Very Short Answer Type Questions

[1 Mark]

Que 1. Is the number $(3-\sqrt{7})(3+\sqrt{7})$ rational or irrational?

Sol. $(3-\sqrt{7})(3+\sqrt{7}) = (3)^2 - (\sqrt{7})^2 = 9 - 7 = 2$ which is rational.

Que 2. Find the rationalising factor for the denominator of the expression $\frac{1}{3+\sqrt{5}}$.

Sol. $3 - \sqrt{5}$

Que 3. Find the value of $8\sqrt{15} \div 2\sqrt{3}$.

Sol. $8\sqrt{15} \div 2\sqrt{3} = \frac{8}{2}\sqrt{\frac{15}{3}} = 4\sqrt{5}.$

Que 4. Find the sum of $2\sqrt{5}$ and $3/\sqrt{7}$

Sol. $2\sqrt{5} + 3\sqrt{7}$

Que 5. Evaluate: $(25)^{\frac{1}{3}} \times (5)^{\frac{1}{3}}$.

Sol. $(25)^{\frac{1}{3}} \times (5)^{\frac{1}{3}}. = (25 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5.$

Que 6. Find the value of $\frac{(16)^{\frac{3}{4}}}{(16)^{\frac{1}{4}}}$.

Sol. $\frac{(16)^{\frac{3}{4}}}{(16)^{\frac{1}{4}}}. = (16)^{\frac{3}{4}-\frac{1}{4}} = (16)^{\frac{1}{2}} = 4$

Que 7. Write the simplified value of $(49)^{-\frac{1}{4}} \div (49)^{\frac{1}{4}}$.

Sol. $(49)^{-\frac{1}{4}} \div (49)^{\frac{1}{4}}. = (49)^{-\frac{1}{4}-\frac{1}{4}} \div (49)^{-\frac{1}{2}}$

$$= \frac{1}{(49)^{\frac{1}{2}}} = \frac{1}{(7^2)^{\frac{1}{2}}} = \frac{1}{7}$$

Que 8. Simplify: $\sqrt[12]{(x^4)^{\frac{1}{3}}}$.

Sol. $\sqrt[12]{(x^4)^{\frac{1}{3}}}. = \left[(x^4)^{\frac{1}{3}}\right]^{\frac{1}{12}} = x^{4\times\frac{1}{3}\times\frac{1}{12}} = x^{\frac{1}{9}}$.

Que 9. Is the product of two irrational numbers always irrational? Justify your answer.

Sol. No; sometimes rational, sometimes irrational.

For example, $\sqrt{5} \times \sqrt{5} = \sqrt{25}$

And $\sqrt{3} \times \sqrt{5} = \sqrt{15.}$

Short Answer Type Questions – I

[2 marks]

Que 1. Is every rational number a whole number? Justify your answer.

Sol. No, for example $\frac{1}{5}$ is a rational number but not a whole number.

Que 2. Classify the following numbers as rational or irrational and give justification of your answer.

(i) 0.05918 (ii) 1.010010001…… (iii) $\sqrt{\frac{9}{27}}$ (iv) $\sqrt{\frac{12}{75}}$

Sol. (i) 0.05918 is a rational number as decimal expansion is terminating.

(ii) 1.010010001…. is an irrational number as decimal expansion is non-terminating non-recurring (non-repeating).

(iii) $\sqrt{\frac{9}{27}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$, Which is a quotient of rational and irrational number therefore it is in irrational number.

(iv) $\sqrt{\frac{12}{75}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$, Which is a rational number.

Que 3. Simplify: $\left(\frac{3125}{243}\right)^{\frac{-4}{5}}$.

Sol.

$$\left(\frac{3125}{243}\right)^{\frac{-4}{5}} = \left(\frac{243}{3125}\right)^{\frac{4}{5}} = \left(\frac{3^5}{5^5}\right)^{\frac{4}{5}} = \left[\left(\frac{3}{5}\right)^5\right]^{\frac{4}{5}} = \left(\frac{3}{5}\right)^4 = \frac{81}{625}$$

Que 4. Rationalise: $\frac{1}{7+5\sqrt{2}}$

$$\textit{Sol.} \frac{1}{7+5\sqrt{2}} = \frac{1}{7+5\sqrt{2}} = \times \frac{7-5\sqrt{2}}{7-5\sqrt{2}} = \frac{7-5\sqrt{2}}{7^2-\left(5\sqrt{2}\right)^2}$$

$$= \frac{7-5\sqrt{2}}{49-50} = \frac{7-5\sqrt{2}}{-1} = -7+5\sqrt{2}$$

Que 5.Which is grater $\sqrt[3]{3}$ *or* $\sqrt[4]{4}$?

Sol. LCM of 3 and 4 = 12

$$\sqrt[3]{3} = \sqrt[12]{3^4} = \sqrt[12]{81} \text{ and } \sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$$

$$Clearly, \sqrt[12]{81} > \sqrt[12]{64}. \sqrt[3]{3} > \sqrt[4]{4}$$

Que 6. Find two rational numbers between -2 and 5.

Sol. A rational number between -2 and 5 is $\frac{1}{2}[-2+5] = \frac{3}{2}$

Further, a rational number between $\frac{3}{2}$ *and* 5 *is* $\frac{1}{2}\left[\frac{3}{2}+5\right] = \frac{1}{2}\left[\frac{3+10}{2}\right] = \frac{13}{4}$

Hence, two rational numbers between -2 and 5 are 3/2 and 13/4.

Que 7. Express $0.\overline{6}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. Let $x = 0.\overline{6}$

Then, $x = 0.666\ldots$ …(i)

$$\Rightarrow 10x = 6.666\ldots\ldots \quad \ldots(ii)$$

Subtracting (i) from (ii), we get

$$9x = 6 \Rightarrow x = \frac{6}{9} \Rightarrow x = \frac{2}{3}$$

Que 8. Multiply: $5\sqrt[3]{4}$ *by* $\sqrt{3}$.

