RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.19

Straight lines Ex 23.19 Q1

Line through the intersection of $4x - 3y = 0$ and $2x - 5y + 3 = 0$ is

$$(4x - 3y) + \lambda(2x - 5y + 3) = 0 \quad \text{---(i)}$$

or, $x(4 + 2\lambda) - y(3 + 5\lambda) + 3\lambda = 0$

And the required line is parallel to $4x + 5y + 6$

$\therefore$ slope of required = slope of $4x + 5y + 6 = \frac{-4}{3}$

$\therefore$ $\frac{-(4 + 2\lambda)}{-(3 + 5\lambda)} = \frac{-4}{3}$

$\Rightarrow$ $5(4 + 2\lambda) = -4(3 + 5\lambda)$

$\Rightarrow$ $20 + 10\lambda = -12 - 20\lambda$

$\Rightarrow$ $30\lambda = -32$

$\Rightarrow$ $\lambda = \frac{-16}{15}$

Putting λ in equation (i)

$(4x - 3y) - \frac{16}{15}(2x - 5y + 3) = 0$

$\Rightarrow$ $60x - 45y - 32x + 80y - 48 = 0$

$\Rightarrow$ $28x + 35y - 48 = 0$

Is the required line

Straight lines Ex 23.19 Q2

The equation of the required line is

$(x + 2y + 3) + \lambda(3x + 4y + 7) = 0$

or, $x(1 + 3\lambda) + y(2 + 4\lambda) + 3 + 7\lambda = 0$

m_1 = slope of the line $= -\left(\frac{1 + 3\lambda}{2 + 4\lambda}\right)$

The line is perpendicular to $x - y + 9 = 0$ whose slope $(m_2 = 1)$

$\therefore \quad m_1 \times m_2 = -1$

$\Rightarrow \quad -\left(\frac{1+3\lambda}{2+4\lambda}\right) \times 1 = -1$

$\Rightarrow \quad 1 + 3\lambda = 2 + 4\lambda$

$\Rightarrow \quad \lambda = -1$

$\therefore$ The required line is

$$x + 2y + 3 - (3x + 4y + 7) = 0$$

$$-2x - 2y - 4 = 0$$

or, $\quad x + y + 2 = 0$

Straight lines Ex 23.19 Q3

The required line is

$$2x - 7y + 11 + \lambda(x + 3y - 8) = 0$$

or, $\quad x(2+\lambda) + y(-7+3\lambda) + 11 - 8\lambda = 0$

(i) When the line is parallel to x-axis. It slope is 0

$\therefore \quad -\frac{(2+\lambda)}{3\lambda - 7} = 0$

$$\lambda = -2$$

$\therefore$ Equation of line is

$$2x - 7y + 11 - 2(x + 3y - 8) = 0$$

$$-13y + 27 = 0$$

(ii) When the line is parallel to y-axis then,

$$\frac{-1}{\text{slope}} = 0$$

i.e $\quad \frac{3\lambda - 7}{2 + \lambda} = 0$

$$\lambda = \frac{7}{3}$$

$\therefore$ Equation of line is

$$2x - 7y + 11 + \frac{7}{3}(x + 3y - 8) = 0$$

$$\Rightarrow \frac{6x - 21y + 33 + 7x + 21y - 56}{3} = 0$$

$\Rightarrow 6x-21y+33+7x+21y-56=0$

$\Rightarrow 13x-23=0$

$\Rightarrow 13x=23$

Straight lines Ex 23.19 Q4

The required line is

$$(2x+3y-1)+\lambda(3x-5y-5)=0$$

or, $x(2+3\lambda)+y(3-5\lambda)-1-5\lambda=0$

Since this lines is equally inclined to both the axes, it slope should be 1. or -1

$\therefore \quad \frac{-2-3\lambda}{3-5\lambda}=1 \quad$ or, $\quad \frac{-2-3\lambda}{3-5\lambda}=-1$

$\Rightarrow \quad 3-5\lambda=-2-3\lambda \quad$ or, $\quad \Rightarrow -2-3\lambda=-3+5\lambda$

$\Rightarrow \quad 5=2\lambda \quad$ or, $\quad \Rightarrow 1=8\lambda$

$\Rightarrow \quad \lambda=\frac{5}{2} \quad$ or, $\quad \Rightarrow \lambda=\frac{1}{8}$

$\therefore$ The required line is

$$2x+3y+1+\frac{5}{2}(3x-5y-5)=0$$

$$4x+6y+2+15x-25y-25=0$$

$$19x-19y-23=0$$

or

$$(2x+3y+1)+\frac{1}{8}(3x-5y-5)=0$$

$$16x+24y+8+3x-5y-5=0$$

$$19x+19y+3=0$$

$\therefore$ The two possible equation are

$19x-19y-23=0 \quad$ or $\quad 19x+19y+3=0$

Straight lines Ex 23.19 Q5

The required line is

$$(x+y-4)+\lambda(2x-3y-1)=0$$

or, $x(1+2\lambda)+y(1-3\lambda)-4-\lambda=0$

And it is perpendicular to $\frac{x}{5}+\frac{y}{6}=1$

$\therefore$ (slope of required line) $\times\left(\text{slope of } \frac{x}{5}+\frac{y}{6}=1\right)=-1$

$\Rightarrow -\left(\frac{1+2\lambda}{1-3\lambda}\right)\times\frac{-6}{5}=-1$

$\Rightarrow \frac{1+2\lambda}{1-3\lambda}=\frac{-5}{6}$

$\Rightarrow 6+12\lambda=-5+15\lambda$

$\Rightarrow 11=3\lambda$ or $\lambda=\frac{11}{3}$

$\therefore$ The required line is

$$(x+y-4)+\frac{11}{3}(2x-3y-1)=0$$

$$3x+3y-12+22x-33y-11=0$$

$$25x-30y-23=0$$

Straight lines Ex 23.19 Q6

$$x(1+\lambda)+y(2-\lambda)+5=0$$

$\Rightarrow x+x\lambda+2y-\lambda y+5=0$

$\Rightarrow \lambda(x-y)+(x+2y+5)=0$

$\Rightarrow (x+2y+5)+\lambda(x-y)=0$

This is of the form $L_1+\lambda L_2=0$

So it represents a line passing through the intersection of $x-y=0$ and $x+2y=-5$.

