RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.11

Straight lines Ex 23.11 Q1(i)

If the lines are concurrent then point of intersection of any two lines satisfies the third line

$15x - 18y + 1 = 0 \qquad ---(1)$

$12x + 10y - 3 = 0 \qquad ---(2)$

$6x + 66y - 11 = 0 \qquad ---(3)$

Solving (1) and (2)

$$x = \frac{18y - 1}{15}$$

$$12\left(\frac{18y - 1}{15}\right) + 10y - 3 = 0$$

$$216y - 12 + 150y - 45 = 0$$

$$366y = 57$$

$$y = \frac{57}{366} = \frac{19}{122}$$

$$\Rightarrow x = \frac{18y - 1}{15}$$

$$= \frac{18 \times \frac{19}{122} - 1}{15}$$

$$= \frac{18 \times 19 - 122}{122 \times 15}$$

$$= \frac{342 - 122}{1730}$$

$$= \frac{220}{1730}$$

$$= \frac{22}{173}$$

Putting x and y in (3)

$$6\left(\frac{22}{173}\right) + 66\left(\frac{19}{122}\right) - 11 = 0$$

$$6 \times 22 \times 122 + 66 \times 19 \times 173 - 11 \times 173 \times 122 = 0$$

$$0 = 0$$

Straight lines Ex 23.11 Q1(ii)

$3x - 5y - 11 = 0,\ 5x + 3y - 7 = 0,\ x + 2y = 0$

$$3x - 5y - 11 \quad ---(1)$$

$$5x + 3y - 7 = 0 \quad ---(2)$$

$$x + 2y = 0 \quad ---(3)$$

Solving (1) and (2)

$$x = -2y$$

$$5(-2y) + 3y - 7 = 0$$

$$-10y + 3y - 7 = 0$$

$$-7y = y$$

$$y = -1$$

$$\Rightarrow x = 2$$

substituting x and y in (1)

$$3(2) - 5(-1) - 11 = 0$$

$$6 + 5 - 11 = 0$$

$$0 = 0$$

Hence, the lines are concurrent

Straight lines Ex 23.11 Q1(iii)

$\frac{x}{a} + \frac{y}{b} = 1,\ \frac{x}{b} + \frac{y}{a} = 1,\ y = x$

$$bx + ay = ab,\ \ ax + by = ab$$

Put $y = x$

$$bx + ax = ab,\ \ ax + bx = ab$$

Hence the lines are concurrent

Straight lines Ex 23.11 Q2

The three lines are concurrent if they have the common point of intersection.

$$2x-5y+3=0 \quad \text{---}(1)$$

$$x-2y+1=0 \quad \text{---}(2)$$

Solving (1) and (2)

$$2x = 5y - 3$$

$$x = \frac{5y-3}{2}$$

$$\frac{5y-3}{2} - 2y + 1 = 0$$

$$5y - 3 - 4y + 2 = 0$$

$$y = 0$$

$$\Rightarrow \quad x = \frac{5y-3}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$$

Substituting x and y is $5x - 9y + \lambda = 0$

$$5(1) - 9(1) + \lambda = 0$$

$$5 - 9 + \lambda = 0$$

$$\lambda = 4$$

Straight lines Ex 23.11 Q3

The three lines are

$$y = m_1x + c_1 \quad \text{---}(1)$$

$$y = m_2x + c_2 \quad \text{---}(2)$$

$$y = m_3x + c_3 \quad \text{---}(3)$$

Collinear or they meet at a point only when they have common point of intersection

Solving (1) and (2) for x and y

$$m_1x + c_1 = m_2x + c_2$$

$$x(m_1 - m_2) = c_2 - c_1$$

$$x = \frac{c_2 - c_1}{m_1 - m_2}$$

$$\Rightarrow \quad y = m_1x + c_1$$

$$= m_1\left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$$

$$= m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1$$

Putting x and y in (3)

$$m_1c_2 - m_1c_1 = m_3\frac{(c_2 - c_1)}{m_1 - m_2} + c_3$$

$$m_1^2c_2 - m_1m_2c_2 - m_1m_2c_1 + m_2^2c_1 = m_3c_2 - m_3c_1 + m_1c_3 - m_2c_3$$

$$\Rightarrow \quad m_1(c_2 - c_3) + m_2(c_3 - c_1) + m_3(c_1 - c_2) = 0$$

Straight lines Ex 23.11 Q4

If the lines are concurrent, then the lines have common point of intersection.

The given line are

$$p_1x + q_1y = 1 \quad ---(1)$$

$$p_2x + q_2y = 1 \quad ---(2)$$

$$p_3x + q_3y = 1 \quad ---(3)$$

Solving (1) and (2)

$$x = \frac{1 - q_1y}{p_1}$$

$$p_2\left(\frac{1 - q_1y}{p_1}\right) + q_2y = 1$$

$$p_2 = p_2q_1y + p_1q_2y = p_1$$

$$y = \frac{p_1 - p_2}{p_1q_2 - p_2q_1} \Rightarrow x = \frac{1 - q_1\left(\frac{p_1 - p_2}{p_1q_2 - p_2q_1}\right)}{p_1}$$

