Very Short Answer Type Questions

[1 MARK]

Que 1.In how many chapters did Euclid divide his famous treatise "The elements"?

Sol. 13. Chapters.

Que 2. To which country does Euclid belong?

Sol. Greece.

Que 3. State Euclid's fifth axiom (as per order given in the Textbook for class IX).

Sol. The whole is greater than the part.

Que 4. It is known that if $x + y = 10$, then $x + y + z = 10 + z$. Which axiom of Euclid does this statement illustrate?

Sol. Second axiom.

Que 5. State Euclid's first postulate.

Sol. A straight line may be drawn from any point to any other point.

Short Answer Type Questions – I

[2MARKS]

State whether the following statements are True of False. Justify your answers

Que 1. The Euclidean geometry is valid only for figures in the plane.

Que 2. If area of a triangle equals the area of a square and the area of the square equals that of a rectangle, then the area of the triangle also equals the area of the rectangle.

Que 3. The statements that are proved are called axioms.

Que 4. Euclid's fourth axiom says that everything equals itself.

Que 5. The edges of a surface are curves.

Que 6. Two distinct intersecting lines cannot be parallel to the same line.

Que 7. In geometry, we take a point, a line and a plane as undefined terms.

Que 8. The things which are double of the same thing are equal to one another.

Que 9. The boundaries of the solids are curves.

Que 10. Attempt to prove Euclid's fifth postulate, using the other postulates and axioms, led to the discovery of several other geometries.

Sol. 1. True, it fails on the curved surfaces. For example, on curved surfaces, the sum of angles of a triangle may be more than 180^0.

2. True, things equal to the same thing are equal.

3. False, statements that are proved are theorems.

4. True, as it is the justification of the principle of super position.

5. False, the edges of the surfaces are line.

6. True, it is an equivalent version of Euclid's fifth postulate.

7. True, to define a point, a line and a plane in geometry we need to define many other things that give a long chain of definitions without an end. Due to these reasons, mathematicians agree to leave these geometric terms undefined.

8. True, one of the Euclid's axioms.

9. False, as boundaries of the solids are surfaces.

10. True, as these geometries are different from Euclidean geometry.

Que 11. If P, Q and R are three points on a line and Q is between P and R, then prove that $PR - QR = PQ$.

Sol.

In the above figure PQ coincides with $PR - QR$.

So, according to axiom, “things” which coincide with one another are equal to ‘one another’. We have,

$$PR - QR = PQ$$

Que 12. Solve the equation $u - 5 = 15$ and state the axiom that you use here.

Sol. $u - 5 = 15$

On adding 5 to both sides, we have

$$u - 5 + 5 = 15 + 5$$

Euclid’s second axiom, when equals are added to equals, the wholes are equal.
Or

$$u = 20$$

Short Answer Type Questions – II

[3 MARKS]

Que 1.In Fig. 5.6, if $AC = BD$, THEN PROVE THAT $AB = CD$.

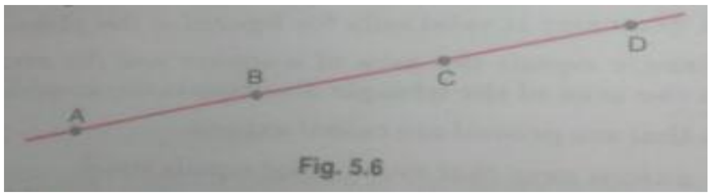

Fig. 5.6

Sol. $AC = BD$ (Given) $\ldots(i)$

$AC = AB + BC$ (Point B lies between A and C) $\ldots(ii)$

$BD = BC + CD$ (Point C lies between B and D) $\ldots(iii)$

Substituting (ii) and (iii) in (i), we get

$AB + BC = BC + CD$

According to Euclid's third axiom, if equals are subtracted from equals, the remainders are equal.

So, $AB = CD$ (Subtracting BC from both sides)

Que 2. In Fig. 5.7, $AC = XD, C$ is the mid-point of AB and D is the mid-point of XY. Using a Euclid's axiom, show that $AB = XY$.

