Very Short Answer Type Questions

[1 mark]

Que 1. What is the relationship between chord of a circle and a perpendicular to it from the centre?

Sol. Perpendicular line from the centre bisect the chord.

Que 2. What is the minimum number of points required to determine a unique circle?

Sol. Three.

Que 3. In Fig. 10.5. if$\angle ABC = 30°$, then find ∠AOC.

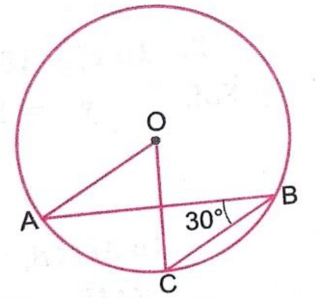

Fig. 10.5

Sol. ∠AOC = ∠ABC = 2 × 30° = 60°

Que 4. In Fig. 10.6, PQRS is a cyclic quadrilateral. If ∠QRS = 110°, then find ∠SPQ.

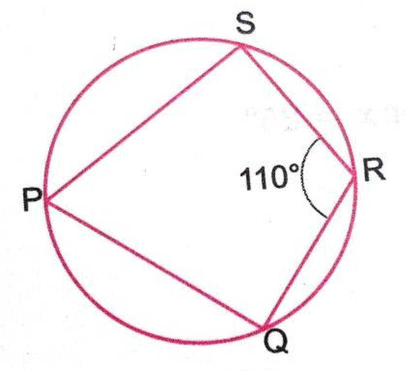

Fig. 10.6

Sol. ∠QRS + $\angle SPQ = 180°$ Opposite angles of cyclic quadrilateral)

110° + ∠SPQ = 180°

⇒ ∠SPQ = 180° - 110° = 70°

Que 5. In Fig. 10.7, if ∠BAC = 75°, find ∠BDC.

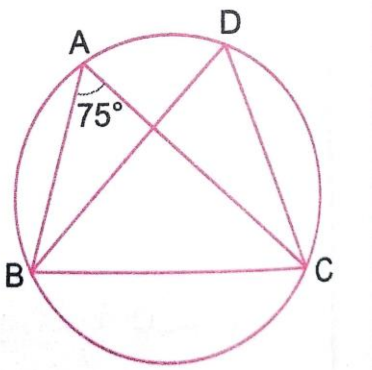

Fig. 10.7

Sol. Angles in the same segment are equal
∴ ∠BAC = ∠BDC = 75°

Que 6. If a circle is divided into eight equal parts, find the angle subtended by each arc at the centre.

Sol. Angles subtended by each arc at the centre of the circle = $\frac{1}{8} \times 360° = 45°$

Que 7. If AOB is a diameter of a circle [Fig. 10.8] and C is appoint on the circle, then prove that $AC^2 + BC^2 = AB^2$.

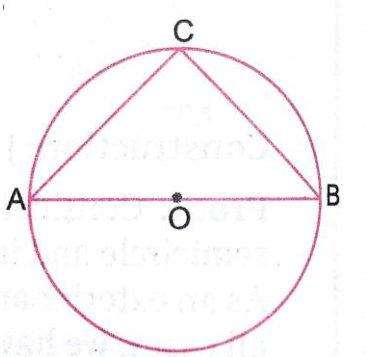

Fig. 10.8

Sol. As, ∠C = 90° (Angles in the semicircle)
∴ $AC^2 + BC^2 = AB^2$ (By Pythagoras Theorem)

Que 8. Two chords AB and CD of a circle are each at a distance 4 cm from the centre. Then prove that AB = CD.

Sol. Since chords equidistance from the centre of the circle are equal.
∴ AB = CD

Short Answer Type Questions – I

[2 marks]

Que 1. In Fig. 10.9, O is the centre of the circle. If ∠ACB = 30°, then find ∠ABC.

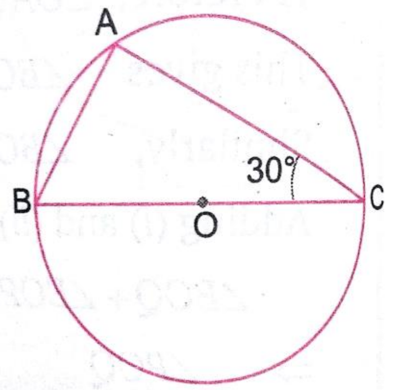

Fig. 10.9

Sol. ∠CAB = 90° (Angles in the semi-circle is 90°)

In ΔABC

∠ABC + ∠BCA + ∠CAB = 180°

∠ABC + 30° + 90° = 180°

∠ABC = 180° - 120°

⇒ ∠ABC = 60°

Que 2. In Fig. 10.10, if O is the centre of the circle then find ∠AOB.

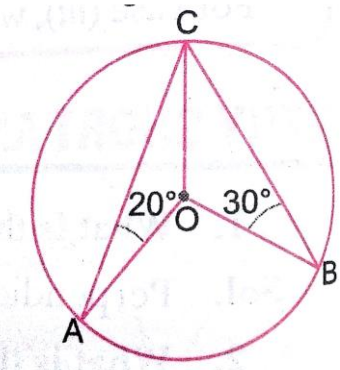

Fig. 10.10

Sol. OA = OC (radii of the same circle)

∴ ∠OCA = ∠OAC ⇒ ∠OCA = 20 °

Also, OB = OC

∴ ∠OCB = ∠OBC⇒ ∠OCB = 30°

Now, ∠ACB = 20° + 30° = 50°

∠AOB = 2 ∠ACB = 2 × 50° = 100°

Que 3. In Fig. 10.11, find the value of x and y.

