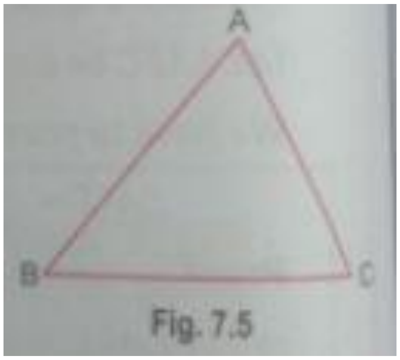
Very Short Answer Type Questions

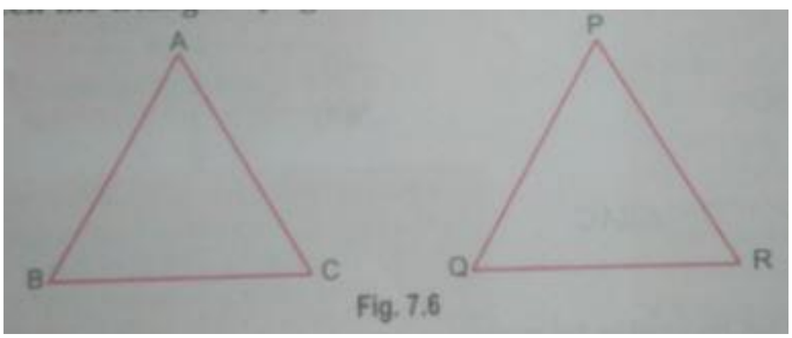
[1 MARK]

Que 1. In ΔABC, if $\angle C > \angle B$, then which two sides of the triangle can you relate? State the relation. [Fig. 7.5]



Sol. $AB > AC$

Que 2. If $AB = PQ, BC = QR$ and $AC = PR$, then write the congruence relation between the triangles. [Fig.7.6]

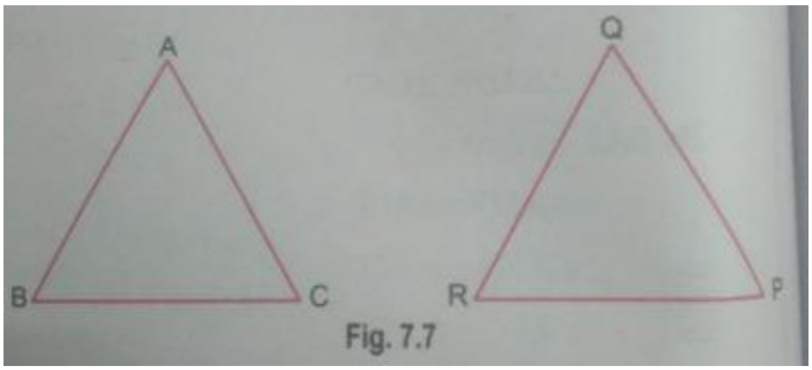


Sol. $\Delta ABC \cong \Delta PQR$

Que 3. It is given that $\Delta ABC \cong \Delta DEF$. Is it true to say that $AB = EF$? Justify your answer.

Sol. No, AB and EF are not corresponding sides in triangles ABC and DEF, Here, AB corresponds to DE.

Que 4. In triangles ABC and $PQR, \angle A = \angle Q$ and $\angle B = \angle R$. Which side of ΔPQR should be equal to side AB of ΔABC so that the two triangles are congruent? Give reason for your answer.



Sol. In triangles ABC and QRP

$\angle A = \angle Q$ (Given)

$\angle B = \angle R$ (Given)

If $AB = QR$,

Then $\Delta ABC \cong$ ΔQRP (By ASA).

Que 5. In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of Δ PQR should be equal to side BC of ΔABC so that two triangles are congruent? Give reason for your answer.

Sol. RP, they will be congruent by AAS congruence criterion.

Que 6. In Δ $PQR, \angle P = 70^0$ and $\angle Q = 30^0$. Which side of this triangle is the longest?

Sol. PQ.

Short Answer Type Questions – I

[2 MARKS]

Que 1. In Fig. 7.8, $\Delta PQR, PQ = PR$ and $\angle Q = 65^0$. Then find $\angle R$.

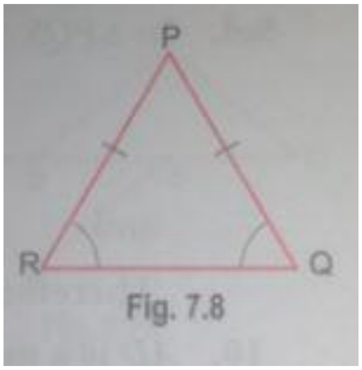


Fig. 7.8

Sol. **In** $\Delta \mathrm{PQR}, \mathrm{PQ} = \mathrm{PR}$, So $\angle Q = 65^0 = \angle R$

[Angles opposite to equal sides of a triangle are equal.]

Que 2. If the corresponding angles of two triangles are equal, then they are always congruent. State true or false and justify your answer.

Sol. False, because two equilateral triangles with sides 3 cm and 6 cm respectively have all angles equal, but the triangles are not congruent.

Que 3. In the Fig. 7.9, PM is the bisector of $\angle P$ and $PQ = PR$. Then ΔPQM and ΔPRM are congruent by which criterion?

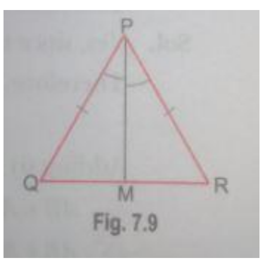


Fig. 7.9

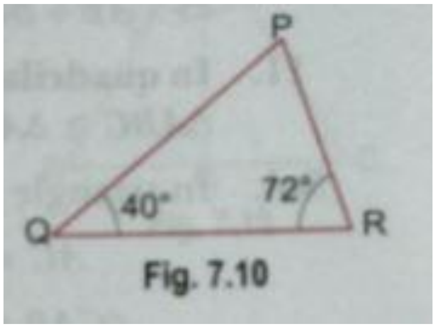
Sol. In ΔPQM and ΔPRM

$PQ = PR$ and $\angle QPM = \angle RPM$ (Given)

PM is common.

So, $\Delta PQM \cong \Delta \mathrm{PQM}$ (By SAS rule)

Que 4. In Fig. 7.10 Δ PQR, if $\angle Q = 40^0$ and $\angle R = 72^0$, then find the shortest and the largest sides of the triangle.