Sol. LCM of 3 and 2 = 6

$$5\sqrt[3]{4} = 5\sqrt[6]{4^2} = 5\sqrt[6]{16}$$

$$\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$5\sqrt[3]{4} \times \sqrt{3} = 5\sqrt[6]{16} \times \sqrt[6]{27}$$

$$= 5\sqrt[6]{16 \times 27} = 5\sqrt[6]{432}$$

Short Answer Type Questions – II

[3 MARKS]

Que 1. Find the irrational numbers between $\frac{1}{2}$ *and* $\frac{2}{7}$.

Sol. $\frac{1}{2} = 0.142857142857...$ $\Rightarrow \frac{1}{7} = 0.\overline{142857}$

$\frac{2}{7} = 2 \times \frac{1}{7} = 0.285714285714... \Rightarrow \frac{2}{7} = 0.\overline{285714}$

To find irrational numbers between $\frac{1}{2}$ *and* $\frac{2}{7}$. we find numbers which are non-terminating and non-repeating (non-recurring). There are infinitely many such numbers between $\frac{1}{2}$ *and* $\frac{2}{7}$. Two of them are:

0.150 1500 15000 150000 …

0.250 2500 25000 250000 …

Que 2. Find six rational numbers between 3 and 4.

Sol. A rational number between 3 and 4 is

$$\frac{1}{2}(3+4) = \frac{7}{2} \ \ i.e., 3 < \frac{7}{2} < 4$$

Now, a rational number between 3 and $\frac{7}{2}$ is

$$\frac{1}{2}\left(3+\frac{7}{2}\right) = \frac{13}{4} \ \ i.e., 3 < \frac{13}{4} < \frac{7}{2}$$

A rational number between $\frac{13}{4}$ *and* $\frac{7}{2}$

$$\frac{1}{7}\left(\frac{13}{4}+\frac{7}{2}\right) = \frac{27}{8} \ \ i.e., \quad 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2}$$

A rational number between $\frac{7}{2}$ and 4 is

$$\frac{1}{2}\left(\frac{7}{2}+4\right) = \frac{15}{4} \ \ i.e., \ 3 < \frac{13}{4} < \frac{27}{8} < \frac{7}{2} < \frac{15}{4} < 4$$

A rational number between $\frac{7}{2}$ and $\frac{15}{4}$ is

$$\frac{1}{2}\left(\frac{7}{2}+\frac{15}{4}\right)=\frac{29}{8} \quad i.e., \quad 3<\frac{13}{4}<\frac{27}{8}<\frac{7}{2}<\frac{29}{8}<\frac{15}{4}<4$$

A rational number between 15/4 and 4 is

$$\frac{1}{2}\left(\frac{15}{4}+4\right)=\frac{31}{8} \quad i.e., \quad 3<\frac{13}{4}<\frac{27}{8}<\frac{7}{2}<\frac{29}{8}<\frac{15}{4}<\frac{31}{8}<4$$

Hence, 6 rational numbers between 3 and 4 are

$$\frac{13}{4},\frac{27}{8},\frac{7}{2},\frac{29}{8},\frac{15}{4}\ and\ \frac{31}{8}$$

Alternative method

We have, $3 \;=\; 3\times\frac{(6+1)}{(6+1)}=\frac{21}{7} \quad and\ 4=4\times\frac{(6+1)}{(6+1)}=\frac{28}{7}$

We know that $21<22<23<24<25<26<27<28$

$$\Rightarrow \frac{21}{7}<\frac{22}{7}<\frac{23}{7}<\frac{24}{7}<\frac{25}{7}<\frac{26}{7}<\frac{27}{7}<\frac{28}{7}$$

Hence, six rational numbers between 3 = $\frac{21}{7}$ $and\ 4=\frac{28}{7} are$

$$\frac{22}{7},\frac{23}{7},\frac{24}{7},\frac{25}{7},\frac{26}{7},and\ \frac{27}{7}$$

Que 3. Find six rational numbers between $\frac{5}{7}$ and $\frac{6}{7}$.

Sol. We have, $\frac{5}{7}=\frac{5(6+1)}{7(6+1)}=\frac{35}{49}\ and\ \frac{6}{7}=\frac{6(6+1)}{7(6+1)}=\frac{42}{49}$

Therefore, six rational numbers between $\frac{5}{7}=\frac{42}{49}\ and\ \frac{6}{7}=\frac{42}{49}\ are$

$$\frac{36}{49},\frac{37}{49},\frac{38}{49},\frac{39}{49},\frac{40}{49},\frac{41}{49}$$

Que 4. Write $4\frac{1}{8}$ in decimal form and find what kind of decimal expansion it has.

Sol. $4\frac{1}{8} = \frac{33}{8}$, By long division, we have

$$\begin{array}{r} 4.125 \\ \hline 8 \div 33.000 \\ -32 \\ \hline 10 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$$\therefore \frac{33}{8} = 4.125, Which\ is\ terminating.$$

Que 5. Write $\frac{3}{13}$ in decimal form and find what kind of decimal expansion it has.

Sol. By long division, we have

$$\begin{array}{c} 0.230769230 \\ \hline 13 \div 3.00 \\ -26 \\ \hline 40 \\ -39 \\ \hline 100 \\ -91 \\ \hline 90 \\ -78 \\ \hline 120 \\ -117 \\ \hline 30 \\ -26 \\ \hline 40 \\ -39 \\ \hline 1 \end{array}$$

$\therefore \frac{3}{13} = \overline{0.230769},$ non-terminating and repeating.

Que 6. Express 0.00323232… in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol.

$$Let\ \ x = 0.00323232\ldots$$

$$Then,\ \ 100x\ 0.323232\ldots \qquad \ldots(i)$$

$$Also\ 10000x = 32.323232\ldots \qquad \ldots(ii)$$

$$Subtracting\ (i)\ from\ (ii), we\ get$$

$$10000x - 100x = 32.32\ 32\ 32\ldots - 0.32\ 32\ 32\ldots$$

$$9900x = 32$$

$$x = \frac{32}{9900} \Rightarrow x = \frac{8}{2475}.$$

Que 7. Express $0.3\overline{57}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol.

$$Let\ \ x = 0.3\overline{57}.$$

$$Then,\ \ x = 0.35777\ldots$$

$$So, 100x = 35.777\ldots \qquad \ldots(i)$$

$$1000x = 357.777\ldots \qquad \ldots(ii)$$

$$Subtracting\ (i) from\ (ii), we\ get$$

$$1000x - 100x = 357.777\ldots - 35.777\ldots$$

$$900x = 322 \Rightarrow x = \frac{322}{9000}$$

$$x = \frac{322}{900} = \frac{161}{450}$$

Que 8. Express: $2.0\overline{15}$ in the $\frac{p}{q}$ Form, Where p and q are integers and $q \neq 0$.

