

## Very Short Answer Type Questions

[1 mark]

**Que 1.** If a triangle and a parallelogram are on the same base and between the same parallels, then find the ratio of the area of the triangle to the area of parallelogram.

**Sol.** The area of a triangle is half the area of a parallelogram, if they are on the same base and between the same parallel lines.

$$\therefore \frac{\text{Area of Triangle}}{\text{Area of parallelogram}} = \frac{1}{2} = 1:2$$

**Que 2.** The area of a rhombus is  $10 \text{ cm}^2$ . If one of its diagonal is  $4 \text{ cm}$ , then find the other diagonal.

**Sol.** Area of a rhombus  $= \frac{1}{2} d_1 d_2$

$$\Rightarrow 10 = \frac{1}{2} \times 4 \times d_2$$

$$\Rightarrow d_2 = 5 \text{ cm}$$

**Que 3.** In Fig. 9.6, ABCD is a parallelogram and P is the point of intersection of its diagonals AC and BD. If the area of  $\Delta APB$  is  $10 \text{ cm}^2$ , then find the area of parallelogram ABCD.

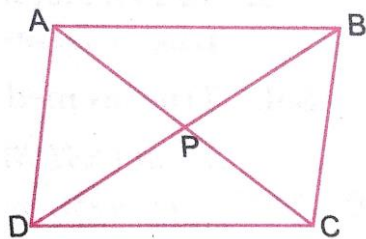


Fig. 9.6

**Sol.**  $\text{ar}(\Delta APB) = \frac{1}{4} \text{ar}(\text{||}^{gm} ABCD)$

$$\Rightarrow \text{ar}(\text{||}^{gm} ABCD) = 4 \times 10 = 40 \text{ cm}^2$$

**Que 4.** What is the area of trapezium?

**Sol.** Area of trapezium  $= \frac{1}{2} \times \text{Sum of the parallel sides} \times \text{height}$ .

**Que 5.** What is the formula of area of triangle?

**Sol.** Area of triangle  $= \frac{1}{2} \times \text{base} \times \text{altitude}$ .

**Que 6. The area of parallelogram ABCD is 25 cm<sup>2</sup>. What is the area of  $\Delta$  ABCD?**

**Sol.** Area of  $\Delta ABC = \frac{25}{2} = 12.5 \text{ cm}^2$  ( $\because$  Area of  $\Delta ABC = \frac{1}{2}$  Area of  $\square ABCD$ )

## Short Answer Type Questions – I

[2 marks]

**Que 1.** The diagonal of a square is 10 cm. Find its area.

**Sol.** Diagonal of a square =  $\sqrt{2}a$

$$\Rightarrow 10 = \frac{10}{\sqrt{2}}$$

$$\text{Area of the square} = a^2 = \left(\frac{10}{\sqrt{2}}\right)^2 = \frac{100}{2} = 50 \text{ cm}^2$$

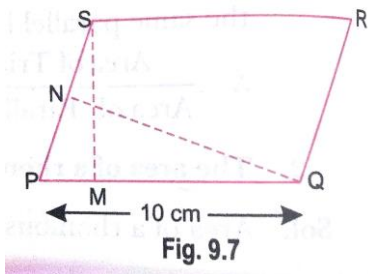
**Que 2.** The area of a stapezium is  $39 \text{ cm}^2$ . The distance between its parallel sides is 6 cm. If one of the parallel sides is 5 cm, then find the other parallel side.

**Sol.** Area of a trapezium =  $\frac{1}{2}$  x sum of parallel sides x height

$$\Rightarrow 39 = \frac{1}{2}(5 + x) \times 6 \text{ (Let other side be } x)$$

$$\Rightarrow 13 = 5 + x \Rightarrow x = 8 \text{ cm}$$

**Que 3.** In parallelogram PQRS, PQ = 10 cm. The altitudes corresponding to the sides PQ and SP are respectively 6 cm and 8 cm. Find SP.



**Sol.** Area of parallelogram = base x height

$$\therefore \text{ar}(\parallel^{\text{gm}} PQRS) = PQ \times SM$$

$$= 10 \times 6 = 60 \text{ cm}^2 \dots (i)$$

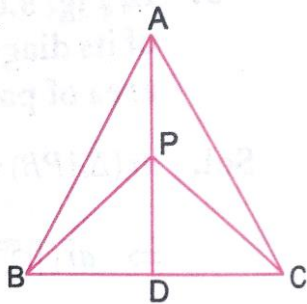
$$\text{Also ar}(\parallel^{\text{gm}} PQRS) = SP \times QN$$

$$= SP \times 8 \dots (ii)$$

From (i) and (ii), we have

$$60 = SP \times 8 \Rightarrow SP = \frac{60}{8} = 7.5 \text{ cm.}$$

**Que 4.** In Fig. 9.8, if P is any point on the median AD of a  $\triangle ABC$ , Then or  $(\triangle ABP) = \text{ar}(\triangle ACP)$ . Write true or false and justify your answer.



**Fig. 9.8**

**Sol.** True. As median of a triangle divides it into two triangles of equal area

$$\therefore \text{ar}(\triangle ADB) = \text{ar}(\triangle ADC) \dots\dots (i)$$

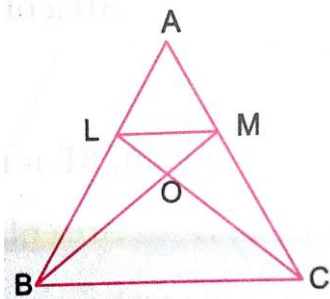
$$\text{and } \text{ar}(\triangle PDB) = \text{ar}(\triangle PDC) \dots\dots (ii)$$

Subtracting (ii) from (i), we have

$$\text{ar}(\triangle ADB) - \text{ar}(\triangle PDB) = \text{ar}(\triangle ADC) - \text{ar}(\triangle PDC)$$

$$\text{ar}(\triangle ABP) = \text{ar}(\triangle ACP)$$

**Que 5.** In  $\triangle ABC$ , if L and M are the points on AB and AC, respectively such that  $LM \parallel BC$ . Prove that  $\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC)$ .



**Fig. 9.9**

**Sol.** Given in  $\triangle ABC$ , L and M are points on AB and AC respectively such that  $LM \parallel BC$ .

**To Prove:**  $\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC)$

**Proof:** We know that, triangle on the same base and between the same parallels are equal in area.

Hence  $\triangle LBC$  and  $\triangle MBC$  lie on the same base BC and between the same parallel BC and LM.