Sol. In ΔPQR, we know

$\angle Q = 40^0$ and $\angle R = 72^0$

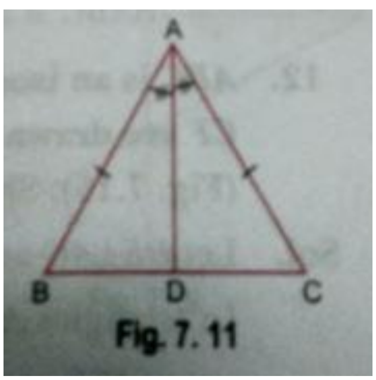
Then, $\angle P = 180^0 - (72^0 + 40^0) = 68^0$

In ΔPQR,

PQ is largest side [Because side opposite to largest angle is largest]

PR is shortest side [Because side opposite to shortest angle is shortest]

Que 5. In Fig. 7.11, if AB = AC and BD = DC, then find $\angle ADB$.



Sol. ΔADB and ΔADC

$AB = AC, BD = DC$ [Given]

$AD = AD$ [Common]

So, ΔADB ≅ ΔADC [By SSS congruence rule]

⇒ $\angle ADB = \angle ADC$ [CPCT]

But $\angle ADB + \angle ADC = 180^0$ [Linear pair]

⇒ $\angle ADB + \angle ADB = 180^0$

$\Rightarrow \qquad 2\angle ADB = 180^0 \Rightarrow \angle ADB = 90^0$

Que 6. Is it possible to construct a triangle with lengths of its sides 5cm, 3cm and 8cm? Give reason for your answer.

Sol. No, since sum of two sides is equal to third side. (5 cm + cm = 8 cm)

Que 7. Is it possible to construct a triangle with lengths of its sides as 7 cm, 8 cm and 5 cm? Give reason for your answer.

Sol. Yes, because in each case sum of two sides is greater than the third side.

Que 8. In $\Delta ABC, \angle A = 65^0$ and $\angle C = 30^0$. Which side of this triangle is the longest? Give reason for your answer.

Sol. $\angle B = 180^0 - 65^0 - 30^0 = 85^0$

$\because AC$ is the longest side as side opposite to the larger angle is longer.

Que 9. In Fig. 7.12, $PQ = PR$ and $\angle Q = \angle R$. Prove that $\Delta PQS \cong \Delta PRT$.

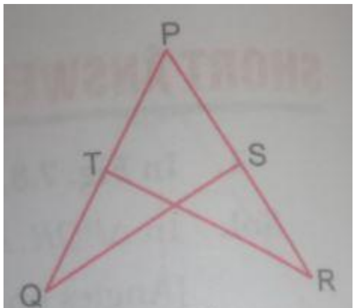


Fig. 7.12

Sol. In ΔPQS and ΔPRT

$PQ = PR$ (Given)

$\angle Q = \angle R$ (Given)

And $\angle P = \angle P$ (Common)

Therefore $\Delta PQS \cong \Delta \text{PRT}$ (ASA Congruence criterion)

Que 10. AD is a median of the $\Delta ABC (Fig. 7.13)$. Is it true that $AB + BC + CA > 2AD$? Give reason for your answer.

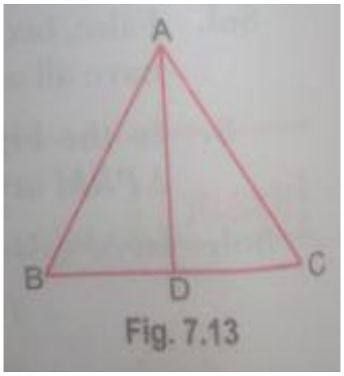


Fig. 7.13

Sol. Yes, since the sum of two sides of a triangle is greater than the third side.

Therefore, $AB + BD > AD$ …(i)

$AC + CD > AD$ …(ii)

Adding (i) and (ii), we get

$$AB + AC + (BD + CD) > AD + AD$$

$$\Rightarrow \; AB + BC + CA > 2AD$$

Que 11. In quadrilateral $ACBD, AC = AD$ and AB bisects $\angle A$ (in Fig.7.14). Show that $\Delta ABC \cong \Delta ABD$. What can you say about BC and BD?

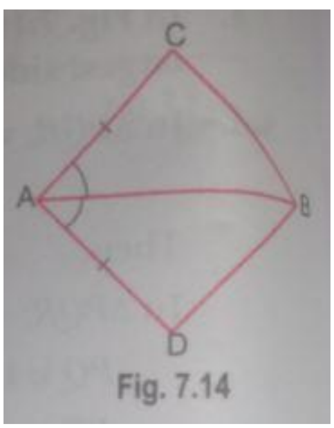


Fig. 7.14

Sol. In triangle ABC and $ABD,$ we have,

$AC = AD$ (Given)

$\angle CAB = \angle BAD$ ($\therefore$ AB bisects $\angle A$)

$AB = AB$ (Common)

And by SAS congruence criterion, we have

$\Delta ABC \cong \Delta ABD$ $\Rightarrow BC = BD$ $(CPCT)$

Que 12. ABC is an isosceles triangle in which altitude BE and CF are drawn to equal sides AC and AB respectively (Fig.7.15). Show that these altitudes are equal.

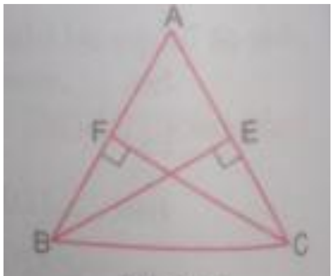


Fig. 7.15

Sol. Let $BE \perp AC$ and $CF \perp AB$.

In triangles ABE and $ACF,$ we have

$\angle AEB = \angle AFC$ (∵ Each 90^0)

$\angle A = \angle A$ (Common)

And $AB = AC$ (Given)

By AAS criterion of congruence, we have

$\Delta ABE \cong \Delta ACF$

So, $BE = CF$ $(CPCT)$

Que 13. In Fig. (7.16) $ABCD$ is a square and P is the midpoint of AD. BP and CP are joined. Prove that $\angle PCB = \angle PBC$.

