Very Short Answer Type Questions

[1MARK]

Que 1. How many zeros does cubic polynomial has?

Sol. A cubic polynomial has three zeros.

Que 2. Write the coefficient of x^2 in the expansion of $(x – 2)^3$

Sol. $(x-2)^3 = x^3 - 3 \times x^2 \times 2 + 3 \times x \times 2^2 - 2^3 = x^3 - 6x^2 + 12x - 8$

Clearly the coefficient of x^2 is -6.

Que 3. Find the value of $\oint (x) = 2x^2 + 7x + 3$ at $x = -2$.

Sol.

$$\oint (x) = 2x^2 + 7x + 3$$

$$\oint (-2) = 2(-2)^2 + 7(-2) + 3$$

$$= 8 - 14 + 3 = 11 - 14 = -3$$

Que 4. Is $x^2 + \frac{4x^{\frac{3}{2}}}{\sqrt{x}}$ a Polynomial? Justify your answer.

Sol. Yes,

$$x^2 + \frac{4x^{\frac{3}{2}}}{\sqrt{x}} = x^2 + 4x^{\frac{3}{2}} \times x^{-\frac{1}{2}}$$

$$= x^2 + 4x^{\frac{3}{2}-\frac{1}{2}} = x^2 + 4x$$

Que 5. Write the coefficient of y in the expansion of $(5 – y)^2$.

Sol.

$$(5-y)^2 = 25 + y^2 - 10y. \textit{Clearly the coefficient of } y \textit{ is } -10.$$

Que 6. Find the value of polynomial $12x^2 - 7x + 1$, $when\ x\ \frac{1}{4}$.

Sol. $Let\ p(x) = 12x^2 - 7x + 1$

$$p\left(\frac{1}{4}\right) = 12 \times \left(\frac{1}{4}\right)^2 - 7 \times \frac{1}{4} + 1 = 12 \times \frac{1}{16} - \frac{7}{4} + 1$$

$$= \frac{3}{4} - \frac{7}{4} + 1 = \frac{3 - 7 + 4}{4} = \frac{7 - 7}{4} = 0$$

Que 7. $\boldsymbol{If\ a+b+c=0, then\ what\ is\ the\ value\ of\ a^3+b^3+c^3}$?

Sol. We know $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$As\ a+b+c = 0$$

$$\therefore\ a^3+b^3+c^3-3abc = 0$$

$$\Rightarrow\ a^3+b^3+c^3 = 3abc$$

Que 8. Find the value of $\mathbf{513^2-512^2}$.

$$513^2-512^2 = (513+512)(513-512)$$

$$= 1025\times 1 = 1025$$

Que 9. Find the zero of the polynomial $\boldsymbol{p(x)=2x+3}$.

Sol. For zero of the Polynomial $p(x), we\ put\ p(x)=0 \Rightarrow 2x+3=0$

$$\Rightarrow 2x=-3 \quad \Rightarrow x=-\frac{3}{2}$$

Que 10. Give an example of a polynomial which is

(i) Monomial of degree 1.

(ii) Binomial of degree 20.

(iii) Trinomial of degree 2.

Sol. (i) Required polynomial should have one term with highest power of the variable 1.

$\therefore x\ or\ 9y\ or\ -4a$ are some of the possible polynomials.

(ii) Required polynomial should have two terms with highest power of the variable 20.

$\therefore\ y^{20}+9\ or\ 8x+x^{20}\ or\ m^3-9m^{20}$ are some of the possible polynomials.

(iii) Required polynomial should have three terms with highest power of the variable 2.

$\therefore x^2+x-1\ or\ y^2+8y+11\ or\ y^2-6y-7$ are some of the possible polynomials.

Short Answer Type Questions – I

[2 marks]

Que 1. If x + 1 Is a factor of the polynomial $3x^2$ –kx, then find the value of k.

Sol. $Let\ p(x) = 3x^2 - kx, as\ (x+1)\ is\ a\ factor\ of\ p(x)$

$$So, p(-1) = 0\ i.e., \quad 3(-1)^2 - k(-1) = 0 \Rightarrow k = -3$$

Que 2. Find the value of k, if y+3 is a factor of $3y^2$ + ky + 6.

Sol. $Let\ p(y) = 3y^2 + ky + 6$

$$As\ y+3\ is\ a\ factor\ of\ p(y), so\ p(-3) = 0$$

$$i.e., \quad 3(-3)^2 + k(-3) + 6 = 0$$

$$\Rightarrow 27 - 3k + 6 = 0 \quad \Rightarrow 33 - 3k = 0$$

$$\Rightarrow -3k = -33 \quad \Rightarrow k = 11$$

Que 3. Find the value of a, if x – a is a factor of $x^3 - ax^2 + 2x + a - 1$.

Sol. $Let\ p(x) = x^3 - ax^2 + 2x + a - 1$

$$As\ (x-a)\ is\ a\ factor\ of\ p(x), so\ p(a) = 0, i.e., a^3 - a.a^2 + 2a + a - 1 = 0$$

$$\Rightarrow 3a - 1 = 0$$

$$\Rightarrow a = \frac{1}{3}$$

Que 4. If $\oint(z) = z^2 - 3\sqrt{2}\,z - 1, then\ find \oint\left(3\sqrt{2}\right)$.