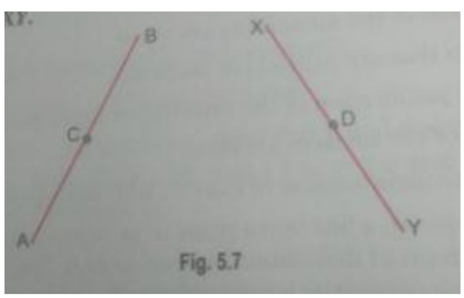

Fig. 5.7

Sol. $AB = 2AC$ (C is the mid-point of AB)

$XY = 2XD$ (D is the mid-point of XY)

Also $AC = XD$ (Given)

Therefore, $AB = XY$, because things which are double of the same things are equal to one another.

Que 3. In the Fig. 5.8, if $\angle 1 = \angle 3, \angle 2 = \angle 4$ and $\angle 3 = \angle 4$, write the relation between $\angle 1$ and $\angle 2$, using a Euclid's axiom.

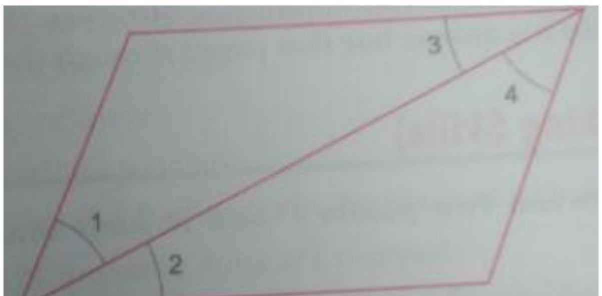

Fig. 5.8

Sol. Here $\angle 3 = \angle 4, \angle 1 = \angle 3$

And $\angle 2 = \angle 4$

According to Euclid's first axiom, the things which are equal to equal things are equal to one another.

Therefore, $\angle 1 = \angle 2$

Long Answer Type Questions

[4 MARKS]

Que 1. Read the following statement:

"A square is a polygon made up of four line segments, out of which, length of three line segments are equal to the length of fourth one and all its angles are right angles." Define the terms used in this definition which you feel are necessary. Are necessary. Are there any undefined terms in this? Can you justify that all angles and sides of a square are equal?

Sol. Undefined terms used: line, point.

The terms that need to be defined are:

Polygon: A simple closed figure made up of three of more line segments.

Line segment: Part of a line with two end points.

Angle: A figure A figure formed by two rays with a common initial point.

Ray: Part of a line with one end point.

Right angle: Angle whose measure is 90^0.

Euclid's fourth postulate says that "all right angles are equal to one another."

In a square, all angles are right angles, therefore, all angles are equal.

Three line segments are equal to fourth line segment (Given).

Therefore, all the four sides of a square are equal. (By Euclid's first axiom, "things which are equal to the same thing are equal to one another."

Que 2. Consider two postulates given below:

$(\boldsymbol{i})$ **Given any two distinct points $\boldsymbol{A}$ and $\boldsymbol{B}$, there exists a third point $\boldsymbol{C}$ which is in between $\boldsymbol{A}$ and $\boldsymbol{B}$.**

$(\boldsymbol{ii})$ **There exists at least three points that are not on the same line.**

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow Euclid's postulates? Explain.

Sol. Undefined terms used: line, point. They are consistent, because they deal with two different situations.

Postulate (i) says that given two points A and B, there is a point C lying on the line in between them.

Postulate (ii) says that given A and B, we cannot take C not lying on the line through A and B. These 'postulates' do not follow Euclid's postulates. However, they follow axiom stated as given two distinct points, there is a unique line that passes through them.

HOTS (Higher Order Thinking Skills)

Que 1. If a point O lies between two points P and R such that PO = OR then prove that PO = $\frac{1}{2}$ PR.

Sol. Proof: From Fig. 5.9,

PO + OR = PR …(i)

PO = OR (Given) …(ii)

PO + PO = PR [Using (ii) in (i)]

2PO = PR

PO = $\frac{1}{2}$ PR

Fig. 5.9

Que 2. Prove that every line segment has one and only one mid-point.

Sol. Proof: Let us prove this statement by contradiction method. Let us assume that the line segment PT has two midpoints R and S.

