RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.13

Straight lines Ex 23.13 Q1(i)

Writing the equation in the form

$$y = mx + c$$

$$3x + y + 12 = 0$$

$$y = -3x - 12$$

$\Rightarrow \quad m_1 = -3$

Also

$$x + 2y - 1 = 0$$

$$2y = 1 - x$$

$$y = \frac{1}{2} - \frac{x}{2}$$

$\Rightarrow \quad m_2 = \frac{-1}{2}$

Angle between the lines

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-3 - \left(\frac{-1}{2}\right)}{1 + (-3)\left(\frac{-1}{2}\right)}\right|$$

$$= \left|\frac{-3 + \frac{1}{2}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-6 + 12}{2}}{\frac{2 + 3}{2}}\right|$$

$$= \left|\frac{-5}{5}\right| = 1$$

$\Rightarrow \quad \text{angle} = \frac{\pi}{4}$

Straight lines Ex 23.13 Q(ii)

Finding slopes of the lines by converting the equation in the form

$$y = mx + c$$

$$3x - y + 5 = 0$$

$$\Rightarrow \quad y = 3x + 5$$

$$\Rightarrow \quad m_1 = 3$$

Also

$$x - 3y + 1 = 0$$

$$3y = x + 1$$

$$y = \frac{x}{3} + \frac{1}{3}$$

$$\Rightarrow \quad m_2 = \frac{1}{3}$$

Thus angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{m_1 m_2}\right|$$

$$= \left|\frac{3 - \frac{1}{3}}{1 + 3 \times \frac{1}{3}}\right| = \left|\frac{\frac{9-1}{3}}{1+1}\right|$$

$$= \left|\frac{\frac{8}{3}}{2}\right| = \left|\frac{8}{6}\right| = \frac{4}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Straight lines Ex 23.13 Q(iii)

To find angle between the lines, convert the equations in the form

$y = mx + c$

$3x + 4y - 7 = 0$

$\Rightarrow \quad 4y = -3x + 7$

$y = \frac{-3}{4}x + \frac{7}{4}$

$\Rightarrow \quad m_1 = \frac{-3}{4}$

Also, $4x - 3y + 5 = 0$

$\Rightarrow \quad 3y = 4x + 5$

$\Rightarrow \quad y = \frac{4}{3}x + \frac{5}{3}$

$\Rightarrow \quad m_1 = \frac{4}{3}$

The angle between the lines is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$= \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 + \frac{(-3)}{4}\left(\frac{4}{3}\right)}\right| = \left|\frac{\frac{-3}{4} - \frac{4}{3}}{1 - 1}\right|$$

$\Rightarrow \quad \theta = \frac{\pi}{2}$ or 90^0

Straight lines Ex 23.13 Q(iv)

To find angle convert the equation in the form $y = mx + c$

$x - 4y = 3$

$\Rightarrow \quad 4y = x - 3$

$y = \frac{x}{4} - \frac{3}{4}$

$\Rightarrow \quad m_1 = \frac{1}{4}$

Also, $6x - y = 11$

$$y = 6x - 11$$

$$\Rightarrow \quad m_2 = 6$$

Thus, angle between the lines is

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{4} - 6}{1 + \frac{1}{4} \times 6}\right|$$

$$= \left|\frac{\frac{-23}{4}}{1 + \frac{3}{2}}\right| = \left|\frac{\frac{-23}{4}}{\frac{5}{2}}\right|$$

$$\theta = \tan^{-1}\left(\frac{23}{10}\right)$$

Straight lines Ex 23.13 Q(v)

Converting the equation in the form

$$y = mx + c$$

$$y = \frac{\left(mn + n^2\right)}{m^2 - mn} x + \frac{n^3}{\left(m^2 - mn\right)}$$

$$\Rightarrow \quad m_1 = \frac{mn + n^2}{m^2 - mn}$$

Also, $y = \frac{\left(mn - n^2\right)}{nm + m^2} x + \frac{m^3}{nm + m^2}$

$$\Rightarrow \quad m_2 = \frac{mn - n^2}{nm + m^2}$$

Thus, angle between 2 lines is $\tan\theta$

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\left(\frac{mn + n^2}{m^2 - mn}\right) - \left(\frac{mn - n^2}{nm + m^2}\right)}{1 + \left(\frac{mn + n^2}{m^2 - mn}\right)\left(\frac{mn - n^2}{nm + m^2}\right)}\right|$$

$$= \left|\frac{m^2n^2 + m^3n + n^3m + n^2m^2 - m^3n + m^2n^2 + n^2m^2 - mn^3}{m^3n + m^4 - m^2n^2 - m^3n + m^2n^2 - mn^3 + mn^3 - n^4}\right|$$

$$= \left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

$$\Rightarrow \quad \theta = \tan^{-1}\left|\frac{4m^2n^2}{m^4 - n^4}\right|$$

Straight lines Ex 23.13 Q2

Slope of line $2x - y + 3 = 0$

$$\text{is } \frac{-2}{-1} = \frac{(\text{coefficient of } x)}{(\text{coefficient of } y)} = 2$$

