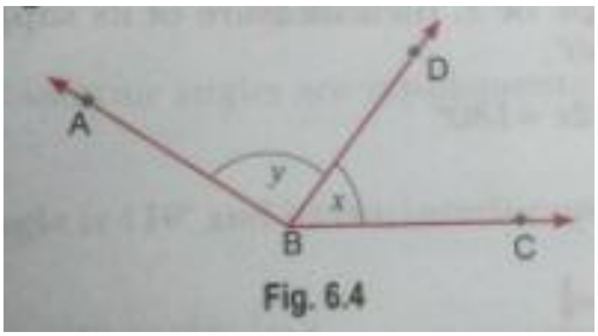
Very Short Answer Type Questions

[1 MARK]

Que 1. A transversal intersects two lines in such a way that the two interior angles on the same side of transversal are equal. Will the two lines always be parallel?

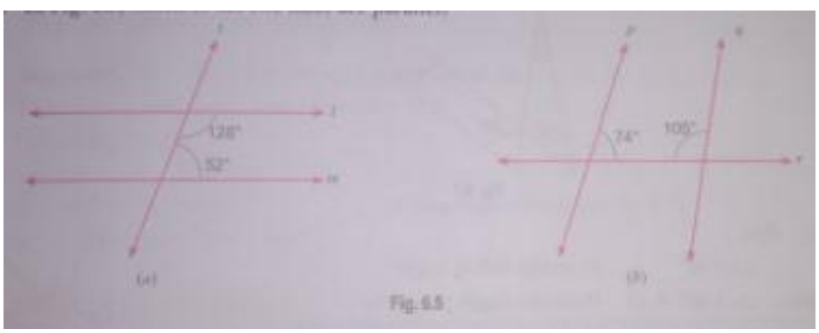
Sol. The two lines will not be always parallel as the sum of the two equal angles will not always be 180^0. Lines will be parallel when each of the equal angles is equal to 90^0

Que 2. For what value of $x + y$ in Rig. 6.4 will ABC be a line?



Sol. For ABC to be a line, the sum of two adjacent angles must be 180^0, $i.e.$, $x + y$ must be equal to180^0 .

Que 3. In Fig. 6.5, which of the two lines are parallel?



Sol. $l \parallel m$, because angles on the same side of the transversal are supplementary, $i.e.$, $128^0 + 52^0 = 180^0$. Therefore p is not parallel to q, because $105^0 + 74^0 = 180^0$.

Que 4. In Fig. 6.6, find the value of x for which the lines l and m are parallel.

Sol. Two lines are parallel when angles on the same side of transversal are supplementary $i.e.$,

$$x + 55^0 = 180^0 \Rightarrow x = 180^0 - 55^0 \Rightarrow x = 125^0$$

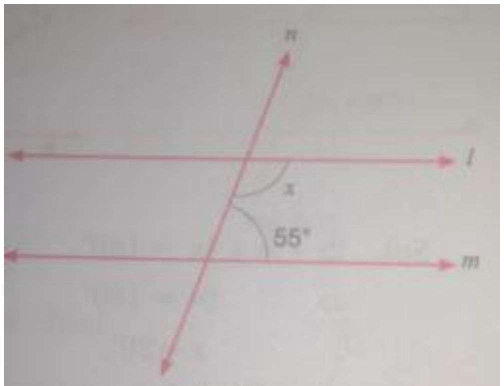


Fig. 6.6

Que 5. Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other?

Sol. No, they are parallel.

Que 6. Can a triangle have two obtuse angles? Give reason.

Sol. No, because sum of angles of a triangle cannot be more than 180^0.

Que 7. Can a triangle have all the angles less than 60^0? Give reason.

Sol. No, because the angle sum will be less than 180^0

Que 8. How many triangles can be drawn having its angles as $60^0, 90^0, 30^0$?

Sol. Infinitely many triangles.

Que 9. Find the angle whose complement is equal to the angle itself.

Sol. Let the measure of an angle be x, then the measure of its complement is also x. We know that the sum of the measures of complementary angles is 90^0.

Therefore, $x + x = 90^0$

$\Rightarrow \quad 2x = 90^0 \quad \Rightarrow \quad x = 45^0$

Que 10. Find the measure of an angle whose supplement is equal to the angle itself.

Sol. Let the measure of an angle be x, then measure of its supplement is also x. Since the sum of supplementary angles is180^0.

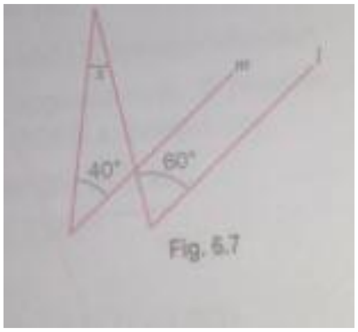
$\therefore \quad x + x = 180^0 \quad \Rightarrow \quad 2x = 180^0$

$\Rightarrow \quad x = 90^0$

Short Answer Type Questions – I

[2 MARKS]

Que 1. In Fig. 6.7, if $l \parallel m$, then find the value of x.



Sol. $\because$ $l \parallel m$

$\therefore$ $\angle 1 = 60^0$ (Corresponding angle)

Now, $\angle x + 40^0 = \angle 1$ (Exterior angle property)

$\Rightarrow$ $\angle x = 60^0 - 40^0 = 20^0$

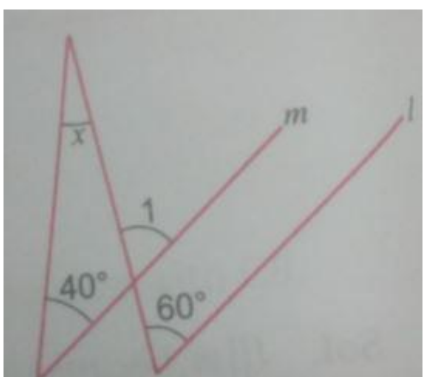


Fig. 6.8

Que 2. In Fig. 6.9, find the value of x.

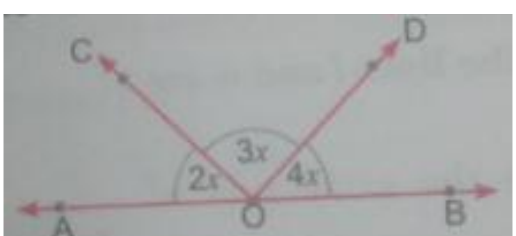


Fig. 6.9

Sol. $2x + 3x + 4x = 180^0$ (Straight angle)

$\Rightarrow \quad 9x = 180^0$

$\therefore \quad x = 20^0$.

Que 3. If the ratio between two complementary angles is 2: 3, then find the angles.