Sol.

$$Let\ \ x = 2.0\overline{15}$$

$$Then,\ \ x = 2.0151515\ldots$$

$$\Rightarrow 10x = 20.151515\ldots$$

$$10x = 20 + 0.151515\ldots \qquad \ldots(i)$$

$$Let\ y = 0.151515\ldots \qquad \ldots(ii)$$

$$\Rightarrow\ 100y\ =\ 15.151515\ldots \quad \ldots(iii)$$

$$Subtracting\ (ii)\ from\ (iii), we\ get$$

$$100y - y\ =\ 15.1515\ldots. - 0.151515\ldots.$$

$$99y\ =\ 15$$

$$y\ =\ \frac{15}{99} \Rightarrow\ y\ =\ \frac{5}{33}$$

$$Now\quad 10x\ =\ 20 + \frac{5}{33} \Rightarrow\ 10x = \frac{660+5}{33}$$

$$\Rightarrow\ 10x = \frac{665}{33} \Rightarrow\ x\ = \frac{665}{330}\ \Rightarrow x = \frac{133}{66}.$$

Que 9. Show that 0.142857142857 … = $\frac{1}{7}$

Sol.

$$Let\ x\ = 0.\overline{142857} \qquad \ldots(i)$$

$$Then\ 10{,}00{,}000x\ =\ 142857.\overline{142857} \qquad \ldots(ii)$$

$$Subtracting\ (i)\ from\ (ii), we\ get$$

$$9{,}99{,}999x\ =\ 142857$$

$$\Rightarrow\ x\ =\ \frac{142857}{999999} = \frac{1}{7}$$

$$Hence, 0.142857142857\ldots = \frac{1}{7}$$

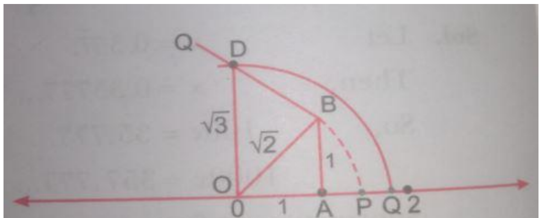


Fig. 1.2

Que 10. Locate $\sqrt{3}$ on the number line.

Sol. Construct a number line and mark a point O, representing zero. Let point A represents 1 as shown in **Fig.1.2** Clearly, OA = 1 unit. Now, draw a right triangle OAB in which AB = OA = 1 unit. Using Pythagoras theorem, we have

$$OB^2 = OA^2 + AB^2 = 1^2 + 1^2$$

$$\Rightarrow \quad OB^2 = 2 \quad \Rightarrow OB = \sqrt{2}$$

Taking O as centre OB as a radius draw an arc intersecting the number line at point P. Then P

corresponds to $\sqrt{2}$ on the number line. Now draw DB of unit length perpendicular to OB. Then using Pythagoras theorem, we have

$$OD^2 = OB^2 + DB^2$$

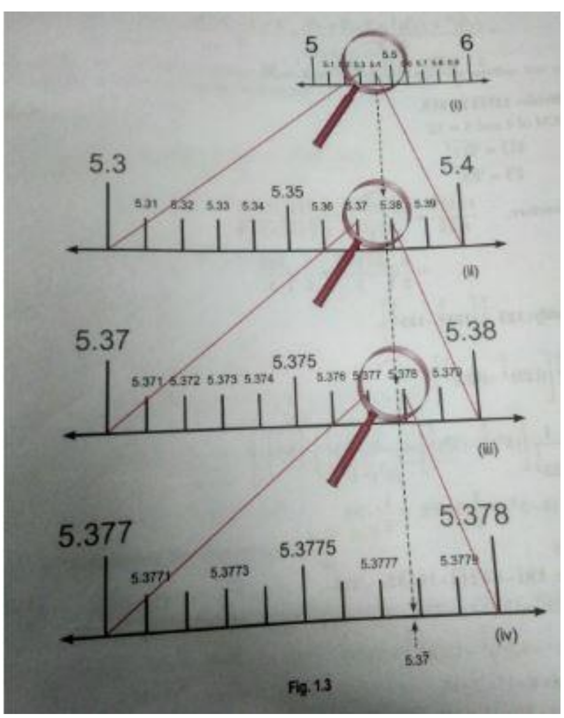
$$OD^2 = \left(\sqrt{2}\right)^2 + 1^2 = 2 + 1 = 3$$

$$OD = \sqrt{3}$$

Taking O as centre and OD as a radius draw an arc which intersects the number line at the point Q Clearly, Q corresponds to $\sqrt{3}$.

Que 11. Visualize the representation of 5. $3\overline{7}$ On the number line up to 5 decimal places, that is, up to 5.37777.

Sol. First, we se that $5.3\overline{7}$ is located between 5 and 6. In the next step, we locate $5.3\overline{7}$ between 5.3 and 5.4 To get a more accurate visualization of the representation, we divide this portion of the number line into 10 equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies between $5.3\overline{7}$ and 5.38. To visualize 5.37 more accurately, we again divide the portion between 5.37 and 5.38 into ten equal parts and use a magnifying glass to visualize that $5.3\overline{7}$ lies between 5.377 and 5.378 into 10 equal parts, and visualize the representation of $5.3\overline{7}$ as in Fig. 1.3 (iv). We notice that $5.3\overline{7}$ is located closer to 5.3778 than to 5.3777.



Que 12. Rationalise the denominator of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$.