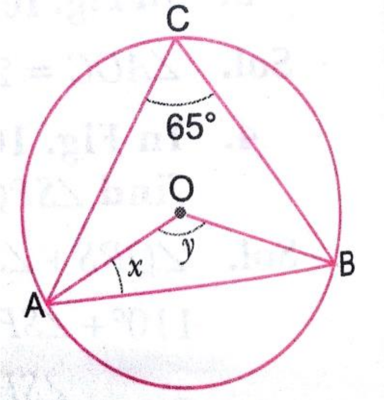

Fig. 10.11

Sol. y = 2∠ABC ⇒ y = 2 × 65° ⇒ y = 130°

OA = OB (radii of the same circle)

∴ ∠OBA = ∠OAB ⇒ ∠OBA = x

In ΔOAB,

∠OA + $\angle OBA$ + y = 180°

x + x + 130° = 180°

or 2x = 50° or x = 25°

Que 4. In Fig. 10.12, ABCD is a cyclic quadrilateral in which AB ||CD. If ∠B = 65°, then find other angles.

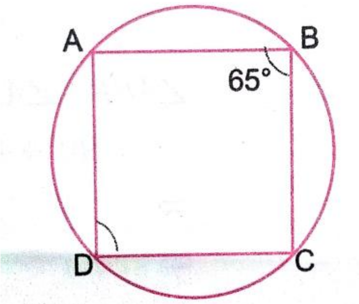

Fig. 10.12

Sol. ∠B + ∠D= 180° (Opposite angles of cyclic quadrilateral)

⇒ 65° + ∠D = 180° - 65° = 115°

Since AB ||C and BC is the transversal

∴ ∠B + ∠C = 180° ⇒ 65° + ∠C = 180°

⇒ ∠C = 180° - 65°

⇒ ∠C = 115°

Now, ∠A + 115° = 180° (Opposite angles of cyclin quadrilateral)

60° + ∠AEC = 180°

⇒ ∠AEC = 180° - 60° = 120°

Que 5. In Fig. 10.15, ∠ABC = 45°, prove that OA ⊥ OC.

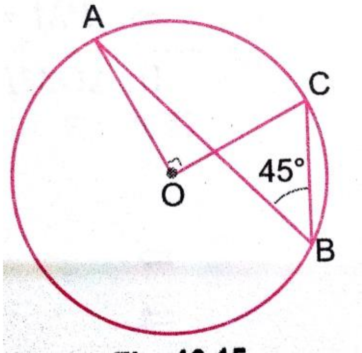

Fig. 10.15

Sol. As the angle subtended by an arc at the centre s twice the angle subtended by it at any point on the remaining part of the circle. Therefor,

$\angle AOC = 2\ \angle ABC$

⇒ $\angle AOC = 2 \times 45° = 90°$

Hence, OA ⊥ OC

Que 6. In Fig. 10.16, ∠AOC = 120°. Find ∠BDC.

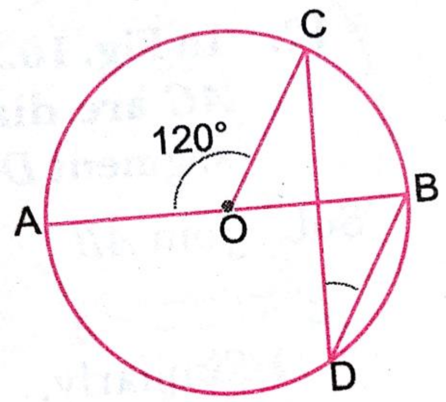

Fig. 10.16

Sol. $\angle AOC + \angle BOC = 180°$ (Linear pair)

⇒ $120° + \angle BOC = 180°$

$\angle BOC = 180° - 120° = 60°$

Now, $\angle BOC = 2\ \angle BDC$

⇒ $60° = 2\ \angle BDC$

⇒ $\angle BDC = 30°$

Que 7. In Fig. 10.17, O is the centre of a circle, find the value of x.

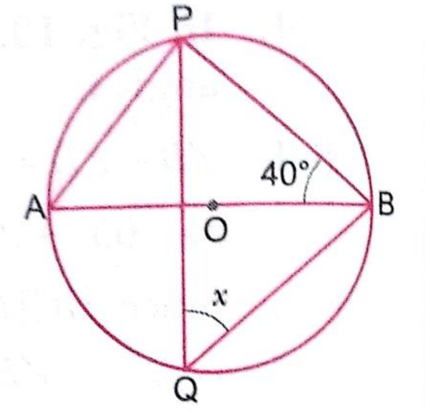

Fig. 10.17

Sol. In Δ APB,

$\angle APB = 90°$ (Angle in the semi-circle)

∠APB + ∠ABP + ∠BAP = 180°

90° + 40° + ∠BAP = 180°

⇒ ∠BAP = 180° - 130°

⇒ ∠ BAP = 50°

Now, ∠BQP = ∠BAP (Angles in the same segment)

∴ x = 50°

Que 8. Find the length of a chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm.

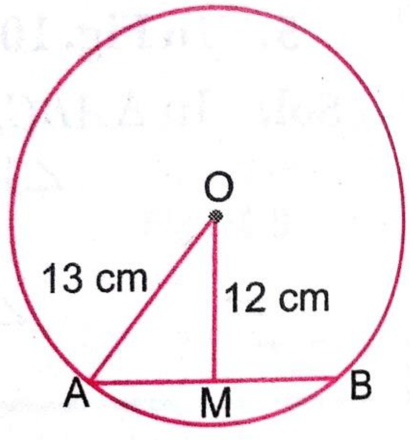

Fig. 10.18

Sol. Let AB be a chord of circle with centre O and radius 13 cm Draw OM ⊥ AB and join OA.

In the right triangle OMA, we have

$OA^2 = OM^2 + AM^2$

⇒ $13^2 = 12^2 + AM^2$

⇒ $AM^2 = 169 – 144 = 25$

As the perpendicular from the centre of a chord bisects the chord. Therefore,

AB = 2AM = 2 × 5 = 10 cm.