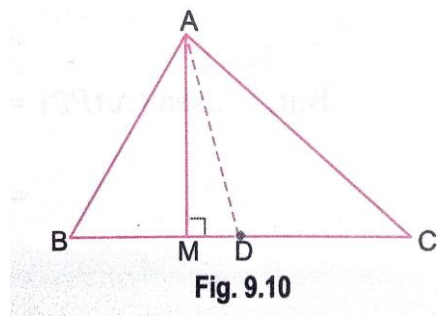
$$\text{So, } \text{ar}(\triangle LBC) = \text{ar}(\triangle MBC)$$

$$\Rightarrow \text{ar}(\triangle LOB) + \text{ar}(\triangle BOC) = \text{ar}(\triangle MOC) + \text{ar}(\triangle BOC)$$

On eliminating  $\text{ar}(\triangle BOC)$  from both sides, we get

$$\text{ar}(\triangle LOB) = \text{ar}(\triangle MOC) \quad \text{Hence proved.}$$

**Que 6. Prove that median of a triangle divides in into two triangles of equal area.**

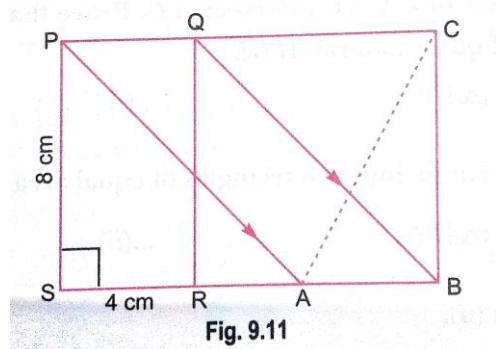


**Sol.** Draw  $AM \perp BC$ , as AD is median  
 $\therefore BD = DC$

$$\frac{\text{ar}(\triangle ABD)}{\text{ar}(\triangle ACD)} = \frac{\frac{1}{2}BD \times AM}{\frac{1}{2}DC \times AM} = \frac{\frac{1}{2}BD \times AM}{\frac{1}{2}BD \times AM} = 1$$

$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD)$

**Que 7. Prove that median of a triangle divides it into two triangles of equal area.**



**Sol.** Area of parallelogram PQRS = Area Of parallelogram PQBA

[Parallelograms on the same base PQ and between the same parallels PC and SB]

$$8 \times 4 = \text{Area of parallelogram PQBA}$$

$$32 \text{ cm}^2 = \text{Area of parallelogram PQBA}$$

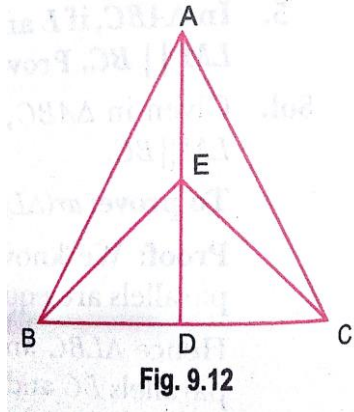
Again,

$$\therefore \text{Area of parallelogram PQBA} = 2(\text{Area of } \triangle ABC)$$

[Area of triangle is half of area of parallelogram, if they are on the same base and between the same parallel.]

$$\therefore 16 \text{ cm}^2 = \text{Area of } \triangle ABC$$

**Que 8.** D and E are mid-Point of BC hence AD respectively. If area of  $\Delta ABC = 10 \text{ cm}^2$ , find area of  $\Delta EBD$ .



**Sol.**  $\because$  D is the midpoint of BC hence AD is the median.

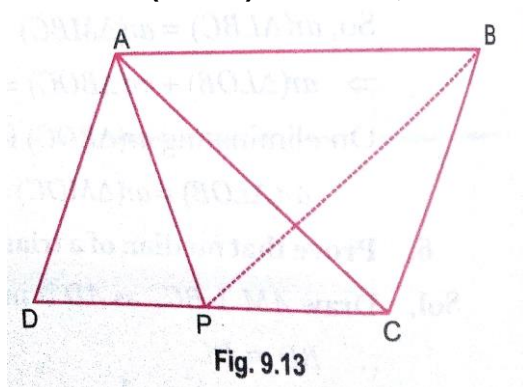
$\therefore$  Area of  $\Delta ABD = \frac{1}{2}$  area of  $\Delta ABC$  [Median divides a triangle into two triangles of equal area]

$$\begin{aligned} \text{Area of } \Delta ABD &= \frac{1}{2} \times 10 \text{ cm}^2 \\ &= 5 \text{ cm}^2 \end{aligned}$$

Again, BE in the median of  $\Delta ABD$ .

$$\begin{aligned} \therefore \text{Area of } \Delta EBD &= \frac{1}{2} \text{ area of } \Delta ABD \\ &= \frac{1}{2} \times 5 \\ &= 2.5 \text{ cm}^2 \end{aligned}$$

**Que 9.** ABCD is a parallelogram. P is any point on CD. If area ( $\Delta DPA$ ) =  $15 \text{ cm}^2$  and area ( $\Delta APC$ ) =  $20 \text{ cm}^2$ , find the area ( $\Delta APB$ ).



**Sol.** area ( $\Delta ADC$ ) = area ( $\Delta ADP$ ) + area ( $\Delta APC$ )  
 $= 15 + 20$

$$= 35 \text{ cm}^2$$

But, area ( $\Delta APB$ ) =  $\frac{1}{2}$  (area of parallelogram ABCD)

$$= \frac{1}{2} [2 (\text{area of } \Delta ADC)]$$

[As diagonal divides a parallelogram into equal areas]

$$= \frac{1}{2} \times 2 (35) = 35 \text{ cm}^2$$

**Que 10. The medians BE and CF of a  $\Delta ABC$  intersect at G. prove that area ( $\Delta BGC$ ) = area of quadrilateral AFGE**

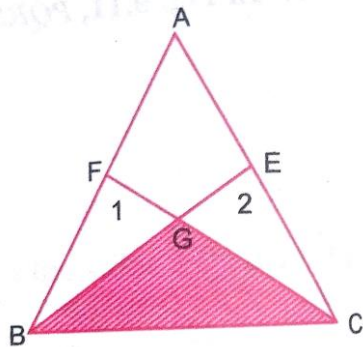


Fig. 9.14

**Sol.** area ( $\Delta FBC$ ) =  $\frac{1}{2}$  area ( $\Delta ABC$ ) .....(i)

[Median divides the triangles into triangles of equal area]

area ( $\Delta EBC$ ) =  $\frac{1}{2}$  area ( $\Delta ABC$ ) .....(ii)

From equation (i) and (ii),

$$\text{area } (\Delta FBC) = \text{area } (\Delta EBC)$$

subtract area ( $\Delta BGC$ ) from both sides

$$\text{area } (\Delta FBC) - \text{area } (\Delta BGC) = \text{area } (\Delta EBC) - \text{area } (\Delta BGC)$$

$$\therefore \text{area } (\Delta FGB) = \text{area } (\Delta EGC) \quad \text{.....(iii)}$$

$$\text{area } (\Delta ABE) = \text{area } (\Delta BEC) \quad [\because BE \text{ is median}]$$

$$\text{area } (\Delta BFG) + \text{area (Quadrilateral AFGE)} = \text{area } (\Delta BGC) + \text{area } (\Delta GEC)$$

$$\Rightarrow \text{area (Quadrilateral AFGE)} = \text{area } (\Delta BGC) \quad [\text{From equation (iii)}]$$