Sol. $\oint(z) = z^2 - 3\sqrt{2}\,z - 1, then\ find \oint\left(3\sqrt{2}\right)$.

$$\Rightarrow \oint\left(3\sqrt{2}\right) = \left(3\sqrt{2}\right)^2 - 3\sqrt{2}\left(3\sqrt{2}\right) - 1 = 9 \times 2 - 9 \times 2 - 1 = -1$$

Que 5. If $x^{11} + 101\ is\ divided\ by\ x + 1, what\ is\ the\ remainder$?

Sol. $Let \quad p(x) = x^{11} + 101$

Using the remainder theorem, we have

$$Remainder = p(-1) = (-1)^{11} + 101$$

$$= -1 + 101 = 100$$

Que 6. Find the factors of (1 - x^3).

Sol. $1 - x^3 = 1^3 - x^3 = (1-x)(1+x+x^2)$

Que 7. Find the factors of $y^3 + y^2 + y + 1$.

Sol. $y^3 + y^2 + y + 1 = y^2(y+1) + 1(y+1) = (y+1)(y^2+1)$

Que 8. If $x + y = 9$ and $xy = 20$, then find the value of $x^2 + y^2$.

Sol. We know that $(x+y)^2 = x^2 + y^2 + 2xy$

$$\Rightarrow 9^2 = x^2 + y^2 + 2 \times 20$$

$$\Rightarrow x^2 + y^2 = 81 - 40 = 41$$

Que 9. If $x + \frac{1}{x} = 4$, then find the value of $x^2 + \frac{1}{x^2}$.

Sol. $x + \frac{1}{x} = 4$

$$\Rightarrow \left(x \frac{1}{x}\right)^2 = 4^2 \Rightarrow x^2 + \frac{1}{x^2} + 2x \times \frac{1}{x} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = 16 - 2 = 14$$

$$\therefore x^2 + \frac{1}{x^2} = 14$$

Que 10. Using factor theorem, show that (x- y) is a factor of $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$.

Sol. $Let\ p(x) = x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

Putting x = y in given polynomial p(x), we get

$$p(y) = y(y^2 - z^2) + y(z^2 - y^2) + z(y^2 - y^2)$$

$$= y(y^2 - z^2) - y(y^2 - z^2) = 0$$

$\therefore$ $(x - y)$ is a factor of given polynomial $p(x)$.

Que 11. If $x^2 - 1$ is a factor of $ax^3 + bx^2 + cx + d$, show that a + c = 0.

Sol. $Since\ x^2 - 1 = (x+1)(x-1)\ is\ a\ factor\ of\ p(x) = ax^3 + bx^2 + cx + d$

$$\therefore p(1) = p(-1) = 0$$

$$\Rightarrow a + b + c + d = -a + b - c + d = 0$$

$$\Rightarrow 2a + 2c = 0 \quad \Rightarrow 2(a + c) = 0$$

$$\Rightarrow a + c = 0$$

Que 12. If $\boldsymbol{x + 2k\ is\ a\ factor\ of \oint (x) = x^5 - 4k^2x^3 + 2x + 2k + 3, find\ k}$.

Sol. $Here, \oint (x) = x^5 - 4k^2x^3 + 2x + 2k + 3$

$$Since\ x + 2k\ is\ a\ factor\ of \oint (x), so\ by\ factor\ theorem,$$

$$\oint (-2k) = 0$$

$$(-2k)^5 - 4k^2(-2k)^3 + 2(-2k) + 2k + 3 = 0$$

$$-32k^5 + 32k^5 - 4k + 2k + 3 = 0$$

$$\Rightarrow \quad -2k + 3 = 0 \Rightarrow -2k = -3 \Rightarrow k = \frac{3}{2}$$

Que 13. Find the remainder when $\boldsymbol{\oint (x) = 4x^3 - 12x + 14x - 3}$ is divided by $\boldsymbol{g(x) = (2x - 1)}$.

Sol. $Taking\ g(x) = 0\ we\ have,$

$$2x - 1 = 0$$

$$\Rightarrow x = \frac{1}{2}$$

By remainder theorem when $\oint (x)$ is divided by $g(x)$, the remainder is equal to $\oint \left(\frac{1}{2}\right)$

Now, $\oint (x) = 4x^3 - 12x^2 + 14x - 3$

$$\oint \left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 14\left(\frac{1}{2}\right) - 3$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 7 - 3$$

$$= \frac{1}{2} - 3 + 7 - 3 = \frac{1}{2} - 6 + 7 = 1 + \frac{1}{2}$$

$$\oint \left(\frac{1}{2}\right) = \frac{3}{2}$$

Hence, required remainder $= \frac{3}{2}$.

Que 14. $\boldsymbol{If\ (x + 1)\ is\ a\ factor\ of\ ax^3 + x^2 - 2x + 4a - 9, find\ the\ value\ of\ a}$.

Sol. $Let \oint (x) = ax^3 + x^2 - 2x + 4a - 9$

$$As\ (x + 1)\ is\ a\ factor\ of\ f(x)$$

$$\therefore \oint (-1) = 0$$

$$\Rightarrow\ a(-1)^3\ +(-1)^2\ -2(-1)+4a-9=0$$

$$\Rightarrow\ -a+1+2+4a-9=0$$

$$3a-6=0\ \Rightarrow\ 3a\ =\ 6$$

$$\Rightarrow\ a=\frac{6}{3}\ \Rightarrow a=2$$

Que 15. For what value of $\boldsymbol{k}, (\boldsymbol{x}+\mathbf{1})\ \boldsymbol{is\ a\ factor\ of\ p(x)\ =\ kx^2\ -x\ -4^2}$

Sol. $As\ x+1\ is\ a\ factor\ of\ p(x), so\ p(-1)\ =\ 0,$

$$i.e.,\quad k(-1)^2\ -(-1)\ -4\ =\ 0$$

$$k+1-4\ =4$$

$$\Rightarrow\ k-3\ =\ 0\ \ \Rightarrow k=3$$

Que 16. Expand using suitable identity (-2x+5y-3z)^2.