Putting x, y in (3)

$$p_3[(p_1q_2 - p_2q_1) - q_1p_1 - q_1p_2][p_1q_2 - p_2q_1] + q_3p_1(p_1 - p_2) = 1$$

$$(p_1p_3q_2 - p_2p_3q_1 - p_1p_3q_1 + p_2p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$(p_1p_3q_2 - p_1p_3q_1)(p_1q_2 - p_2q_1) + q_3p_1^2 - q_3p_1p_2 = 1$$

$$p_1^2p_3q_2^2 - p_1p_2p_3q_1q_2 - p_1^2p_3q_1q_2 + p_1p_2p_3q_1^2 + q_3p_1^2 - q_3p_1p_2 = 1 \quad ---(1)$$

Also if $(p_1q_1)(p_2q_2)(p_3q_3)$ are collinear

Then,

$$p_1(q_2 - q_3) + p_2(q_3 - q_1) + p_3(q_1 - q_3) = 0$$

From (1)

$$p_1\left[p_1p_3q_2^2 - p_2p_3q_1q_2 - p_1p_3q_1q_2 + p_2p_3q_1^2 + q_3p_1 - q_3p_2\right] = 1$$
$$p_1\left[p_3q_2\left(p_1q_2 - p_2q_1\right) - p_3q_1\left(p_1q_2 - p_2q_1\right) + q_3\left(p_1 - p_2\right)\right] = 1$$
Hence, the points are collinear

Straight lines Ex 23.11 Q5

The three lines are concurrent if they have the common point of intersection

$$(b+c)x + ay + 1 = 0$$
$$(c+a)x + by + 1 = 0$$
$$(a+b)x + cy + 1 = 0$$

Solving (1) and (2)

$$y = \frac{-1-(b+c)x}{a}$$

Putting in (2)

$$(c+a)x + b\frac{(-1-(b+c)x)}{a} + 1 = 0$$
$$acx + a^2x + b - b^2x - bcx + a = 0$$
$$x\left(ac + a^2 - b^2 - bc\right) = b - a$$
$$x\left(ac - bc + a^2 - b^2\right) = b - a$$
$$x\left(c(a-b) + (a-b)(a+b)\right) = b - a$$
$$x(c+a+b) = -1 \qquad \left[\text{Cancelling}(a-b) \text{ both sides}\right]$$
$$x = \frac{-1}{a+b+c}$$
$$y = \frac{-1 + \frac{(b+c)(-1)}{a+b+c}}{a} = \frac{-a-b-c-b-c}{a(a+b+c)}$$

Putting the value of x, y in (3);

$$(a+b)\left(\frac{-1}{a+b+c}\right) + c\left(\frac{-a-2b-2c}{a(a+b+c)}\right) + 1 = 0$$
$$-a^2 - ba - ac - 2bc - 2c^2 + a^2 + ab + ac = 0$$
$$0 = 0$$

Hence, the lines are concurrent

Straight lines Ex 23.11 Q6

If the three lines are concurrent then the point of intersection of (1) and (2) should verify the (3) line, where

$$ax + a^2y + 1 = 0 \qquad ---(1)$$

$$bx + b^2y + 1 = 0 \qquad ---(2)$$

$$cx + c^2y + 1 = 0 \qquad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1 - a^2y}{a} \Rightarrow b\left(\frac{-1 - a^2y}{a}\right) + b^2y + 1 = 0$$

$$-b - a^2by + ab^2y + a = 0$$

$$y = \frac{b - a}{ab(b - a)} = \frac{1}{ab}$$

$$\Rightarrow \qquad x = \frac{1 - a^2 \times \frac{1}{ab}}{a} = \frac{1 - \frac{a}{b}}{a} = \frac{b - a}{ab}$$

Putting in (3)

$$c\left(\frac{b - a}{ab}\right) + c^2\left(\frac{1}{ab}\right) + 1 = 0$$

$$bc - ac + c^2 + ab = 0$$

$$bc + c^2 - ac + ab = 0$$

$$c(b + c) - a(c - b) = 0$$

$\Rightarrow$ Either $c = b \Rightarrow 2bc = 0 \Rightarrow 2c^2 = 0 \Rightarrow c = 0$

Straight lines Ex 23.11 Q7

If a, b, c are in $A.P.$

$b - a = c - b$

$2b = a + c$ [Common difference]

To prove that the straight lines are concurrent then they have the common point of intersection.

$$ax + 2y + 1 = 0 \quad ---(1)$$

$$bx + 3y + 1 = 0 \quad ---(2)$$

$$cx + 4y + 1 = 0 \quad ---(3)$$

Solving (1) and (2)

$$x = \frac{-1 - 2y}{a}$$

Put in (2)

$$b\left(\frac{-1-2y}{a}\right) + 3y + 1 = 0$$

$$y = \frac{b - a}{3a - 2b} \Rightarrow x = \frac{-1 - \frac{2(b-a)}{3a-2b}}{a} = \frac{-3a + 2b - 2b + 2a}{a(3a - 2b)}$$

$$x = \frac{-1}{3a - 2b}$$

Putting x, y in (3)

$$c\left(\frac{-1}{3a-2b}\right) + 4\left(\frac{b-a}{3a-2b}\right) + 1 = 0$$

$$-c + 4b - 4a + 3a - 2b = 0$$

$$-a + 2b - c = 0$$

$$-a + a + c - c = 0$$

$$0 = 0$$

Hence, Proved