Sol. We have,

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(2+\sqrt{3})^2}{2^2-(\sqrt{3})^2}$$

$$= \frac{2^2+(\sqrt{3})^2+2\times 2\sqrt{3}}{4-3}$$

$$= \frac{4+3+4\sqrt{3}}{1} = 7+4\sqrt{3}$$

Que 13. Rationalise the denominator of $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.

Sol.

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2.\sqrt{5}\times\sqrt{3}}{5-3} = \frac{5+3+2\sqrt{15}}{2}$$

$$= \frac{8+2\sqrt{15}}{2} = \frac{2(4+\sqrt{15})}{2} = 4+\sqrt{15}.$$

Que 14. Divide: $12\sqrt[4]{15}$ *by* $8\sqrt[3]{3}$.

Sol. LCM of 4and 3 = 12

$$\therefore\ \sqrt[4]{15} = \sqrt[12]{15^3}$$

$$\sqrt[3]{3} = \sqrt[12]{3^4}$$

$$Therefore, \frac{12\sqrt[4]{15}}{8\sqrt[3]{3}} = \frac{12\sqrt[12]{15^3}}{8\sqrt[12]{3^4}} = \frac{3}{2}\sqrt[12]{\frac{15\times 15\times 15}{3\times 3\times 3\times 3}}$$

$$= \frac{3}{2}\sqrt[12]{\frac{5\times 5\times 5}{3}} = \frac{3}{2}\sqrt[12]{\frac{125}{3}}$$

Que 15. Simplify: $\mathbf{125^{-\frac{1}{3}}\left[125^{\frac{1}{3}} - 125^{\frac{2}{3}}\right]}$.

Sol.

$$125^{-\frac{1}{3}}\left[(125)^{\frac{1}{3}} - (125)^{\frac{2}{3}}\right]$$

$$= \frac{1}{125^{\frac{1}{3}}}\left[(5^3)^{\frac{1}{3}} - (5^3)^{\frac{2}{3}}\right] = \frac{1}{(5^3)^{\frac{1}{3}}}\left[\left(5^{\frac{3}{3}}\right) - \left(5^{\frac{2}{3}}\right)^2\right]$$

$$= \frac{1}{5}(5 - 5^2) = \frac{1}{5}(5 - 25) = \frac{1}{5}(5 - 25) = \frac{1}{5}(-20)$$

$$= -4$$

Que 16. Simplify: $\mathbf{\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}}$.

Sol.

$$\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$$

$$= (3^4)^{\frac{1}{4}} - 8(6^3)^{\frac{1}{3}} + 15(2^5)^{\frac{1}{5}} + (15^2)^{\frac{1}{2}}$$

$$= 3 - 8 \times 6 + 15 \times 2 + 15$$

$$= 3 - 48 + 30 + 15 = 48 - 48 = 0$$

Que 17. Locate $\sqrt{10}$ on the number line.

Sol. We write 10 as the sum of the squares of two natural numbers.

10 = 9 + 1 = $3^2 + 1^2$

Take OA = 3 units, on the number line

Draw BA = 1 unit, perpendicular to OA. Join OB (Fig. 1.4).

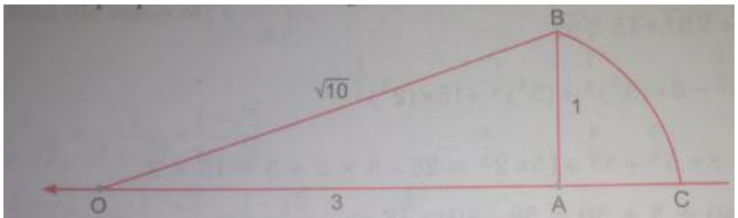


Fig. 1.4

Now, by Pythagoras theorem,

$OB^2 = AB^2 + OA^2$

$OB^2 = 1^2 + 3^2 = 10 \quad \Rightarrow \quad OB = \sqrt{10}$

Taking O as Centre and OB as a radius, draw an arc which intersects the number line at point C. Clearly, C corresponds to $\sqrt{10}$ on the number line.

Que 18. Simplify: $\frac{7+3\sqrt{5}}{3+\sqrt{5}} + \frac{7-3\sqrt{5}}{3-\sqrt{5}}$.

Sol.

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}} + \frac{7-3\sqrt{5}}{3-\sqrt{5}} = \frac{(3-\sqrt{5})(7+3\sqrt{5}) + (3+\sqrt{5})(7-3\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{21 + 9\sqrt{5} - 7\sqrt{5} - 15 + 21 - 9\sqrt{5} + 7\sqrt{5} - 15}{3^2 - (\sqrt{5})^2}$$

$$= \frac{42 - 30}{9 - 5} = \frac{12}{4} = 3$$

Que 19. Simplify: $\frac{2+\sqrt{3}}{2-\sqrt{3}} - \frac{2-\sqrt{3}}{2+\sqrt{3}}$.

Sol.

$$\frac{2+\sqrt{3}}{2-\sqrt{3}} - \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})^2 - (2-\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{2^2 + (\sqrt{3})^2 + 2 \times 2\sqrt{3} - \left[(2)^2 + \left(\sqrt{3}\right)^2 - 2 \times 2\sqrt{3}\right]}{2^2 - (\sqrt{3})^2}$$

$$= \frac{4 + 3 + 4\sqrt{3} - \left(4 + 3 - 4\sqrt{3}\right)}{4 - 3} = \frac{7 + 4\sqrt{3} - \left(7 - 4\sqrt{3}\right)}{1}$$

$$= 7 + 4\sqrt{3} - 7 + 4\sqrt{3} = 8\sqrt{3}.$$

Que 20. Simplify by rationalizing the denominator: $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$.

Sol.

$$= \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}} = \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{16 \times 3} + \sqrt{9 \times 2}} = \frac{4\sqrt{3}+5\sqrt{2}}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}$$

$$= \frac{16\left(\sqrt{3}\right)^2 + 20\sqrt{6} - 12\sqrt{6} - 15\left(\sqrt{2}\right)^2}{\left(4\sqrt{3}\right)^2 - \left(3\sqrt{2}\right)^2}$$

$$= \frac{16 \times 3 + (20 - 12)\sqrt{6} - 15 \times 2}{16 \times 3 - 9 \times 2}$$

$$= \frac{48 + 8\sqrt{6} - 30}{48 - 18} = \frac{18 + 8\sqrt{6}}{30}$$

$$= \frac{2(9 + 4\sqrt{6})}{30} = \frac{9 + 4\sqrt{6}}{15}$$

Que 21. Simplify: $\sqrt{625} - 8\sqrt[3]{125} + \sqrt[4]{81} + 15\sqrt[5]{32}$.

