RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.18

Straight lines Ex 23.18 Q1

Let the required equation be $ax + by = c$ but here it passes through origin $(0,0)$.

$\therefore \quad c = 0$

$\therefore$ Equaiton is $ax + by = 0$

Slope of the line $(m_1) = \frac{-a}{b}$ and $m_2 = \frac{-\sqrt{3}}{1}$

$\Rightarrow \quad$ Angle between $\sqrt{3}x + y = 11$ and $ax + by = 0$ is $45°$

$\therefore \quad \tan 45° = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$

$$1 = \frac{\frac{-a}{b} \pm \left(-\sqrt{3}\right)}{1 \mp \frac{a}{b} \times \sqrt{3}}$$

$$1 - \frac{\sqrt{3}a}{b} = \frac{-a}{b} - \sqrt{3} \text{ and } 1 + \frac{a}{b}\sqrt{3} = \frac{-a}{b} + \sqrt{3}$$

$$b - \sqrt{3}a = -a - \sqrt{3}b \text{ and } b + a\sqrt{3} = -a + b\sqrt{3}$$

$$a\left(1 - \sqrt{3}\right) = b\left(-\sqrt{3} - 1\right) \text{ and } a\left(\sqrt{3} + 1\right) = b\left(\sqrt{3} - 1\right)$$

$$\frac{a}{b} - \frac{1 - \sqrt{3}}{\sqrt{3} - 1} = \frac{\left(\sqrt{3} - 1\right)^2}{2} = 2 - \sqrt{3}$$

or

$$\frac{a}{b} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = -2 - \sqrt{3}$$

$\therefore$ Required lines are $\frac{y}{x} = \sqrt{3} \pm 2$ or $y = \left(\sqrt{3} \pm 2\right)x$

Straight lines Ex 23.18 Q2

Let the required equation be $y = mx + c$

But, $c = 0$ as it passes through origin $(0,0)$

$\therefore \quad$ Equation of the lines is $y = mx$.

Slope of $x + y + \sqrt{3}y = \sqrt{3}x = a$

or $\left(\sqrt{3}+1\right)x + \left(1-\sqrt{3}\right)y = a$ is

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}.$$

The angle between $x + y + \sqrt{3}y - \sqrt{3} = a$ and $y = mx$ is $75°$

$$\tan\left(75°\right) = \frac{m_1 \pm m_2}{1 \mp m_1 m_2}$$

$$\tan\left(30° + 45°\right) = \frac{m \pm \left(2-\sqrt{3}\right)}{1 - m\left(2-\sqrt{3}\right)}$$

$$\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}\times 1} = \frac{m \pm 2-\sqrt{3}}{1-m\left(2-\sqrt{3}\right)}$$

$$2+\sqrt{3} = \frac{m+2-\sqrt{3}}{1+m\left(\sqrt{3}-2\right)} \text{ and } 2+\sqrt{3} = \frac{m+\sqrt{3}-2}{1+m\left(2-\sqrt{3}\right)}$$

$\therefore$ $\frac{1}{m} = 0$ or $m = -\sqrt{3}$

$\therefore$ $y = mx$ $y = -\sqrt{3}x$ and $x = 0$ are the required equatins.

Straight lines Ex 23.18 Q3

Given equation is 6x + 5y - 8=0.

Slope of given line $= m = -\frac{6}{5}$

Equations of required line is,

$$y+1 = \frac{-\frac{6}{5}-1}{1+\frac{6}{5}}(x-2)$$

$$y+1=\frac{-11}{11}(x-2)$$

$$y+1=-x+2$$

$$x+y-1=0$$

$$y+1=\frac{-\frac{6}{5}+1}{1-\frac{6}{5}}(x-2)$$

$$y+1=\frac{-1}{-1}(x-2)$$

$$y+1=x-2$$

$$x-y=3$$

Straight lines Ex 23.18 Q4

The required equation is

$$y-k=m'(x-h)$$

And this line is inclined at $\tan^{-1} m$ to straight line $y=mx+c$.