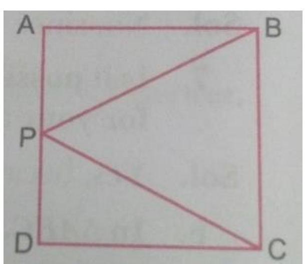


Fig. 7.16

Sol. In triangles PAB and $PDC,$

$PA = PD$ (Given

$AB = CD$ (Side of square)

$\angle PAB = \angle PDC = 90^0$ (By RHS, $\Delta PAB \cong \Delta$PDC

∴ $PC = PB \Rightarrow \angle PCB = \angle PBC$

SHORT ANSWER QUESTIONS-II [3 marks]

Que 1. In Fig. 7.17, it is given that $AB = CF$, $EF = BD$ and $\angle AFE = \angle CBD$. Prove that $\Delta AFE \cong \Delta CBD$.

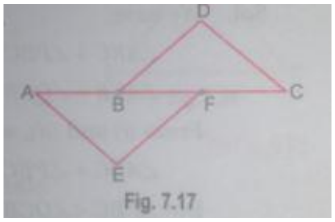


Fig. 7.17

Sol. In triangles AFE and CBD, we have

$$AB = CF$$

Adding BF on both the sides

$$AB + BF = CF + BF$$

$$AF = BC$$

Now in triangles AFE and CBD, we have $AF = CB$ (Proved above)

$\angle AFE = \angle CBD$ (Given)

And $EF = BD$ (Given)

$\therefore$ $\Delta AFE \cong \Delta CBD$ (SAS congruence criterion)

Que 2. Prove that angles opposite to equal sides of a triangle are equal.

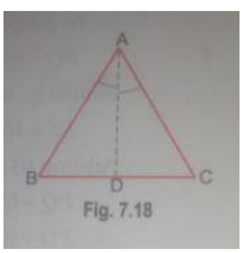


Fig. 7.18

Sol. **Given:** A ΔABC in which $AB = AC$.

To prove: $\angle B = \angle C$

Construction: Draw AD, the bisector of $\angle A$, to meet BC at D.

Proof: In ΔABD and ΔACD, we have

$AB = AC$ (Given)

$\angle BAD = \angle CAD$ (By Construction)

$AD = AD$ (Common)

$\therefore \quad \Delta ABD \cong \Delta ACD$ (SAS Congruence criterion)

Hence, $\angle B = \angle C$ (CPCT)

Que 3. In Fig. 7.19, AD and BC are equal perpendicular to a line segment AB. Show that CD bisects AB.

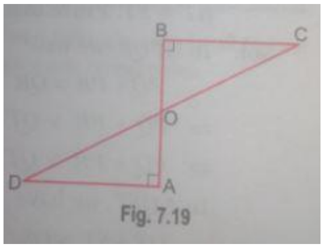


Fig. 7.19

Sol. In ΔOAD and ΔOBC, we have

$\angle AOD = \angle BOC$ (Vertically opposite angles)

$\angle OAD = \angle OBC$ (Each90^0)

And, $AD = BC$

$\therefore \quad \Delta AOD \cong \Delta BOC$ (AAS congruence criterion)

$\Rightarrow \quad OA = OB$ (CPCT)

Thus, CD bisects AB.

Que 4. In Fig. 7.20, ABC and DBC are two isosceles triangles on the same base BC. Show that $\angle ABD = \angle ACD$.

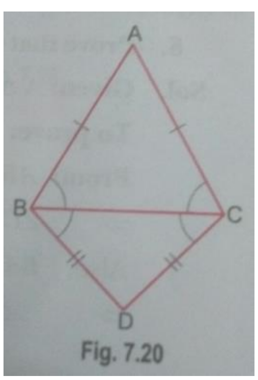


Fig. 7.20

Sol. In ΔABC, we have, $AB = AC$

$\Rightarrow \angle ACB = \angle ABC$ (Angles opposite to equal sides) …(i)

In $\Delta\ DBC$, we have

$BD = CD$

$\Rightarrow\ \angle DCB = \angle DBC$ (Angles opposite to equal sides) ...(ii)

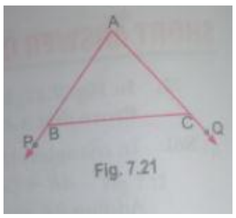
Adding (i) and (ii), we get

$$\angle ACB + \angle DCB = \angle ABC + \angle DBC$$

$$\angle ACD = \angle ABD$$

Hence, $\angle ABD = \angle ACD$

Que 5. In Fig. 7.21, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



Sol. We have,

$\angle ABC + \angle PBC = 180^0$ (Linear Pair) ...(i)

$\angle ACB + \angle QCB = 180^0$ (Linear Pair) ...(ii)

From (i) and (ii), we have

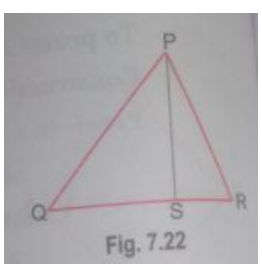
$$\angle ACB + \angle QPBC = \angle ACB + \angle QCB$$

But $\angle PBC < \angle QCB$ (Given)

∴ $\angle ABC > \angle ACB$

$\Rightarrow$ $AC > AB$ (∵ Side opposite to greater angle is larger)

Que 6. S is any point on side QR of a ΔPQR. Show that: $PQ + QR + RP > 2PS$.



Sol. Since sum of the two sides of a triangle is greater than the third side

∴ In ΔPQS, we have

$PQ + QS > PS$ $\quad ...(i)$

Similarly, in ΔPRS, we have

$RS + RP > PS \qquad ...(ii)$

Adding (i) and (ii), we get

$PQ + QS + RS + RP > PS + PS$

$\Rightarrow \quad PQ + (QS + RS) + RP > 2PS$

$\Rightarrow \quad PQ + QR + RP > 2PS$

Que 7. In Fig. 7.23, T is a point on side QR of ΔPQR and S is a point such that $RT = ST$. Prove that $PQ + PR > QS$.