Sol.

$$\sqrt{625} - 8\sqrt[3]{125} + \sqrt[4]{81} + 15\sqrt[5]{32}$$

$$= (25^2)^{\frac{1}{2}} - 8 \times (5^3)^{\frac{1}{3}} + (3^4)^{\frac{1}{4}} + 15 \times (2^5)^{\frac{1}{5}}$$

$$= 25^{\frac{2}{2}} - 8 \times 5^{\frac{3}{3}} + 3^{\frac{4}{4}} + 15 \times 2^{\frac{5}{5}} = 25 - 8 \times 5 + 3 + 15 \times 2$$

$$= 25 - 40 + 3 + 30 = 58 - 40 = 18$$

Que 22. Rationalise the denominator of $\frac{1}{\sqrt{3}+\sqrt{2}}$ and hence evaluate by taking $\sqrt{2}$ =1.414 and $\sqrt{3}$ = 1.732, up to three places of decimal.

Sol. Rationalising the denominator, we get

$$\frac{1}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{3 - 2} = \sqrt{3} - \sqrt{2}$$

Substituting the values of $\sqrt{3}$ and $\sqrt{2}$, we get 1.732 – 1.414 = 0.318.

Que 23. Simplify: $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

Sol.

$$\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}} = \frac{9^{\frac{1}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times 27^{\frac{1}{7}}} = \frac{(3^2)^{\frac{1}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times (3^3)^{\frac{1}{2}}} = \frac{3^{\frac{2}{3}} \times 3^{\frac{2}{3}}}{3^{\frac{1}{6}} \times 3^2}$$

$$= \frac{3^{\frac{2}{3}+\frac{2}{3}}}{3^{\frac{1}{6}+\frac{3}{2}}} = \frac{3^{\frac{4}{3}}}{3^{\frac{1+9}{6}}} = \frac{3^{\frac{4}{3}}}{3^{\frac{10}{6}}} = 3^{\frac{4}{3}-\frac{5}{3}} = 3^{-\frac{1}{3}} = \frac{1}{3^{\frac{1}{3}}}$$

Que 24. Simplify: $\left[9\left(64^{\frac{1}{3}}+125^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}}$.

Sol.

$$\left[9\left(64^{\frac{1}{3}}+125^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}} = \left[9\left((4^3)^{\frac{1}{3}}+(5^3)^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}}$$

$$= [9(4+5)^3]^{\frac{1}{4}} = (9\times 9^3)^{\frac{1}{4}} = (9^4)^{\frac{1}{4}} = 9$$

Que 25. If a=2 + $\sqrt{3}$, Find the value of a-$\frac{1}{a}$.

Sol.

$$a = 2+\sqrt{3} \quad \Rightarrow \frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$\therefore \frac{1}{a} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$Hence, a-\frac{1}{a} = 2+\sqrt{3}-\left(2-\sqrt{3}\right) = 2+\sqrt{3}-2+\sqrt{3} = 2\sqrt{3}$$

Que 26. If x = 1 + $\sqrt{2}$, find the value of $\left(x-\frac{1}{x}\right)^{3}$.

Sol.

$$x = 1+\sqrt{2}$$

$$and, \frac{1}{x} = \frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}}\times\frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$\frac{1}{x} = \frac{1-\sqrt{2}}{1^2-\left(\sqrt{2}\right)^2} = \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} \Rightarrow \frac{1}{x} = \sqrt{2}-1$$

$$Now, x-\frac{1}{x} = 1+\sqrt{2}-\left(\sqrt{2}-1\right) = 1+\sqrt{2}-\sqrt{2}+1 = 2$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^{3} = 2^3 = 8$$

Que 27. Find the value of x, if $\left(\frac{6}{5}\right)^{x}\left(\frac{5}{6}\right)^{2x} = \frac{125}{216}$

Sol.

$$\left(\frac{6}{5}\right)^{x}\left(\frac{5}{6}\right)^{2x} = \frac{125}{216} \Rightarrow \left(\frac{6}{5}\right)^{x}\left(\frac{5}{6}\right)^{x}\left(\frac{5}{6}\right)^{x} = \frac{125}{216}$$

$$\Rightarrow \left(\frac{6}{5}\times\frac{5}{6}\right)^{x}\left(\frac{5}{6}\right)^{x} = \frac{5^3}{6^3}$$

$$\Rightarrow 1^{x}\left(\frac{5}{6}\right)^{x} = \left(\frac{5}{6}\right)^{3} \Rightarrow \left(\frac{5}{6}\right)^{x} = \left(\frac{5}{6}\right)^{3}$$

$$\Rightarrow x = 3$$

Que 28. Represent $\sqrt{4.5}$ on the number line.

Sol. Consider a line segment AB = 4.5 units. Extend AB upto point C such that BC = 1 unit.

AC = 4.5 + 1 5.5 units. Now mark O as the midpoint of AC. With O as centre and radius OC draw a semicircle. Draw Perpendicular BD on AC which intersect the semicircle at D.

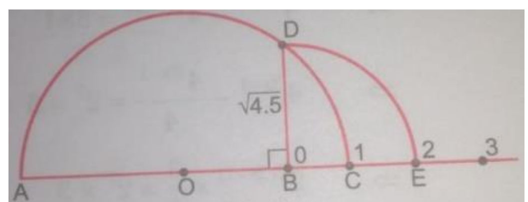


Fig. 1.5

This length BD = $\sqrt{4.5}$ units.

To show BD on the number line, consider line ABC as number line with point B as zero. Therefore, BC = 1 unit.

With B as centre and radius BD draw an arc which intersects number line ABC at E. So this point E represents $\sqrt{4.5}$ on number line.