**Que 11.** ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then prove that  $\text{area}(\Delta BDE) = \frac{1}{4} \text{area}(\Delta ABC)$

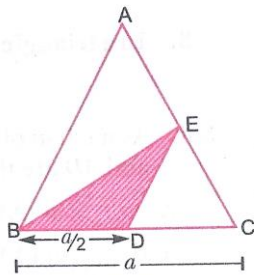


Fig. 9.15

**Sol.** Let the side of triangle,  $BC = a \Rightarrow BD = \frac{a}{2}$

$$\begin{aligned} \text{Area}(\Delta BDE) &= \frac{\sqrt{3}}{4} \left(\frac{a}{2}\right)^2 \\ &= \frac{\sqrt{3} a^2}{4 \cdot 4} = \frac{1}{4} \left(\frac{\sqrt{3}}{4} a^2\right) \end{aligned}$$

$$\text{area}(\Delta BDE) = \frac{1}{4} \text{area}(\Delta ABC)$$

**Que 12.** PQRS is parallelogram whose area is  $180 \text{ cm}^2$  and A is any point on the diagonal Qs. The area of  $\Delta ASR = 90 \text{ cm}^2$ . Find this statement is true or false.

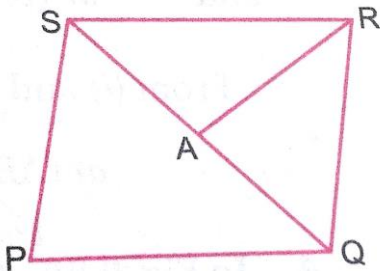


Fig. 9.16

**Sol.** As diagonal of the parallelogram divides it into triangles equal area.

$$\therefore \text{area}(\Delta SRQ) = \frac{1}{2} \text{area}(\text{PQRS})$$

$$\begin{aligned} \text{area}(\Delta SRQ) &= \frac{1}{2} \times 180 \\ &= 90 \text{ cm}^2 \end{aligned}$$

But  $\text{area}(\Delta ASR) = 90 \text{ cm}^2$  (Given)

This is not possible unless  $\text{area}(\Delta SRQ) = 90 \text{ cm}^2$

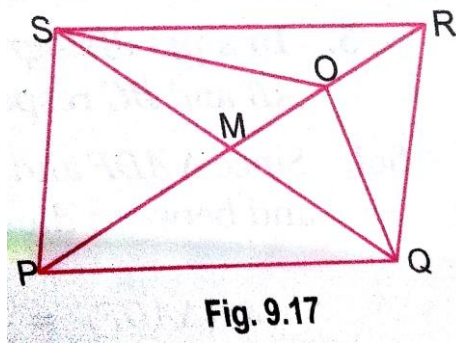
So, the given statement is false.



## Short Answer Type Questions – II

[3 marks]

**Que 1.** O is any point on the diagonals PR of parallelogram PQRS. Prove that  $\text{ar}(\Delta PSO) = \text{ar}(\Delta PQO)$ .



**Sol.** Join SQ. Since diagonals of a parallelogram bisect each other. Therefore, M is the mid-point of PR as well as SQ.

In  $\Delta SOQ$ , OM is a median

$$\therefore \text{ar}(\Delta SOM) = \text{ar}(\Delta QOM) \quad \dots\dots(i)$$

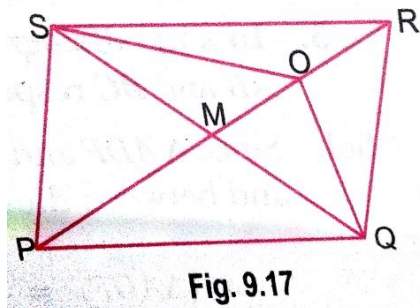
In  $\Delta SPQ$ , PM is the median

$$\therefore \text{ar}(\Delta PSM) = \text{ar}(\Delta PQM) \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$\begin{aligned} \text{ar}(\Delta SOM) + \text{ar}(\Delta PSO) &= \text{ar}(\Delta PQO) \\ \text{ar}(\Delta PSO) &= \text{ar}(\Delta PQO) \end{aligned}$$

**Que 2.** In Fig. 9.18, x and Y are points on the side LN of the triangle LMN such that  $LX = XY = YN$ . Through X, a line is drawn parallel to LM to meet MN at Z. Prove that  $\text{ar}(\Delta LZY) = \text{ar}(\square MZYX)$ .



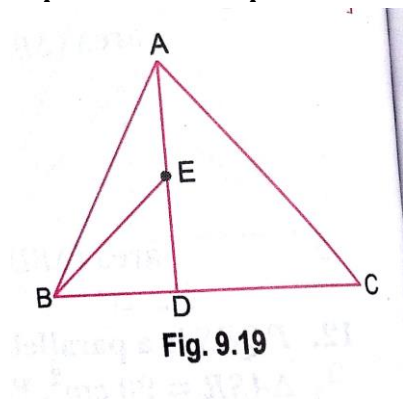
**Sol.** Since,  $\Delta LXZ$  and  $\Delta MXY$  lie on the same base XZ and between the same parallels XZ and LM.

$$\therefore \text{ar}(\Delta LXZ) = \text{ar}(\Delta MXY)$$

Adding  $\text{ar}(\Delta XYZ)$  to both sides, we get

$$\begin{aligned} \text{ar}(\Delta LXZ) + \text{ar}(\Delta XYZ) &= \text{ar}(\Delta MXY) + \text{ar}(\Delta XYZ) \\ \Rightarrow \text{ar}(\Delta LZY) &= \text{ar}(\square MZYX) \end{aligned}$$

**Que 3.** In a triangle  $ABC$ ,  $E$  is the mid-point of median  $AD$ . Show that  $\text{ar}(\Delta BED) = \frac{1}{4} \text{ar}(\Delta ABC)$ .