Sol. Let the two complementary angles be $2x$ and $3x$.

$\therefore\ 2x + 3x = 90^0 \ \Rightarrow\ 5x = 90^0 \ \Rightarrow x = 18^0$

$\therefore$ The angles are $2 \times 18^0 = 36^0$ and $3 \times 18^0 = 54^0$.

Que 4. If the difference between two supplementary angles is 40^0 , then find the angles.

Sol. Let the two supplementary angles be x and $x + 40^0$.

$\therefore\ x + x + 40^0 = 180^0 \quad \Rightarrow \quad 2x = 180^0 - 40^0$

$\Rightarrow \quad 2x = 140^0 \quad \Rightarrow \quad x = 70^0$

Also $x + 40^0 = 70^0 + 40^0 = 110^0$.

Que 5. In Fig. 6.10, if $m \parallel n$, then find the value of x.

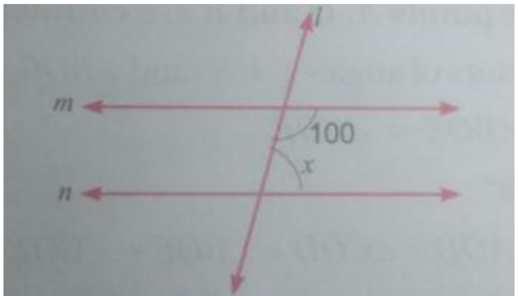


Fig. 6.10

Sol. $100^0 + x = 180^0$ (Cointerior angles are supplementary)

$\Rightarrow \quad x = 180^0 - 100^0 = 80^0$.

Que 6. An exterior angle of a triangle is 110^0 and its two interior opposite angles are equal. Find each of these equal angles.

Sol. Let each of the interior opposite angles be x.

$\therefore$ An exterior angle is equal to sum of its two interior opposite angles.

Then $x + x = 100^0$ or $x = \frac{110^0}{2} = 55^0$

Que 7. In a $\Delta ABC, \angle A + \angle B = 110^0, \angle C + \angle A = 135^0$. Find $\angle A$.

Sol. Given $\angle A + \angle B = 110^0, \angle C + \angle A = 135^0$

On adding, we get

$\angle A + \angle B + \angle C + \angle A = 110^0 + 135^0$

$\Rightarrow \quad 180^0 + \angle A = 245^0$ (Using angle sum property of Δ)

$\Rightarrow \quad \angle A = 245^0 - 180^0 = 65^0$.

Short Answer Type Questions – II

[3 MARKS]

Que 1. For what value of x will the lines l and m be parallel to each other? [Fig. 6.11].

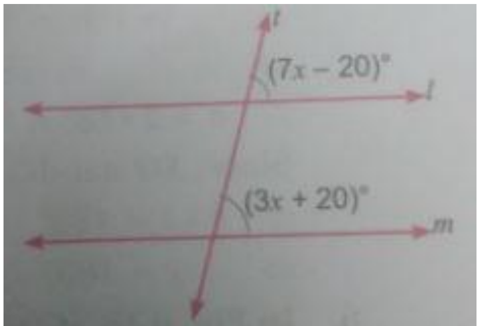


Fig. 6.11

Sol. $l \parallel m$ only when a pair of corresponding angles is equal.

$\therefore \quad 7x - 20 = 3x + 20$

$7x - 3x = 20 + 20$

$\Rightarrow \quad 4x = 40$

$\Rightarrow \quad x = \frac{40^0}{4} = 10^0$

Que 2. In Fig. 6.12, $\angle AOC$ and $\angle BOC$ form a linear pair. If $a - b = 20^0$, find the values of a and b.

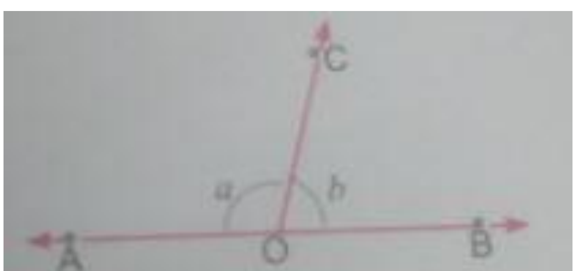


Fig. 6.12

Sol. $a + b = 180^0$ (Linear pair) $\ldots(i)$

$a - b = 20^0$ (Given) $\ldots(ii)$

Adding (i) and (ii), we get

$$2a = 200^0 \Rightarrow a = \frac{200^0}{2} \quad a = 100^0$$

Putting the value of a in equation (i), we get

$$100^0 + b = 180^0 \quad \Rightarrow \quad b = 80^0$$

Que 3. In Fig. 6.13, OD is the bisector of $\angle AOC$, OE is the bisector of $\angle BOC$ and $OD \perp OE$. Show that the points A, O and B are collinear.

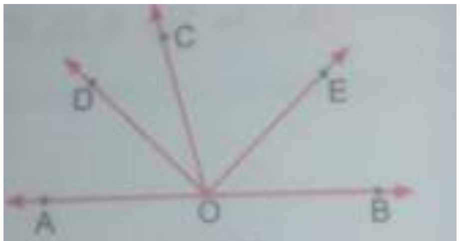


Fig. 6.13

Sol. Since OD and OE are the bisectors of angles $\angle AOC$ and $\angle BOC$ respectively

$\therefore \quad \angle AOD = \angle COD$ and $\angle BOE = \angle COE$

Also $\quad \angle DOE = 90^0$

Now, $\angle AOC + \angle BOC = \angle AOD + \angle COD + \angle BOE + \angle COE$

$$= \angle COD + \angle COD + \angle COE + \angle COE$$

$\Rightarrow \quad \angle AOC + \angle BOC = 2\angle COD + 2\angle COE = 2(\angle COD + \angle COE)$

$$= 2\angle DOE = 2 \times 90^0 = 180^0$$

Hence, points A, O and B are collinear.

Que 4. In Fig. 6.14, if $x + y = w + z$, then prove that AOB is a line.

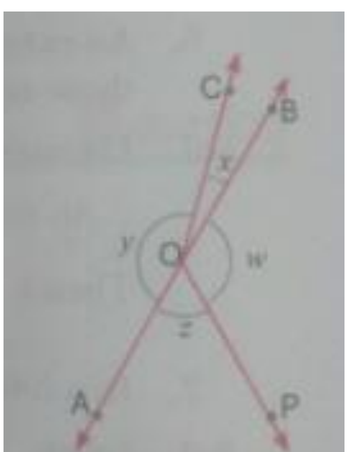


Fig. 6.14

Sol. As sum of all the angles about a point is equal to 360^0

Therefore, $x + y + z + w = 360^0$

$\Rightarrow$ $(x + y) + (z + q) = 360^0$

Also, $z + w = x + y$ (Given)

$\therefore$ $(x + y) + (x + y) = 360^0$ $\Rightarrow$ $2x + 2y = 360^0$

$\Rightarrow$ $2(x + y) = 360^0$ $\Rightarrow$ $(x + y) = 180^0$

$\therefore$ AOB is a straight line.