AB = 4.5 units

BC = 1 unit

BD = BE = $\sqrt{4.5}$ units

Que 29. Multiply: $5\sqrt[3]{4}$ *by* $\sqrt{3}$.

Sol. LCM of 3 and 2 = 6

$$\therefore\ 5\sqrt[3]{4} = 5\sqrt[6]{4^2} = 5\sqrt[6]{16}$$

$$\sqrt{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$$

$$5\sqrt[3]{4}\times\sqrt{3} = 5\sqrt[6]{16}\times\sqrt[6]{27}$$

$$= 5\sqrt[6]{16 \times 27} = 5\sqrt[6]{432.}$$

Que 30. If $2^{5x} \div 2^{x} = \sqrt[5]{2^{20}}, find\ x.$

Sol.

$$\frac{2^{5x}}{2^{x}} = (2^{20})^{\frac{1}{5}}$$

$$\Rightarrow 2^{5x-x} = 2^{20\times\frac{1}{5}} \quad \Rightarrow 2^{4x} = 2^{4}$$

$$\Rightarrow 4x = 4 \quad \Rightarrow x = 1$$

Que 31. Simplify: $\frac{\sqrt{25}}{\sqrt[3]{64}} + \left(\frac{256}{625}\right)^{-\frac{1}{4}} + \frac{1}{\left(\frac{64}{125}\right)^{\frac{2}{3}}}.$

Sol.

$$\frac{\sqrt{25}}{\sqrt[3]{64}} + \left(\frac{256}{625}\right)^{-\frac{1}{4}} + \frac{1}{\left(\frac{64}{125}\right)^{\frac{2}{3}}}.$$

$$= \frac{\sqrt{5\times 5}}{\sqrt[3]{4\times 4\times 4}} + \left(\frac{625}{256}\right)^{\frac{1}{4}} + \left(\frac{125}{64}\right)^{\frac{2}{3}}$$

$$= \frac{5}{4} + \left(\frac{5^4}{4^4}\right)^{\frac{1}{4}} + \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} = \frac{5}{4} + \left(\frac{5}{4}\right)^{4\times\frac{1}{4}} + \left(\frac{5}{4}\right)^{3\times\frac{2}{3}}$$

$$= \frac{5}{4} + \frac{5}{4} + \left(\frac{5}{4}\right)^{2} = \frac{5}{4} + \frac{5}{4} + \frac{25}{16}$$

$$= \frac{20+20+25}{16} = \frac{65}{16}$$

Que 32. If $4^{2x-1} - 16^{x-1} =$ 384, find the value of x.

Sol. $4^{2x-1} - 16^{x-1} = 384,$

$$\Rightarrow 4^{2x-1} - 4^{2(x-1)} = 384 \quad \Rightarrow 4^{2x-1} - \frac{4^{2x-2+1}}{4} = 384$$

2	384
2	192
2	96
2	48
2	24
2	12
2	6
	3

$$\Rightarrow\ 4^{2x-1} - \frac{4^{2x-1}}{4} = 2^7 \times 3 \quad \Rightarrow\ 4^{2x-1}\left(1 - \frac{1}{4}\right) = 2^7 \times 3$$

$$\Rightarrow\ 2^{2(2x-1)} \times \frac{3}{4} = 2^7 \times 3 \quad \Rightarrow\ 2^{4x-2} = 2^7 \times 3 \times \frac{2^2}{3} = 2^9$$

Equating the exponents, we get

$$4x - 2 = 9 \ or\ x = \frac{11}{4}.$$

Long Answer Type Questions

[4 MARKS]

Que 1. Find the value of:

$$\frac{\mathbf{4}}{(\mathbf{216})^{\frac{-2}{3}}} + \frac{\mathbf{1}}{(\mathbf{256})^{-\frac{3}{4}}} + \frac{\mathbf{2}}{(\mathbf{243})^{-\frac{1}{5}}}.$$

Sol.

$$\frac{4}{(216)^{\frac{-2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}.$$

$$= 4(216)^{\frac{2}{3}} + (256)^{\frac{3}{4}} + 2(243)^{\frac{1}{5}} = 4(6^3)^{\frac{2}{3}} + (4^4)^{\frac{3}{4}} + 2(3^5)^{\frac{1}{5}}$$

$$= 4 \times 6^2 + 4^3 + 2 \times 3 = 4 \times 36 + 64 + 6$$

$$= 144 + 64 + 6 = 214$$

Que 2. Find the values of a and b:

$$\frac{\mathbf{7} + \sqrt{\mathbf{5}}}{\mathbf{7} - \sqrt{\mathbf{5}}} - \frac{\mathbf{7} - \sqrt{\mathbf{5}}}{\mathbf{7} + \sqrt{\mathbf{5}}} = \boldsymbol{a} + \frac{\mathbf{7}}{\mathbf{11}}\sqrt{\mathbf{5}}\boldsymbol{b}.$$

Sol. LHS

$$\frac{7 + \sqrt{5}}{7 - \sqrt{5}} - \frac{7 - \sqrt{5}}{7 + \sqrt{5}}$$

Rationalising the denominator, we get

$$\frac{7 + \sqrt{5}}{7 - \sqrt{5}} \times \frac{7 + \sqrt{5}}{7 + \sqrt{5}} - \frac{7 - \sqrt{5}}{7 + \sqrt{5}} \times \frac{7 - \sqrt{5}}{7 - \sqrt{5}}$$

$$= \frac{\left(7 + \sqrt{5}\right)^2}{7^2 - \left(\sqrt{5}\right)^2} - \frac{\left(7 - \sqrt{5}\right)^2}{7^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{7^2 + \left(\sqrt{5}\right)^2 + 2 \times 7 \times \sqrt{5}}{49 - 5} - \frac{7^2 + \left(\sqrt{5}\right)^2 - 2 \times 7\sqrt{5}}{49 - 5}$$

$$= \frac{49 + 5 + 14\sqrt{5}}{44} - \frac{49 + 5 - 14\sqrt{5}}{44} = \frac{54 + 14\sqrt{5}}{44} - \frac{54 - 14\sqrt{5}}{44}$$

$$= \frac{54 + 14\sqrt{5} - 54 + 14\sqrt{5}}{44} = \frac{28\sqrt{5}}{44} = \frac{7\sqrt{5}}{11} = 0 + \frac{7\sqrt{5}}{11}$$

Hence,

$$0+\frac{7\sqrt{5}}{11} = a+\frac{7\sqrt{5}b}{11}$$

$$\Rightarrow a=0, b=1$$

Que 3. Simplify: $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

Sol.