**Sol.** As median of a triangle divides it into two triangles of equal area and  $BE$  and  $AD$  are the is a medians of the  $\Delta ABD$  and  $\Delta ABC$  respectively

$$\therefore \text{ar}(\Delta ABD) = \text{ar}(\Delta ADC)$$

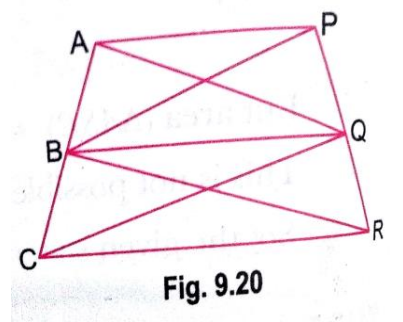
$$\Rightarrow \text{ar}(\Delta BED) = \frac{1}{2} \text{ar}(\Delta ABD) \quad \dots\dots (i)$$

$$\text{And } \text{ar}(\Delta ABD) = \frac{1}{2} \text{ar}(\Delta ABC) \quad \dots\dots (ii)$$

from (i) and (ii), we have

$$\text{ar}(\Delta BED) = \frac{1}{2} \left( \frac{1}{2} \text{ar}(\Delta ABC) \right) = \frac{1}{4} \text{ar}(\Delta ABC)$$

**Que 4.** In Fig. 9.20,  $AP \parallel BQ \parallel CR$ . Prove that  $\text{ar}(\Delta AQC) = \text{ar}(\Delta PBR)$ .



**Sol.** Since  $\Delta ABQ$  and  $\Delta PBQ$  are on the same base  $BQ$  and between the same parallels  $AP$  and  $BQ$ .

$$\therefore \text{ar}(\Delta ABQ) = \text{ar}(\Delta PBQ) \quad \dots\dots(i)$$

Similarly,  $\Delta BCQ$  and  $\Delta BRQ$  are on the same base  $BQ$  and between the same parallels  $BQ$  and  $CR$ .

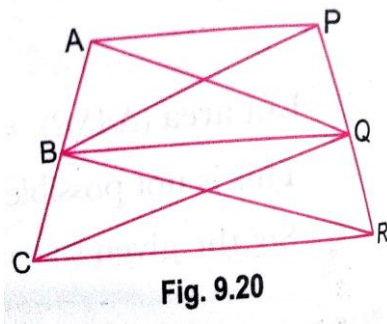
$$\therefore \text{ar}(\Delta BCQ) = \text{ar}(\Delta BRQ) \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$\text{ar}(\Delta ABQ) + \text{ar}(\Delta BCQ) = \text{ar}(\Delta PBQ) + \text{ar}(\Delta BRQ)$$

$$\Rightarrow \text{ar}(\Delta AQC) = \text{ar}(\Delta PBR)$$

**Que 5.** In a parallelogram, ABCD, E, F are any two points on the sides AB and BC respectively. Show that  $\text{ar}(\triangle ADF) = \text{ar}(\triangle DCE)$



**Fig. 9.20**

**Sol.** Since  $\triangle ADF$  and parallelogram ABCD are on the same base AD and between the same parallels AD and BC.

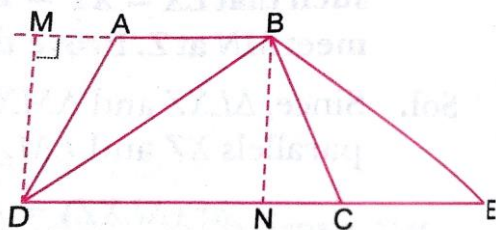
$$\therefore \text{ar}(\triangle ADF) = \frac{1}{2} \text{ar}(\text{||}^{gm} ABCD) \quad \dots (i)$$

Also,  $\triangle DCE$  and  $\text{||}^{gm} ABCD$  are on the same base DC and between the same parallels DC and AB.

$$\therefore \text{ar}(\triangle DCE) = \frac{1}{2} \text{ar}(\text{||}^{gm} ABCD) \quad \dots (ii)$$

From (i) and (ii), we get  
 $\text{ar}(\triangle ADF) = \text{ar}(\triangle DCE)$

**Que 6.** ABCD is a trapezium in which  $AB \parallel DC$ . DC is produced to E such that  $CE = AB$ , Prove that  $\text{ar}(\triangle ABD) = \text{ar}(\triangle BCE)$ .



**Fig. 9.22**

**Sol.** Produce BA to M Such that  $DM \perp BM$  and draw  $BN \perp DC$ .

$$\text{Now, ar}(\triangle ABD) = \frac{1}{2}(AB \times DM) \quad \dots (i)$$

$$\text{Ar}(\triangle BCE) = \frac{1}{2}(CE \times BN) \quad \dots (ii)$$

Since, triangle ABD and BCE are between the same parallels, Therefore,

$$DM = BN \quad \dots (iii)$$

Also,  $AB = CE$  (Given)  $\dots (iv)$

From (iii) and (iv), we get

$$\frac{1}{2}(AB \times DM) = \frac{1}{2}(CE \times BN)$$

$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle BCE)$  (Using (i) and (ii))

**Que 7.** In Fig. 9.23, ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F. If area of  $\triangle BDF = 3 \text{ cm}^2$ , find the area of parallelogram ABCD.

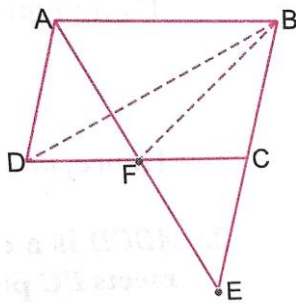


Fig. 9.23

**Sol.** In  $\triangle ADF$  and  $\triangle ECF$ , we have

$$\angle ADF = \angle ECF$$

(Alternate interior angles)

$$AD = CE$$

( $\because AD=BC$  and  $=CE$ )

$$\angle DFA = \angle CFE$$

(Vertically opposite angles)

$$\therefore \triangle ADF \cong \triangle ECF$$

(AAS congruence criterion)

$$\Rightarrow \text{ar}(\triangle ADF) = \text{ar}(\triangle ECF)$$

$$\text{Also, } DF = CF$$

(CPCT)

$\Rightarrow$  BF is the median in  $\triangle BCD$

$$\Rightarrow \text{ar}(\triangle BCD) = 2 \text{ ar}(\triangle BDF)$$

$$\Rightarrow \text{ar}(\triangle BCD) = 2 \times 3 \text{ cm}^2 = 6 \text{ cm}^2$$

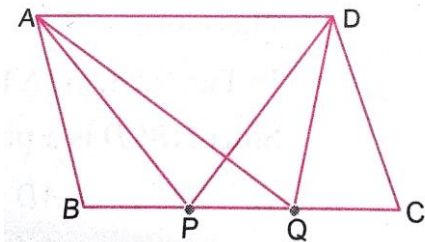
$$\text{ar}(\text{||}^{\text{gm}} \text{ABCD}) = 2 \text{ ar}(\triangle BCD)$$

$$2 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$$

## Long Answer Type Questions

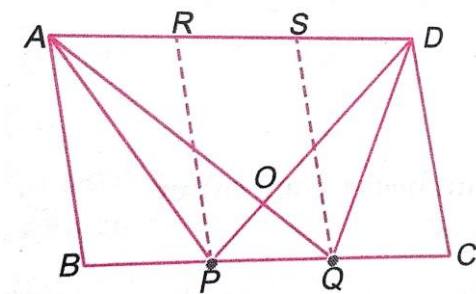
[4 Marks]

**Que 1.** In Fig. 9.24, ABCD is a parallelogram. Point P and Q on BC trisects BC. Prove that  $ar(\Delta APQ) = ar(\Delta DPQ) = \frac{1}{6} ar(\parallel^{gm} ABCD)$ .