Que 9. The radius of a circle is 13 cm and the length of one of its chords is 24 cm. Find the distance of the chord from the centre.

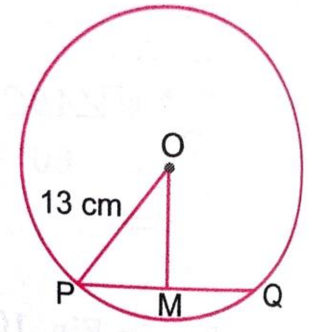

Fig. 10.19

Sol. Let PQ be a chord of a circle with centre O and radius 13 cm such that PQ = 24 cm.

From O, draw OM ⊥ PQ and join OP.

As, the Perpendicular from the centre of a circle to a chord bisects the chord.

∴ PM = MQ = $\frac{1}{2}$ PQ = $\frac{1}{2} \times 24$

⇒ PM = 12 cm

In ΔPMP, we have

$OP^2 = OM^2 + PM^2$

$13^2 = OM^2 + 12^2$

⇒ $OM^2 = 169 - 144 = 25$

⇒ OM = 5 cm

Hence, the distance of the chord from the centre is 5 cm.

Que 10. In Fig. 10.20, two circles intersects at two points A and B. AD and AC are diameters to the circles. Prove that B lies on the line segment DC.

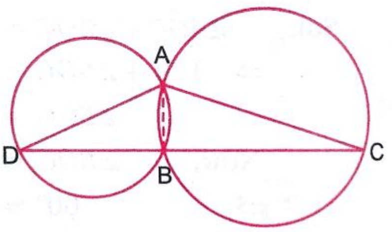

Fig. 10.20

Sol. Join AB ∠ABD = 90°

(Angles in a semicircle

Similarly, ∠ABC = 90°

So, ∠ABD + $\angle ABC = 90°$ + 90° = 180°

Therefore, DBC is a line i.e., B lies on the light segment DC.

Que 11. In Fig. 10.21, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of ∠ACD + ∠BED.

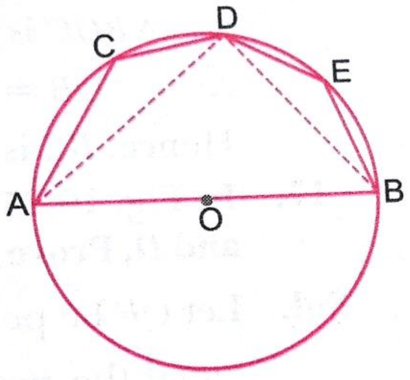

Fig. 10.21

Sol. Join BC,

Then, ∠ACB = 90° (Angle in the semi-circle)

Since DCBE is a cyclic quadrilateral.

∠BCD + ∠BED = 180°

Adding ∠ACB both the sides, we get

∠BCD + ∠BED + ∠ACB = ∠ACB + 180°

(∠BCD + ∠ACB) + ∠BED = 90° + 180°

∠ACD + ∠BED = 270°

Que 12. In Fig. 10.22, A, B, C and D are four points on a circle. AC and BD intersect at point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.

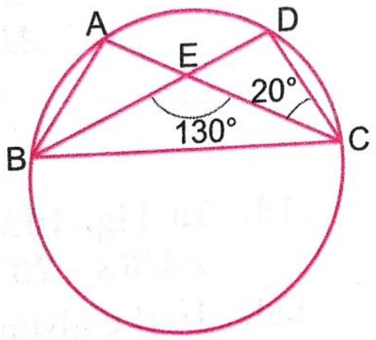

Fig. 10.22

Sol. Since the exterior angle of a triangle is equal to the sum of the interior opposite angles,

∴ ∠BEC = ∠ECD + ∠CDE

⇒ 130° = 20° + ∠CDE

⇒ ∠CDE = 130° - 20° = 110°

∠BDC = 110°

Now, ∠BAC = ∠BDC (Angles in the same segment)

∴ ∠BAC = 110°

Que 13. In Fig. 10.23, ∠ACB = 40°. Find ∠OAB.

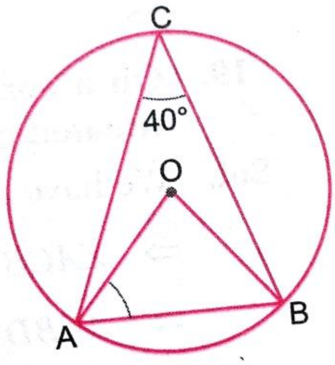

Fig. 10.23

Sol. Since OA = OB (Radii of the same circle)

∴ ∠OAB = ∠OBA

As the angle form by the act at the centre is twice the angle formed at any point in remaining part of the circle.

∴ ∠OAB = 2 ∠ACB = 2 × 40°

⇒ ∠AOB = 80°

In Δ AOB, we have

∠AOB + ∠OAB + ∠OBA = 180°

80° + ∠OAB + ∠OAB = 180° (∵ ∠OBA =$\angle OAB$)

⇒ 2 ∠OAB = 100° ⇒ ∠OAB = 50°

Que 14. In Fig. 10.24, ∠BAC = 30°. Show that BC is equal to the radius of the circumcircle of ΔABC whose centre is O.

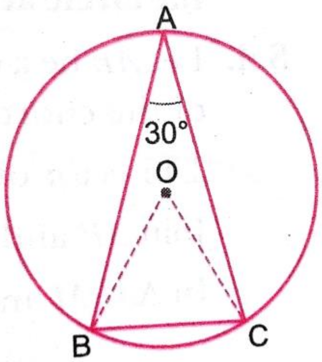

Fig. 10.24

Sol. ∠BOC = 2 ∠BAC

⇒ ∠BOC = 2 × 30° = 60°

Also, OC = OB (Radii of the same circle)

∴ ∠OCB = ∠OBC

In ΔOBC, we have

∠OBC + ∠OCB + ∠BOC = 180°

∠OBC + ∠OBC + 60° = 180°

⇒ 2∠OBC = 120° ⇒ ∠OBC = 60°

So, $\angle OBC = \angle OCB = \angle BOC = 60°$

∴ ΔBOC is an equilateral triangle.