Que 5. In Fig. 6.15, lines XY and MN intersect at O. If $\angle POY = 90^0$ and $a : b = 2 : 3$, find c.

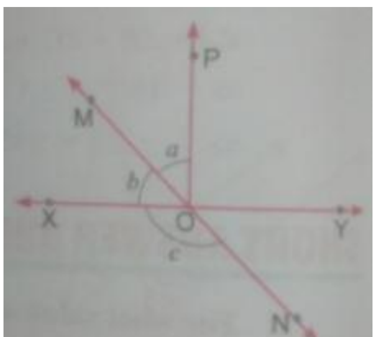


Fig. 6.15

Sol. $\angle POX + \angle POY = 180^0$

$\Rightarrow$ $\angle POX + 90^0 = 180^0$

$\therefore$ $\angle POX = 90^0$

Let $a = 2x$ and $b = 3x$

$\therefore$ $2x + 3x = 90^0$

$\Rightarrow$ $5x = 90^0$

$\therefore$ $a = 2 \times 18^0 = 36^0$ and $b = 3 \times 18^0 = 54^0$

Since, XO stands on MN

$\therefore$ $b + c = 180^0$ $\Rightarrow$ $54^0 + c = 180^0$

$\Rightarrow$ $c = 180^0 - 54^0 = 126^0$

Que 6. In Fig. 6.16, if $OP \parallel RS, \angle OPQ = 110^0$ and $\angle QRS = 130^0$, then determine $\angle PQR$.

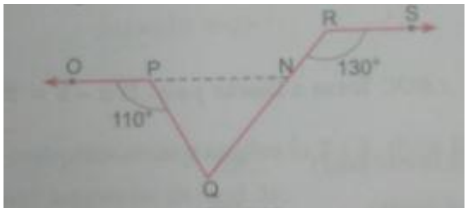


Fig. 6.16

Sol. Produce OP TO intersect RQ at point N.

Now, $OP \parallel RS$ and transversal RN intersects them at N and R respectively

$\therefore \quad \angle RNP = \angle SRN$ (Alternate interior angles)

$\Rightarrow \quad \angle RNP = 130^0$

$\therefore \quad \angle PNQ = 180^0 - 130^0 = 50^0$ (Linear pair)

$\angle OPQ = \angle PNQ + \angle PQN$ (Exterior angle property)

$\Rightarrow \quad 110^0 = 50^0 + \angle PQN$

$\Rightarrow \quad \angle PQN = 110^0 - 50^0 = 60^0 = \angle PQR$

Que 7. In Fig. 6.17, $AB \parallel CD$. Find the value of x.

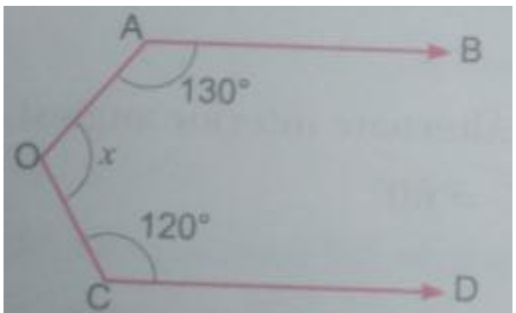


Fig. 6.17

Sol. Through O, draw a line POQ parallel to AB, [Fig. 6.18]

Now $PQ \parallel AB$ and $CD \parallel AB$

So, $CD \parallel PQ$

$\because \quad AB \parallel PQ$ and AO is a transversal

We have,

$\angle AOQ + \angle OAB = 180^0$ (Cointerior angles are supplementary)

$\Rightarrow \quad \angle AOQ + 130^0 = 180^0$

$\Rightarrow \quad \angle AOQ = 180^0 - 120^0 = 60^0$

$\therefore \quad \angle AOC = \angle AOQ + \angle QOC = 50^0 + 60^0 = 110^0$

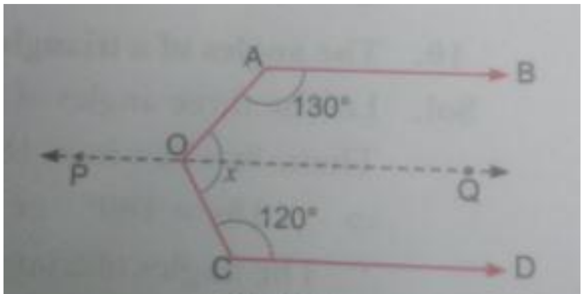


Fig. 6.18

Que 8. In Fig. 6.19, $AB \parallel CD$. Determine x.

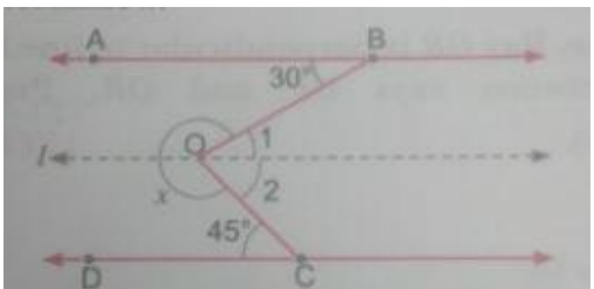


Fig. 6.19

Sol. Through O,draw a line l parallel to both AB and CD.

Then, $\angle 1 = \angle ABO = 30^0$ (Alternate interior angles)

And $\angle 2 = \angle DCO = 45^0$ (Alternate interior angles)

Now, $\angle BOC = \angle 1 + \angle 2 \quad \Rightarrow \quad \angle BOC = 30^0 + 45^0 = 75^0$

So, $x = 360^0 - \angle BOC = 360^0 - 75^0 = 285^0$

Hence, $x = 285^0$

Que 9. In Fig. 6.20, find x if $AB \parallel CD \parallel EF$.