Slope $= m = \tan\theta$

Passing through $\underset{(x_1,\, y_1)}{(h,k)}$

$\therefore$ Equation of line is

$$y-y_1=m(x-x_1) \qquad \text{---(i)}$$

Also, $\tan\theta=\left|\frac{m-m'}{1+mm'}\right|$

Here, $m=m'$

$\therefore \quad \tan\theta=\frac{m-m}{1+m^2}$ or $\left|\frac{-m-m}{1-m^2}\right|$

$= 0$ or $\frac{+2m}{1-m^2}$

Substituting in (i)

$$y-k=0$$

$\Rightarrow \quad y=k \qquad$ or

$$y-k=\frac{+2m}{1-m^2}(x-h)$$

$$(1-m^2)(y-k)=+2m(x-h)$$

Straight lines Ex 23.18 Q5

Here, $x_1 = 2,\ y_1 = 3,\ \alpha = 45°$

m = slope of line $3x + y - 5 = 0$

$$= \frac{-\text{coeff of } x}{\text{coeff of } y} = -3$$

The equations of the required line are

$$y - y_1 = \frac{-3 - \tan 45°}{1 + (-3)\tan 45°}(x - 2)$$

$$y - 3 = \frac{-3 - 1}{1 + (-3)(1)}(x - 2)$$

$$y - 3 = \frac{-4}{2}(x - 2) = 2x - 4$$

$$2x - y - 1 = 0$$

Also, $$y - 3 = \frac{-3 + \tan 45°}{1 - (-3)\tan 45}(x - 2)$$

$$y - 3 = \frac{-3 + 1}{1 + 3}(x - 2)$$

$$y - 3 = \frac{-2}{4}(x - 2) = \frac{-x}{2} + 1$$

$$x + 2y - 8 = 0$$

Straight lines Ex 23.18 Q6

Let the isosceles right triangle be.

$AC = 3x + 4y = 4$

$c(2,2)$

Then, slope of $AC = \frac{-3}{4}$

$AB = BC$ [$\because$ It is an isoscales right triangle]

Then, angle between $(AB \text{ and } AC)$ and $(BC \text{ and } AC)$ is 45°.

$$\tan\frac{\pi}{4} = \frac{m_1 - \left(\frac{-3}{4}\right)}{1 + \left(\frac{-3}{4}\right)m_1}$$ [when m_1 = slope of BC]

$$1 = \frac{m_1 + \frac{3}{4}}{1 - \frac{3}{4}m}$$

$4 - 3m_1 = 4m_1 + 3$

$7m_1 = 1$ $m_1 = \frac{1}{7}$

and, $AB \perp BC$

$\therefore$ (slope of AB) $\times$ (slope of BC) $= -1$

$m_2 \times \frac{1}{7} = -1$

$m_2 = -7.$

The equation of BC is

$(y - 2) = \frac{1}{7}(x - 2)$

$7y - 14 = x - 2$

$x - 7y + 12 = 0$

and

The equation of AB is

$(y - 2) = -7(x - 2)$

$y - 2 = -x + 14$

$y + 7x - 16 = 0$

Straight lines Ex 23.18 Q7

Let $C\left(2+\sqrt{3},5\right)$ be one vertex and $x = y$ be the opposite side of equilateral triangle ABC.

The other two sides makes an angle of $60°$ with other two sides.
slope of $x - y = 0$ is 1.

$$\therefore \quad y-5 = \frac{1 \pm \tan 60°}{1 \mp \tan 60°}\left(x-2-\sqrt{3}\right)$$

$$y-5 = \frac{1+\sqrt{3}}{1-\sqrt{3}}\left(x-2-\sqrt{3}\right) \text{ and } y-5 = \frac{1-\sqrt{3}}{1+\sqrt{3}}\left(x-2-\sqrt{3}\right)$$

$$y-5 = \left(\sqrt{3}-2\right)\left(x-2-\sqrt{3}\right) \text{ and } y-5 = \left(\sqrt{3}-2\right)\left(x-2-\sqrt{3}\right)$$

$$y+\left(2+\sqrt{3}\right)x = 12+4\sqrt{3} \text{ and } y+\left(2-\sqrt{3}\right)x = 6$$

Hence proved the 2nd side of ΔABC is $y+\left(2-\sqrt{3}\right)x = 6$

and the 3rd side is $y+\left(2+\sqrt{3}\right)x = 12+4\sqrt{3}$.

Straight lines Ex 23.18 Q8

Let $ABCD$ be a square whose diagnal BD is $4x + 7y = 12$

Then, slope of $BD = \frac{-4}{7}$

Let slope of $AB = m$

$$\text{Then, } \tan 45° = \frac{m+\frac{4}{7}}{1-\frac{4}{7}m}$$

$$7 - 4m = 7m + 4$$

$$11m = 3$$

$$\therefore \quad m = \frac{3}{11}$$

$$\therefore \text{ Slope of } BC = \frac{-1}{\text{slope of } AB}$$

$$= \frac{-11}{3}$$

$\therefore$ Equationf of AB is

$$\left(y-2\right) = \frac{3}{11}\left(x-1\right)$$

$$11y - 22 = 3x - 3$$

$$3x - 11y + 19 = 0$$

and

Equation of BC is

$$\left(y-2\right) = \frac{-11}{3}\left(x-1\right)$$

$$11x + 3y - 17 = 0$$