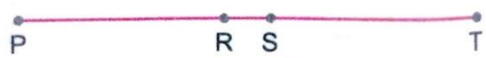

Fig. 5.10

⇒ PR = $\frac{1}{2}$ PT

PS = $\frac{1}{2}$ PT (∵ R and S are mid-point according to assumption)

⇒ PR = PS

But this is possible only if R and S coincide.

Que 3. Does Euclid's fifth postulate imply the existence of parallel lines? Explain.

Sol. If straight line l falls on two straight lines m and n such that the sum of interior angles on same side of l is 180^0, then by Euclid's 5^{th} postulates the lines will not meet on this side of l.

Also, the sum of interior angles on other side of l will be 180^0, they will not meet on the other side also.

⇒ m and n never meet ⇒ m and n are parallel.

Value Based Questions

Que 1. Teacher held two sticks AB and CD of equal length in her hands and marked their mid points M and N respectively. She then asked the students whether AM is equal to ND or not. Aprajita answered yes. Is Aprajita correct? State the axiom of Euclid that supports her answer.
Which values of Aprajita are depicted here?

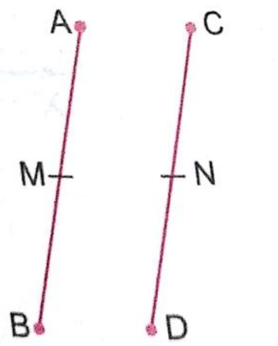

Fig. 5

Sol. Yes
Things which are halves of the same things are equal to one another.
Curiosity, knowledge, truthfulness.

Que 2. For her records, a teacher asked the students about their heights. Manav said his height is same as that of Arnav. Raghav also answered the same, way that his height is same as that of Arnav. She then asked the students to relate the height of Manav and Raghav. Arnav answered they both have same height. Is Arnav correct? If yes, state Euclid's axiom which supports his answer.
Which values of Arnav are depicted here?

Sol. Yes, Things which are equal to the same thing are equal to one another.
Knowledge, curiosity, truthfulness.

Que 3. The number of members of society A who participated in 'Say No to Crackers' campaign is double the number of members from society B. Also, the number of members from society C is double the number of members from society B. Can you relate the number of participants from society A and C? Justify your answer using Euclid's axiom.
Which values are depicted here?

Sol. The number of participants from society A and C is equal. Things which are double of the same thing are equal to one another.
Social service, helpfulness, cooperation, environmental concern.

Que 4. In a society, the number of persons using CNG instead of petrol for their vehicles has increased by 15 and now the number is 25. Form a linear equation to find the original number of persons using CNG and solve it using Euclid's axiom.
Which values are depicted in the question?

Sol. X + 15 = 25
⇒ $x + 15 - 15 = 25 - 15$ (Using Euclid's third axiom)
⇒ x = 10
Environmental care, responsible citizens, futuristic.

Que 5. Teacher asked the students to find the value of x in the following figure if l|| m.
Shalini answered 35°. Is she correct? Which values are depicted here?

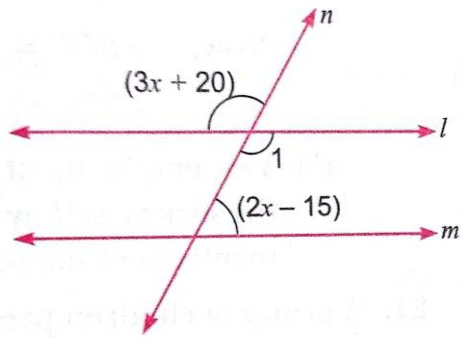

Fig. 6

Sol. ∠1 = 3x + 20 (Vertically opposite angles)
∴ 3x + 20 2x – 15 = 180 ° (Co-interior angles are supplementary)
⇒ $5x + 5 = 180°$ ⇒ $5x = 180° - 5°$
⇒ 5x = 175 ° ⇒ $x = \frac{175}{5} = 35°$

Yes, Knowledge, truthfulness.

Que 6. For spreading the message 'Save Environment Save Future' a rally was organised by some students of a school. They were given triangular cardboard pieces which they divided into two parts by drawing bisectors of base angles (say ∠B and ∠C) intersecting at O in the given figure. Prove that ∠BOC = $90 + \frac{1}{2}\angle A$

Which values are depicted by these students?