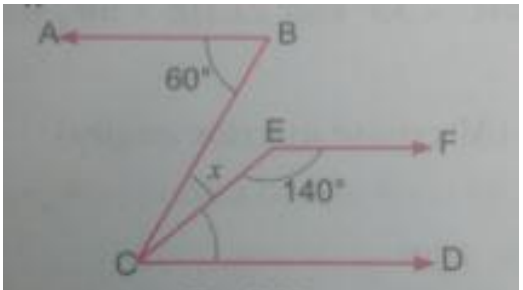


Fig. 6.20

Sol. As $EF \parallel CD$ and EC is the transversal

$\therefore$ $\angle DCE + \angle FEC = 180^0$ (Cointerior angles are supplementary)

$\angle DCE + 140^0 = 180^0$

$\Rightarrow$ $\angle DCE = 180^0 - 140^0 = 40^0$

Also, $\angle BCD = \angle ABC$ (Alternate interior angles)

$\therefore$ $\angle BCD = 60^0 \Rightarrow X + \angle DCE = 60^0$

$\Rightarrow$ $x = 60^0 - 40^0 = 20^0$

Que 10. The angles of a triangle are in the ratio 3: 7: 8. Find the angles of the triangle.

Sol. Let the three angles of the triangle be $3x, 7x$ and $8x$.

Then, $3x + 7x + 8x = 180^0$ (By angle sum property of Δ)

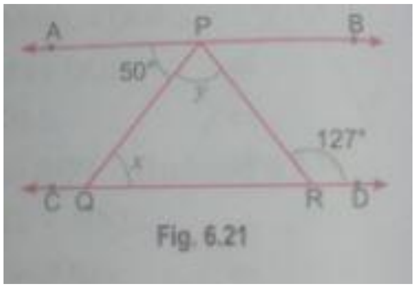
$\Rightarrow$ $18x = 180^0$ or $x = 10^0$

$\therefore$ The angles of triangle are

$3 \times 10^0 = 30^0, 7 \times 10^0 = 70^0, 8 \times 10^0 = 80^0$

Hence, the angles of triangle are $30^0, 70^0$ and 80^0

Que 11. In Fig. 6.21, if $AB \parallel CD, \angle APQ = 50^0$ and $\angle PRD = 127^0$, find x and y.



Sol. As $AB \parallel CD$ and PQ is a transversal.

$\therefore$ $\angle APQ = \angle PQR$ (Alternate interior angles)

$\Rightarrow$ $50^0 = x$

Also $\angle APR = \angle PRD$ (Alternate interior angles)

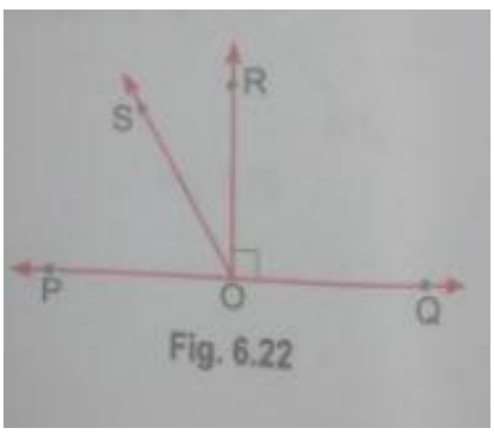
$\Rightarrow$ $50^0 + \angle QPR = 127^0$

$\Rightarrow$ $50^0 + y = 127^0$

Or $y = 127^0 - 50^0 = 77^0$

Hence, $x = 50^0, y = 77^0$

Que 12. In Fig. 6.22, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Sol. As $RO \perp PQ$

$\therefore$ $\angle POR = \angle QOR = 90^0$

Now, $\angle QOS = \angle QOR + \angle ROS$

$\Rightarrow$ $\angle QOS = 90^0 + \angle ROS$ $\quad \ldots(i)$

Since $\angle POR = 90^0 \Rightarrow \angle POS + \angle ROS = 90^0$

$\angle POS = 90^0 - \angle ROS$

Subtracting (ii) from (i), we get

$$\angle QOS - \angle POS = 90^0 + \angle ROS - (90^0 - \angle ROS)$$

$$\angle QOS - \angle POS = 2\angle ROS$$

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$

Que 13. In Fig. 6.23, if $AB \parallel DE, \angle BAC = 35^0$ and $\angle CDE = 50^0$, find $\angle DCE$.

Sol. Since $AB \parallel DE$

$\therefore$ $\angle AED = \angle BAE = 35^0$ (Alternate interior angles)

In $\Delta\, CDE$

$$\angle CED + \angle EDC + \angle DCE = 180^0$$

$$35^0 + 50^0 + \angle DCE = 180^0$$

$$85^0 + \angle DCE = 180^0$$

$\angle DCE = 180^0 - 85^0$ $\therefore$ $\angle DCE = 95^0$

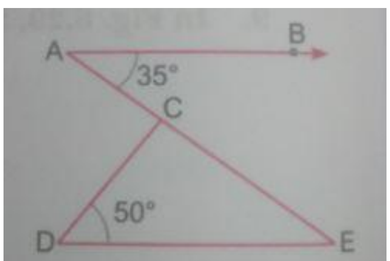


Fig. 6.23

Que 14. In Fig. 6.24, $AB \parallel DC$ AND $AD \parallel BC$. Prove that $\angle DAB = \angle DCB$.

Sol. As $AB \parallel DC$ and BC is a transversal intersecting them at B and C respectively.

$\therefore$ $\angle ABC + \angle DCB = 180^0$ $\quad \ldots(i)$

Also $AD \parallel BC$ and AB is a transversal intersecting them at A and B respectively.

$\therefore$ $\angle DAB + \angle ABC = 180^0$ $\quad \ldots(ii)$

From (i) and (ii), we get

$$\angle ABC + \angle DCB = \angle DAB + \angle ABC$$

$\Rightarrow$ $\angle DCB = \angle DAB$

Hence, $\angle DAB = \angle DCB$

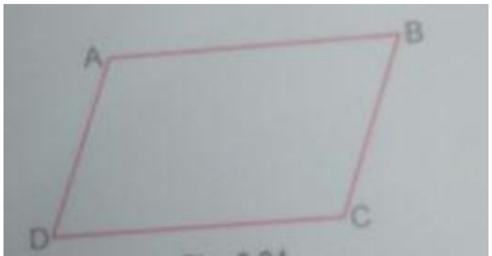


Fig. 6.24

Que 15. In Fig. 6.25, if $PQ \perp PS, PQ \parallel SR,\ \angle SQR = 28^0$ and $\angle QRT = 65^0$, then find the values of x and y.