**Fig. 9.24**

**Sol.** Through P and Q, draw PR and QS parallel to AB [Fig. 9.25]. Now, PQSR is a parallelogram and its base  $PQ = \frac{1}{3} BC$ .



**Fig. 9.25**

Since  $\Delta APQ$  and  $\Delta DPQ$  are on the same base PQ, and between the same parallel AD and BC.

$$\therefore ar(\Delta APQ) = ar(\Delta DPQ) \quad \dots\dots(i)$$

Since  $\Delta APQ$  and  $\Delta PQSR$  are on the same base PQ, and between same parallel PQ and AD.

$$\therefore ar(\Delta APQ) = \frac{1}{2} ar(\parallel^{gm} PQRS) \quad \dots\dots(ii)$$

Now,

$$\frac{ar(\parallel^{gm} ABCD)}{ar(\parallel^{gm} PQRS)} = \frac{BC \times height}{PQ \times height}$$

$$= \frac{3PQ}{PQ} \quad (\because \text{height of the two } \parallel^{gm} \text{ is same})$$

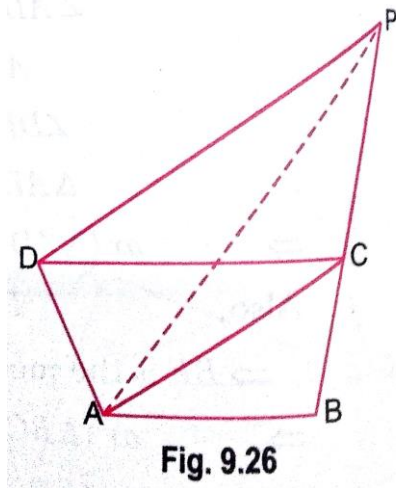
$$\Rightarrow ar(\parallel^{gm} PQRS) = \frac{1}{3} ar(\parallel^{gm} ABCD) \quad \dots\dots(iii)$$

Using equation (ii) and (iii), we have

$$ar(\Delta APQ) = \frac{1}{2} ar(||^{gm} PQRS) = \frac{1}{2} \times \frac{1}{3} ar(||^{gm} ABCD)$$

Hence,  $ar(\Delta APQ) = ar(\Delta DPQ) = \frac{1}{6} ar(||^{gm} ABCD)$ . [Using (i)]

**Que 2.** ABCD is a quadrilateral [Fig. 9.26]. A line through D, parallel to AC meets BC produced in P. Prove  $ar(\Delta ABP) = ar(\text{quad. } ABCD)$ .



**Fig. 9.26**

**Sol.** Given: A quadrilateral ABCD in which  $DP \parallel AC$

To Prove:  $ar(\Delta ABP) = ar(\text{quad. } ABCD)$

Proof:  $\Delta ACP$  and  $\Delta ACD$  are on same base AC and between same parallels AC and DP.

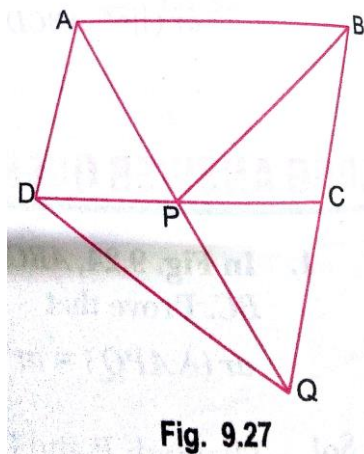
$$\Rightarrow ar(\Delta ACP) = ar(\Delta ACD)$$

Adding,  $ar(\Delta ABC)$  on both sides,

$$\Rightarrow ar(\Delta ABC) + ar(\Delta ACP) = ar(\Delta ABC) + ar(\Delta ACD)$$

$$ar(\Delta ABP) = ar(\text{quad. } ABCD)$$

**Que 3.** In Fig. 9.27, ABCD is a parallelogram and BC is produced to point Q such that  $BC = CQ$ . If AQ intersects DC at P. Show that  $ar(\Delta BPC) = ar(\Delta DPQ)$ .



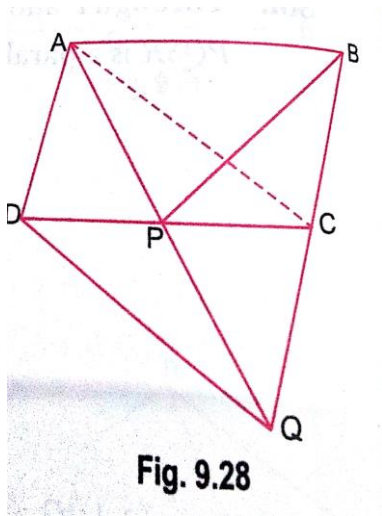
**Fig. 9.27**

**Sol.** Join AC. As triangle APC and BPC are on the same base PC and between the same parallels PC and AB.

Therefore,

In Fig. 9.28,  $\text{ar}(\Delta APC) = \text{ar}(\Delta BPC)$  ....(i)

Since ABCD is a Parallelogram,



**Fig. 9.28**

$\therefore$   $AD = BC$  (Opposite sides of parallelogram)

Also,  $CQ = BC$  (Given)

$\Rightarrow AD = CQ$

Now,  $AD \parallel CQ$  and  $AD = CQ$

$\therefore$  ACQD is a parallelogram.

As diagonals of a parallelogram bisect each other.

$\therefore AP = PQ$  and  $CP = DP$

In  $\Delta APC$  and  $\Delta DPQ$ , we have

$$AP = PQ$$

$$\angle APC = \angle DPQ$$

(Vertically opposite angles)

and  $PC = PD$

$\therefore \Delta APC \cong \Delta DPQ$

(SAS congruence criterion)

$\Rightarrow \text{ar}(\Delta APC) = \text{ar}(\Delta DPQ)$

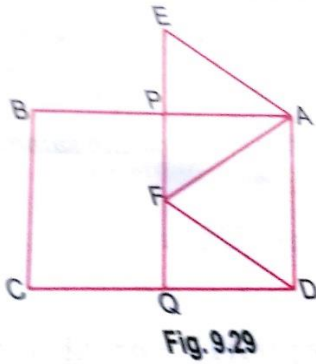
.....(ii)

From (i) and (ii), we get

$$\text{ar}(\Delta BPC) = \text{ar}(\Delta DPQ)$$

## HOTS (Higher Order Thinking Skills)

**Que 1.** In Fig. 9.29, ABCD and AEFD are two parallelograms. Prove that  $\text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$ .



