RD Sharma Solutions Class 11 Maths Chapter 24 Ex 24.2

Circles Ex 24.2 Q1(i)

The genral equation of circles is

$x^2 + y^2 + 2gx + 2fy + c = 0$ .... (i)

$$\left.\begin{array}{l}\text{centre} = (-g, -f) \\ \text{radius} = \sqrt{g^2 + f^2 - c}\end{array}\right] \ldots\ldots\ldots (A)$$

i) $x^2 + y^2 + 6x - 8y - 24 = 0$

Hence $g = 3,\ f = -4,\ c = -24$

Thus,

$\text{centre} = (-3, 4)$

$\text{radius} = \sqrt{g^2 + f^2 - c} = \sqrt{9 + 16 + 24}$

$= \sqrt{49}$

$\therefore \quad \text{radius} = 7$

Circles Ex 24.2 Q1(ii)

$2x^2 + 2y^2 - 3x + 5y - 7 = 0$

$\Rightarrow x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$

Hence $g = \frac{-3}{4}, \quad f = \frac{5}{4}\ c = \frac{-7}{2}$

Thus,

$\text{centre} = \left(\frac{3}{4}, -\frac{5}{4}\right)$

$\text{radius} = \sqrt{\left(\frac{-3}{4}\right)^2 + \left(-\frac{5}{4}\right)^2 + \frac{7}{2}}$

$= \sqrt{\frac{9}{16} + \frac{25}{16} + \frac{7}{2}} = \frac{\sqrt{90}}{4}$

$\therefore \quad \text{radius} = \frac{3\sqrt{10}}{4}$

Circles Ex 24.2 Q1(iii)

$\frac{1}{2}(x^2 + y^2) + x\cos\theta + y\sin\theta - 4 = 0$

$\Rightarrow \quad x^2 + y^2 + 2x\cos\theta + 2y\sin\theta - 8 = 0$

Comparing with (i)

Hence $g = \cos\theta, \quad f = \sin\theta, \quad c = -8$

Thus,

$\text{centre} = (-\cos\theta,\ -\sin\theta)$

$\text{radius} = \sqrt{\cos^2\theta + \sin^2\theta + 8} = \sqrt{1 + 8} = 3$

$\text{radius} = 3$

Circles Ex 24.2 Q1(iv)

iv) $x^2 + y^2 - ax - by = 0$

Hence $g = -\frac{a}{2}, \quad f = -\frac{b}{2}, \quad c$

Thus,

$$\text{centre} = \left(\frac{a}{2}, \frac{b}{2}\right)$$

$\therefore \quad \sqrt{a^2 \quad b^2 \quad -} \quad \sqrt{a}$

Circles Ex 24.2 Q2(i)

We know that the general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$........(i)

We have,

$P(5, \ 7)$, $Q(8,1)$ and $R(1,3)$

Since P, Q and R lies on (i)

so,

$$25 + 49 + 10g + 14f + c = 0 \text{..........}(ii)$$

$$64 + 1 + 16g + 2f + c = 0 \text{ ....}(iii)$$

$$1 + 9 + 2g + 6f + c = 0 \text{ ....}(iv)$$

Solving (ii), (iii) & (iv), we get,

$$g = -\frac{29}{6}, \quad f = -\frac{19}{6}, \quad c = \frac{56}{3}$$

Thus, the equation of circle is on putting g, f & c on (i)

$$x^2 + y^2 - \frac{29}{3}x - \frac{19}{3}y + \frac{56}{3} = 0$$

$$\Rightarrow \quad 3\left(x^2 + y^2\right) - 29x - 19y + 56 = 0$$

Circles Ex 24.2 Q2(ii)

We know that the general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$........(i)

We have,

$P(1,2)$, $Q(3,-4)$ and $R(5,-6)$

Since P, Q & R lies on (i)

$$\therefore \quad x^2 + y^2 + 2gx + 2fy + c = 0 \text{..........}(1)$$

$$1 + 4 + 2g + 4f + c = 0 \text{..........}(2)$$

$$9 + 16 + 6g - 8f + c = 0 \text{..........}(3)$$

$$25 + 36 + 10g - 12f + c = 0 \text{..........}(4)$$

Solving (ii), (iii) & (iv), we get,

$$g = -11, \quad f = -2 \ \& \ c = 25$$

from (i)