∴ $OB = BC = OC$

Hence, BC is equal to the radius of the circumcircle.

Que 15. In Fig. 10.25, a line intersect two concentric circles with O at A, B, C and D, Prove that AB = CD.

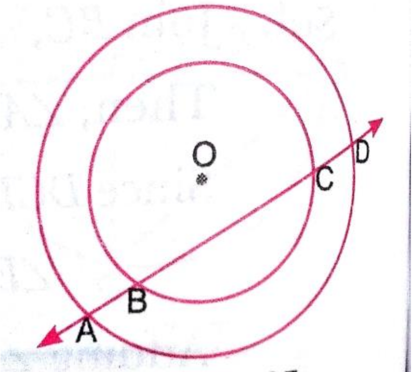

Fig. 10.25

Sol. Let OP be perpendicular from O on line l.

Since the perpendicular from the centre of a circle to a chord, bisects the chord. Therefore,

$$AP = DP \quad \text{.....(i)}$$

$$BP = CP \quad \text{.....(ii)}$$

Subtracting (ii) from (i), we get

$$AP - BP = DP - CP$$

$$\Rightarrow \quad AP = CD$$

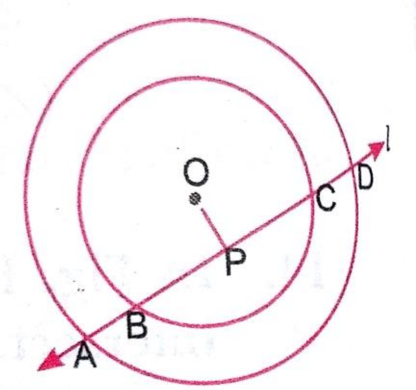

Fig. 10.26

Que 16. In Fig. 10.27, RS is diameter of the circle, NM is parallel to RS and ∠MRS = 29°, find ∠RNM.

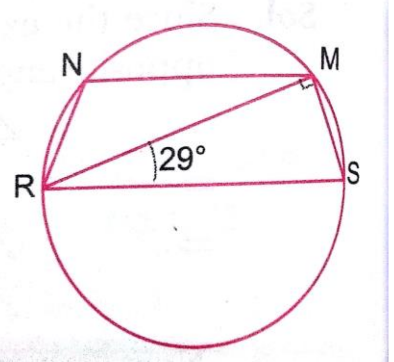

Fig. 10.27

Sol. In the Given figure ∠RMS = 90°

(Angle in the semi-circle as RS is diameter)

∴ ∠RSM = 180° - (29° + 90°) = 61°

∠RNM + ∠ ∠RSM = 180°

(Opposite angles if cyclin quadrilateral are supplementary)

∠RNM + 61° = 180°

⇒ ∠RNM = 119°

Que 17. On a common hypotenuse AB, two right triangle ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.

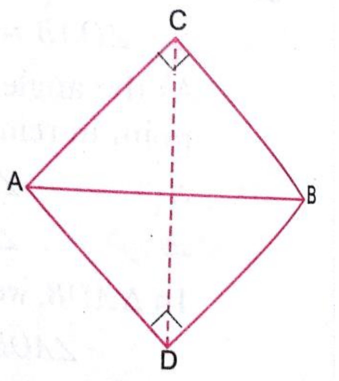

Fig. 10.28

We have ∠ACB = ∠ADB (Each 90°)

⇒ ∠ACB + ∠ADB = 90° + 90° 180°

⇒ ACBD is a cyclic quadrilateral

⇒ ∠BAC = ∠BDC (Angles in the same segment)

Que 18. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, then prove that arc PXA ≅ arc PYB.

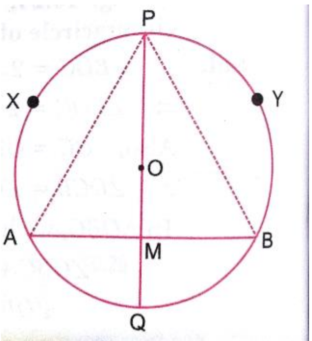

Fig. 10.29

Sol. Let AB be a chord of a circle having centre at O. Let PQ be the ⊥ bisector of the chord AB intersects it say at M.

⊥ Bisectors of the chord passes through the centre of the circle, i.e., O.

Join AP and BP.

In ΔAPM and ΔBPM

	AM= MB	(Given)
	∠PMA = ∠PMB	(90° each)
	PM = PM	(Common)
∴	ΔAPM ≅ ΔBPM	(SAS)
	PA = PB	(CPCT)

Hence. Arc PXA ≅ arc PYB

Que 19. In Fig. 10.30, P is the centre of the circle. Prove that ∠XPZ = 2(∠XYZ + ∠YXZ)

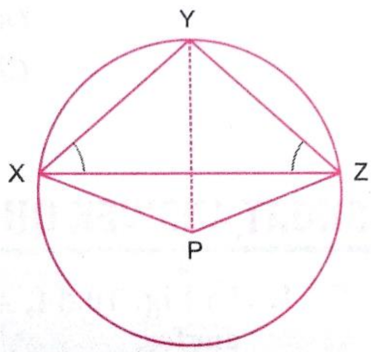

Fig. 10.30

Sol. Since arc XY subtends ∠XPY at the centre and ∠XZY at a point Z in the remaining part of the circle.

∴ ∠XPY = 2∠XZY …....(i)

Similarly, arc YZ subtends ∠YPZ at the centre and ∠YXZ at a point Y in the remaining part of the circle.