**Sol.** In triangles PEA and QFD, we have

$$\angle APE = \angle DQF \quad (\text{Corresponding angles})$$

$$AE = DF \quad (\text{Opposite sides of } \parallel^{\text{gm}} \text{ AEFD})$$

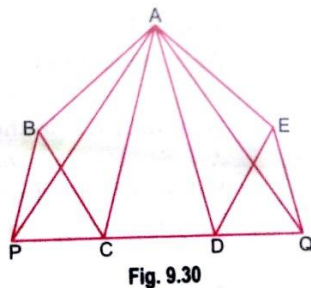
$$\angle AEP = \angle DFQ \quad (\text{Corresponding angles})$$

$$\therefore \triangle PEA \cong \triangle QFD \quad (\text{AAS congruence criterion})$$

As congruent triangles have equal area.

$$\therefore \text{ar}(\triangle PEA) = \text{ar}(\triangle QFD)$$

**Que 2.** In Fig. 9.30, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that  $\text{ar}(ABCDE) = \text{ar}(APQ)$ .



**Sol.** Since,  $\triangle ABC$  and  $\triangle APC$  are on the same base AC and between the same parallels BP and AC



$$\therefore \text{ar}(\Delta ABC) = \text{ar}(\Delta APC) \quad \dots(i)$$

(Triangles on the same base and between the same parallels are equal in area)

Similarly,  $EQ \parallel AD$

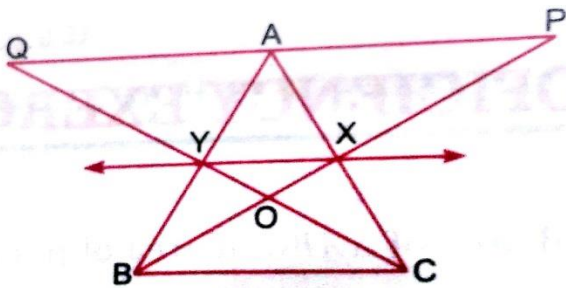
$$\therefore \text{ar}(\Delta AED) = \text{ar}(\Delta AQD) \quad \dots(ii)$$

Adding (i) and (ii), and then adding  $\text{ar}(\Delta ACD)$  to both the sides, we get

$$\text{Ar}(\Delta ABC) + \text{ar}(\Delta AED) + \text{ar}(\Delta ACD) = \text{ar}(\Delta APC) + \text{ar}(\Delta AQD) + \text{ar}(\Delta ACD)$$

$$\Rightarrow \text{ar}(\text{ABCDE}) = \text{ar}(\text{APQ}).$$

**Que 3.** In Fig. 9.31, X and Y are the mid-point of AC and AB respectively,  $QP \parallel BC$  and  $CYQ$  and  $BXP$  are straight lines. Prove that  $\text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$ .



**Fig. 9.31**

**Sol.** As X and Y are the mid-point of AC and AB respectively.

$$\therefore XY \parallel BC$$

Since  $\Delta BYC$  and  $\Delta BXY$  are on the same base BC and between the same parallels XY and BC.

$$\therefore \text{ar}(\Delta BYC) = \text{ar}(\Delta BXC)$$

$$\Rightarrow \text{ar}(\Delta BYC) - \text{ar}(\Delta BOC) = \text{ar}(\Delta BXC) - \text{ar}(\Delta BOC)$$

$$\Rightarrow \text{ar}(\Delta BOY) = \text{ar}(\Delta COX)$$

$$\Rightarrow \text{ar}(\Delta BOY) + \text{ar}(\Delta XOY) = \text{ar}(\Delta COX) + \text{ar}(\Delta XOY)$$

$$\Rightarrow \text{ar}(\Delta BXY) = \text{ar}(\Delta CXY) \quad \dots(i)$$

Since quadrilaterals XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

$$\therefore \text{ar}(\text{XYAP}) = \text{ar}(\text{XYQA}) \quad \dots(ii)$$

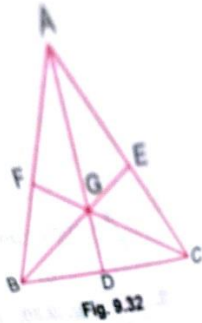
Adding (i) and (ii), we get

$$\text{Ar}(\Delta BXY) + \text{ar}(\text{XYAP}) = \text{ar}(\Delta CXY) + \text{ar}(\text{XYQA})$$

$$\Rightarrow \text{ar} (\Delta ABP) = \text{ar} (\Delta ACQ)$$

**Que 4. If the medians of a  $\Delta ABC$  intersect at G. Show that**

$$\text{Ar} (\Delta AGC) = \text{ar} (\Delta AGB) = \text{ar} (\Delta BGC) = \frac{1}{3} \text{ar} (\Delta ABC)$$



**Sol. Give:** A  $\Delta ABC$  in which medians AD, BE and CF intersect at G.

**To prove:**

$$\text{Ar} (\Delta AGB) = \text{ar} (\Delta BGC) = \text{ar} (\Delta CGA) = \frac{1}{3} \text{ar} (\Delta ABC)$$

**Proof:** In  $\Delta ABC$ , AD is the median.

As a median of a triangle divides it into two triangles of equal area.

$$\therefore \text{ar} (\Delta ABD) = \text{ar} (\Delta ACD) \quad \dots(i)$$

In  $\Delta GBC$ , GD is the median

$$\therefore \text{Ar} (\Delta GBD) = \text{ar} (\Delta GCD) \quad \dots(ii)$$

Subtracting (ii) from (i), we get

$$\begin{aligned} \text{Ar} (\Delta ABD) - \text{ar} (\Delta GBD) &= \text{ar} (\Delta ACD) - \text{ar} (\Delta GCD) \\ \text{ar} (\Delta AGB) &= \text{ar} (\Delta AGC) \quad \dots(iii) \end{aligned}$$

$$\text{Similarly,} \quad \text{ar} (\Delta AGB) = \text{ar} (\Delta BGC) \quad \dots(iv)$$

From (iii) and (iv), we get

$$\text{ar} (\Delta AGB) = \text{ar} (\Delta BGC) = \text{ar} (\Delta AGC) \quad \dots(v)$$

$$\text{But,} \quad \text{ar} (\Delta AGB) + \text{ar} (\Delta BGC) + \text{ar} (\Delta AGC) = \text{ar} (\Delta ABC) \quad \dots(vi)$$

From (v) and (vi), we get

$$3\text{ar} (\Delta AGB) = \text{ar} (\Delta ABC)$$

$$\Rightarrow \text{ar} (\Delta AGB) = \frac{1}{3} \text{ar} (\Delta ABC)$$

$$\text{Hence,} \quad \text{ar} (\Delta AGB) = \text{ar} (\Delta AGC) = \text{ar} (\Delta BGC) = \frac{1}{3} \text{ar} (\Delta ABC)$$

## Value Based Questions

**Que 1.** Teacher held two sticks AB and CD of equal length in her hands and marked their mid points M and N respectively. She then asked the students whether AM is equal to ND or not. Aprajita answered yes. Is Aprajita correct? State the axiom of Euclid that supports her answer. Which values of Aprajita are depicted here?