Sol. As $PQ \parallel SR$ AND QR IS A TRANSVERSAL

$\therefore \quad \angle PQR = \angle QRT$ (Alternate interior angles)

$\Rightarrow \quad x + 28^0 = 65^0$

$\Rightarrow \quad x = 65^0 - 28^0$

$\therefore \quad x = 37^0$

Now, in ΔPQS, we have

$\angle QPS + \angle PQS + \angle PSQ = 180^0$

$\Rightarrow \quad 90^0 + 37^0 + y = 180^0$

$\Rightarrow \quad 127^0 + y = 180^0$

$\Rightarrow \quad y = 180^0 - 127^0$

$\therefore \quad y = 53^0$

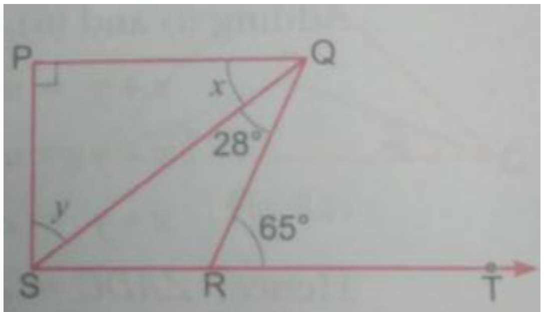


Fig. 6.25

Long Answer Type Questions

[4 MARKS]

Que 1. If a transversal intersects two parallel lines, prove that the bisectors of any pair of corresponding angles so formed are parallel.

Sol. **Given:** A transversal EF cuts two parallel lines AB and CD at point G and H respectively. GL and HM are respectively the bisectors of a pair of corresponding angles $\angle EGB$ and $\angle GHD$ respectively [Fig. 6.26].

To prove: $GL \parallel HM$

Proof: Since $AB \parallel CD$ and EF is a transversal

$\therefore$ $\angle EGB = \angle GHD$ (Corresponding angles)

$\Rightarrow$ $\frac{1}{2}\angle EGB = \frac{1}{2}\angle GHD$

$\Rightarrow$ $\angle EGL = \angle GHM$

But these are corresponding angles formed by the lines GL and HM

$\therefore$ $GL \parallel HM$

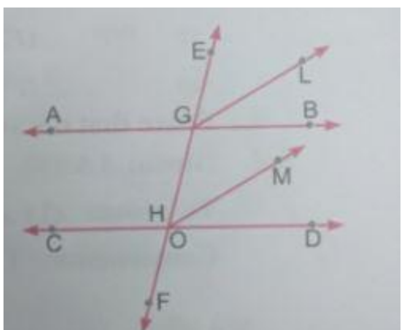


Fig. 6.26

Que 2. In Fig. 6.27, prove that $\angle ADC = \angle A + \angle B + \angle C$.

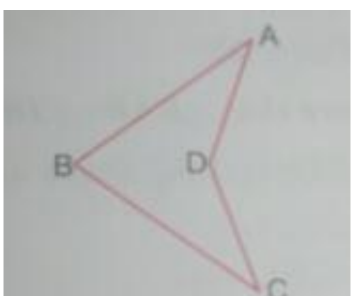


Fig. 6.27

Sol. $Join$ B and D and produce BD to E (Fig. 6.28). Since the exterior angle of a triangle is equal to sum of the two interior opposite angles.

Therefore, in ΔABD

$$x = w + \angle A \quad \dots(i)$$

In $\Delta CBD,$ $\quad y = z + \angle C \quad \dots(ii)$

Adding (i) and (ii), we get

$$x + y = w + \angle A + z + \angle C$$

$$x + y = w + z + \angle A + \angle C$$

$$x + y = \angle B + \angle A + \angle C$$

Hence, $\angle ADC = \angle A + \angle B + \angle C$

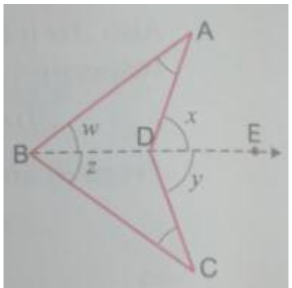


Fig. 6.28

Que 3. In Fig. 6.29, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$ respectively. Find $\angle APB$.

Sol. Since interior angles on the same side of transversal are supplementary

$\therefore \quad \angle EAB + \angle RBA = 180^0$

$\Rightarrow \quad \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = \frac{1}{2} \times 180^0 \quad \dots(i)$

As AP and BP are bisectors of $\angle EAB$ and $\angle RBA,$ respectively

$\therefore \quad \angle 1 = \frac{1}{2}\angle EAB$ and $\angle 2 = \frac{1}{2}\angle RBA \quad \dots(ii)$

From (i) and (ii), we get

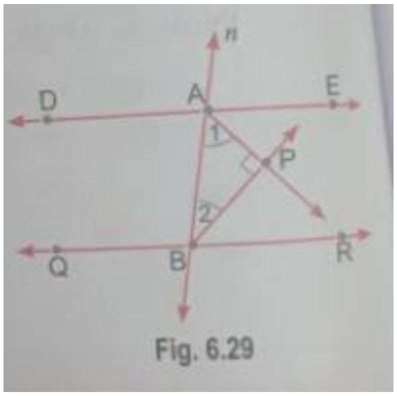
$$\angle 1 + \angle 2 = 90^0 \quad \dots(iii)$$

In Δ APB, we have

$$\angle 1 + \angle 2 + \angle APB = 180^0$$

$\Rightarrow \quad 90^0 + \angle APB = 180^0 \quad$ [Using (iii)]

$\Rightarrow \quad \angle APB = 180^0 - 90^0 \quad \therefore\ \angle APB = 90^0$

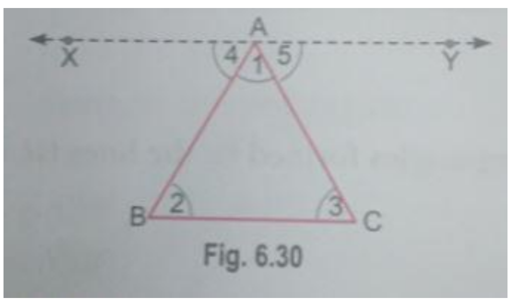


Que 4. Prove that the sum of the angles of a triangle is 180^0.

Sol. **Given:** $\angle 1 + \angle 2 + \angle 3 = 180^0$

To prove: $\angle 1 + \angle 2 + \angle 3 = 180^0$

Construction: Through A, draw a line $XY \parallel BC$ (Fig. 6.30).



Proof: Since $XY \parallel BC$ and AB is the transversal

$\therefore \quad \angle 4 = \angle 2$ (Alternate interior angles) $\dots(i)$

Similarly, $XY \parallel BC$ and AC is the transversal $\dots(ii)$

$\therefore \quad \angle 5 = \angle 3$

Adding (i) and (ii), we get

$\angle 4 + \angle 5 = \angle 2 + \angle 3$

Adding $\angle 1$ on both the sides

$\angle 4 + \angle 1 + \angle 5 = \angle 1 + \angle 2 + \angle 3$

But $\angle 4 + \angle 1 + \angle 5 = 180^0$ ($\therefore$ XAY is a straight line)

$\therefore \quad \angle 1 + \angle 2 + \angle 3 = 180^0$

Hence, the sum of the angles of a triangle is 180^0

Que 5. If the bisector of angles $\angle B$ and $\angle C$ of a triangle ABC meet at a point O, then prove that $\angle BOC = 90^0 + \frac{1}{2}\angle A$.