∴ ∠YPZ = 2∠YXZ ……..(ii)

Adding (i) and (ii),

$\angle XPY + \angle YPZ = 2\angle XYZ + 2\angle YXZ$

$\angle XPZ = 2(\angle XZY + \angle YXZ)$

Hence Prove.

Que 20. If BM and CN are the perpendiculars, drawn on the sides AB and AC of the ΔABC, then prove that the points B, C, M and N are cyclic.

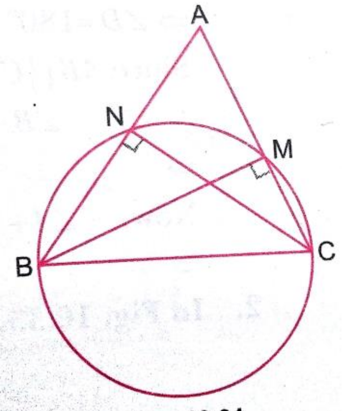

Fig. 10.31

Sol. Let us consider BC as a diameter of the circle.

Angles subtended by the diameter in a semicircle is 90°.

Given, $\angle BNC = \angle BMC = 90°$

So, the points M and N should be on the same circle.

Hence, BCMN form a cyclin quadrilateral.

Que 21. In Fig. 10.32, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.

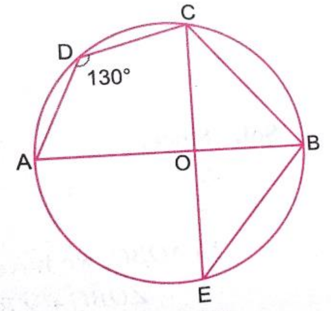

Fig. 10.32

Sol. Consider the points A, B, C and D. They formed a cyclin quadrilateral.

∴ $\angle ADC + \angle ABC = 180°$ (Opposite angles of cyclin quadrilateral)

$130° + \angle ABC = 180°$

$\angle ABC = 50°$

In ΔBOC and ΔBOE,

BC = BE (Equal chords)

OC = OE (Radii)

OB = OB (Common)

$\Delta BOC \cong \Delta BOE$ (SSS rule)

∴ $\angle OBC = \angle OBE = 50°$ (CPCT)

∴ $\angle CBE = \angle CBO + \angle EBO$

$= 50° + 50° = 100°$

Que 22. In Fig. 10.33, if OA = 10 cm, AB = 16 cm and OD ⊥ to AB. Find the value of CD.

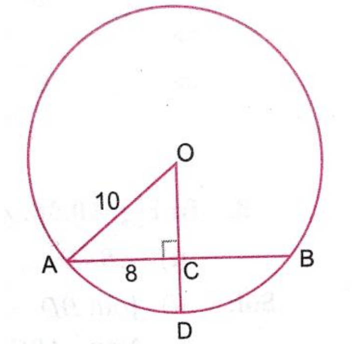

Fig. 10.33

Sol. As OD **is** ⊥ to AB

⇒ AC = CB

(⊥ from the centre to the chord bisects the chord)

∴ $AC = \frac{AB}{2} = 8cm$

In right ΔOCA,

$OA^2 = AC^2 + OC^2$

$(10)^2 = 8^2 + OC^2$

$OC^2 = 100 - 64$

$OC^2 = 36$

∴ $OC = \sqrt{36}$

OC = 6 cm

CD = OD – OC

= 10 – 6 = 4 cm. [∵ OA = OD = 10 cm (radii)]

Short Answer Type Questions – II

[3 marks]

Que 1. In Fig. 10.34, ABCD is a cyclic quadrilateral in which AB || CD. If ∠ = 65°, then find other angles.

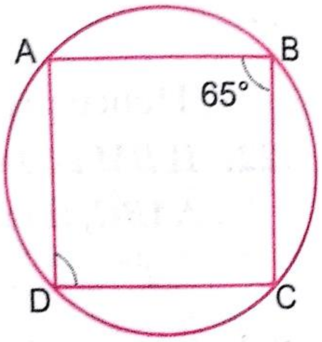

Fig. 10.34

Sol. ∠B + ∠D = 180° (OPP. angles of cyclin quadrilateral)

⇒ 65° + ∠D = 180°

⇒ ∠D = 180° - 65° = 115°

Since AB || CD and BC is the transversel

∴ ∠B + ∠C = 180° ⇒ 65° + ∠C = 180°

⇒ ∠C = 180° -65° ⇒ ∠C = 115°

Now, ∠A + 115° = 180° (Opposite angles of cyclin quadrilateral)

⇒ ∠A = 180° - 115° ⇒ ∠A = 65°

Que 2. In Fig. 10.35, ∠OAB = 30° and ∠OCB = 57°. Find ∠BOC and ∠AOC.

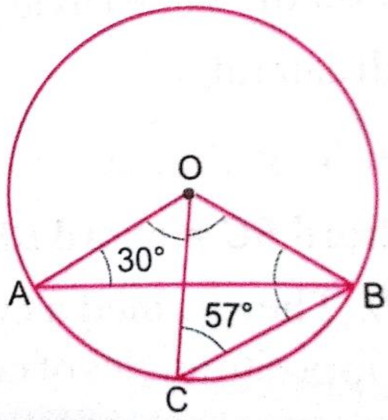

Fig. 10.35

Sol. Since, OC = OB (Radii of the same circle)

∴ ∠OBC = ∠OCB

⇒ $\angle OBC = 57°$

In ΔOBC, we have

$\angle OBC + \angle BOC + \angle OCB = 180°$

$57° + \angle BOC + 57° = 180°$

⇒ $\angle BOC = 66°$

In ΔOAB, we have

$\angle OAB + \angle OBA + \angle AOB = 180°$ (∵ AO = OB ∴ $\angle OAB = \angle OBA$)