The equation of circle is

$$x^2 + y^2 - 22x - 4y + 25 = 0$$

Circles Ex 24.2 Q2(iii)

We know that the general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$........(i)

We have,
$P = (5,-8)$, $Q(-2,9)$ and $R = (2,1)$

P, Q & R lies on (i), so,

$$25 + 64 + 10g - 16f + c = 0 \text{..........(ii)}$$
$$4 + 81 - 4g + 18f + c = 0 \text{..........(iii)}$$
$$4 + 1 + 4g + 2f + c = 0 \text{..........(iv)}$$

Solving (ii), (iii) & (iv), we get,
$g = 58$, $f = 24$, $c = -285$

Thus, equation of circle is,
$x^2 + y^2 + 116x + 48y - 285 = 0$ from (i)

Circles Ex 24.2 Q2(iv)

We know that the general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$........(i)

We have,
$P = (0,0)$, $Q = (-2,1)$ and $R = (-3,2)$

P, Q & R lies on (i), so

$$0 + 00 + 0 + c = 0 \text{............(ii)}$$
$$4 + 1 - 4x + 2y + c = 0 \text{......(iii)}$$
$$9 + 4 - 6x + 4y + c = 0 \text{......(iv)}$$

Solving (ii), (iii) & (iv), we get,
$g = -\frac{3}{2}$, $f = -\frac{11}{2}$, $c = 0$

from (i),
The equation of circle is
$x^2 + y^2 - 3x - 11y = 0$

Circles Ex 24.2 Q3

A circle passing through $P(3,-2)$ *and* $Q(-2,0)$ *and having its centre on* $2x - y = 3$.
Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$.
Since the circle passes through $(3,-2)$ *andAlso* $(-2,0)$ *therefore*
$9 + 4 + 6g - 4f + c = 0$.......(i)
$4 + 0 - 4g + 0 + c = 0$........(ii)
Also the centre of the circle lies on $2x - y = 3$
$-2g + f = 3$.........(iii)
Solving equations (i),(ii) and (iii), we get
$g = \frac{3}{2}$, $f = 6$ *and* $c = 2$
Therefore the equation of the circle is
$x^2 + y^2 + 3x + 12y + 2 = 0$

Circles Ex 24.2 Q4

The circle passes through P & Q and the centre lies on

$$x - 4y = 1 \ldots\ldots\ldots (i)$$

The genral equation of circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots\ldots (ii)$$

$\because$ P & Q lies on (ii), so,

$$9 + 49 + 6g + 14f + c = 0 \ldots\ldots\ldots (iii)$$
$$25 + 25 + 10g + 10f + c = 0 \ldots\ldots\ldots (iv)$$

Also, centre $(-g, -f)$ lies on (i)

$\therefore$ $-g + 4f = 1 \ldots\ldots\ldots\ldots (v)$

Solving (iii)(iv)& (v,) we get

$$g = 3, \quad f = 1 \quad \& \quad c = -90$$

from (ii)

The equation of circle is

$$x^2 + y^2 + 6x + 2y - 90 = 0$$

Circles Ex 24.2 Q5

We have,

$$P = (3, -2) \quad Q = (1, 0) \quad R = (-1, -2) \text{ and } S = (1, -4)$$

let us consider A circle $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots\ldots (i)$

Passes through P, Q & R

$\therefore$
$$9 + 4 + 6g - 4f + c = 0 \ldots\ldots\ldots (ii)$$
$$1 + 0 + 2g - 0 + c = 0 \ldots\ldots\ldots (iii)$$
$$1 + 4 - 2g - 4f + c = 0 \ldots\ldots\ldots (iv)$$

Solving (ii), (iii) & (iv) we get,

$$g = -1, \quad f = 2 \quad \& \quad c = 1$$

from (i)

The required equation of circle is

$$x^2 + y^2 - 2x + 4y + 1 = 0 \ldots\ldots\ldots\ldots (v)$$

Clearly $s = (1, -4)$ satisfy (v)

Thus,

P, Q, R & S are concyclic

Circles Ex 24.2 Q6

The genral equation of circle is

$$x^2+y^2+2gx+2fy+c=0 \text{ ....} (1)$$

centre = $(-g,-f)$ and

radius = $\sqrt{g^2+f^2-c}$

$\because$ $P=(5,5)$, $Q(6,4)$ & $R=(-2,4)$ lies on (i)