Sol. In $\Delta\ ABC$ (Fig. 6.31), we have

$$\angle A + \angle B + \angle C = 180^0$$

($\because$ Sum of the angles of a Δ is 180^0)

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180^0}{2}$

$\Rightarrow \quad \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90^0$

$\therefore \quad \angle 1 + \angle 2 = 90^0 - \frac{1}{2} \quad \ldots(i)$

Now, in ΔOBC, we have:

$\angle 1 + \angle 2 + \angle BOC = 180^0$ ($\because$ Sum of the angles of Δ is 180^0)

$\Rightarrow \quad \angle BOC = 180^0 - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle BOC = 180^0 - (90^0 - \frac{1}{2}\angle A)$ [Using (i)]

$\Rightarrow \quad \angle BOC = 180^0 - 90^0 + \frac{1}{2}\angle A$

$\therefore \quad \angle BOC = 90^0 + \frac{1}{2}\angle A$

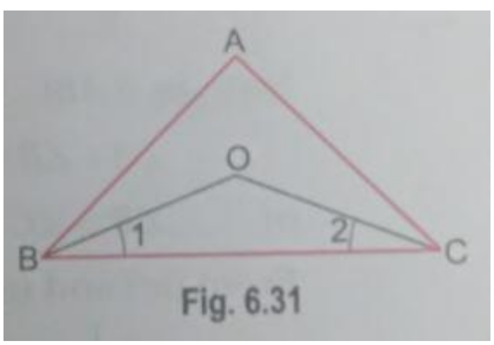


Fig. 6.31

Que 6. In ΔABC (Fig. 6.32), the sides AB and ΔABC are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90^0 - \frac{1}{2}\angle A$.

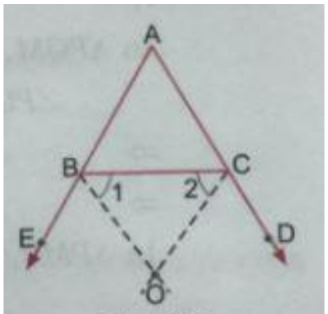


Fig. 6.32

Sol. As $\angle ABC$ and $\angle CBE$ form a linear pair

$\therefore \quad \angle ABC + \angle CBE = 180^0$

As BO is the bisector of $\angle CBE$

$\therefore \quad \angle CBE = 2\angle 1$

Therefore, $\angle ABC + 2\angle 1 = 180^0$

$\Rightarrow \quad 2\angle 1 = 180^0$

$\Rightarrow \quad \angle 1 = 90^0 - \frac{1}{2}\angle ABC \qquad \ldots(i)$

Again, $\angle ACB$ and $\angle BCD$ form a linear pair

$\therefore \quad \angle ACB + \angle BCD = 180^0$

As, CO is the bisector of $\angle BCD$, therefore, $\angle BCD = 2\angle 2$

So, $\angle ACB + 2\angle 2 = 180^0$

$\Longrightarrow \quad 2\angle 2 = 180^0 - 180^0 - \angle ACB$

$\Longrightarrow \quad \angle 2 = 90^0 - \frac{1}{2}\angle ACB$

In ΔOBC, we have

$\angle 1 + \angle 2 + \angle BOC = 180^0$ (Angle sum property of triangle)

From, $(i), (ii)$ and (iii), We have

$$90^0 - \frac{1}{2}\angle ABC + 90^0 - \frac{1}{2}\angle ACB + \angle BOC = 180^0$$

Now, in $\Delta\ ABC$, we have

$\angle A + \angle B + \angle C = 180^0$

Or $\quad \angle B + \angle C = 180^0 - \angle A$

From (iv) and (V), we have

$$180^0 - \frac{1}{2}(180^0 - \angle A) + \angle BOC = 180^0$$

$\Rightarrow \quad \angle BOC = 180^0 - 180^0 + \frac{1}{2}(180^0 - \angle A$

$\Rightarrow \quad BOC = \frac{1}{2}(180^0 - \angle A$

Hence, $\quad \angle BOC = 90^0 - \frac{1}{2}\angle A$

HOTS (Higher Order Thinking Skills)

Que 1. If Fig. 6.33, ∠Q> ∠R, PA is the bisector Of ∠QPR and PM ⊥ QR. Prove that ∠APM = $\frac{1}{2}$ (∠Q - ∠R).

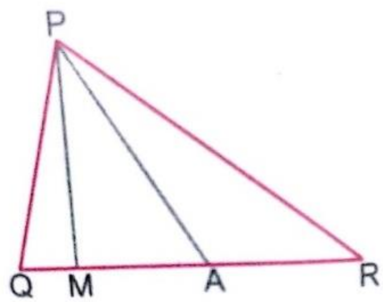


Fig. 6.33

Sol. Since PA is the bisector of ∠QPR

∴ ∠QPA = ∠APR …(i)

In ΔPQM, we have

∠PQM + ∠PMQ + ∠QPM = 180^0 (Angle sum property)

⇒ ∠PQM + 90^0 + ∠QPM = 180^0

⇒ ∠PQM + 90^0 - ∠QPM …(ii)

In ΔPMR, we have

∠PMR + ∠PRM + ∠RPM = 180^0

⇒ 90^0 + ∠PRM + ∠RPM = 180^0

⇒ ∠PRM = 180^0 - 90^0 - ∠RPM

⇒ ∠R = 90^0 - ∠RPM …(iii)

Subtracting (iii) from (ii), we get

∠Q - ∠R = (90^0 - ∠QPM) – (90^0 - ∠RPM)

∠Q - ∠R = ∠RPM - ∠QPM

∠Q - ∠R = (∠RPA + ∠APM) – (∠QPA - ∠APM) …(iv)

∠Q - ∠R = ∠QPA + ∠APM - ∠QPA + ∠APM [Using (i)]

⇒ ∠Q - ∠R = 2 ∠APM

Hence, ∠APM = $\frac{1}{2}$ (∠Q - ∠R).

Que 2. In Fig. 6.34, the side QR of ΔPQR is produced to point S. If the bisector of ∠PQR and ∠PRS meet at point T. then prove that ∠QTR = $\frac{1}{2}$ ∠QPR.

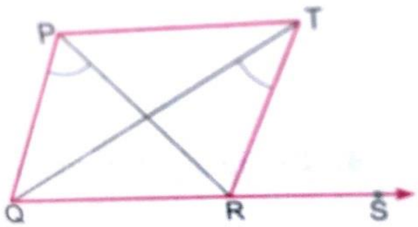


Fig. 6.34

Sol. Given: A Δ PQR, whose side QR is produced to S. The bisectors of ∠PQR and ∠PRS meet at point T.