Solving the two equations, we get $\left(\frac{-5}{3},\ \frac{-5}{3}\right)$ which is the fixed point through which the given family of lines passes for any value of λ.

Straight lines Ex 23.19 Q7

$$(2+k)x+(1+k)y=5+7k$$

or, $(2x+y-5)+k(x+y-7)=0$

It is of the form $L_1+kL_2=0$ i.e., the equation of line passing through the intersection of 2 lines L_1 and L_2.

So, it represents a line passing through $2x+y-5=0$ and $x+y-7=0$.

Solving the two equation we get, $(-2,9)$. Which is the fixed point through which the given line pass. For any value of k.

Straight lines Ex 23.19 Q8

$L_1 + \lambda l_2 = 0$ is the equation of line passing through two lines. L_1 and L_2.

$\therefore \quad (2x + y - 1) + \lambda(x + 3y - 2) = 0$ is the required equation. ---(i)

or, $\quad x(2+\lambda) + y(1+3\lambda) - 1 - 2\lambda = 0$

$$\frac{x}{\frac{1+2\lambda}{2+\lambda}} + \frac{4}{\frac{1+2\lambda}{1+3\lambda}} = 1$$

Area of $\Delta = \frac{1}{2} \times OB \times OA$

$$\frac{8}{3} = \frac{1}{2} \times (y \text{ intercept}) \times (x \text{ intercept})$$

$$\frac{8}{3} = \frac{1}{2} \times \left(\frac{1+2\lambda}{1+3\lambda}\right) \times \left(\frac{1+2\lambda}{2+\lambda}\right)$$

$$\frac{16}{3} = \frac{1+4\lambda^2+4\lambda}{2+3\lambda^2+7\lambda}$$

$$32 + 48\lambda^2 + 112\lambda = -3 - 12\lambda^2 - 12\lambda$$

$$60\lambda^2 + 124\lambda + 35 = 0$$

$$\lambda = \frac{-124 \pm \sqrt{(124)^2 - 4 \times 60 \times 35}}{2 \times 60}$$

$$= \frac{-124 \pm \sqrt{15376 - 8400}}{120}$$

Approximately = 1

$\therefore$ Subtituting in (i) $\Rightarrow 3x + 4y - 3 = 0,\ 12x + y - 3 = 0$

Straight lines Ex 23.19 Q9

The required line is

$$(3x - y - 5) + \lambda(x + 3y - 1) = 0$$

or, $\quad (3+\lambda)x + (-1+3\lambda)y - 5 - \lambda = 0$

or, $$\frac{x}{\left(\frac{5+\lambda}{3+\lambda}\right)} + \frac{y}{\frac{5+\lambda}{3\lambda-1}} = 1$$

And the line makes equal and positive intercepts with the line (given)

$$\therefore \quad \frac{5+\lambda}{3+\lambda} = \frac{5+\lambda}{3\lambda-1}$$

$$3\lambda - 1 = 3 + \lambda$$

$$2\lambda = 4$$

$$\lambda = 2$$

$\therefore$ The required line is

$$3x - y - 5 + 2x + 6y - 2 = 0$$

or, $\quad 5x + 5y = 7$

Straight lines Ex 23.19 Q10

The required line is

$$x - 3y + 1 + \lambda(2x + 5y - 9) = 0$$

or, $(1 + 2\lambda)x + (-3 + 5\lambda)y + 1 - 9\lambda = 0$

Distance from origin of this line is

$$\left|\frac{(1+2\lambda)0 + (-3+5\lambda)0 + 1 - 9\lambda}{\sqrt{(1+2\lambda)^2 + (5\lambda - 3)^2}}\right| \qquad \left[\text{using } \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right]$$

$$\sqrt{5} = \left|\frac{1 - 9\lambda}{\sqrt{1 + 4\lambda^2 + 4\lambda + 25\lambda^2 + 9 - 30\lambda}}\right|$$

$\Rightarrow \quad \sqrt{5} = \left|\frac{1 - 9\lambda}{\sqrt{10 + 29\lambda^2 - 26\lambda}}\right|$

$\Rightarrow \quad 5\left(10 + 29\lambda^2 - 26\lambda\right) = (1 - 9\lambda)^2$

$\Rightarrow \quad 50 + 145\lambda^2 - 130\lambda = 1 + 81\lambda^2 - 18\lambda^2$

$\Rightarrow \quad 64\lambda^2 - 112\lambda + 49 = 0$

$\Rightarrow \quad (8\lambda - 7)^2 = 0 \quad$ or, $\quad \lambda = \frac{7}{8}$

$\therefore$ Required line is

$$x - 3y + 1 + \frac{7}{8}(2x + 5y - 9) = 0$$

$\Rightarrow \quad 8x - 24y + 8 + 14x + 35y - 63 = 0$

$\Rightarrow \quad 22x + 11y - 55 = 0$

$\Rightarrow \quad 2x + y - 5 = 0$