$\therefore$ $25+25+10g+10f+c=0$ .... (ii)

$36+16+12g+8f+c=0$.......... (iii)

$4+16+-4g+8f+c=0$.......... (iv)

Solving (ii), (iii) & (iv), we get,

$g=-2,\quad f=-1,\quad c=-20$

from (i)

The equation of circle is

$$x^2+y^2-4x-2y-20=0 \text{ ....} (A)$$

Clearly $s=(7,1)$ Satisfy (A)

Hence P,Q,R,S are concyclic

Now

centre = $(-g,-f)=(2,1)$

radius = $\sqrt{g^2+f^2-c}=\sqrt{4+1+20}=\sqrt{25}=5$

Circles Ex 24.2 Q7(i)

The given equation of lines

$x+y=-3$ .... (i)

$x-y=-1$.......... (ii)

$x=3$............. (iii)

Let A,B,C are the point of intersection of lines (i) & (ii), (ii) & (iii) and (iii) & (i) respectively

$\therefore$ $A=(-2,-1)$, $B=(3,4)$ & $C=(3,-6)$

Now A circle $x^2+y^2+2gx+2fy+c=c$........ (A)

circumscribing the $\triangle ABC$

$\therefore$ $4+1-4g-2f+c=0$ .... (iv)

$9+16+6g+8f+c=0$......... (v)

$9+36+6g-12f+c=0$........ (vi)

Solving (iv), (v) & (vi) we get,

$g=-3,\quad f=1,\quad c=-15$

from (A),

The equation of required circle is

$$x^2+y^2-6x+2y-15=0$$

Circles Ex 24.2 Q7(ii)

The equation of lines are

$2x + y = 3 \ldots\ldots (i)$

$x + y = 1 \ldots\ldots (ii)$

$3x + 2y = 5 \ldots\ldots (iii)$

Let A, B & C are the points of intersection of the lines (i) & (ii), (ii) & (iii) and (iii) & (i) respectively.

$\therefore \quad A = (2, -1)$

$B = (3, -2)$

$C = (1, 1)$

Let $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots\ldots (A)$

be the circle that circumscribing $\triangle ABC$.

$\therefore \quad 4 + 1 + 4g - 2f + c = 0 \ldots\ldots\ldots (iv)$

$9 + 4 + 6g - 4f + c = 0 \ldots\ldots\ldots (v)$

$1 + 1 + 2g + 2f + c = 0 \ldots\ldots\ldots (vi)$

Solving (iv), (v) & (vi), we get,

$g = -\frac{13}{2}, \quad f = -\frac{5}{2} \& \quad c = 16$

from (A),

The required circle is

$x^2 + y^2 - 13x - 5y + 16 = 0$

Circles Ex 24.2 Q7(iii)

The given equation of lines are

$x + y = 2 \ldots\ldots\ldots (i)$

$3x - 4y = 6 \ldots\ldots (ii)$

$x - y = 0 \ldots\ldots (iii)$

Let A, B & C are the points of intersection of the lines (i) & (ii), (ii) & (iii) and (iii) & (i) respectively.

$\therefore \quad A = (2, 0)$

$B = (-6, -6)$

$C = (1, 1)$

Let $x^2 + y^2 + 2gx + 2fy + c = 0 \ldots\ldots\ldots (A)$

be the circle circumscribing $\triangle ABC$.

$\therefore \quad 4 + 4g + c = 0 \ldots\ldots\ldots (iv)$

$36 + 36 - 12g - 12f + c = 0 \ldots\ldots\ldots (v)$

$1 + 1 + 2g + 2f + c = 0 \ldots\ldots\ldots (vi)$

Solving (iv), (v) & (vi), we get,

$g = 2, \quad f = 3 \; \& \quad c = -12$

from (A),

The required circle is

$x^2 + y^2 + 4x + 6y - 12 = 0$

Circles Ex 24.2 Q7(iv)

Solving equations $y = x + 2$ and $3y = 4x$ we get,
$x = 6$ and $y = 8$

Solving equations $y = x + 2$ and $2y = 3x$ we get,
$x = 4$ and $y = 6$

Solving equations $3y = 4x$ and $2y = 3x$ we get,
$x = 0$ and $y = 0$

So, the vertices of the triangle are (6, 8), (4, 6) and (0, 0).