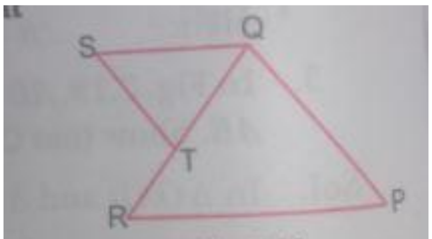


Fig. 7.23

Sol. In ΔPQR, we have

$PQ + PR > QR$

$\Rightarrow \quad PQ + PR > QT + RT \qquad (\because QR = QT + RT)$

$\Rightarrow \quad PQ + PR > QT + ST \qquad (\because RT = ST) \qquad$ ...(i)

In ΔQST, we have

$QT + ST > QS \qquad$ ...(ii)

From (i) and (ii), we have

$PQ + PR > QS$

Que 8. Prove that each angle of an equilateral triangle is 60^0.

Sol. **Given:** A ΔABC in which $AB = BC = CA$ (Fig. 7.24)

To prove: $\angle A = \angle B = \angle C = 60^0$

Proof: $AB = AC$

$\Rightarrow \quad \angle C = \angle B$ (Angles opposite to equal sides are equal) ... (i)

Also, $BA = BC$

$\Rightarrow \quad \angle C = \angle A$ (Angles opposite to equal sides are equal) ... (ii)

From (i) and (ii), we have

$\angle A = \angle B = \angle C$

Now, $\angle A + \angle B + \angle C = 180^0 \quad \Rightarrow \angle A + \angle A + \angle A = 180^0$

$\Rightarrow \quad 3\angle A = 180^0 \quad \Rightarrow \angle A = 60^0$

Hence, $\angle A = \angle B = \angle C = 60^0$

Que 9. Show that in a quadrilateral $ABCD, AB + BC + CD + DA > AC + BD$.

Sol. Since the sum of any two sides of a triangle is greater than the third side.

Therefore, in ΔABC, we have

$$AB + BC > AC \quad \text{...}(i)$$

In ΔBCD, we have

$$BC + CD > BD \quad \text{...}(iii)$$

In ΔCDA, we have

$$CD + DA > AC \quad \text{...}(iv)$$

Adding: (i), (ii), (iii) and (iv), we get

$$2AB + 2BC + 2CD + 2DA > 2AC + 2BD$$

$\Rightarrow \quad 2(AB + BC + CD + DA) > 2(AC + BD)$

$\Rightarrow \quad AB + BC + CD + DA > AC + BD$

LONG ANSWER QUESTIONS [4 Marks]

Que 1. Prove that if in two angles and the included side of one triangle are equal to two angles and the included side of the other triangle, then two triangles are congruent.

Sol. Given: two triangles ABC and DEF

Such that $\angle B = \angle E, \angle C = \angle F$ and $BC = EF$.

To prove: $\Delta ABC \cong \Delta \mathrm{DEF}$

Proof: For proving the congruence of two triangles, three cases arise.

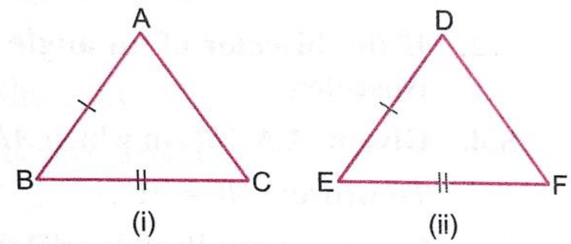


Fig. 7.26

Case I: When $AB = DE$

In this case

$AB = DE$ and $\angle B = \angle E$

$BC = EF$

∴ $\Delta \mathrm{ABC} \cong \Delta \mathrm{DEF}$ (SAS congruence criterion)

Case II: When $AB < ED$

In this case, take a point P on ED such that $PE = AB$. Join FP.

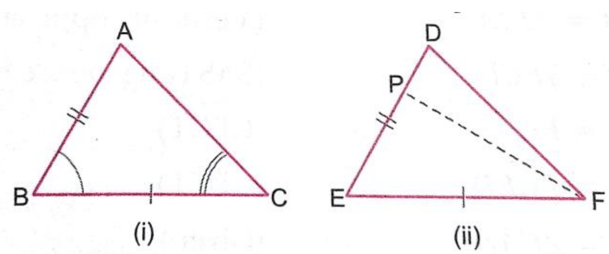


Fig. 7.27

In triangles ABC and PEF, we have

$AB = PE$ (By supposition)

$\angle B = \angle E$ (Given)

And $BC = EF$ (Given)

∴ ΔABC ≅ ΔPEF (SAS criterion of congruence)

⟹ ∠ACB = ∠PFE (CPCT)

But ∠ACB = ∠DFE (Given)

∴ ∠PFE = ∠DFE

This is possible only when P and D coincide.
Therefore, AB must be equal to DE.
Thus, in triangle ABC and DEF, we have

AB = DE (Proved above)
∠B = ∠ (Given)
and BC = EF (Given)
∴ ΔABC ≅ ΔDEF (SAS congruence criterion)

Case III: When AB > ED

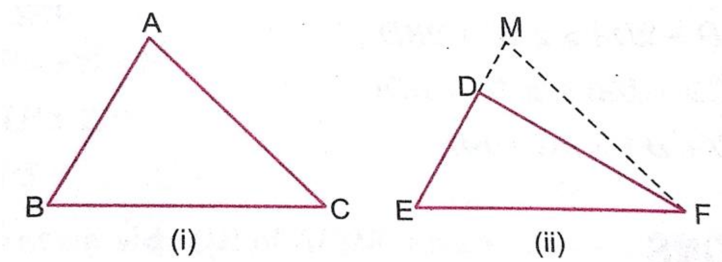


Fig. 7.28

In this case, take a point M on ED produced such that ME = AB. Join FM. Now, repeating the arguments as given in case (II), we can conclude that AB = DE and

So, ΔABC ≅ ΔDEF
Hence, in all the three cases, we have
ΔABC ≅ ΔDEF

Que 2. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Sol. Given: A ΔABC in which AD is the bisector of ∠A which meets BC in D such that BD = DC
To prove: AB = AC
Construction: Produce AD to E such that AD = DE and then join CE.
Proof: In ΔABD and ΔECD, we have

BD = CD (Given)
AD = ED (By construction)

and	∠ADB = ∠EDC	(Vertically opposite angles)	
Therefore,	ΔABD ≅ ΔECD	(SAS congruence criterion)	
So,	AB = EC	(CPCT)	.....(i)
and	∠BAD = ∠CED	(CPCT)	......(ii)
Also,	∠BAD = ∠CAD	(Given)	......(iii)

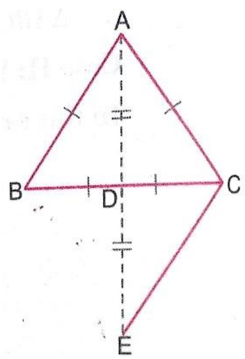


Fig. 7.29

Therefore, from (ii) and (iii)

∠CAD = ∠CED

So, AC = EC (Sides opposite to equal angles) .....(iv)

From (i) and (iv), we get

AB = AC

Que 3. In Fig. 7.30, two sides AB and BC and median AM of two triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR. Show that ΔABC ≅ ΔPQR.