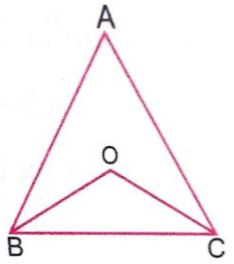

Fig. 7

Sol. In ΔABC, we have

$\angle A + \angle B + \angle C = 180°$ (∵ sum of the angles of a Δ is 180 °)

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2}$

$\Rightarrow \quad \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90°$

$\therefore \quad \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$ ….(i)

Now, in ΔOBC, we have:

$\angle 1 + \angle 2 + \angle BOC = 180°$ [∵ sum of the angles of Δ is 180 °]

$\Rightarrow \quad \angle BOC = 180° - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle BOC = 180° - (90° - \frac{1}{2}\angle A)$ [using (i)]

$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{1}{2}\angle A$

$\therefore \quad \angle BOC = 90° + \frac{1}{2}\angle A$

Environmental care, social, futuristic.

Que 7. Three bus stops situated at A, B and C in the figure are operated by handicapped persons. These 3 bus stops are equidistant from each other. OB is the bisector of ∠ABC and OC is the bisector of ∠ACB.
(a) Find ∠BOC.
(b) Do you think employment provided to handicapped persons is important for the development of the society? Express your views with relevant points.

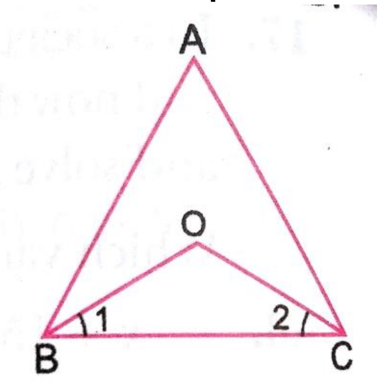

Fig. 9

Sol. (a) Since, A, B, C are equidistant from each other.
∴ ∠ABC is an equilateral triangle.
⇒ ∠ABC = ∠ABC = 60°

$\Rightarrow \quad \angle OBC = \angle OCB = \frac{1}{2} \times 60° = 30°$ (∵ OB and OC are angle bisectors)

Now, ∠BOC = 180° - ∠OBC - ∠OCB (Using angle sum property of triangle)

⇒ ∠BOC = 180°- 30°-30° = 120°

(b) Yes, employment provided to the handicapped persons is important for the development of the society as they would become independent, self-reliant, confident, social, helpful and useful members of the society.

Que 8. A group of children prepared some decorative pieces in the shape of a star for the orphans in an orphanage. Show that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Which values of the children are depicted here?

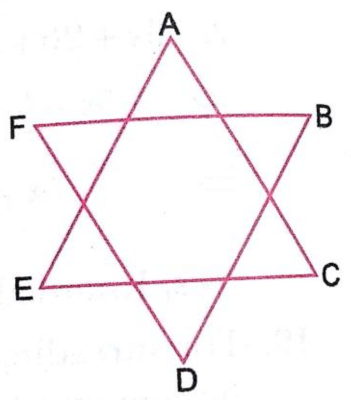

Fig. 10

Sol. In ΔAEC,
$\angle A + \angle E + \angle C = 180°$ … (i) (Angle sum property of a triangle)

Similarly, in ΔBDF,
$\angle B + \angle D \angle F = 180°$ …. (ii)

Adding (i) and (ii), we get
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Social, caring, cooperative, hardworking.

Que 9. For annual day, Sakshi and Nidhi were asked to make one rangoli each on two different places. They started it with triangles (say ABC and ΔPQR) and their medians (AM and PN). If two sides (AB and BC) and a median (AM) of one triangle are respectively equal to two sides (PQ and QR) and a median (PN) of other triangle, prove that the two triangles (ΔABC and ΔPQR) are congruent. Which values of the girls are depicted here?

