RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.12

Straight lines Ex 23.12 Q1

Equation of line through $(2,3)$ is

$$y-y_1 = m(x-x_1) \qquad ---(1)$$

$(2,3)$ is (x_1y_1)

Since the line is parallel to $3x-4y+5=0$

$\Rightarrow$ The slope will be equal

Slope of $3x-4y+5=0$

$$4y = 3x+5$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

$\Rightarrow$ $$m = \frac{3}{4}$$

Substituting m and (x_1y_1) is (1)

$$y-3 = \frac{3}{4}(x-2)$$

$$4y-12 = 3x-6$$

$$3x-4y = -12+6 = -6$$

$$3x-4y+6=0$$

Straight lines Ex 23.12 Q2

Any equation passing through $(3,-2)$ and perpendicular to givven line is

$$y-y_1 = -\frac{1}{m}(x-x_1) \qquad ---(1)$$

Where (x_1-y_1) is $(3,-2)$ and m is slope of line.

$\frac{-1}{m}$ is taken as lines are perpendicular

Finding slope of line $x-3y+5=0$

$$3y = x+5$$

$$y = \frac{x}{3} + \frac{5}{3}$$

$\Rightarrow$ $$m = \frac{1}{3}$$

Substituting the value of m and $(x_1 - y_1)$ in (1)

$$y-(-2)=-\frac{1}{\frac{1}{3}}(x-3)$$

$$y+2=-3(x-3)=-3x+9$$

$$3x+y=7$$

Straight lines Ex 23.12 Q3

Any line which is perpendicular bisector means line is perpendicular to the given line and one end point is the mid-point of that line.

The line joining $\underset{(x_1,y_1)}{(1,3)}$ and $\underset{(x_2,y_2)}{(3,1)}$.

Has the mid-point

$$x=\frac{x_1+x_2}{2},\ y=\frac{y_1+y_2}{2}$$

$$\Rightarrow \quad (x_1y_1)=\left(\frac{1+3}{2},\ \frac{3+1}{2}\right)=(2,2)$$

Also slope of line is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{1-3}{3-1}=\frac{-2}{2}=-1$$

So, the slope of required line is 1 (negative reciprocal of slope)

Thus, the equation of perpendicular bisector is

$$y-y_1=\frac{-1}{m}(x-x_1)$$

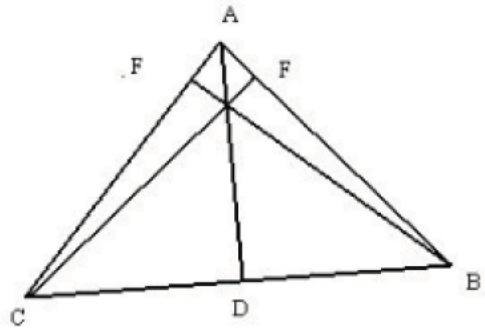
$$y-2=1(x-2)$$

$$y-2=x-2$$

$$y=x$$

Straight lines Ex 23.12 Q4

Let the perpendiculars of the triangle on the side AB, BC and AC be CF, AD and FB respectively.



Slope of the side AB $= \frac{4-2}{1+3} = \frac{2}{4} = \frac{1}{2}$

Corresponding slope of CF $= -\frac{1}{1/2} = -2$

[since $m_1 \times m_2 = -1$]

Equation of CF, $y - y_1 = m(x - x_1)$

$y + 3 = -2(x+5)$ [Putting co-ordinates of C in place of x_1 and y_1]

$y + 3 = -2x - 10$

$y = -2x - 13$

Slope of the side BC $= \frac{2+3}{-3+5} = \frac{5}{2}$

Corresponding slope of AD $= -\frac{1}{5/2} = -\frac{2}{5}$

Equation of AD,

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{2}{5}(x - 1)$

$5y - 20 = -2x + 2$

$5y = -2x - 22$

Slope of the side AC $= \frac{4+3}{1+5} = \frac{7}{6}$

Corresponding slope of FB $= -\frac{1}{7/6} = -\frac{6}{7}$

Equation of FB,

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-6}{7}(x+3)$

$7y - 14 = -6x - 18$

$7y = -6x - 4$

Equation of AD, 2x+5y+22=0
Equation of CF, 2x+y+13=0
Equation of FB, 6x+7y+4=0

Straight lines Ex 23.12 Q5

Required equation of line is

$y - y_1 = m'(x - x_1)$ ---(1)