To prove: ∠PRS = $\frac{1}{2}$ ∠QPR

Proof: Side QR of Δ PQR is produced to S.

∴ ∠PRS = ∠P + ∠Q ⇒ $\frac{1}{2}$ ∠PRS = $\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q

⇒ ∠TRS = $\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q …(i)

Again, side QR of Δ TQR is produced to S

∴ ∠TRS = ∠QTR + ∠RQT

⇒ ∠TRS = ∠T + $\frac{1}{2}$ ∠Q …(ii)

From (i) and (ii), we get

$\frac{1}{2}$ ∠P + $\frac{1}{2}$ ∠Q = ∠T + $\frac{1}{2}$ ∠Q

⇒ ∠T = $\frac{1}{2}$ ∠P or ∠QTR = $\frac{1}{2}$ ∠QPR

Que 3. Prove that a triangle must have atleast two acute angles.

Sol. Let us assume a triangle ABC which has only one acute angle (say ∠A)

Then we have the following three cases:

(i) The other two angles (∠B and ∠C) are right angle.

Then ∠A + ∠B + ∠C = ∠A + 90^0 + 90^0 = ∠A + 180^0 > 180^0 which is not possible.

(ii) The other two angles (∠B and ∠C) are obtuse angles.

Then ∠A + ∠B + ∠C > 180^0 which is not possible.

(iii) One angle (say ∠B) is right and the other angle (∠C) is obtuse.

Then ∠A + ∠B + ∠C > 180^0 which is not possible as we know that sum of the three angles of a triangle is 180^0 by angle sum property of a triangle.

Thus, a triangle must have atleast two acute angles.

Value Based Questions

Que 1. Teacher held two sticks AB and CD of equal length in her hands and marked their mid points M and N respectively. She then asked the students whether AM is equal to ND or not. Aprajita answered yes. Is Aprajita correct? State the axiom of Euclid that supports her answer. Which values of Aprajita are depicted here?

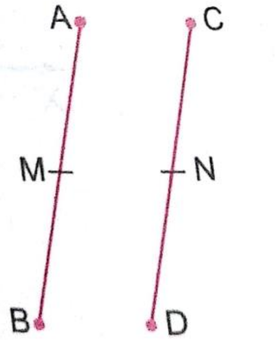


Fig. 5

Sol. Yes, Things which are halves of the same things are equal to one another. Curiosity, knowledge, truthfulness.

Que 2. For her records, a teacher asked the students about their heights. Manav said his height is same as that of Arnav. Raghav also answered the same, way that his height is same as that of Arnav. She then asked the students to relate the height of Manav and Raghav. Arnav answered they both have same height. Is Arnav correct? If yes, state Euclid's axiom which supports his answer.
Which values of Arnav are depicted here?

Sol. Yes, Things which are equal to the same thing are equal to one another. Knowledge, curiosity, truthfulness.

Que 3. The number of members of society A who participated in 'Say No to Crackers' campaign is double the number of members from society B. Also, the number of members from society C is double the number of members from society B. Can you relate the number of participants from society A and C? Justify your answer using Euclid's axiom.
Which values are depicted here?

Sol. The number of participants from society A and C is equal. Things which are double of the same thing are equal to one another. Social service, helpfulness, cooperation, environmental concern.

Que 4. In a society, the number of persons using CNG instead of petrol for their vehicles has increased by 15 and now the number is 25. Form a linear equation to find the original number of persons using CNG and solve it using Euclid's axiom.
Which values are depicted in the question?

Sol. X + 15 = 25

⇒ $x + 15 - 15 = 25 - 15$ (Using Euclid's third axiom)

⇒ x = 10

Environmental care, responsible citizens, futuristic.

Que 5. Teacher asked the students to find the value of x in the following figure if l|| m. Shalini answered 35°. Is she correct? Which values are depicted here?

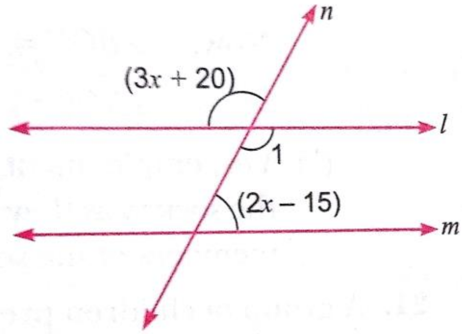


Fig. 6

Sol. ∠1 = 3x + 20 (Vertically opposite angles)

∴ 3x + 20 2x – 15 = 180 ° (Co-interior angles are supplementary)

⇒ $5x + 5 = 180°$ ⇒ $5x = 180° - 5°$

⇒ 5x = 175 ° ⇒ $x = \frac{175}{5} = 35°$

Yes, Knowledge, truthfulness.

Que 6. For spreading the message 'Save Environment Save Future' a rally was organised by some students of a school. They were given triangular cardboard pieces which they divided into two parts by drawing bisectors of base angles (say ∠B and ∠C) intersecting at O in the given figure. Prove that ∠BOC = $90 + \frac{1}{2}\angle A$

Which values are depicted by these students?

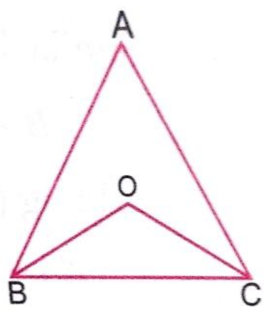


Fig. 7

Sol. In ΔABC, we have

∠A + ∠B + ∠C = 180 ° (∵ sum of the angles of a Δ is 180 °)

⇒ $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2}$

$\Rightarrow \quad \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90°$

$\therefore \quad \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$ ....(i)

Now, in ΔOBC, we have:

$\angle 1 + \angle 2 + \angle BOC = 180°$ [∵ sum of the angles of Δ is 180 °]

$\Rightarrow \quad \angle BOC = 180° - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle BOC = 180° - (90° - \frac{1}{2}\angle A)$ [using (i)]

$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{1}{2}\angle A$

$\therefore \quad \angle BOC = 90° + \frac{1}{2}\angle A$

Environmental care, social, futuristic.

Que 7. Three bus stops situated at A, B and C in the figure are operated by handicapped persons. These 3 bus stops are equidistant from each other. OB is the bisector of ∠ABC and OC is the bisector of ∠ACB.
(a) Find ∠BOC.
(b) Do you think employment provided to handicapped persons is important for the development of the society? Express your views with relevant points.