$30° + 30° + \angle AOB = 180°$

⇒ $\angle AOB = 180° - 60°$

⇒ $\angle AOC = 120°$

$\angle AOC = \angle AOB - \angle BOC$

$= 120° - 66° = 54°$

Que 3. In Fig. 10.36, AD is a diameter of the circle. If $\angle BCD = 150°$, calculate (i) $\angle BAD$ (ii) $\angle ADB$

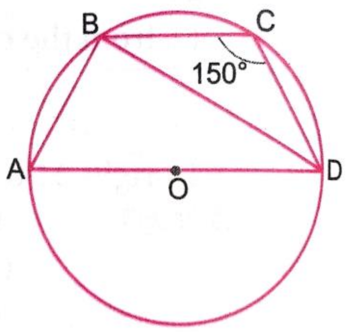

Fig. 10.36

Sol. Join BD

Now, ABCD is a cyclic quadrilateral

∴ $\angle BAD + \angle BCD = 180°$ (Opposite angles of a cyclin quadrilateral)

⇒ $\angle BAD + 150° = 180°$

⇒ $\angle BAD = 180° - 150° = 30°$

(ii) $\angle ABD = 90°$ (Angle in a semi-circle)

Now, in Δ ABD, we have

$\angle ABD + \angle BAD + \angle ADB = 180°$

$90° + 30° + \angle ADB = 180°$

⇒ $\angle ADB = 180° - 120° = 60°$

Que 4. In Fig. 10.37, RS is diameter of the circle, PM is parallel to RS and ∠MRS = 29°, find ∠RPM.

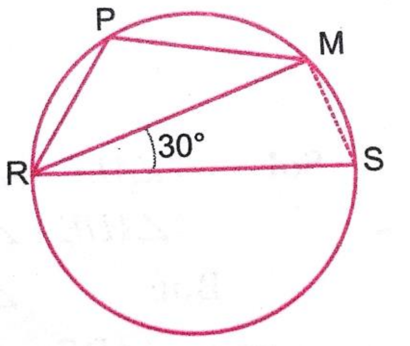

Fig. 10.37

Sol. In the given figure ∠RMS = 90°

(Angles in the semi-circle as RS is diameter)

∴ ∠RSM = 180° - (30° + 90°) = 60°

∠RPM + ∠RSM = 180°

(Opposite angles of cyclin quadrilateral are supplementary)

∠RPM + 60° = 180°

⇒ ∠RPM = 120°

Que 5. If circle are drawn talking two sides of a triangle as diameter, prove that the point of intersection of these circles lie on the third side.

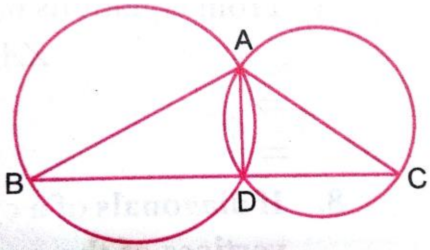

Fig. 10.38

Sol. Given: Two circles are drawn on sides AB and AC of a ΔABC as diameters. The circles intersects at D.

To prove: D lies on BC

Construction: Join A and D

Proof: ∠ADB = 90° (Angles in the semi-circle) …..(i)

and ∠ADC = 90° (Angles in the semi-circle) …..(ii)

Adding (i) and (ii), we get

∠ADB + ∠ADC = 90° + 90°

⇒ ∠ADB + ∠ADC = 180°

⇒ BDC is a straight line.

Hence, D lies on third side BC.

Que 6. In Fig. 10.39, O is the circumcenter of the triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.

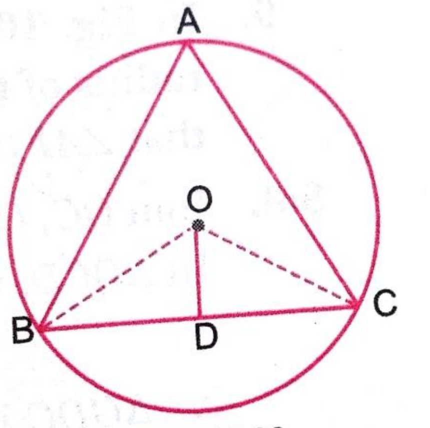

Fig. 10.39

Sol. As line drawn through the centre of a circle bisecting a chord is perpendicular to the chord.

∴ $OD \perp BC$

In the right triangles OBD and OCD, We have

$OB = OC$ (Radii of the same circle)

$OD = OD$ (Common)

$\angle ODB = \angle ODC$ (Each 90°)

∴ $\Delta OBD \cong \Delta OCD$ (By RHS congruence criterion)

⇒ $\angle BOD = \angle COD$ (CPCT)

⇒ $\angle BOD = \frac{1}{2}\angle BOC$ …….(i)

Also, $\angle A = \frac{1}{2}\angle BOC$ ……..(ii)

From (i) and (ii), we have

∴ $\angle BOD = \angle A$

Que 7. ABCD is a parallelogram. The circle through A, B and C intersect CD (Produce if necessary) at E. Prove that AE = AD.

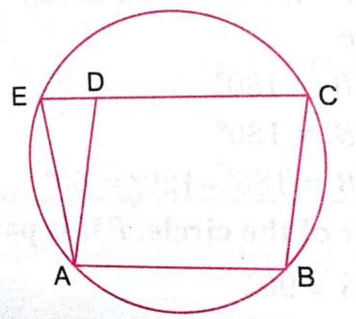

Fig. 10.40

Sol. $\angle ABC + \angle AEC = 180°$ (Opposite angles of cyclin quadrilateral) ……(i)

$\angle ADE + \angle ADC = 180°$ (Linear Pair)

But $\angle ADC = \angle ABC$ (Opposite angles of $||^{gm}$)

$\therefore$ $\angle ADE + \angle ABC = 180°$

From equations (i) and (ii), we have

$$\angle ABC + \angle AEC = \angle ADE + \angle ABC$$

$\Rightarrow$ $\angle AEC = \angle ADE$

$\Rightarrow$ $AD = AE$ (Sides opposite to equal angles)