Sol. $(-2x+5y-3z)^2$

$$=(-2x)^2\ +(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)\ (-3z)+2(-3z)(-2x)$$

$$=\ 4x^2\ +25y^2\ +9z^2\ -20xy-30yz+12zx$$

Que 17. Find: $\boldsymbol{x+\frac{1}{x}, if\ x^2\ +\frac{1}{x^2}=62}$.

Sol.

$$\left(x+\frac{1}{x}\right)^2\ =x^2\ +\frac{1}{x^2}+2.x.\frac{1}{x}$$

$$=x^2+\frac{1}{x^2}+2=62+2\ =\ 64\quad \Rightarrow\left(x+\frac{1}{x}\right)^2\ =\ 64$$

Taking square root on both sides, we get $x+\frac{1}{x}$ = 8.

Que 18. Factorise: $\boldsymbol{\frac{25}{4}x^2\ -\frac{y^2}{9}}$.

$$Sol.\frac{25x^2}{4}-\frac{y^2}{9}=\left(\frac{5}{2}x\right)^2\ -\left(\frac{y}{3}\right)^2\ =\left(\frac{5}{2}x+\frac{y}{3}\right)\left(\frac{5}{2}x-\frac{y}{3}\right)$$

Que 19. Find the value of k if (x – 2) is a factor of polynomial $\boldsymbol{p(x)\ =\ 2x^3\ -6x^2\ + 5x+k}$.

Sol. As (x – 2) is a factor of polynomial $p(x)\ =\ 2x^3\ -6x^2\ +5x+k, so, p(2)\ =\ 0$

$$\Rightarrow 2(2)^3\ -6(2)^2\ +5\times 2\ +k\ =\ 0$$

$$\Rightarrow\ 16 - 24 + 10 + k = 0$$

$$26 - 24 + k = 0 \ \Rightarrow\ k + 2 = 0 \Rightarrow\ k = -2$$

Que 20. Factorise: $\boldsymbol{a^2 + b^2 - 2bc + 2bc - 2ca}$

Sol. $a^2 + b^2 - 2bc + 2bc - 2ca = (a-b)^2 + 2c(b-a) = (a-b)^2 - 2c(a-b)$

$= (a-b)(a-b-2c) \quad [Taking\ common\ (a-b)]$

Que 21. $\boldsymbol{Evaluate\ 185 \times 185 - 15 \times 15}$

Sol. $185 \times 185 - 15 \times 15$

$\Rightarrow (185)^2 - (15)^2 \quad [\ Using\ a^2 - b^2 = (a-b)(a+b)]$

$\Rightarrow (185 + 15)\,(185 - 15)$

$\Rightarrow 200 \times 170 = 34000$

Short Answer Type Questions – II

[3 MARKS]

Que 1. What must be subtracted from $x^4 + 3x^3 + 4x^2 - 3x - 6$ to get $3x^3 + 4x^2 - x + 3$?

Sol. Let $p(x)$ be the required polynomial.

$$Then, x^4 + 3x^3 + 4x^2 - 3x - 6 - p(x) = 3x^3 + 4x^2 - x + 3$$

$$\therefore\ p(x) = x^4 + 3x^3 + 4x^2 - 3x - 6 - 3x^3 - 4x^2 + x - 3$$

$$= x^4 - 2x - 9$$

Que 2. What must be added to $2x^2 - 5x + 6$ *to get* $x^3 - 3x^2 + 3x - 5$?

Sol. Let $p(x)$ be added.

$$Than, 2x^2 - 5x + 6 + p(x) = x^3 - 3x^2 + 3x - 5$$

$$\therefore\ p(x) = x^3 - 3x^2 + 3x - 5 - 2x^2 + 5x - 6$$

$$= x^3 - 5x^2 + 8x - 11$$

Que 3. If x $x + 2k$ *is a factor of* $f(x) = x^4 - 4k^2x^2 + 2x + 3k + 3$, *find* k.

Sol. Here, $f(x) = x^4 - 4k^2x^2 + 2x + 3k + 3$

Since (x – 2k) is a factor of f(x), so by factor theorem,

$$f(-2k) = 0$$

$$(-2k)^4 - 4k^2(-2k)^2 + 2(-2k) + 3k + 3 = 0$$

$$16k^4 - 16k^4 - 4k + 3k + 3 = 0$$

$$\Rightarrow -k + 3 = 0 \qquad \Rightarrow -k = -3 \qquad \Rightarrow k = 3$$

Que 4. Find the remainder when $f(x) = 9x^3 - 3x^2 + 14x - 3$ *is divided by* $g(x) = (3x - 1)$.

Sol. Taking g(x) = 0 we have,

$$3x - 1 = 0 \qquad \Rightarrow x = \frac{1}{3}$$

By remainder theorem when $f(x)$ is divided by $g(x)$, the remainder is equal to $f\left(\frac{1}{3}\right)$

Now,

$$f(x) = 9x^3 - 3x^2 + 14x - 3$$

$$f\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)^2 + 14\left(\frac{1}{3}\right) - 3$$

$$= 9 \times \frac{1}{27} - 3 \times \frac{1}{9} + \frac{14}{3} - 3 = \frac{1}{3} - \frac{1}{3} + \frac{14}{3} - 3 \Rightarrow f\left(\frac{1}{3}\right) = \frac{5}{3}$$

Hence, required remainder = $= \frac{5}{3}$

Que 5. Check whether polynomial $p(x) = 2x^3 - 9x^2 + x + 12$ is a multiple of 2x – 3 or not.