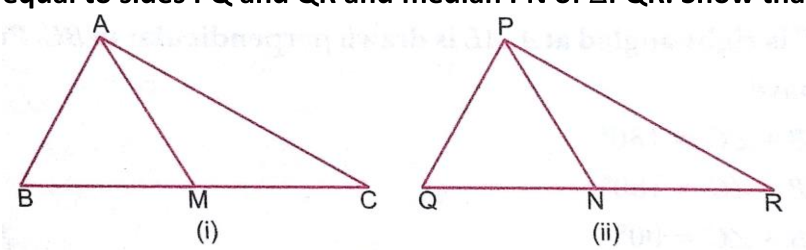


Fig. 7.30

Sol. In ΔABC and ΔPQR,

BC = QR (Given)

⇒ $\frac{1}{2}BC = \frac{1}{2}QR$

⇒ BM = QN

In triangle ABM and PQN, we have

AB = PQ	(Given)
BM = QN	(Proved above)
AM = PN	(Given)

∴ ΔABM ≅ ΔPQN (SSS congruence criterion)
⇒ ∠B = ∠Q (CPCT)

Now, in triangle ABC and PQR, we have

∠B = ∠Q (Proved above)
BC = QR (Given)
∴ ΔABC ≅ ΔPQR (SAS congruence criterion)

Que 4. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle.

Sol.

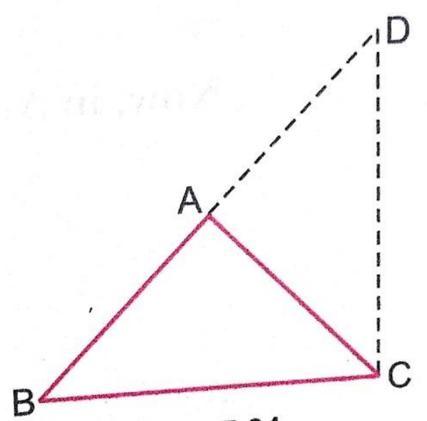


Fig. 7.31

Given: A ΔABC in which AB = AC, side BA is produced to D such that AD = AB

Construction: Join CD
To prove: ∠BCD, we have
Proof: In ΔABC, we have

AB = AC (Given)
∴ ∠ACB = ∠ABC (Angles opposite to equal sides) …. (i)

Also, AB = AD ⇒ AC = AD
In ΔADC, we have AD = AC
⇒ ∠ACD = ∠ADC (Angles opposite to equal sides) …..(ii)

Adding (i) and (ii), we get

∠ACB + ∠ACD = ∠ABC + ∠ADC
∠BCD = ∠ABC + ∠BDC

Adding ∠BCD on both sides

$\angle BCD + BCD = \angle ABC + \angle BDC + \angle BCD$

$\Rightarrow \quad 2\angle BCD = 180^\circ \quad \Rightarrow \angle BCD = 90^\circ$

Hence, ∠BCD is a right angle

Que 5. A triangle ABC is right-angled at A. AL is drawn perpendicular to BC. Prove that ∠BAL = ∠ACB.

Sol.

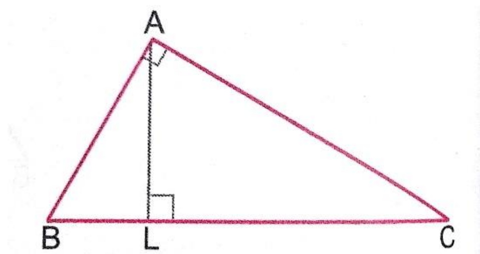


Fig. 7.32

In ΔABC, we have

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \angle B + \angle C = 90^\circ$

$\Rightarrow \quad \angle C = 90^\circ - \angle B \quad$ ....(i)

In ΔABL, we have

$\angle ALB + \angle BAL + \angle B = 180^\circ$

$\Rightarrow \quad 90^\circ + \angle BAL + \angle B = 180^\circ \quad \Rightarrow \angle BAL + \angle B = 90^\circ$

$\angle BAL = 90^\circ - \angle B \quad$ ....(ii)

From (i) and (ii), we get

$\angle BAL = \angle ACB$

Que 6. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, AB = AC and DB = DC. Show that AD is the perpendicular bisector of BC.

Sol.

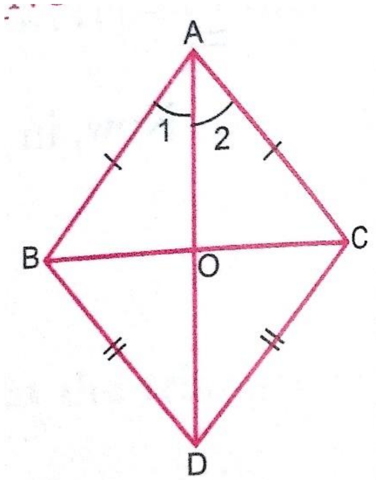


Fig. 7.33

Let AD intersect BC at O

Then we have to prove $\angle AOB = \angle AOC = 90°$

and BO = OC

In ΔABD and ΔACD

AB = AC (Given)

AD = DA (Common)

BD = DC (Given)

$\therefore$ $\Delta ABD \cong \Delta ACD$ (By SSS congruence)

$\Rightarrow$ $\angle 1 = \angle 2$

Now, in ΔAOB and ΔAOC

AB = AC (Given)

AO = OA (Common)

$\angle 1 = \angle 2$ (Proved above)

$\therefore$ $\Delta AOB \cong \Delta AOC$ (By SAS congruence)

$\Rightarrow$ BO = OC and $\angle AOC$

But $\angle AOB + \angle AOC = 180°$ (Linear Pair)

$\Rightarrow$ $\angle AOB + \angle AOB = 180°$

$\Rightarrow$ $\angle AOB = 90°$

Hence, $AD \perp BC$ and AD bisects BC, i.e., AD is the perpendicular bisector of BC.