Que 8. If diagonals of a cyclin quadrilateral are diameter of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

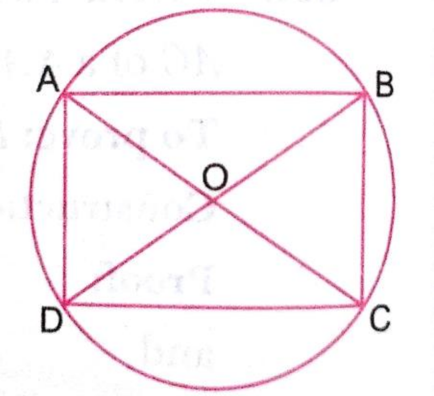

Fig. 10.41

Sol. Let, ABCD be a cyclin quadrilateral such that its diagonal AC and BD are the diameters of the circle through the vertices A, B, C and D.

As angle in a semi-circle is 90°

$\therefore$ $\angle ABC = 90°$ and $\angle ADC = 90°$

$\angle DAB = 90°$ and $\angle BCD = 90°$

So, $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90°$

Hence, ABCD is a rectangle.

Que 9. In Fig. 10.42, AB is a diameter of the circle, CD is a chord equal to the radius of the circle. AC and BD when extended intersect at point E. Prove that

∠AEB = 60°.

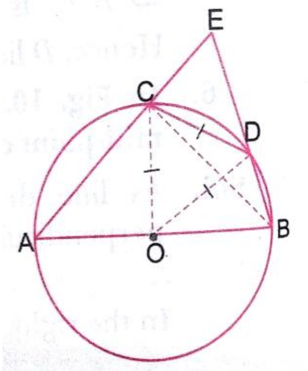

Fig. 10.42

Sol. Join OC, OD and BC

In ΔOCD, we have

OC = OD = CD (Each equal to radius)

∴ ΔODC is an equilateral triangle.

⇒ ∠COD = 60°

Also, ∠COD = 2 ∠CBD

⇒ 60° = 2 ∠CBD ⇒ ∠CBD = 30°

Since ∠ACB is angle in a semi-circle.

∠ACB = 90°

⇒ ∠BCE = 180° - ∠ACB = 180° - 90° = 90°

Thus, in ΔBCE, we have

∠BCE = 90° and ∠CBE = 30°

∴ ∠BCE + ∠CEB + ∠CBE = 180°

⇒ 90° + ∠CEB + 30° = 180° ⇒ ∠CEB = 60°

Hence, ∠AEB = ∠CEB = 60°

Que 10. In Fig. 10.43 two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively. Prove that ∠ACP = ∠QCD.

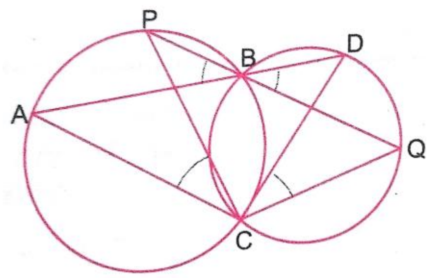

Fig. 10.43

Sol. As angles in the same segment of circle are equal

∠ABP = ∠ACP …….(i)

∠ABP = ∠QBD (Vertically opposite angles)

Also, ∠QCD = ∠QBD (Angles in the same segment)

∴ $\angle ABP = \angle QCD$ ......(ii)

From (i) and (ii), we have

$\angle ACP = \angle QCD$

Que 11. A circle has radius $\sqrt{2}$ cm. It is divided into two segment by a chord of length 2 cm. Prove that the angle subtended by chord a point in major segment is 45°.

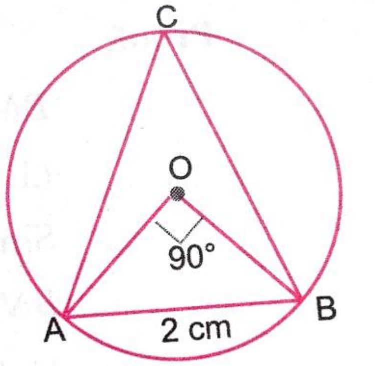

Fig. 10.44

Sol. Given: A chord AB of length 2 cm and radius of the circle is $\sqrt{2}$ cm

Proof: In ΔAOB,

$$OA^2 + OB^2 = (\sqrt{2})^2 + (\sqrt{2})^2 = 2 + 2 = 4 = AB^2$$

⇒ ΔAOB is a right triangle right angled at O.

i.e. $\angle AOB = 90°$

As the angle subtended by an arc at the centre is double the angle subtended by it at remaining part of the circle.

∴ $\angle AOB = 2\angle ACB$

⇒ $\angle ACB = \frac{1}{2} \times 90° = 45°$

Que 12. Two congruent circles intersect each other at point A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that

BP = BQ.

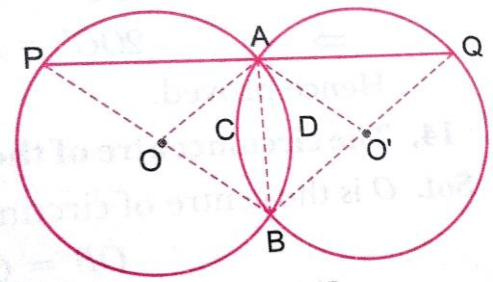

Fig. 10.45

Sol. Let, O and O’ be the centres of two congruent circles. As, AB is the common chord of these circles.

∴ ACB = ADB

As congruent arcs subtend equal angles at the centre.

∠AOB = ∠AO’B

⇒ $\frac{1}{2}\angle AOB = \frac{1}{2}\angle AO'B$

⇒ ∠BPA = ∠BQA

⇒ BP = BQ (Sides opposite to equal angles)

Que 13. Two circles with centre O and O’ intersect at two points A and B. A line PQ is drawn parallel to OO’ through B intersecting the circles at P and Q. Prove that PQ = 2OO’.