Sol. The polynomial p(x) will be a multiple of $2x - 3$ *if* $(2x - 3)$ divides p(x) completely.

Now, 2x – 3 = 0 $\Rightarrow x = \frac{3}{2}$

Also,

$$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12$$

$$= 2 \times \frac{27}{8} - 9 \times \frac{9}{4} + \frac{3}{2} + 12$$

$$= \frac{54}{8} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{54-162+12+96}{8} = \frac{162-162}{8} = \frac{0}{8}$$

$$p\left(\frac{3}{2}\right) = 0$$

As (2x – 3) divides p(x) completely, therefore p(x) is a multiple of (2x – 3).

Que 6. Show that $2x + 1$ is a factor of polynomial $2x^3 - 11x^2 - 4x + 1$.

Sol.

$$Let, p(x) = 2x^3 - 11x^2 - 4x + 1 \quad and \ g(x) = 2x + 1$$

By factor theorem (2x – 1) will be a factor of p(x) if $p\left(\frac{-1}{2}\right) = 0$

Now,

$$p(x) = 2x^3 - 11x^2 - 4x + 1$$

$$\Rightarrow p\left(\frac{-1}{2}\right) = 2\left(\frac{-1}{2}\right)^3 - 11\left(\frac{-1}{2}\right)^2 - 4\left(\frac{-1}{2}\right) + 1$$

$$= 2\left(\frac{-1}{8}\right) - 11 \times \frac{1}{4} + 4 \times \frac{1}{2} + 1 = -\frac{1}{4} - \frac{11}{4} + 2 + 1$$

$$= \frac{-1 - 11 + 8 + 4}{4} = \frac{-12 + 12}{4} \Rightarrow p\left(\frac{-1}{2}\right) = 0$$

As $p\left(\frac{-1}{2}\right) = 0$, therefore (2x – 1) is a factor of $2x^3 - 11x^2 - 4x + 1$.

Que 7. By actual division, find the quotient and remainder when $3x^4 - 4x^3 - 3x - 1$ is divided by x + 1.

Sol. By long division, we have

$$x+1\,\big)\,3x^4 - 4x^3 - 3x - 1\,\big(\,3x^3 - 7x^2 + 7x - 10$$
$$\underline{-3x^4 \pm 3x^3}$$
$$-7x^3 - 3x - 1$$
$$\underline{\mp 7x^3 \quad \mp 7x^2}$$
$$7x^2 - 3x - 1$$
$$\underline{-7x^2 \pm 7x}$$
$$-10x - 1$$
$$\underline{\mp 10x \mp 10}$$
$$9$$

Quotient = $= 3x^3 - 7x^2 + 7x - 10, Remainder = 9$

Que 8. If $\sqrt{m} + \sqrt{n} - \sqrt{p} = 0$, then find the value of $(m + n - p)^2$.

Sol. We have $\sqrt{m} + \sqrt{n} - \sqrt{p} = 0$

$\Rightarrow \quad \sqrt{m} + \sqrt{n} = \sqrt{p}$

Squaring both the sides, we get

$$\left(\sqrt{m} + \sqrt{n}\right)^2 = \left(\sqrt{p}\right)^2$$

$\Rightarrow \quad m + n + 2\sqrt{m}\sqrt{n} = p$

$\Rightarrow \quad m + n - p = -2\sqrt{mn}$

Again squaring both the sides, we get $(m + n - p)^2 = 4mn$

Que 9. Expand: $\left(\frac{1}{x} + \frac{y}{3}\right)^3$

Sol.

$$\left[\frac{1}{x} + \frac{y}{3}\right]^3 = \left(\frac{1}{x}\right)^3 + 3\left(\frac{1}{x}\right)^2 \frac{y}{3} + 3\,\frac{1}{x}\left(\frac{y}{3}\right)^2 + \left(\frac{y}{3}\right)^3$$

$$= \left(\frac{1}{x}\right)^3 + 3.\left(\frac{1^2}{x^2}\right)\frac{y}{3} + 3.\frac{1}{x}\frac{y^2}{3^2} + \frac{y^2}{3^3}$$

$$= \frac{1}{x^3} + \frac{y}{x^2} + \frac{y^2}{3x} + \frac{y^3}{27}$$

Que 10. Evaluate: $(104)^3$ using a suitable identity.

Sol. $(104)^3 = (100+4)^3$

Using identity $(x+y)^3 = x^3 + 3xy\,(x+y) + y^3$

We get,

$$(100+4)^3 = (100)^3 + 3 \times 100 \times 4(100+4) + 4^3$$

$$= 10{,}00{,}000 + 1{,}200 \times 104 + 64$$

$$= 10{,}00{,}000 + 1{,}24{,}800 + 64$$

$$= 11{,}24{,}864$$

Que 11. Evaluate 105×108 without multiplying directly.

Sol. $105 \times 108 = (100 + 5)\,(100 + 8)$

Using identity $(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$

$$We\ get,\ \ 105 \times 108 = 100^2 + (5+8)\,100 + 5 \times 8$$

$$= 10000 + 1300 + 40 = 11340$$

Que 12. Find the value of $x^2 + \frac{1}{x^2}, if\ x - \frac{1}{x} = \sqrt{3}$.