Sol. In ΔABC and ΔPQR

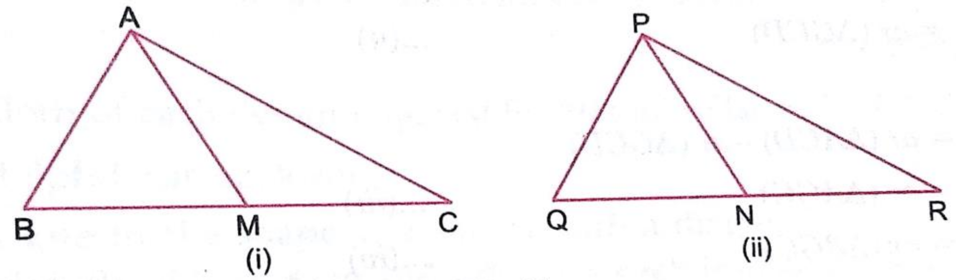

Fig. 11

BC = QR

$\Rightarrow \quad \frac{1}{2}BC = \frac{1}{2}QR$

$\Rightarrow$ BM = QN

In triangle ABM and PQN, we have

AB = PQ (Given)

BM = QN (Proved above)

AM = PN (Given)

$\therefore \quad \Delta ABM \cong \Delta PQN$ (SSS congruence criterion)

$\Rightarrow \quad \angle B = \angle Q$ (CPCT)

Now, in triangles ABC and PQR, we have

AB = PQ (Given)

$\angle B = \angle Q$ (Proved above)

BC = QR (Given)

$\therefore$ ΔABC ≅ ΔPQR (SSS congruence criterion)

Participation, beauty, hardworking.

Que 10. Triangular pieces of cardboards were cut out by some people who were organising 'No Pollution' campaign in their area. If the three angles of one cutout are respectively equal to the three angles of the other cutout, can we say the two cutouts are congruent? Justify your answer.
Which values of these people are depicted here?

Sol. The two cutouts may not be congruent. For example all equilateral triangles have equal angles but may have different sides.
Environmental concern, cooperative, caring, social.

Que 11. Anya wants to prepare a poster on education of girlchild for a campaign. She takes a triangular sheet and divides it into three equal parts by drawing its medians which intersect at the point G (see Fig. 12).

Show that ar (ΔAGC) = ar (ΔAGC) = ar (ΔAGB) = (ΔBGC) = $\frac{1}{3}ar\ (\Delta ABC)$

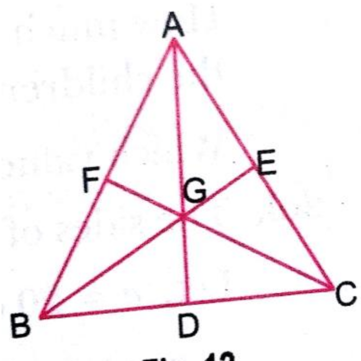

Fig. 12

Do you think education of a girl child is important for the development of a society? Justify your answer.

Sol. Given: A ΔABC in which medians AD, BE and CF intersects at G.

Proof: (ΔAGB) = ar (ΔBGC) = ar (ΔCGA) = $\frac{1}{3}$ ar (ΔABC)

Proof: In ΔABC, AD is the median. As a median of a triangle divides it into two triangles of equal area.

∴ ar (ΔABD) = ar (ΔACD) … (i)

In ΔGBC, GD is the median

∴ aq (ΔGBD) = ar (ΔGCD) …. (ii)

Subtracting (ii) from (i), we get

ar (ΔABD) – ar (ΔGBD) = ar (ACD) – ar (ΔGCD)

ar (ΔAGB) = ar (ΔAGC) … (iii)

Similarly, ar (ΔAGB) = ar (ΔBGC) … (iv)

From (iii) and (iv), we get

ar (ΔAGB) = ar (ΔBGC) = ar (ΔAGC) …. (v)

But, ar (ΔAGB) + ar (ΔBGC) + ar (ΔAGC) = ar (ΔABC) …. (vi)

From (v) and (vi), we get

3 ar (ΔAGB) = ar (ΔABC)

⇒ ar (ΔAGB) = $\frac{1}{3} ar(\Delta ABC)$

Hence, ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

Yes, for the development of a society, education of each girl child is essential. An educated society always progresses.