$\therefore \quad m_1 = 2 \qquad$ ---(i)

Slope of line $x + y + 2 = 0$

$$\text{is } \frac{-1}{1} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } y)}$$

$\therefore \quad m_2 = -1 \qquad$ ---(ii)

Acute angle between lines

$$\theta = \tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \tan^{-1}\left|\frac{2 - (-1)}{1 - (2)(-1)}\right|$$

$$= \tan^{-1}\left|\frac{3}{1 - 2}\right| = \tan^{-1}\left|\frac{3}{1}\right| = \tan^{-1}|3|$$

Straight lines Ex 23.13 Q3

Let $ABCD$ be a quadrilateral

$$AB = \sqrt{(0-2)^2 + (2+1)^2}$$

Using distance formula

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$BC = \sqrt{(2-0)^2 + (3-2)^2} = \sqrt{4+1} = \sqrt{5}$$

$$CD = \sqrt{(4-2)^2 + (0-3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$DA = \sqrt{(4-2)^2 + (0+1)^2} = \sqrt{4+1} = \sqrt{5}$$

Since opposite sides $(AB$ and $CD)$ and $(BC$ and $DA)$ are equal

$\therefore$ The given quadrilateral is a parallelogram.

Straight lines Ex 23.13 Q4

The equation between the points

$\underset{(x_1, y_1)}{(2,0)}$ and $\underset{(x_2, y_2)}{(0,3)}$

Slope of line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m_1 = \frac{3-0}{0-2} = \frac{-3}{2}$$

Also, slope of line $x + y = 1$

Converting in the form $y = mx + c$

$$y = 1 - x$$

$\Rightarrow \quad m_2 = -1$

Thus, $\tan\theta$ = angle between the lines

$$\Rightarrow \quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{-3}{2} - (-1)}{1 + \left(\frac{-3}{2}\right)(-1)}\right| = \left|\frac{\frac{-3}{2} + 1}{1 + \frac{3}{2}}\right|$$

$$= \left|\frac{\frac{-3+2}{2}}{\frac{2+3}{2}}\right| = \left|\frac{\frac{-1}{2}}{\frac{5}{2}}\right| = \frac{1}{5}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{5}\right)$$

Straight lines Ex 23.13 Q5

Let l_1, be the line joining AO and
Let l_2 be the line joining BO

Then, line l_1 is $y - 0 = \left(\frac{0 - x_1}{0 - y_1}\right)(x - 0)$

$$yy_1 - x_1x = 0$$

Then, $m_1 = \frac{x_1}{y_1}$

Then line l1 is $y - 0 = \left(\frac{0 - x_2}{0 - y_2}\right)(x - 0)$

Then, $m_2 = \frac{x_2}{y_2}$

Then, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| = \left|\frac{\frac{x_1}{y_1} - \frac{x_2}{y_2}}{1 + \frac{x_1}{y_1}\frac{x_2}{y_2}}\right|$

$$= \left|\frac{x_1y_2 - y_1x_2}{y_1y_2 + x_1x_2}\right|$$

From triangle.

$$AC = \sqrt{(AB)^2 + (BC)^2}$$

$$= \sqrt{\left(m_1^2 + m_2^2 - 2m_1m_2\right) + (1 + m_1m_2)^2}$$

$$= \sqrt{m_1^2 + m_2^2 - 2m_1m_2 + 1 + m_1^2m_2^2 + 2m_1m_2}$$

$$= \sqrt{m_1^2 + m_2^2 + 1 + m_1^2m_2^2}$$

$$\cos\theta = \frac{BC}{AC} = \frac{1 + m_1m_2}{\sqrt{m_1^2 + m_2^2 + m_1^2m_2^2 + 1}}$$

$$= \frac{1 + \frac{x_1}{y_1}\frac{x_2}{y_2}}{\sqrt{\frac{x_1^2}{y_1^2} + \frac{x_2^2}{y_2^2} + \frac{x_1^2x_2^2}{y_1^2y_2^2} + 1}}$$

$$= \frac{\frac{y_1y_2 + x_1x_2}{y_1y_2}}{\sqrt{\frac{x_1^2y_2^2 + x_2^2y_1^2 + x_1^2x_2^2 + y_1^2y_2^2}{y_1^2y_2^2}}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2\left(y_2^2 + x_2^2\right) + y_1^2\left(y_2^2 + x_2^2\right)}}$$

$$= \frac{y_1y_2 + x_1x_2}{\sqrt{x_1^2 + y_1^2}\sqrt{y_2^2 + x_2^2}}$$

Hence proved.