Sol. $x - \frac{1}{x} = \sqrt{3}$

Squaring both the sides, we get $\left(x - \frac{1}{x}\right)^2 = \left(\sqrt{3}\right)^2$

$$\Rightarrow\ x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 3$$

$$\Rightarrow\ x^2 + \frac{1}{x^2} = 3 + 2 \Rightarrow x^2 + \frac{1}{x^2} = 5$$

Que 13. Factorise: $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$ by splitting the middle term.

Sol. $5\sqrt{5}x^2 + 30x + 8\sqrt{5} = 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$

$$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}\,(\sqrt{5}x + 4)$$

$$= (\sqrt{5}x + 4)\,(\sqrt{5}x + 2\sqrt{5}) = \sqrt{5}(\sqrt{5}x + 2)(\sqrt{5}x + 4)$$

Que 14. Factorise: $2x^5 + 432x^2 y^3$.

Sol. We have, $2x^5 + 432x^2y^3 = 2x^2(x^3 + 216y^3)$

$$= 2x^2(x^3 + 6^3y^3) = 2x^2[x^3 + (6y)^3]$$

$$= 2x^2(x + 6y)[x^2 - x.6y + (6y)^3]$$

$$= 2x^2(x + 6y)(x^2 - 6xy + 36y^2)$$

Que 15. Factorise: $125x^3 + 27y^3 + 8z^3 - 90xyz$.

Sol. $125x^3 + 27y^3 + 8z^3 - 90xyz$

$$= 5^3x^3 + 3^3y^3 + 2^3z^3 - 90xyz$$

$$= (5x)^3 + (3y)^3 + (2z)^3 - 3 \times 5x \times 3y \times 2z$$

$$= (5x + 3y + 2z)[(5x)^2 + (3y)^2 + (2z)^2 - (5x)(3y) - (3y)(2z) - (2z)(5x)]$$

$$= (5x + 3y + 2z)(25x^2 + 9y^2 + 4z^2 - 15xy - 6yz - 10zx)$$

Que 16. Factorise: $\frac{r^3}{8} - \frac{s^3}{343} - \frac{t^3}{216} - \frac{1}{28}rst$.

Sol.

$$\frac{r^3}{8} - \frac{s^3}{343} - \frac{t^3}{216} - \frac{1}{28}rst.$$

$$\left(\frac{r}{8}\right)^3 + \left(\frac{-s}{7}\right)^3 + \left(\frac{-t}{6}\right)^3 - 3\left(\frac{r}{2}\right)\left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right)$$

$$= \left[\frac{r}{2} + \left(\frac{-s}{7}\right) + \left(\frac{-t}{6}\right)\right]\left[\left(\frac{r}{2}\right)^2 + \left(\frac{-s}{7}\right) + \left(\frac{-t}{6}\right)^2 - \frac{r}{2}\left(\frac{-s}{7}\right) - \left(\frac{-s}{7}\right)\left(\frac{-t}{6}\right) - \left(\frac{-t}{6}\right)\left(\frac{r}{2}\right)\right]$$

$$= \left(\frac{r}{2} - \frac{s}{7} - \frac{t}{6}\right)\left(\frac{r^2}{4} + \frac{s^2}{49} + \frac{t^2}{36} + \frac{rs}{14} - \frac{st}{42} + \frac{tr}{12}\right)$$

Que 17. Factorise: $2x^2 - 7x - 15$ by splitting the middle term.

Sol. $2x^2 - 7x - 15$

$$= 2x^2 - 10x + 3x - 15$$

$$= 2x(x - 5) + 3(x - 5)$$

$$= (x - 5)(2x + 3)$$

Que 18. Factorise: $125x^3 – 343y^3$.

Sol. $125x^3 – 343y^3$.

$$= 5^3x^3 - 7^3x^3 = (5x)^3 - (7y)^3$$

$$= (5x - 7y)[(5x)^2 + 5x.7y + (7y)^2]$$

$$= (5x - 7y)(25x^2 + 35xy + 49y^2)$$

Que 19. Factorise: $\mathbf{3x^2 + 4y^2 + 25z^2 - 4\sqrt{3xy} - 20yz + 10\sqrt{3}zx}$.

Sol. $3x^2 + 4y^2 + 25z^2 - 4\sqrt{3xy} - 20yz + 10\sqrt{3}zx$.

$$= \left(\sqrt{3}x\right)^2 + (-2y)^2 + (5z)^2 + 2\left(\sqrt{3x}\right)(-2y) + 2(-2y)(5z) + 2(5z)\left(\sqrt{5}x\right)$$

$$= (\sqrt{3}x - 2y + 5z)^2 = \left(\sqrt{3}x - 2y + 5z\right)\left(\sqrt{3}x - 2y + 5z\right)$$

Que 20. Factorise: $\left(5r + \frac{2}{3}\right)^2 - \left(2r - \frac{1}{3}\right)^2$.