Que 7. ΔABC is a right triangle such that AB = AC and bisector of angle C intersects the side AB at D. Prove that AC + AD = BC.

Sol.

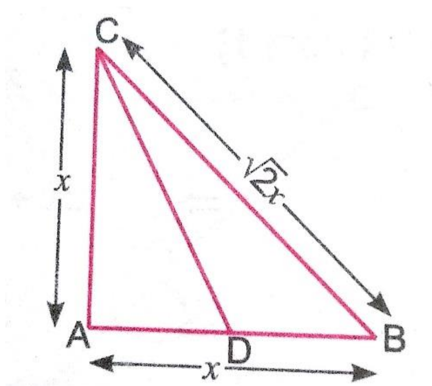


Fig. 7.34

Let AB = AC = X
By Pythagoras theorem

$$BC = \sqrt{AB^2 + AC^2} = \sqrt{x^2 + x^2} \Rightarrow BC = \sqrt{2x}$$

Again by Bisector theorem

$$\frac{AC}{BC} = \frac{AD}{BD} \quad \Rightarrow \quad \frac{BC}{AC} = \frac{BD}{AD}$$

$$\Rightarrow \frac{BC}{AC} + 1 = \frac{BD}{AD} + 1 \quad \Rightarrow \quad \frac{BC + AC}{AC} = \frac{BD + AD}{AD}$$

$$\Rightarrow \frac{BC + AC}{AC} = \frac{AB}{AD} \quad \Rightarrow \quad \frac{\sqrt{2x} + x}{x} = \frac{x}{AD}$$

$$\Rightarrow \frac{\sqrt{2} + 1}{1} = \frac{x}{AD} \quad \Rightarrow \quad AD = \frac{x}{\sqrt{2} + 1}$$

$$\therefore AC + AD = x + \frac{x}{\sqrt{2} + 1} = \frac{\sqrt{2x} + x + x}{\sqrt{2} + 1} = \frac{\sqrt{2x} + 2x}{\sqrt{2} + 1} = \frac{\sqrt{2x}(1 + \sqrt{2})}{(\sqrt{2} + 1} = \sqrt{2x} = BC$$

HOTS (Higher Order Thinking Skills)

Que 1. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.

Sol. Given: A Δ ABC in which AD is a median.

To prove: AB + AC > 2AD

Construction: Produce AD to E such that

AD = DE. Join EC

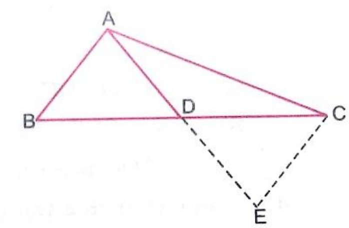


Fig. 7.35

Proof: In triangles ADB and EDC, we have

AD = DE (By construction)

BD = DC (∴ AD is the median)

and, ∠ADB = ∠EDC (Vertically opposite angles)

∴ Δ ADB ≅ Δ EDC (SAS congruence criterion)

⇒ AB = EC (CPCT) ...(i)

In Δ AFC, we have

AC + EC > AE ...(ii)

[As sum of the two sides of a triangle is greater than the third side]

Also, AE = 2AD (by construction) ...(iii)

Using (i) and (iii) in (ii), we get

AC + AB>2AD

Que 2. ABC is a triangle with ∠B = 2∠C.D is a point on BC such that AD bisects ∠BAC and AD = CD. Prove that ∠BAC = 72^0.

Sol. Given, In ΔABC, ∠B = 2∠C, AD = CD

And AD bisects ∠BAC.

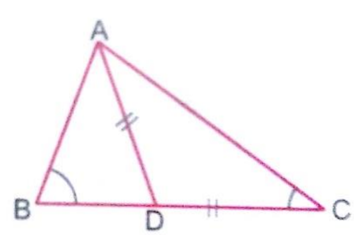


Fig. 7.36

Since AD = CD ⇒ ∠C = ∠DAC

But ∠B = 2∠C ⇒ ∠B = 2∠DAC

⇒ ∠B = ∠A = x (say) [∴ AD in bisector of ∠BAC]

Now, $\angle A + \angle B + \angle C = 180^0$ [Angle Sum Property]

$$x + x + \frac{\angle B}{2} = 180^0$$

$\Rightarrow$ $2x + \frac{x}{2} = 180^0$ $\Rightarrow$ $\frac{4x+x}{2} = 180^0$

$\Rightarrow$ $\frac{5x}{2} = 180^0$ $\Rightarrow$ $x = \frac{180^0 \times 2}{5}$

$\Rightarrow$ $\angle A = 72^0$ $\Rightarrow$ $\angle BAC = 72^0$

Que 3. O is a point in the interior of a square ABCD such that OAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle.

Sol. Given: ΔOAB is an equilateral triangle

To prove: ΔCOD is an isosceles triangle

Since ΔAOB is an equilateral triangle

∴ $\angle OAB = \angle OBA = 60^0$ ...(i)

Also, $\angle DAB = \angle CBA = 90^0$ ...(ii) (∵ ABCD is a square)

Subtracting (i) from (ii), we get

$\angle DAB - \angle OAB = \angle CBA - \angle OBA = 90^0 - 60^0$

i.e., $\angle DAO = \angle CBO = 30^0$

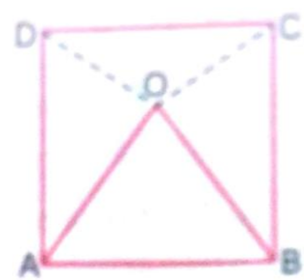


Fig. 7.37

Now, in ΔAOD and ΔBOC

AO = BO (given)

$\angle DAO = \angle CBO$ (proved above)

AD = BC (ABCD is a square)

∴ ΔAOD ≅ ΔBOC (By SAS congruence)

⇒ DO = OC (CPCT)

Since, in ΔCOD, CO = OD

∴ ΔCOD is an isosceles triangle.

Que 4. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.

Sol. Let ΔABC be a triangle in which AC is longest side.