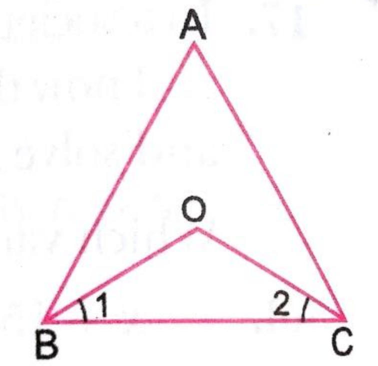


Fig. 9

Sol. (a) Since, A, B, C are equidistant from each other.

∴ ∠ABC is an equilateral triangle.

⇒ ∠ABC = ∠ABC = 60°

⇒ ∠OBC = ∠OCB = $\frac{1}{2} \times 60° = 30°$ (∵ OB and OC are angle bisectors)

Now, ∠BOC = 180° - ∠OBC - ∠OCB (Using angle sum property of triangle)

⇒ ∠BOC = 180°- 30°-30° = 120°

(b) Yes, employment provided to the handicapped persons is important for the development of the society as they would become independent, self-reliant, confident, social, helpful and useful members of the society.

Que 8. A group of children prepared some decorative pieces in the shape of a star for the orphans in an orphanage. Show that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Which values of the children are depicted here?

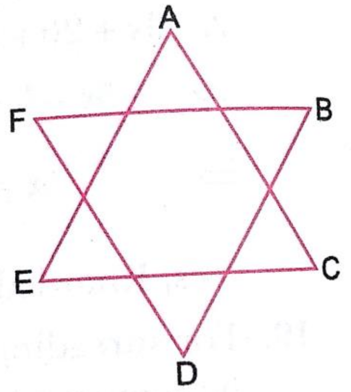


Fig. 10

Sol. In ΔAEC,
$\angle A + \angle E + \angle C = 180°$ … (i) (Angle sum property of a triangle)

Similarly, in ΔBDF,
$\angle B + \angle D \angle F = 180°$ …. (ii)

Adding (i) and (ii), we get
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Social, caring, cooperative, hardworking.

Que 9. For annual day, Sakshi and Nidhi were asked to make one rangoli each on two different places. They started it with triangles (say ABC and ΔPQR) and their medians (AM and PN). If two sides (AB and BC) and a median (AM) of one triangle are respectively equal to two sides (PQ and QR) and a median (PN) of other triangle, prove that the two triangles (ΔABC and ΔPQR) are congruent. Which values of the girls are depicted here?

Sol. In ΔABC and ΔPQR

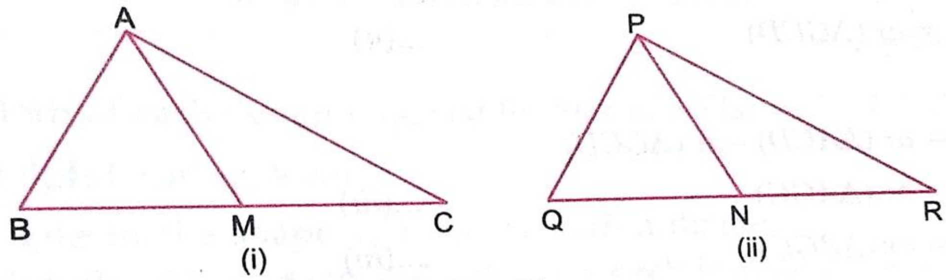


Fig. 11

BC = QR

⇒ $\frac{1}{2} BC = \frac{1}{2} QR$

⇒ BM = QN

In triangle ABM and PQN, we have

AB = PQ	(Given)
BM = QN	(Proved above)
AM = PN	(Given)

∴ $\Delta ABM \cong \Delta PQN$ (SSS congruence criterion)
⇒ $\angle B = \angle Q$ (CPCT)

Now, in triangles ABC and PQR, we have

AB = PQ	(Given)
$\angle B$ = ∠Q	(Proved above)
BC = QR	(Given)

∴ ΔABC ≅ ΔPQR (SSS congruence criterion)

Participation, beauty, hardworking.

Que 10. Triangular pieces of cardboards were cut out by some people who were organising 'No Pollution' campaign in their area. If the three angles of one cutout are respectively equal to the three angles of the other cutout, can we say the two cutouts are congruent? Justify your answer.
Which values of these people are depicted here?

Sol. The two cutouts may not be congruent. For example all equilateral triangles have equal angles but may have different sides.
Environmental concern, cooperative, caring, social.

Que 11. Anya wants to prepare a poster on education of girlchild for a campaign. She takes a triangular sheet and divides it into three equal parts by drawing its medians which intersect at the point G (see Fig. 12).

Show that ar (ΔAGC) = ar (ΔAGC) = ar (ΔAGB) = (ΔBGC) = $\frac{1}{3} \boldsymbol{ar}\ (\Delta \boldsymbol{ABC})$

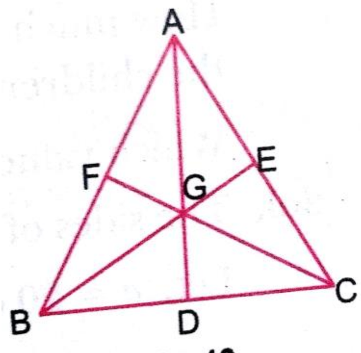


Fig. 12

Do you think education of a girl child is important for the development of a society? Justify your answer.

Sol. Given: A ΔABC in which medians AD, BE and CF intersects at G.

Proof: (ΔAGB) = ar (ΔBGC) = ar (ΔCGA) = $\frac{1}{3}$ ar (ΔABC)

Proof: In ΔABC, AD is the median. As a median of a triangle divides it into two triangles of equal area.

∴ ar (ΔABD) = ar (ΔACD) … (i)

In ΔGBC, GD is the median

∴ aq (ΔGBD) = ar (ΔGCD) …. (ii)

Subtracting (ii) from (i), we get

ar (ΔABD) – ar (ΔGBD) = ar (ACD) – ar (ΔGCD)

ar (ΔAGB) = ar (ΔAGC) … (iii)

Similarly, ar (ΔAGB) = ar (ΔBGC) … (iv)

From (iii) and (iv), we get

ar (ΔAGB) = ar (ΔBGC) = ar (ΔAGC) …. (v)

But, ar (ΔAGB) + ar (ΔBGC) + ar (ΔAGC) = ar (ΔABC) …. (vi)

From (v) and (vi), we get

3 ar (ΔAGB) = ar (ΔABC)

⇒ ar (ΔAGB) = $\frac{1}{3} ar(\Delta ABC)$

Hence, ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

Yes, for the development of a society, education of each girl child is essential. An educated society always progresses.