RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.7

Straight lines Ex 23.7 Q1(i)

$P = 5, \alpha = 60^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 60^\circ + y\sin 60^\circ = 5$

$\Rightarrow \quad x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 5$

$\Rightarrow \quad x + \sqrt{3}y = 10$

Straight lines Ex 23.7 Q1(ii)

$P = 4, \alpha = 150^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 150^\circ + y\sin 150^\circ = 4$

$\Rightarrow \quad -x \times \frac{\sqrt{3}}{2} + y \times \frac{1}{2} = 4$

$\Rightarrow \quad -\sqrt{3}x + y = 8$

Straight lines Ex 23.7 Q1(iii)

$P = 8, \alpha = 225^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 225^\circ + y\sin 225^\circ = 8$

$\Rightarrow \quad -x \times \frac{1}{\sqrt{2}} - y \times \frac{1}{\sqrt{2}} = 8$

$\Rightarrow \quad x + y + 8\sqrt{2} = 0$

Straight lines Ex 23.7 Q1(iv)

$P = 8, \alpha = 300^\circ$

$x\cos\alpha + y\sin\alpha = P$

$\Rightarrow \quad x\cos 300^\circ + y\sin 300^\circ = 8$

$\Rightarrow \quad x \times \frac{1}{2} - y \times \frac{\sqrt{3}}{2} = 8$

$\Rightarrow \quad x - \sqrt{3}y = 16$

Straight lines Ex 23.7 Q2

Given, Inclination of perpendicular line (L) passing through origin is 30°

$\Rightarrow$ Slope=Tan $30^\circ = \frac{1}{\sqrt{3}}$

Slope of perpedicular line (M) which is perpendicular to line L is $-\sqrt{3}$

So equation of line M is $y=-\sqrt{3}x+c$

Given perpendicular distance from origin to line M is 4

$4=\frac{c}{2} \Rightarrow c=8$

So equation of line M is $y=-\sqrt{3}x+8$

Straight lines Ex 23.7 Q3

Here,

$p = 4$ and $\alpha = 15^\circ$

The equation of line is

$x\cos\alpha + y\sin\alpha = p \quad ---(1)$

$x\cos 15^\circ + y\sin 15^\circ = 4$

$\cos 15^\circ = \cos(45 - 30) = c\,os\,45\cos 30 + \sin 45\sin 30$

$\left(\because \cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi\right)$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{1}{2\sqrt{2}}\left(\sqrt{3} + 1\right)$

$\sin 15^\circ = \sin(45 - 30) = \sin 45\cos 30 \qquad \cos 45\sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{1}{2\sqrt{2}}\left(\sqrt{3} - 1\right)$

Putting in (1)

$x \times \frac{1}{2\sqrt{2}}\left(\sqrt{3} + 1\right) + y \times \frac{1}{2\sqrt{2}}\left(\sqrt{3} - 1\right) = 4$

$x\left(\sqrt{3} + 1\right) + y\left(\sqrt{3} - 1\right) = 8\sqrt{2}$

Straight lines Ex 23.7 Q4

Here $P = 3$

and $\alpha = \tan^{-1}\left(\frac{5}{12}\right)$

$\Rightarrow \cos\alpha = \frac{12}{13}, \sin\alpha = \frac{5}{13}$

Equation of straight line is:

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{12}{13}\right) + y\left(\frac{5}{13}\right) = 3$

$12x + 5y = 39$

Straight lines Ex 23.7 Q5

Here $P = 2$, $\sin\alpha = \frac{1}{3}$

$\Rightarrow \cos\alpha = \frac{2\sqrt{2}}{3}$

The equation of straight line is

$x\cos\alpha + y\sin\alpha = P$

$x\left(\frac{2\sqrt{2}}{3}\right) + y\left(\frac{1}{3}\right) = 2$

$2\sqrt{2}x + y = 6$

Straight lines Ex 23.7 Q6

Given:

$p = \pm 2$

$\tan\alpha = \frac{5}{12}$

The equation of line is

$x\cos\alpha + y\sin\alpha = \pm p$

$x\frac{12}{13} + y\frac{5}{13} = \pm 2$

$12x + 5y \pm 26 = 0$

Straight lines Ex 23.7 Q7

Here,

p = perpendicular distance from origin = 7

Angle made with y-axis is 150°,

∴ Angle made with x-axis is 30°

$\cos\alpha = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\sin\alpha = \sin 30^\circ = \frac{1}{2}$

The equation of line is

$x\cos\alpha + y\sin\alpha = p$

$x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 7$

$\sqrt{3}x + y = 14$

Straight lines Ex 23.7 Q8

We have,

$$\sqrt{3}x+y+2=0$$

$$-\sqrt{3}x-y=2$$

$$\left(\frac{-\sqrt{3}}{2}\right)x+\left(\frac{-1}{2}\right)y=1$$

This same as $x\cos\theta+y\sin\theta=p$

Therefore, $\cos\theta=\frac{-\sqrt{3}}{2}, \sin\theta=-\frac{1}{2}$ and $p=1$

$\theta=210^{\circ}$ and $p=1$

$\theta=\frac{7\pi}{6}$ and $p=1$

Straight lines Ex 23.7 Q9

Perpendicular from origin makes an angle of 30° with y-axis, thus making 60° woth x-axis

Area of triangle is = $96\sqrt{3}$

$$\frac{1}{2}\times 2p\times\frac{2p}{\sqrt{3}}=96\sqrt{3}$$

$$p^2=\frac{96\sqrt{3}\times\sqrt{3}}{2}=48\times 3=2\times 2\times 2\times 2\times 3\times 3$$

$p = 12$

$x\cos\alpha+y\sin\alpha=p$

$x\cos 60^{\circ}+y\sin 60^{\circ}=12$

$$x\times\frac{1}{2}+y\frac{\sqrt{3}}{2}=12$$

$x+\sqrt{3}y=24$