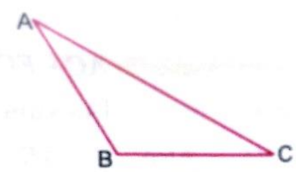


Fig. 7.38

⇒ $\angle B$ is largest angle

⇒ $\angle B > \angle A$ ...(i)

And $\angle B > \angle C$ ...(ii)

Adding (i) and (ii), we get

⇒ $\angle B + \angle B > \angle A + \angle C$

⇒ $2\angle B > \angle A + \angle C$

⇒ $2\angle B + \angle B > \angle A + \angle B + \angle C$

⇒ $3\angle B > 180^0$ ⇒ $\angle B > 60^0$

⇒ $\angle B > \frac{2}{3}$ x right angle. [**Note:** $60^0 = \frac{2}{3} \times 90^0$]

Value Based Questions

Que 1. Teacher held two sticks AB and CD of equal length in her hands and marked their mid points M and N respectively. She then asked the students whether AM is equal to ND or not. Aprajita answered yes. Is Aprajita correct? State the axiom of Euclid that supports her answer. Which values of Aprajita are depicted here?

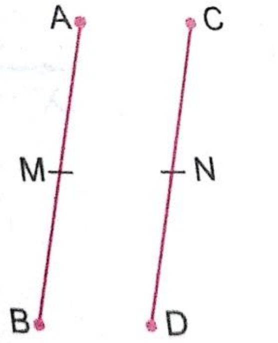


Fig. 5

Sol. Yes, Things which are halves of the same things are equal to one another. Curiosity, knowledge, truthfulness.

Que 2. For her records, a teacher asked the students about their heights. Manav said his height is same as that of Arnav. Raghav also answered the same, way that his height is same as that of Arnav. She then asked the students to relate the height of Manav and Raghav. Arnav answered they both have same height. Is Arnav correct? If yes, state Euclid's axiom which supports his answer.
Which values of Arnav are depicted here?

Sol. Yes, Things which are equal to the same thing are equal to one another. Knowledge, curiosity, truthfulness.

Que 3. The number of members of society A who participated in 'Say No to Crackers' campaign is double the number of members from society B. Also, the number of members from society C is double the number of members from society B. Can you relate the number of participants from society A and C? Justify your answer using Euclid's axiom. Which values are depicted here?

Sol. The number of participants from society A and C is equal. Things which are double of the same thing are equal to one another. Social service, helpfulness, cooperation, environmental concern.

Que 4. In a society, the number of persons using CNG instead of petrol for their vehicles has increased by 15 and now the number is 25. Form a linear equation to find the original number of persons using CNG and solve it using Euclid's axiom.
Which values are depicted in the question?

Sol. X + 15 = 25

⇒ $x + 15 - 15 = 25 - 15$ (Using Euclid's third axiom)

⇒ x = 10

Environmental care, responsible citizens, futuristic.

Que 5. Teacher asked the students to find the value of x in the following figure if l|| m. Shalini answered 35°. Is she correct? Which values are depicted here?

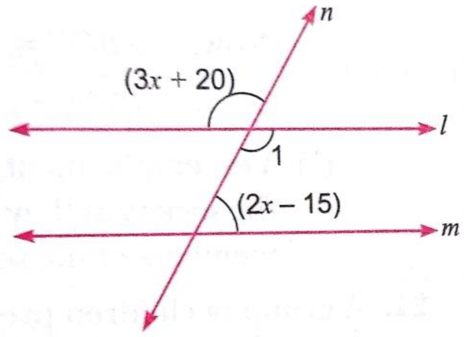


Fig. 6

Sol. ∠1 = 3x + 20 (Vertically opposite angles)

∴ 3x + 20 2x – 15 = 180 ° (Co-interior angles are supplementary)

⇒ $5x + 5 = 180°$ ⇒ $5x = 180° - 5°$

⇒ 5x = 175 ° ⇒ $x = \frac{175}{5} = 35°$

Yes, Knowledge, truthfulness.

Que 6. For spreading the message 'Save Environment Save Future' a rally was organised by some students of a school. They were given triangular cardboard pieces which they divided into two parts by drawing bisectors of base angles (say ∠B and ∠C) intersecting at O in the given figure. Prove that ∠BOC = $90 + \frac{1}{2}\angle A$.

Which values are depicted by these students?

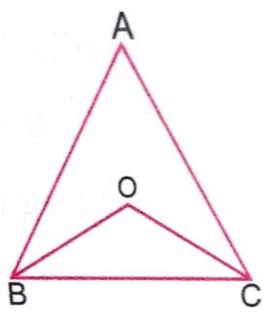


Fig. 7

Sol. In ΔABC, we have

∠A + ∠B + ∠C = 180 ° (∵ sum of the angles of a Δ is 180 °)

⇒ $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2}$

$\Rightarrow \quad \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90°$

$\therefore \quad \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$ ….(i)

Now, in ΔOBC, we have:

$\angle 1 + \angle 2 + \angle BOC = 180°$ [$\because$ sum of the angles of Δ is 180 °]

$\Rightarrow \quad \angle BOC = 180° - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle BOC = 180° - (90° - \frac{1}{2}\angle A)$ [using (i)]

$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{1}{2}\angle A$

$\therefore \quad \angle BOC = 90° + \frac{1}{2}\angle A$

Environmental care, social, futuristic.

Que 7. Three bus stops situated at A, B and C in the figure are operated by handicapped persons. These 3 bus stops are equidistant from each other. OB is the bisector of ∠ABC and OC is the bisector of ∠ACB.
(a) Find ∠BOC.
(b) Do you think employment provided to handicapped persons is important for the development of the society? Express your views with relevant points.

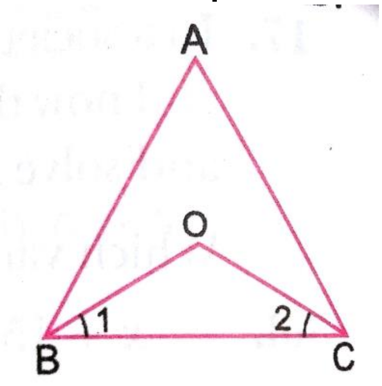


Fig. 9

Sol. (a) Since, A, B, C are equidistant from each other.