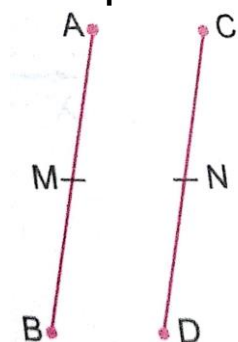


Fig. 5

**Sol.** Yes, Things which are halves of the same things are equal to one another.  
Curiosity, knowledge, truthfulness.

**Que 2.** For her records, a teacher asked the students about their heights. Manav said his height is same as that of Arnav. Raghav also answered the same, way that his height is same as that of Arnav. She then asked the students to relate the height of Manav and Raghav. Arnav answered they both have same height. Is Arnav correct? If yes, state Euclid's axiom which supports his answer.

Which values of Arnav are depicted here?

**Sol.** Yes, Things which are equal to the same thing are equal to one another.  
Knowledge, curiosity, truthfulness.

**Que 3.** The number of members of society A who participated in 'Say No to Crackers' campaign is double the number of members from society B. Also, the number of members from society C is double the number of members from society B. Can you relate the number of participants from society A and C? Justify your answer using Euclid's axiom. Which values are depicted here?

**Sol.** The number of participants from society A and C is equal. Things which are double of the same thing are equal to one another. Social service, helpfulness, cooperation, environmental concern.

**Que 4.** In a society, the number of persons using CNG instead of petrol for their vehicles has increased by 15 and now the number is 25. Form a linear equation to find the original number of persons using CNG and solve it using Euclid's axiom.

Which values are depicted in the question?

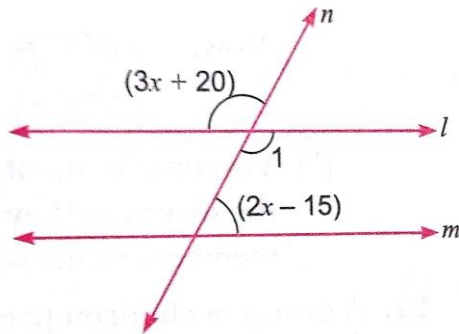
**Sol.**  $X + 15 = 25$

$\Rightarrow x + 15 - 15 = 25 - 15$  (Using Euclid's third axiom)

$\Rightarrow x = 10$

Environmental care, responsible citizens, futuristic.

**Que 5.** Teacher asked the students to find the value of  $x$  in the following figure if  $l \parallel m$ . Shalini answered  $35^\circ$ . Is she correct? Which values are depicted here?



**Fig. 6**

**Sol.**  $\angle 1 = 3x + 20$  (Vertically opposite angles)

$\therefore 3x + 20 + 2x - 15 = 180^\circ$  (Co-interior angles are supplementary)

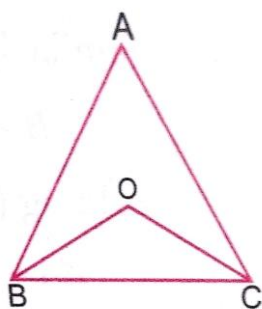
$\Rightarrow 5x + 5 = 180^\circ \quad \Rightarrow 5x = 180^\circ - 5^\circ$

$\Rightarrow 5x = 175^\circ \quad \Rightarrow x = \frac{175}{5} = 35^\circ$

Yes, Knowledge, truthfulness.

**Que 6.** For spreading the message 'Save Environment Save Future' a rally was organised by some students of a school. They were given triangular cardboard pieces which they divided into two parts by drawing bisectors of base angles (say  $\angle B$  and  $\angle C$ ) intersecting at  $O$  in the given figure. Prove that  $\angle BOC = 90^\circ + \frac{1}{2}\angle A$

Which values are depicted by these students?



**Fig. 7**

**Sol.** In  $\triangle ABC$ , we have

$\angle A + \angle B + \angle C = 180^\circ$

( $\because$  sum of the angles of a  $\triangle$  is  $180^\circ$ )

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180^\circ}{2}$

$$\Rightarrow \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90^\circ$$

$$\therefore \angle 1 + \angle 2 = 90^\circ - \frac{1}{2}\angle A \quad \dots(i)$$

Now, in  $\triangle OBC$ , we have:

$$\angle 1 + \angle 2 + \angle BOC = 180^\circ \quad [\because \text{sum of the angles of } \triangle \text{ is } 180^\circ]$$

$$\Rightarrow \angle BOC = 180^\circ - (\angle 1 + \angle 2)$$

$$\Rightarrow \angle BOC = 180^\circ - (90^\circ - \frac{1}{2}\angle A) \quad [\text{using (i)}]$$

$$\Rightarrow \angle BOC = 180^\circ - 90^\circ + \frac{1}{2}\angle A$$

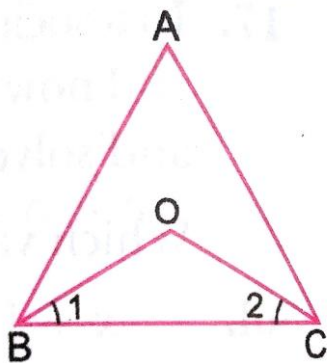
$$\therefore \angle BOC = 90^\circ + \frac{1}{2}\angle A$$

Environmental care, social, futuristic.

**Que 7. Three bus stops situated at A, B and C in the figure are operated by handicapped persons. These 3 bus stops are equidistant from each other. OB is the bisector of  $\angle ABC$  and OC is the bisector of  $\angle ACB$ .**

**(a) Find  $\angle BOC$ .**

**(b) Do you think employment provided to handicapped persons is important for the development of the society? Express your views with relevant points.**



**Fig. 9**

**Sol.** (a) Since, A, B, C are equidistant from each other.

$\therefore \triangle ABC$  is an equilateral triangle.

$$\Rightarrow \angle A = \angle B = \angle C = 60^\circ$$

$$\Rightarrow \angle OBC = \angle OCB = \frac{1}{2} \times 60^\circ = 30^\circ \quad (\because OB \text{ and } OC \text{ are angle bisectors})$$

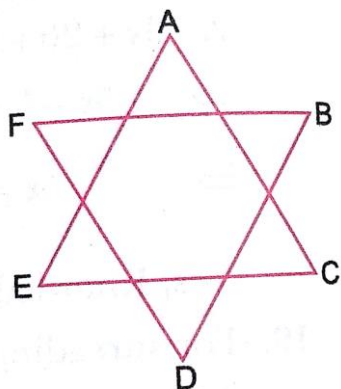
Now,  $\angle BOC = 180^\circ - \angle OBC - \angle OCB$  (Using angle sum property of triangle)

$$\Rightarrow \angle BOC = 180^\circ - 30^\circ - 30^\circ = 120^\circ$$

(b) Yes, employment provided to the handicapped persons is important for the development of the society as they would become independent, self-reliant, confident, social, helpful and useful members of the society.