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the required circle which circumscribes the triangle.
$\therefore x^2 + y^2 + 2gx + 2fy + c = 0$ passes through (6, 8), (4, 6) and (0, 0).
$12g + 16f + c = -100$......(i)
$8g + 12f + c = -52$.......(ii)
$c = 0$.......(iii)

Solving (i),(ii) and (iii) we get
$f = 11$ and $g = -23$

$\therefore$ The required equation of the circle is, $x^2 + y^2 - 46x + 22y = 0$.

Circles Ex 24.2 Q8

The given equation of circle are.

$$x^2 + y^2 - 4x - 6y - 12 = 0 \text{........(i)}$$
$$x^2 + y^2 + 2x + 4y - 10 = 0 \text{........(ii)}$$
$$x^2 + y^2 - 10x - 16y - 1 = 0 \text{......(iii)}$$

Let $C_1 C_2$ & C_3 are the centres of (i)(ii)&(iii)

$\therefore$
$$C_1 = (-g, -f) = (2, 3)$$
$$C_2 = (-g, -f) = (-1, -2)$$
$$C_3 = (-g, -f) = (5, 8)$$

$C_1 C_2$ & C_3 will be collinear if $ar(\Delta C_1 C_2 C_3) = 0$

$$ar(\Delta C_1 C_2 C_3) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -3 & -5 & 0 \\ 3 & 5 & 0 \end{vmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= \frac{1}{2}(-15 + 15) = \frac{1}{2} \times 0 = 0$$

$\therefore$ $C_1 C_2$ & C_3 are collinear

Circles Ex 24.2 Q9

The given equation of circles are.

$x^2+y^2=1 \ldots\ldots (i)$

$x^2+y^2-2x-6y-6=0 \ldots\ldots (ii)$

$x^2+y^2-4x-12y-9=0 \ldots\ldots (iii)$

Let R_1 R_2 & R_3 are the radii of (i)(ii)&(iii) respectively

$\therefore \quad R_1 = 1$

$R_2 = \sqrt{g^2+f^2-c} = \sqrt{1^2+3^2+6} = \sqrt{16} = 4$

$R_3 = \sqrt{g^2+f^2+c} = \sqrt{2^2+6^2+9} = \sqrt{49} = 7$

If a, b, c are in AP, then $b = \frac{a+c}{2}$

For $a = 1, b = 4, c = 7$, $\frac{1+7}{2} = 4 = b$, *therefore 1, 4, 7 are in AP.*

The centres of the three circles lie in AP.

Circles Ex 24.2 Q10

We have a circle thet passes through origin $O(0,0)$ and cut off on intersept of length 4 units on $x-axis$ & 6 units on $y-axis$.

That is, $OA = 4$

$OB = 6$

$C-$ be the centre of the circle and CM & CN are perpendicular line drawn on OA & OB respectively.

Coordinates of A $= (4,0)$ & $B = (0,6)$

$\therefore$ Coordinates of $M = (2,0)$ & $N = (0,3)$

Thus coordinates of $C = (2,3)$

Now in $\triangle OCM$

$OC^2 = OM^2 + CM^2$

$= 2^2 + 3^2 \quad [\because CM = ON = 3]$

$= 4 + 9$

$\therefore \quad OC = \sqrt{13}$

Thus, the required circle is

$(x-2)^2 + (y-3)^2 = 13$

$x^2 + y^2 - 4x - 6y = 0$

Circles Ex 24.2 Q11

The given equation of circle is

$$x^2+y^2-6x+12y+15=0 \text{ ....(i)}$$

$\therefore$ centre $=(-g,-f)=(3,-6)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{9+36-15}=\sqrt{30}$

Now,

the required equation of circle in concentric with (i)

which means both have same centre $(3,-6)$

Also,

Area of required circle $= 2\times$ Area of (i)

$$\pi r^2 = 2\times\pi\left(\sqrt{30}\right)^2$$

$\Rightarrow \quad R^2=60$

$\Rightarrow \quad R=2\sqrt{15}$

Thus,

The required circle is

$$(x-3)^2+(y+6)^2=60$$

$$x^2+y^2-6x+12y-15=0$$

Circles Ex 24.2 Q12

Let $x^2+y^2+2gx+2fy+c=0$ ....(i)

be the required circle.