Sol.

$$\left(5r + \frac{2}{3}\right)^2 - \left(2r - \frac{1}{3}\right)^2$$

$$= \left(5r + \frac{2}{3} + 2r - \frac{1}{3}\right)\left[5r + \frac{2}{3} - \left(2r - \frac{1}{3}\right)\right]$$

$$= \left(7r + \frac{2}{3} - \frac{1}{3}\right)\left(5r - 2r + \frac{2}{3} + \frac{1}{3}\right)$$

$$= \left(7r + \frac{1}{3}\right)(3r + 1).$$

Que 21. Without actually calculating the cubes, find the value of:

$$\left(\frac{-3}{4}\right)^3 + \left(\frac{-5}{8}\right)^3 + \left(\frac{11}{8}\right)^3.$$

Sol. Let

$$a = \frac{-3}{4}, b = \frac{-5}{8}, c = \frac{11}{8}$$

$$\therefore\ a + b + c = \frac{-3}{4} - \frac{5}{8} + \frac{11}{8}$$

$$= \frac{-6 - 5 + 11}{8} = 0$$

$$If\ a + b = 0, then\ a^3 + b^3 + c^3 = 3abc$$

$$\therefore \left(\frac{-3}{4}\right)^3 + \left(\frac{-5}{8}\right)^3 + \left(\frac{11}{8}\right)^3 = 3\left(\frac{-3}{4}\right)\left(\frac{-5}{8}\right)\left(\frac{11}{8}\right) = \frac{495}{256}$$

Que 22. Without finding the cubes, factorise: $(2r - 3s)^3 + (3s - 5t)^3 + (5t - 2r)^3$.

Sol. Let $a = 2r - 3s, b = 3s - 5t + 5t - 2r = 0$

$$\therefore a + b + c = 2r - 3s + 3s - 5t + 5t - 2r = 0$$

$$If\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc$$

$$\Rightarrow (2r - 3s)^3 + (3s - 5t)^3 + (5t - 2r)^3 = 3(2r - 3s)(3s - 5t)(5t - 2r)$$

Que 23. Find the value of $x^3 - 8y^3 - 36xy - 216\ When\ x = 2y + 6$.

Sol.

$$x^3 - 8y^3 - 216 - 36xy = x^3 + (-2y)^3 + (-6)^3 - 3.x(-2y)(-6)$$

$$= (x - 2y - 6)(x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$$

$$\therefore = 0 \times (x^2 + 4y^2 + 36 + 2xy - 12y + 6x)[\because x = 2y + 6 \Rightarrow x - 2y - 6 = 0]$$

$$= 0$$

Que 24. If a, b, c are all non-zero and a + b + c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.

Sol. We have,

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$$

$$LHS \quad \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$$

$$= \frac{3abc}{abc} = 3(if\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc) = RHS$$

Que 25. Simplify: $(2x - 5y)^3 - (2x + 5y)^3$.

Sol. $(2x - 5y)^3 - (2x + 5y)^3$

$$= [8x^3 - 125y^3 - 3 \times 2x \times 5y(2x - 5y)] - [8x^3 + 125y^3 + 3 \times 2x \times 5y(2x + 5y)]$$

$$[using\ the\ identity\ (a + b)^3 = a^3 + b^3 + 3ab(a + b)\ and\ (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$$

$$= (8x^3 - 125y^3 - 60x^2y + 150xy^2) - (8x^3 + 125y^3 + 60x^2y + 150xy^2)$$

$$= -250y^3 - 120x^2y$$

Que 26. Factorise $\left(9x-\frac{1}{5}\right)^2-\left(x+\frac{1}{3}\right)^2$.

Sol. We have, $\left(9x-\frac{1}{5}\right)^2-\left(x+\frac{1}{3}\right)^2$.

$$=\left[\left(9x-\frac{1}{5}\right)-\left(x+\frac{1}{3}\right)\right]\left[\left(9x-\frac{1}{5}\right)+\left(x\frac{1}{3}\right)\right] \qquad [\because a^2-b^2 = (a-b)(a+b)]$$

$$=\left(9x-\frac{1}{5}-x-\frac{1}{3}\right)\left(9x-\frac{1}{5}+x+\frac{1}{3}\right)$$

$$=\left(8x-\frac{1}{5}-\frac{1}{3}\right)\left(10x-\frac{1}{5}+\frac{1}{3}\right)$$

$$=\left(\frac{120x-3-5}{15}\right)\left(\frac{150x-3+5}{15}\right)$$

$$=\left(\frac{120x-8}{15}\right)\left(\frac{150x+2}{15}\right)$$

Long Answer Type Questions

[4 MARKS]

Que 1. Using factor theorem, factorise the polynomial $\boldsymbol{x^3 + x^2 - 4x - 4}$.

Sol. $Let\ p(x) = x^3 + x^2 - 4x - 4$.

The constant term in $p(x)$ is equal to -4 and factors of -4 are $\pm 1, \pm 2$,

Putting $x\ x = -1\ in\ p(x), we\ have$

$$p(-1) = (-1)^3 + (-1)^2 - 4 \times (-1) - 4$$

$$= -1 + 1 + 4 - 4 = 0$$

$\therefore$ $(x + 1)$ is a factor of $p(x)$

Putting $x\ \ x = 2\ in\ p(x)$, we have

$$p(2) = 2^3 + 2^2 - 4 \times 2 - 4$$

$$= -8 + 4 + 8 - 4$$

$$p(-2) = 0$$

$\therefore$ $(x + 2)$ is a factor of $p(x)$.

As $p(x)$ is a polynomial of degree 3, so it cannot have more than three linear factors.

$$\therefore \quad p(x) = k(x + 1)(x + 2)(x - 2)$$

$$x^2 + x^2 - 4x - 4 = 1(x + 1)(x + 2)(x - 2)$$

$$= (x + 1)(x + 2)(x - 2)$$

Que 2. Factorise: $\boldsymbol{x^8 - y^8}$.