**Que 8.** A group of children prepared some decorative pieces in the shape of a star for the orphans in an orphanage. Show that  $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^\circ$

Which values of the children are depicted here?



**Fig. 10**

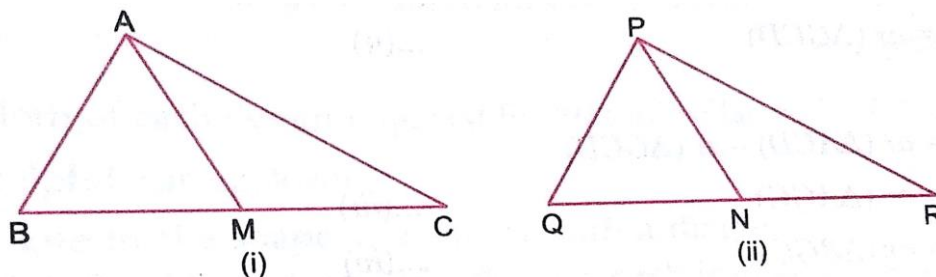
**Sol.** In  $\triangle AEC$ ,  
 $\angle A + \angle E + \angle C = 180^\circ$  ... (i) (Angle sum property of a triangle)

Similarly, in  $\triangle BDF$ ,  
 $\angle B + \angle D + \angle F = 180^\circ$  .... (ii)

Adding (i) and (ii), we get  
 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^\circ$   
 Social, caring, cooperative, hardworking.

**Que 9.** For annual day, Sakshi and Nidhi were asked to make one rangoli each on two different places. They started it with triangles (say  $\triangle ABC$  and  $\triangle PQR$ ) and their medians ( $AM$  and  $PN$ ). If two sides ( $AB$  and  $BC$ ) and a median ( $AM$ ) of one triangle are respectively equal to two sides ( $PQ$  and  $QR$ ) and a median ( $PN$ ) of other triangle, prove that the two triangles ( $\triangle ABC$  and  $\triangle PQR$ ) are congruent. Which values of the girls are depicted here?

**Sol.** In  $\triangle ABC$  and  $\triangle PQR$



**Fig. 11**

$$BC = QR$$

$$\Rightarrow \frac{1}{2}BC = \frac{1}{2}QR$$

$$\Rightarrow BM = QN$$

In triangle ABM and PQN, we have

$$AB = PQ \quad (\text{Given})$$

$$BM = QN \quad (\text{Proved above})$$

$$AM = PN \quad (\text{Given})$$

$$\therefore \triangle ABM \cong \triangle PQN \quad (\text{SSS congruence criterion})$$

$$\Rightarrow \angle B = \angle Q \quad (\text{CPCT})$$

Now, in triangles ABC and PQR, we have

$$AB = PQ \quad (\text{Given})$$

$$\angle B = \angle Q \quad (\text{Proved above})$$

$$BC = QR \quad (\text{Given})$$

$$\therefore \triangle ABC \cong \triangle PQR \quad (\text{SSS congruence criterion})$$

Participation, beauty, hardworking.

**Que 10. Triangular pieces of cardboards were cut out by some people who were organising 'No Pollution' campaign in their area. If the three angles of one cutout are respectively equal to the three angles of the other cutout, can we say the two cutouts are congruent? Justify your answer.**

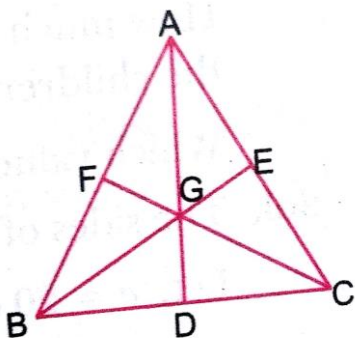
**Which values of these people are depicted here?**

**Sol.** The two cutouts may not be congruent. For example all equilateral triangles have equal angles but may have different sides.

Environmental concern, cooperative, caring, social.

**Que 11. Anya wants to prepare a poster on education of girlchild for a campaign. She takes a triangular sheet and divides it into three equal parts by drawing its medians which intersect at the point G (see Fig. 12).**

**Show that  $ar(\triangle AGC) = ar(\triangle AGC) = ar(\triangle AGB) = ar(\triangle BGC) = \frac{1}{3}ar(\triangle ABC)$**



**Fig. 12**

**Do you think education of a girl child is important for the development of a society? Justify your answer.**

**Sol. Given:** A  $\triangle ABC$  in which medians AD, BE and CF intersects at G.

**Proof:**  $ar(\triangle AGB) = ar(\triangle BGC) = ar(\triangle CGA) = \frac{1}{3}ar(\triangle ABC)$

**Proof:** In  $\triangle ABC$ , AD is the median. As a median of a triangle divides it into two triangles of equal area.

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \quad \dots (i)$$

In  $\triangle GBC$ , GD is the median

$$\therefore \text{ar}(\triangle GBD) = \text{ar}(\triangle GCD) \quad \dots (ii)$$

Subtracting (ii) from (i), we get

$$\begin{aligned} \text{ar}(\triangle ABD) - \text{ar}(\triangle GBD) &= \text{ar}(\triangle ACD) - \text{ar}(\triangle GCD) \\ \text{ar}(\triangle AGB) &= \text{ar}(\triangle AGC) \quad \dots (iii) \end{aligned}$$

Similarly,  $\text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) \quad \dots (iv)$

From (iii) and (iv), we get

$$\text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) = \text{ar}(\triangle AGC) \quad \dots (v)$$

But,  $\text{ar}(\triangle AGB) + \text{ar}(\triangle BGC) + \text{ar}(\triangle AGC) = \text{ar}(\triangle ABC) \quad \dots (vi)$

From (v) and (vi), we get

$$\begin{aligned} 3 \text{ar}(\triangle AGB) &= \text{ar}(\triangle ABC) \\ \Rightarrow \text{ar}(\triangle AGB) &= \frac{1}{3} \text{ar}(\triangle ABC) \end{aligned}$$

Hence,  $\text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ar}(\triangle ABC)$

Yes, for the development of a society, education of each girl child is essential. An educated society always progresses.