$\therefore$ ∠ABC is an equilateral triangle.

$\Rightarrow$ ∠ABC = ∠ABC = 60°

$\Rightarrow \quad \angle OBC = \angle OCB = \frac{1}{2} \times 60° = 30°$ ($\because$ OB and OC are angle bisectors)

Now, ∠BOC = 180° - ∠OBC - ∠OCB (Using angle sum property of triangle)

$\Rightarrow$ ∠BOC = 180°- 30°-30° = 120°

(b) Yes, employment provided to the handicapped persons is important for the development of the society as they would become independent, self-reliant, confident, social, helpful and useful members of the society.

Que 8. A group of children prepared some decorative pieces in the shape of a star for the orphans in an orphanage. Show that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Which values of the children are depicted here?

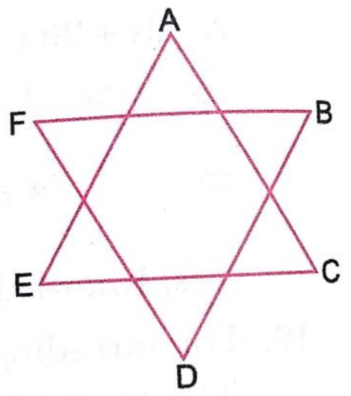


Fig. 10

Sol. In ΔAEC,
$\angle A + \angle E + \angle C = 180°$ … (i) (Angle sum property of a triangle)

Similarly, in ΔBDF,
$\angle B + \angle D \angle F = 180°$ …. (ii)

Adding (i) and (ii), we get
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Social, caring, cooperative, hardworking.

Que 9. For annual day, Sakshi and Nidhi were asked to make one rangoli each on two different places. They started it with triangles (say ABC and ΔPQR) and their medians (AM and PN). If two sides (AB and BC) and a median (AM) of one triangle are respectively equal to two sides (PQ and QR) and a median (PN) of other triangle, prove that the two triangles (ΔABC and ΔPQR) are congruent. Which values of the girls are depicted here?

Sol. In ΔABC and ΔPQR

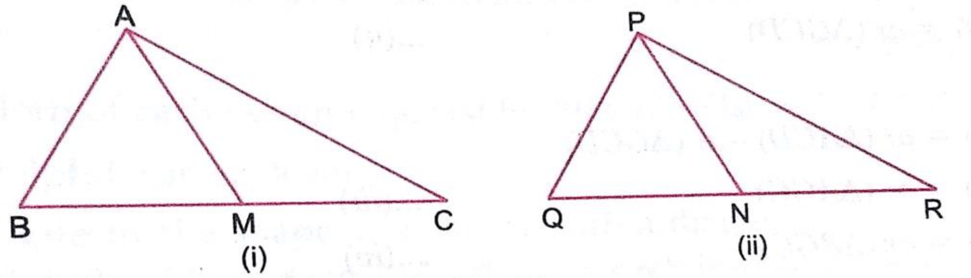


Fig. 11

BC = QR

⇒ $\frac{1}{2}BC = \frac{1}{2}QR$

$\Rightarrow$ BM = QN

In triangle ABM and PQN, we have

AB = PQ	(Given)
BM = QN	(Proved above)
AM = PN	(Given)

$\therefore$ $\Delta ABM \cong \Delta PQN$ (SSS congruence criterion)

$\Rightarrow$ $\angle B = \angle Q$ (CPCT)

Now, in triangles ABC and PQR, we have

AB = PQ	(Given)
$\angle B = \angle Q$	(Proved above)
BC = QR	(Given)

$\therefore$ ΔABC ≅ ΔPQR (SSS congruence criterion)

Participation, beauty, hardworking.

Que 10. Triangular pieces of cardboards were cut out by some people who were organising 'No Pollution' campaign in their area. If the three angles of one cutout are respectively equal to the three angles of the other cutout, can we say the two cutouts are congruent? Justify your answer.
Which values of these people are depicted here?

Sol. The two cutouts may not be congruent. For example all equilateral triangles have equal angles but may have different sides.
Environmental concern, cooperative, caring, social.

Que 11. Anya wants to prepare a poster on education of girlchild for a campaign. She takes a triangular sheet and divides it into three equal parts by drawing its medians which intersect at the point G (see Fig. 12).

Show that ar (ΔAGC) = ar (ΔAGC) = ar (ΔAGB) = (ΔBGC) = $\frac{1}{3} ar\ (\Delta ABC)$

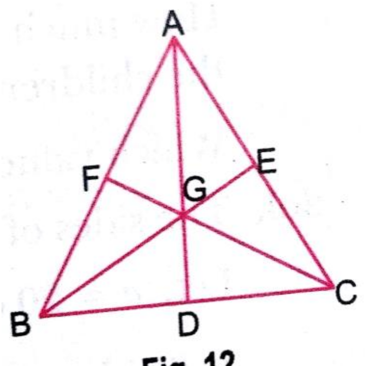


Fig. 12

Do you think education of a girl child is important for the development of a society? Justify your answer.

Sol. Given: A ΔABC in which medians AD, BE and CF intersects at G.

Proof: (ΔAGB) = ar (ΔBGC) = ar (ΔCGA) = $\frac{1}{3}$ ar (ΔABC)

Proof: In ΔABC, AD is the median. As a median of a triangle divides it into two triangles of equal area.

∴ ar (ΔABD) = ar (ΔACD) … (i)

In ΔGBC, GD is the median

∴ aq (ΔGBD) = ar (ΔGCD) …. (ii)

Subtracting (ii) from (i), we get

ar (ΔABD) – ar (ΔGBD) = ar (ACD) – ar (ΔGCD)

ar (ΔAGB) = ar (ΔAGC) … (iii)

Similarly, ar (ΔAGB) = ar (ΔBGC) … (iv)

From (iii) and (iv), we get

ar (ΔAGB) = ar (ΔBGC) = ar (ΔAGC) …. (v)

But, ar (ΔAGB) + ar (ΔBGC) + ar (ΔAGC) = ar (ΔABC) …. (vi)

From (v) and (vi), we get

3 ar (ΔAGB) = ar (ΔABC)

⇒ ar (ΔAGB) = $\frac{1}{3} ar(\Delta ABC)$

Hence, ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

Yes, for the development of a society, education of each girl child is essential. An educated society always progresses.