Point is $(x_1 y_1) = (0, -4)$

It is perpendicular to line $\sqrt{3}x - y + 5 = 0$

$\Rightarrow$ Slope is $y = mx + c$

$y = \sqrt{3}x + 5$

$m = \sqrt{3}$

$m' = \frac{-1}{m} = \frac{-1}{\sqrt{3}}$

Putting m' and $(x_1 y_1)$ in (1)

$y - (-4) = \frac{-1}{\sqrt{3}}(x - 0)$

$y + 4 = \frac{-x}{\sqrt{3}}$

$x + \sqrt{3}y + 4\sqrt{3} = 0$

Straight lines Ex 23.12 Q6

Here,

Let l be line mirror and B is image of A

Let m be slope of line l

So,

$$m(\text{slope of } AB) = -1$$

$$m\left(\frac{2-1}{5-2}\right) = -1$$

$$m\left(\frac{1}{3}\right) = -1$$

$$m = -3$$

M is mid point of AB

$$M = \left(\frac{2+5}{2}, \frac{2+1}{2}\right)$$

$$M = \left(\frac{7}{2}, \frac{3}{2}\right)$$

Equation line l is,

$$y - y_1 = m(x - x_1)$$

$$y - \frac{3}{2} = (-3)\left(x - \frac{7}{2}\right)$$

$$\frac{2y-3}{2} = -3x + \frac{21}{2}$$

$$2y - 3 = -6x + 21$$

$$6x + 2y = 24$$

$$3x + y = 12$$

Straight lines Ex 23.12 Q7

Any line is given by equation

$$y-y_1 = m(x-x_1) \qquad ---(1)$$

Where $(x_1 y_1)$ is (α, β)

And m is negative reciprocal of slope of line $lm+my+n=0$.

i.e; $y = \frac{-lx}{m} - \frac{n}{m}$

$\Rightarrow$ Slope of line $= \frac{-l}{m}$

Putting the data in (i), we get

$$y-\beta = \frac{m}{l}(x-\alpha)$$

$$ly+mx = m\alpha + l\beta$$

$$m(x-\alpha) = l(y-\beta)$$

Straight lines Ex 23.12 Q8

Let the equation of the required line be $y-y_1=m(x-x_1)$, where 'm' denotes the slope of the line and (x_1,y_1) be the point through which the line passes.
Since the x-intercept of the line is 1 on the positive direction of the x-axis therefore the line passes through (1,0)
Also, $2x-3y=5$

$$3y = 2x-5$$

$$y = \frac{2x}{3} - \frac{5}{3}$$

Therefore, the slope of the given line is 2/3.

Slope of the required line $= \frac{-1}{2/3} = -\frac{3}{2}$

Therefore the equation of the required line is

$$y-y_1 = m(x-x_1)$$

$$y-0=\frac{2}{3}(x-1)$$

$$y=-\frac{3}{2}(x-1)$$

$$2y=-3x+3$$

The equation of the required line is 3x+2y -3=0

Straight lines Ex 23.12 Q9

Slope of line through the points $\underset{(x_1,\ y_1)}{(a,\ 2a)}$, $\underset{(x_2,\ y_2)}{(-2,\ 3)}$

$$\Rightarrow \quad m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{3-2a}{-2-a}$$

Also, slope of line $x-ay=1$ in the form $y=mx+c$

$$4x+3y+5=0$$

$$y=\frac{-4}{3}x-\frac{5}{3}$$

$$\Rightarrow \quad m_2=\frac{-4}{3}$$

If two lines are perpendicular then, $m_1m_2=-1$

$$\left(\frac{3-2a}{-2-a}\right)\left(\frac{-4}{3}\right)=-1$$

$$-12+8a=6+3a$$

$$5a=18$$

$$a=\frac{18}{5}$$