RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.15

Straight lines Ex 23.15 Q1

Distance of a point (x_1, y_1) from $ax + by + c = 0$ is

$$= \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Here, $a = 3,\ b = -5,\ c = 7,\ x_1 = 4,\ y_1 = 5$

$$\therefore \text{ Distance } = \left|\frac{3(4) - 5(5) + 7}{\sqrt{3^2 + 5^2}}\right|$$

$$= \left|\frac{12 - 25 + 7}{\sqrt{9 + 25}}\right| = \left|\frac{6}{\sqrt{34}}\right| \text{ units.}$$

Straight lines Ex 23.15 Q2

Equation of line passing through $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ is

$$y - \sin\phi = \left(\frac{\sin\phi - \sin\theta}{\cos\phi - \cos\theta}\right)(x - \cos\phi)$$

$$y - \sin\phi = \left(\frac{2\cos\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}{-2\sin\frac{\theta+\phi}{2}\sin\frac{\phi-\theta}{2}}\right)(x - \cos\phi)$$

$$y - \sin\phi = -\cot\left(\frac{\theta+\phi}{2}\right)(x - \cos\phi)$$

$$x\cot\left(\frac{\theta+\phi}{2}\right) + y - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right) = 0$$

Distance of this line from origin,

$$= \left|\frac{ax_1 + by_1 + c}{a^2 + b^2}\right|$$

$$= \left|\frac{0 + 0 - \sin\phi - \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\sqrt{\left(\cos\left(\frac{\theta+\phi}{2}\right)\right)^2 + 1}}\right|$$

$$= \frac{\sin\phi + \cos\phi\cot\left(\frac{\theta+\phi}{2}\right)}{\operatorname{cosec}\left(\frac{\theta+\phi}{2}\right)}$$

$$= \sin\phi \sin\left(\frac{\theta+\phi}{2}\right) + \cos\phi \, \text{co}\left(\frac{\theta+\phi}{2}\right)$$

$$= \cos\left(\frac{\theta+\phi}{2} - \phi\right)$$

$$= \cos\left(\frac{\theta+\phi-2\phi}{2}\right)$$

$$D = \cos\left(\frac{\theta-\phi}{2}\right)$$

Straight lines Ex 23.15 Q3

Line formed from joining $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - a\sin\beta = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha} \times x - a\cos\beta$$

$$\Rightarrow \quad y - a\sin\beta = \frac{2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)}{-2\sin\left(\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)} \times (x - a\cos\beta)$$

$$\Rightarrow \quad y - a\sin\beta = -\cot\left(\frac{\beta+\alpha}{2}\right)(x - a\cos\beta)$$

$$\Rightarrow \quad y + \cot\left(\frac{\alpha+\beta}{2}\right)x - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta = 0$$

Then, the length of perpendicular

$$\Rightarrow \quad \left|\frac{0(y) + 0 - a\cos\beta\cot\left(\frac{\beta+\alpha}{2}\right) - a\sin\beta}{\sqrt{1+\cot^2\left(\frac{\alpha+\beta}{2}\right)}}\right|$$

$$\Rightarrow \quad \frac{a\cos\beta\cot\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta}{\operatorname{cosec}\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow \quad a\cos\beta\cos\left(\frac{\alpha+\beta}{2}\right) + a\sin\beta\sin\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow \quad a\cos\left(\frac{\alpha-\beta}{2}\right) \qquad \left[\text{using } \cos A\cos B + \sin A\sin B = \cos(A-B)\right]$$

Hence, proved.

Straight lines Ex 23.15 Q4

Let (h, k) be the point on the line $2x + 11y - 5 = 0$

$$\Rightarrow \quad 2h + 11k - 5 = 0 - - - - - (1)$$

Let p and q be length of perpendicular from (h, k) on lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$ so,

$$p = q$$

$$\frac{24h + 7k - 20}{\sqrt{(24)^2 + (7)^2}} = \frac{4h - 3k - 2}{\sqrt{(4)^2 + (-3)^2}}$$

$$\frac{24h + 7k - 20}{\sqrt{576 + 49}} = \frac{4h - 3k - 2}{\sqrt{25}}$$

$$\frac{24h + 7k - 20}{25} = \frac{4h - 3k - 2}{5}$$

$$24h + 7k - 20 = 20h - 15k - 10$$

$$4h = -22k + 10$$

$$4\left(\frac{5 - 11k}{2}\right) = -22k + 10 \qquad [\text{Using equation } (1)]$$

$10 - 22k = -22k + 10$
$LHS = RHS$

So,

Distance $24x + 7y = 20$ and $4x - 3y - 2 = 0$ from any point on the line $2x + 11y - 5 = 0$ is equal.

Straight lines Ex 23.15 Q5

The point of intersection of two lines can be calculated by solving the equations

Solving $2x + 3y = 21$ and $3x - 4y + 11 = 0$, we get the point of intersection as $P(3,-5)$

Distance of P from $8x - 6y + 5 = 0$ is

Here, $a = 8,\ b = -6,\ c = 5,\ x_1 = 3,\ y_1 = 5$

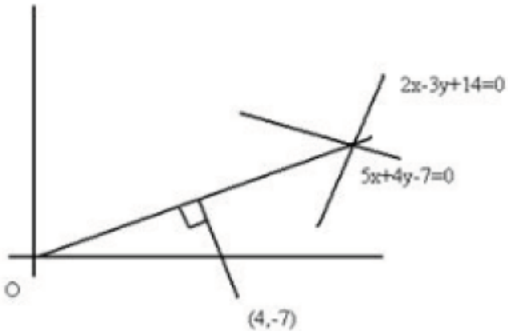
$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\Rightarrow \quad \frac{|8(3) - 6(-5) + 5|}{\sqrt{64 + 36}}$$

$$\Rightarrow \quad \frac{|24 + 30 + 5|}{\sqrt{100}} = \frac{|59|}{10}$$

$$\Rightarrow \quad \frac{59}{10}$$

Straight lines Ex 23.15 Q6



The point of intersection of the lines $2x-3y+14=0$ and $5x+4y-7=0$ can be found out by solving these equations.

Solving these equations we get, $x=-\frac{35}{23}$ and $y=\frac{252}{69}$

Equation of line joining origin and the point $\left(-\frac{35}{23},\frac{252}{69}\right)$

is $y=mx$, where m $=\frac{\frac{252}{69}}{-\frac{35}{23}}=-\frac{12}{5}$

Therefore the equation of required line is $y=-\frac{12x}{5}$

$12x+5y=0$

Perpendicular distance from (4,-7) to $12x+5y=0$ is

$$p=\left|\frac{12(4)+5(-7)}{\sqrt{12^2+(-5)^2}}\right|=\frac{13}{13}=1$$

Straight lines Ex 23.15 Q7

Any point on x-axis is $\underset{(x_1, y_1)}{(\pm a, 0)}$

Perpendicular distance from a line $bx + ay = ab$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = a$$

Where,

$$a = b,\ b = a,\ c = -ab,\ x_1 = \pm a,\ y_1 = 0$$

$$= \left|\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}}\right| = a$$

$$a = 0 \quad \text{or}$$

$$\frac{b(x) + a(0) - ab}{\sqrt{a^2 + b^2}} = a$$

$$\frac{b}{a}x = \pm\sqrt{a^2 + b^2} + b$$

$$x = \frac{a}{b}\left(b \pm \sqrt{a^2 + b^2}\right)$$

$$x = 0$$