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}\times\frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}} = \frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}\right)^2-\left(\sqrt{3}\right)^2} = \frac{7\left(\sqrt{30}-3\right)}{10-3}$$

$$\therefore \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\left(\sqrt{30}-3\right)}{7} = \sqrt{30}-3$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}\times\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{30}-2\times 5}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}$$

$$\therefore \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\left(2\sqrt{30}-10\right)}{6-5} = 2\sqrt{30}-10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}\times\frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}} = \frac{3\sqrt{30}-18}{15-18} = \frac{3\sqrt{30}-18}{-3}$$

$$\therefore \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\left(\sqrt{30}-6\right)}{-3} = -\left(\sqrt{30}-6\right) = 6-\sqrt{30}$$

Therefore,

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$=\sqrt{30}-3-\left(2\sqrt{30}-10\right)-\left(6-\sqrt{30}\right)$$

$$=\sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$$

$$=10-9+2\sqrt{30}-2\sqrt{30}=1$$

Que 4. If $\boldsymbol{a} = \frac{2+\sqrt{5}}{2-\sqrt{5}}$ *and* $\boldsymbol{b} = \frac{2-\sqrt{5}}{2+\sqrt{5}}$, then find the value of $a^2 - b^2$.

Sol.

$$a = \frac{2+\sqrt{5}}{2-\sqrt{5}}$$

$$a = \frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}, \qquad (by\ rationalising\ the\ denominator)$$

$$= \frac{(2+\sqrt{5})^2}{2^2-\left(\sqrt{5}\right)^2} = \frac{4+5+4.\sqrt{5}}{4-5} = \left(9+4\sqrt{5}\right)$$

$$Also, b = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{\left(2-\sqrt{5}\right)^2}{2^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{2^2 + \left(\sqrt{5}\right)^2 - 2.2\sqrt{5}}{4-5} = \frac{4+5-4\sqrt{5}}{-1}.$$

$$\therefore\ b = -\left(9-4\sqrt{5}\right) \Rightarrow b = 4\sqrt{5}-9$$

$$We\ know, \qquad a^2-b^2 = (a+b)(a-b)$$

$$Here,\ \ a+b = -9-4\sqrt{5}+4\sqrt{5}-9 = -18$$

$$a-b = -9-4\sqrt{5}-\left(4\sqrt{5}-9\right) = -8\sqrt{5}$$

$$Hence, a^2-b^2 = (a+b)(a-b) = -18\left(-8\sqrt{5}\right)$$

$$\Rightarrow a^2-b^2 = 144\sqrt{5}$$

Que 5. If $a = \frac{1}{7-4\sqrt{3}}$ $and\ b = \frac{1}{7+4\sqrt{3}}$, then find the value of:

(i) **$A^2 + b^2$** **(ii) $a^3 + b^3$**

Sol.

$$a = \frac{1}{7-4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{7^2 - \left(4\sqrt{3}\right)^2}$$

$$= \frac{7+4\sqrt{3}}{49-16\times 3} = \frac{7+4\sqrt{3}}{49-48}$$

$$\therefore\ a = \frac{1}{7-4\sqrt{3}} = 7+4\sqrt{3}$$

$$b = \frac{1}{7+4\sqrt{3}} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{7-4\sqrt{3}}{7^2 - \left(4\sqrt{3}\right)^2}$$

$$= \frac{7 - 4\sqrt{3}}{49 - 16 \times 3} = \frac{7 - 4\sqrt{3}}{49 - 48}$$

$$\therefore\ b = \frac{1}{7 + 4\sqrt{3}} = 7 - 4\sqrt{3}$$

$$\therefore\ a + b = 7 + 4\sqrt{3} + 7 - 4\sqrt{3} = 14\ and\ ab = \left(7 + 4\sqrt{3}\right)\left(7 - 4\sqrt{3}\right)$$

$$= 7^2 - \left(4\sqrt{3}\right)^2 = 49 - 16 \times 3 = 49 - 48$$

$$\Rightarrow\ ab = 1$$

$$Now, a^2 + b^2 = (a + b)^2 - 2ab = (14)^2 - 2 \times 1 = 196 - 2$$

$$\therefore\ a^2 + b^2 = 194$$

$$Also, a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (14)^3 - 3 \times 1\ (14)$$

$$a^3 + b^3 = 2744 - 42 = 2702$$

HOTS (Higher Order Thinking Skills)

Que 1. Rationalise: $\frac{\mathbf{1}}{\sqrt{\mathbf{7}}+\sqrt{\mathbf{3}})-\sqrt{\mathbf{2}}}$.

Sol. $\frac{1}{\sqrt{7}+\sqrt{3})-\sqrt{2}} = \frac{1}{\sqrt{7}+\sqrt{3})-\sqrt{2}} \times \frac{\sqrt{7}+\sqrt{3})+\sqrt{2}}{\sqrt{7}+\sqrt{3})+\sqrt{2}}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{\sqrt{7}+\sqrt{3})^2-\sqrt{2})^2} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{\sqrt{7})^2+\sqrt{3})^2+2\sqrt{21-2}}$

$= \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{7+3+2\sqrt{21-2}} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{8+2\sqrt{21}} = \frac{\sqrt{7}+\sqrt{3}+\sqrt{2}}{2(4+\sqrt{21})} \times \frac{4-\sqrt{21}}{4-\sqrt{21}}$

$= \frac{4\sqrt{7}+4\sqrt{3}+4\sqrt{2}-7\sqrt{3}-3\sqrt{7}-\sqrt{42}}{2(16-21)}$

$= \frac{\sqrt{7}-3\sqrt{3}+4\sqrt{2}-\sqrt{42}}{-10} = \frac{3\sqrt{3}-4\sqrt{2}+\sqrt{42}-\sqrt{7}}{10}$

Que 2. If a = $\frac{\sqrt{\mathbf{2}}+\mathbf{1}}{\sqrt{\mathbf{2}}-\mathbf{1}}$ and b = $\frac{\sqrt{\mathbf{2}}-\mathbf{1}}{\sqrt{\mathbf{2}}+\mathbf{1}}$, then find the value of $a^2 + b^2 - 4ab$.

