\ (7-1),\ (7+d)+1$ are in G.P.

$(7-d),\ 6,\ (8+d)$ are in G.P.

$$(6)^2=(7-d)(8+d)$$
$$36=56+7d-8d-d^2$$
$$d^2+d-20=0$$
$$(d+5)(d-4)=0$$

$d = 4, \ -5$

So,

Numbers are $3, 7, 11$ or $12, 7, 2$.

Geometric Progressions Ex 20.5 Q 10

Here,

a, b, c are in A.P.

Let $a = A - d, \ b = A, \ c = A + d$

Here,

$$a + b + c = 18$$
$$A - d + A + A + d = 18$$
$$3A = 18$$
$$A = 6$$

And,

$(a + 4), \ (b + 4), \ (c + 36)$ are in G.P.

$(6 - d + 4), \ (6 + 4), \ (6 + d + 36)$ are in G.P.

$(10 - d), \ (10), \ (42 + d)$ are in G.P.

$$(10)^2 = (10 - d)(42 + d)$$
$$100 = 420 + 10d - 42d - d^2$$
$$d^2 + 32d - 320 = 0$$
$$(d + 40)(d - 8) = 0$$
$$d = -40, \ 8$$

So,

Numbers of $-2, 6, 14$ or $46, 6, -34$.

Geometric Progressions Ex 20.5 Q 11

Let numbers are a, ar, ar^2

$$a + ar + ar^2 = 56 \text{ - - - - - } (1)$$

$(a-1), (ar-7), (ar^2-21)$ are in AP

$$\Rightarrow \quad 2(ar-7) = a - 1 + ar^2 - 21$$

$$= (ar^2 + a) - 22$$

$$2ar - 14 = (56 - ar) - 22 \qquad [\text{using equation } (1)]$$

$$2ar - 14 = 34 - ar$$

$$3ar = 48$$

$$ar = 16 \text{ - - - - - - - } (2)$$

$$a = \frac{16}{r}$$

Put a in equation (1),

$$\frac{16 + 16r + 16r^2}{r} = 56$$

$$16 + 16r + 16r^2 = 56r$$

$$16r^2 - 40r + 16 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2, \ \frac{1}{2}$$

Put r in equation (2),

$$ar = 16$$

for $r = \frac{2}{a} = 8$

for $r = \frac{1}{2}$, $a = 32$

thus, there numbers are

$$8, 16, 32$$

in both cases.