Sol. $x^8 - y^8 = (x^4)^2 - (y^4)^2$

$$= (x^4 + y^4)(x^4 - y^4) \quad [Using\ a^2 - b^2 = (a + b)(a - b)]$$

$$= (x^4 + y^4)[(x^2)^2 - (y^2)^2]$$

$$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$

$$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

Que 3. Factorise: $\boldsymbol{x^3 + 13x^2 + 32x + 20}$.

Sol. Let $p(x) = x^3 + 13x^2 + 32x + 20$

The constant term in   is equal to 20 and the factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$.

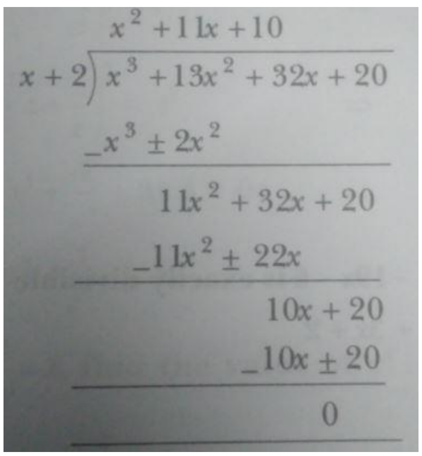
Putting $x = -2\ in\ p(x)$, we have

$$p(-2) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$$

$$= -8 + 52 - 64 + 20 = -72 + 72 = 0$$

$$p(-2) = 0$$

As $p(-2) = 0, so (x + 2)$ is a factor of $p(x)$. Now, divide $p(x)$ by $(x + 2)$



$\therefore\ p(x) = (x + 2)(x^2 + 11x + 10)$

$= (x + 2)[x^2 + 10x + x + 10]$

$= (x + 2)[x(x + 10) + 1(x + 10)] = (x + 2)[(x + 10)(x + 1)]$

$= (x + 1)(x + 2)(x + 10)$

Que 4. Factorise $2x^2 - 3x^2 - 17x + 30$.

Sol. Let, $p(x) = 2x^3 - 3x^2 - 17x + 30$.

$\therefore \quad p(2) = 2 \times 2^3 - 3 \times 2^2 - 17 \times 2 + 30 = 16 - 12 - 34 + 30$

$p(2) = 46 - 46 = 0$

As $p(2) = 0$, Therefore (x – 2) is a factor of p(x)

Let us divide $p(x)\ by\ (x - 2)$ by long division method as given below:

$$\begin{array}{r} 2x^2 + x - 15 \\ x-2 \overline{\big) 2x^3 - 3x^2 - 17x + 30} \\ \underline{-2x^3 \mp 4x^2} \\ x^2 - 17x + 30 \\ \underline{-x^2 \mp 2x} \\ -15x + 30 \\ \underline{\mp 15x \pm 30} \\ 0 \end{array}$$

$\therefore\ p(x) = 2x^3 - 3x^2 - 17x + 30 = (x-2)(2x^2 + x - 15)$

$$= (x-2)(2x^2 + 6x - 5x - 15) = (x-2)[2x(x+3) - 5(x+3)]$$

$$= (x-2)[(x+3)(2x-5)] = (x-2)(x+3)(2x-5)$$

Que 5. If both $(x-2)$ and $\left(x-\frac{1}{2}\right)$ are factors of $px^2 + 5x + r$, Show that p = r.

Sol. $Let\ f(x) = px^2 + 5x + r,$

As $(x - 2$ is a factor of $f(x), So\ f(2) = 0$

$$p \times 2^2 + 5 \times 2 + r = 0$$

$$\Rightarrow\ 4p + 10 + r = 0 \qquad \ldots\ldots\ldots.(i)$$

Also $\left(x - \frac{1}{2}\right)$ is a factor of $f(x), so\ f(\frac{1}{2})$ = 0

$$p\left(\frac{1}{2}\right)^2 + 5.\frac{1}{2} + r = 0$$

$$\Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0 \qquad \Rightarrow p + 10 + 4r = 0$$

From equations (i) and (ii), we have

$$4p + 10 + r = p + 10 + 4r$$

$$4p - p = 10 + 4r - 10 - r$$

$$\Rightarrow\ 3p = 3r \quad \Rightarrow\ p = r$$

Que 6. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.

Sol. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6\ and\ q(x) = x^2 + 3x + 2$

Then, $q(x) = x^2 + 3x + 2 = x^2 + 2x + x + 2$

$= x(x+2) + 1(x+2) = (x+2)(x+1)$

Now, $p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6$

$= 2 - 1 - 14 + 19 - 6 = 21 - 21$

$p(-1) = 0$

And, $p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6$

$= 32 - 8 - 56 + 38 - 6 = 70 - 70$

$p(-2) = 0$

$\Rightarrow (x+1)\ and\ (x+2)$ are the factors of $p(x), so\ p(x)$ is divisible by $(x+1)$ and $(x+2)$.

Hence, $p(x)$ is divisible by $(x+1)(x+2) = x^2 + 3x + 2$.

Que 7. Find the value of $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst, when\ s = \frac{r}{3} + 5t$.