Sol. Here, a = $\frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2-1^2} = \frac{(\sqrt{2})^2+1+2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

∴ $a = 3 + 2\sqrt{2}$ …(i)

$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-1} = \frac{(\sqrt{2})^2+1^2-2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3 + 2\sqrt{2}$

∴ $b = 3 - 2\sqrt{2}$ …(ii)

From equation (i) and (ii)

$a + b = 3 + 2\sqrt{2} + 3-2\sqrt{2} = 6$

$ab = (3 + 2\sqrt{2})(3-2\sqrt{2}) = 3^2 - (2\sqrt{2})^2 = 9 - 4 \times 2 = 9 - 8 = 1$

∴ $a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab = (a + b)^2 - 6ab = 6^2 - 6 \times 1 = 36 - 6 = 30$

Que 3. If a = 3 + 2$\sqrt{2}$, then find the value of:

(i) $a^2 + \frac{1}{a^2}$ **(ii)** $a^3 + \frac{1}{a^3}$

Sol. (i) We have, $a = 3 + 2\sqrt{2}$ and $\frac{1}{a} = \frac{1}{3+2\sqrt{2}}$

$$\frac{1}{a} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{3^2-(2\sqrt{2^2}} = \frac{3-2\sqrt{2}}{9-8}$$

∴ $\frac{1}{a} = 3 - 2\sqrt{2}$ $a + \frac{1}{a} = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

Putting the value of $a + \frac{1}{a}$, we get

$6^2 = a^2 + \frac{1}{a^2} + 2$ ⇒ $a^2 + \frac{1}{a^2} = 36 - 2$ ⇒ $a^2 + \frac{1}{a^2} = 34$

(ii) We have

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3 \times a^2 \times \frac{1}{a} + 3 \times a \times \frac{1}{a^2}$$

⇒ $\left(a + \frac{1}{a}\right)^3 = \left(a^3 + \frac{1}{a^3}\right) + 3\left(a + \frac{1}{a}\right)$

Putting the value of $a + \frac{1}{a}$, we get

$6^3 = a^3 + \frac{1}{a^3} + 3 \times 6$ ⇒ $a^3 + \frac{1}{a^3} = 216 - 18 = 198$

Value Based Questions

Que .1 Aman was facing some difficulties in simplifying$\frac{1}{\sqrt{7}-\sqrt{3}}$. His classmate, Sonia gave him a clue to rationalise the denominator for simplification. Aman simplified the expression and thanked Sonia for this goodwill. How did Aman simplify$\frac{1}{\sqrt{7}-\sqrt{3}}$? What values does it indicate?

Sol. $\frac{1}{\sqrt{7}-\sqrt{3}} = \frac{1}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$$= \frac{\sqrt{7}+\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} = \frac{\sqrt{7}+\sqrt{3}}{4}$$

Helpfulness, cooperativeness, knowledge.

Que 2. In a school, 5 out of every 7 children participated in 'Save Wild Life' campaign organised by the school authorities. What fraction of the students participated in the campaign? Find what kind of decimal expansion it has. What value do the participating students possess?

Sol.$\frac{5}{7} = 0.\overline{714285}$, Non-terminating repeating decimal.
Caring, social, helpful, environmental concern.

Que 3. In a survey, it was found that 9 out of every 11 households are donating some amount of their income to an orphanage or old age homes or institutions for physically handicaps. What fraction of households are not donating? Write it in decimal form and find what kind of decimal expansion it has. What value of society are depicted here?

Sol. $\frac{2}{11} = 0.\overline{18}$, Non terminating repeating.

People are becoming more social, helpful, cooperative and caring.

Que 4. Teacher asked the students "Can we write $0.\overline{47}$ *in* $\frac{p}{q}$ form as $\frac{47}{100}$? Mukta answered, "No, it is $\frac{43}{90}$". Is Mukta correct? Justify her answer. Which values of Mukta are depicted here?

Sol. Yes,
Let x = 0.477777…. (i)
10x = 4.77777….. (ii)

Subtracting (i) from (ii), we get

9x = 4.3 or x = $\frac{43}{90}$

Scientific temper, knowledge, curiosity.

Que 5. The number of trees planted on Van Mahotsav in a region was 103 × 98. Find the number of trees planted without actual multiplication. Which values of the people living in this region are depicted here?

Sol. $103 \times 98 = (100 + 3)(100 - 2)$
$= 100^2 + (3 - 2) \times 100 - 3 \times 2$
$= 10{,}000 + 100 - 6 = 10{,}094$
Environmental care, social, happiness.

Que 6. 95 students each from 102 schools participated in the 'Republic Day Celebration' in Delhi. Find the number of students participated without actual multiplication. Which values of the students are depicted here?

Sol. $95 \times 102 = (100 - 5)(100 + 2)$
$= 100^2 + (-5 + 2)\,100 + (-5 \times 2) = 10{,}000 - 300 - 10 = 9{,}690$
Fraternity, patriotism.

Que 7. Two classmates Anya and Madhur simplified two different expressions during the revision of $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ and Madhur explains simplifications of $\sqrt{25} + \sqrt{98} + \sqrt{147}$.
write both the simplifications. What values does it depict?

Sol. $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{2}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$$= \frac{\sqrt{10}-\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{10}-\sqrt{6}}{2}$$

Also, $\sqrt{28} + \sqrt{98} + \sqrt{147} = \sqrt{2 \times 2 \times 7} + \sqrt{2 \times 7 \times 7} + \sqrt{3 \times 7 \times 7}$
$= 2\sqrt{7} + 7\sqrt{2} + 7\sqrt{3}$

Cooperativeness, knowledge.