Now (i) passes through $P=(1,1)$ & $\theta(2,2)$

$\therefore \quad 1+1+2g+2f+c=0$ ....(ii)

$4+4+4g+4f+c=0$.........(iii)

Also radius = 1

$\Rightarrow \quad \sqrt{g^2+f^2-c}=1$

$\Rightarrow \quad g^2+f^2-c=1$..........(iv)

from (ii) & (iii)

$$g+f+\frac{c}{2}=-1 \text{ \&}$$

$$g+f+\frac{c}{4}=-2$$

on subtraction

$c=4$

and $\quad g+f=-3$ ....(v)

from (iv) $g^2+f^2=5$ $\qquad \left[\because (g+f)^2=g^2+f^2+2gf\right.$

$\therefore \quad 2gf=4$ $\qquad \left.\Rightarrow 9=5+2gf\right]$

$gf=2$

so, $(g-f)^2=(g+f)^2-4gf=9-8=1$

$\therefore \ g-f=\pm1$ ....(vi)

Solving (v) & (vi) we get $\quad g=-1$ or -2 &

$f=-2$ or -1

Thus, required circle

$$x^2+y^2-2x-4y+4=0, \qquad x^2+y^2-4x-2y+4=0$$

Circles Ex 24.2 Q13

The given equation of circle is

$$x^2+y^2-4x-6y-3=0 \quad \ldots\ldots\ldots(i)$$

$\therefore$ centre $= (-g, -f) = (2, 3)$

The required circle is concentric with (i) so, they have same centre $= (2, 3)$

Also, the required circle touches $y-axis$ at A

$\therefore$ CA = radius = 2

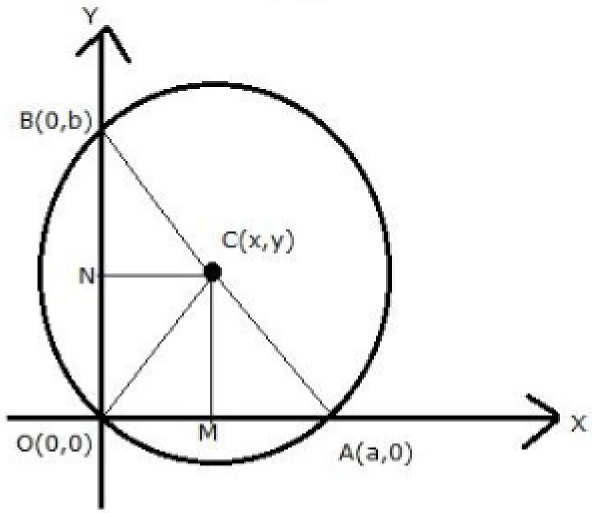
$\therefore$ equation of circle is

$$(x-2)^2+(y-3)^2=4$$

$$\Rightarrow \quad x^2+y^2-4x-6y+9=0$$

Circles Ex 24.2 Q14

Consider the following figure.



In the above diagram, CA, CO and CB are equal radii of the circle and hence we have,

CA=CO=CB=r.

Also the triangle ΔOCA is an isosceles triangle, and CM is the perpendicular bisector to the base OA.

Hence $OM=\frac{a}{2}$

Similarly, CN is the perpendicular bisector to the base OB

Thus, $ON=\frac{b}{2}$

Thus, from the diagram it is clear that

$$OM=x=\frac{a}{2}$$

and

$$ON=y=\frac{b}{2}$$

Hence the centre of the circle is $C\left(\frac{a}{2}, \frac{b}{2}\right)$

Circles Ex 24.2 Q15

Find the equation of the circle which passes through the point (2, 3) and (4, 5) and the centre lies on the straight line $y - 4x + 3 = 0$.

Let the equation of required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ which passes through the point (2, 3) and (4, 5).

$\therefore 13 + 4g + 6f + c = 0 \ldots\ldots(i)$

$41 + 8g + 10f + c = 0 \ldots\ldots(ii)$

Centre (-g, -f) lies on $y - 4x + 3 = 0$

$\therefore -f + 4g = -3 \ldots\ldots\ldots(iii)$

Subtracting (i) from (ii), we get

$28 + 4g + 4f = 0 \ldots\ldots\ldots(iv)$

Solving (iii) and (iv) we get,

$f = -5$ and $g = -2$

Substituting values of f and g in (ii) we get,

$41 - 16 - 50 + c = 0$

$c = 25$

$\therefore$ The required equation of the circle is, $x^2 + y^2 - 4x - 10y + 25 = 0$.