Sol. $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst$

$= \frac{1}{3^3}r^3 + (-s)^3 + 5^3t^3 + 5rst = \left(\frac{r}{3}\right)^3 + (-s)^3 + (5t)^3 - 3\left(\frac{r}{3}\right)(-s)(5t)$

$= \left(\frac{r}{3} + (-s) + 5t\right)\left[\left(\frac{r}{3}\right)^2 + (-s)^2 + (5t)^2 - \frac{r}{3}.(-s) - (-s)(5t) - \frac{r}{3}(5t)\right]$

$= \left(\frac{r}{3} - s + 5t\right)\left(\frac{r2}{9} + s^2 + 25t^2 + \frac{rs}{3} + 5st - \frac{5rt}{3}\right)$

Now, $s = \frac{r}{3} + 5t$ (Given) $\Rightarrow \frac{r}{3} - s + 5t = 0$

$$\therefore \frac{1}{27}r^3 - s^3 + 125t^3 + 5rst = 0 \times \left(\frac{r^2}{9} + s^2 + 25t^2 + \frac{rs}{3} + 5st - \frac{5rt}{3}\right) = 0$$

HOTS (Higher Order Thinking Skills)

Que 1. If $z^2 + \frac{1}{z^2} = 14$, find the value of $z^3 + \frac{1}{z^3}$.

Sol. We have,

$$\left(z + \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2z\frac{1}{z}$$

$\Rightarrow \quad \left(z + \frac{1}{z}\right)^2 = z^2 + \frac{1}{z^2} + 2 \qquad \Rightarrow \quad \left(z + \frac{1}{z}\right)^2 = 14 + 2 = 16$

$\Rightarrow \quad \left(z + \frac{1}{z}\right)^2 = 4^2 \qquad \Rightarrow \quad z^3 + \frac{1}{Z} = 4$

$\Rightarrow \quad \left(z + \frac{1}{z}\right)^3 = 4^3 \qquad \Rightarrow \quad z^3 + \frac{1}{z^3} + 3 \times z \times \frac{1}{z}\left(z + \frac{1}{z}\right) = 64$

$\Rightarrow \quad z^3 + \frac{1}{z^3} + 3 \times 4 = 64 \qquad \Rightarrow \quad z^3 + \frac{1}{z^3} = 64 - 12$

$\Rightarrow \quad z^3 + \frac{1}{z^3} = 52$

Que 2. If $x + \frac{1}{x} = 3$, find the value of $x^4 + \frac{1}{x^4}$.

Sol. We have $x + \frac{1}{x} = 3$

Squaring both the sides, we get

$\left(x + \frac{1}{x}\right)^2 = 3^2 \qquad \Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = 9$

$\Rightarrow \; x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9 \qquad \Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = 9$

$\Rightarrow \; x^2 + \frac{1}{x^2} = 9 - 2 \qquad \Rightarrow \quad x^2 + \frac{1}{x^2} = 7$

Now, $x^2 + \frac{1}{x^2} = 7 \qquad \Rightarrow \quad \left(x^2 + \frac{1}{x^2}\right)^2 = 7^2 \qquad \Rightarrow \quad (x^2)^2 +$

$\Rightarrow x^4 + \frac{1}{x^4} + 2 = 49 \qquad \Rightarrow \quad x^4 + \frac{1}{x^4} = 49 - 2 \qquad \Rightarrow \quad x^4 + \frac{1}{x^4} = 47$

Que 3. If the polynomials $az^3 + 4z^2 + 3z - 4$ and $z^3 - 4z + a$ leave the same remainder when divided by $z - 3$, find the value of a.

Sol. Let $\quad p(z) = az^3 + 4z^2 + 3z - 4$ and $q(z) = z^3 - 4z + a$

When $\quad p(z)$ is divided by $z - 3$ the remainder is given by, P (3) = a x 3^3 + 4 x 3^2 + 3 x 3 – 4 = 27a + 36 + 9 – 4

P (3) = 27a + 41 …(i)

When q(z) is divided by z – 3 the remainder is given by,

q (3) = 3^3 – 4 x 3 + a = 27 – 12 + a

q (3) = 15 + a …(ii)

According to question, p (3) = q (3)

⇒ 27a + 41 = 15 + a ⇒ 27a – a = - 41 + 15

26a = - 26

⇒ a = $\frac{-26}{26}$ ⇒ a = - 1

Que 4. If $x^2 + \frac{1}{x^2} = 34$, find $x^3 + \frac{1}{x^3} - 9$.

Sol. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 34 + 2 = 36$

$\left(x + \frac{1}{x}\right) = 6$

On cubing, we get

$\left(x + \frac{1}{x}\right)^3 = 6^3$ ⇒ $x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 216$

⇒ $x^3 + \frac{1}{x^3} + 3 \times 6 = 216$ ⇒ $x^3 + \frac{1}{x^3} = 198$

⇒ $x^3 + \frac{1}{x^3} - 9 = 198 - 9 = 189$

Value Based Questions

Que 1. On her birthday, Amisha donated 2 toffees to each children of an orphanage and 15 chocolates to adults working there. Taking the total items distributed as x and the number of children y, write a linear equation in two variables for the above situation.
(a) Write the equation in standard for.
(b) How many children are there if total 61 items were distributed?
(c) What values does Amisha possess?

Sol. $X = 2y + 15$
(a) $x – 2y – 15 = 0$
(b) $61 = 2y + 15 \quad \Rightarrow \; y = \frac{46}{2} = 23$ children

(c) Caring, kind, socially active.

Que 2. The number of sincere students (x) in a class is two more than twice the number of careless students (y). Write a linear equation in two variables for this situation. How does sincerity help and carelessness harm a student?

Sol. $X = 2 + 2y$
sincere students always progress in life as they value time and channelize their talent in productive activities while a careless student always wastes him talent and time.