Sol. Construction: Draw two circles having centres O and O’ intersecting at point A and B.
Draw a parallel line PQ to OO’
Join OO’, OP, O’Q, OM and O,N

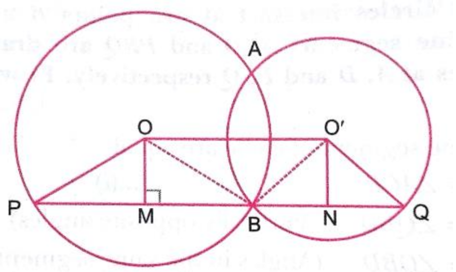

Fig. 10.46

To Prove: PQ = 2OO’

Proof: In ΔOPB

BM = MP ……(i)

(⊥ from the centre to the circle bisects the chord)

Similarly in ΔO, BQ

BN = NQ ……(ii)

(⊥ from the centre to the circle bisects the chord)

Adding (i) and (ii),

BM + BN = PM + NQ

Adding BM + BN to both the sides

BM + BN +BM + BN = BM + PM + NQ + BN

2BM + 2BN = PQ

2(BM + BN) = PQ ……(iii)

Again,

OO' = MN [As OO' NM is a rectangle] ……..(iv)

⇒ 2OO' = PQ

Hence Proved.

Que 14. The circumcenter of the ΔABC is O. Prove that ∠OBC + ∠BAC = 90°

Sol.

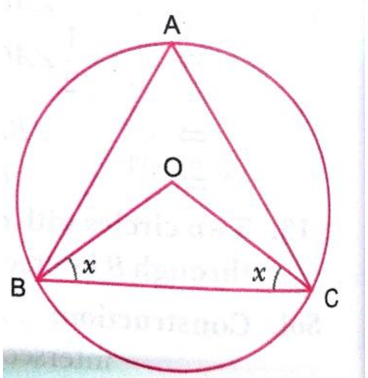

Fig. 10.47

O is the centre of circumscribed circle.

OB = OC = radii

⇒ ∠OBC = ∠OCB = x

∴ x + x + ∠BOC = 180° (Angle sum property of ΔOBC)

2x + ∠BOC = 180°

∠BOC = 180° - 2x

Also, ∠BOC = 2∠BAC

180 – 2x = 2∠BAC

⇒ 90 – x = ∠BAC

∴ ∠BAC + ∠OBC = (90 – x) + x

∠BAC + ∠OBC = 90°

Long Answer Type Questions

[5 Marks]

Que 1. Two chords AB and CD of length 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Sol.

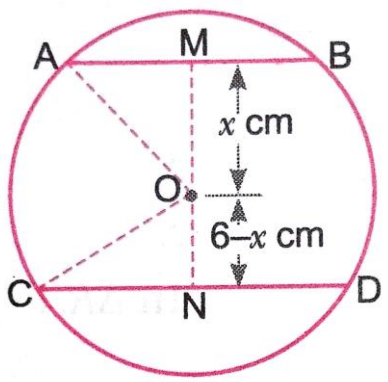

Fig. 10.48

Let, r be the radius of given circle and its centre be O. Draw OM ⊥ AB and ON ⊥ CD
Since, OM ⊥ AB, ON ⊥ CD and AB || CD
Therefore, points M, O and N are collinear. So, MN = 6 cm
Let, OM = x cm. Then, ON = (6 – x) cm.
Join OA and OC. Then OA = OC = r.
As the perpendicular from the centre to a chord of the circle bisects the chord.

∴ AM = BM = $\frac{1}{2}AB = \frac{1}{2} \times 5 = 2.5\ cm.$

CN = DN = $\frac{1}{2}CD = \frac{1}{2} \times 11 = 5.5\ cm.$

In right triangles OAM and OCN, we have

$OA^2 = OM^2 + AM^2$ and $OC^2 = ON^2 + CN^2$

$r^2 = x^2 + \left(\frac{5}{2}\right)^2$ ……(i)

$r^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2$ …….(ii)

From (i) and (ii), we have

$$X^2 + \left(\frac{5}{5}\right)^2 = (6 - x)^2 + \left(\frac{11}{2}\right)^2$$

$$X^2 + \frac{25}{4} = 36 + x^2 - 12x + \frac{121}{4}$$

$$\Rightarrow \quad 4x^2 + 25 = 144 + 4x^2 - 48x + 121$$

$$\Rightarrow \quad 48x = 240$$

$$\Rightarrow \quad x = \frac{240}{48} \quad \Rightarrow x = 5$$

Putting the value of x in equation (i), we get

$$r^2 = 5^2 + \left(\frac{5}{2}\right)^2 \Rightarrow r^2 = 25 + \frac{25}{4}$$

$$\Rightarrow \quad r^2 = \frac{125}{4} \quad \Rightarrow r = \frac{5\sqrt{5}}{2} cm$$

Que 2. Three girls Reshma, Salma and Mandeep are playing a game by standing on a circle of radius 5 cm drawn in a park. Reshma throws a ball to Salma, Salma to Mandeep to Reshma. If the distance between Reshma and Salma and between Salma and Mandeep is 6 cm each, what is the distance between Reshma and Mandeep?

Sol.

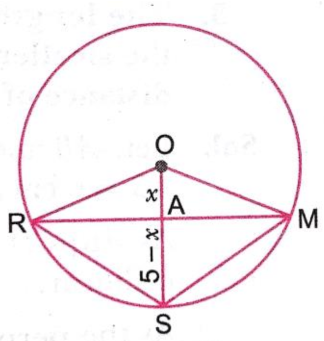

Fig. 10.49

Let R, S and M represent the position of Reshma, Salma and Mandeep respectively. Clearly ΔRSM is an isosceles triangle as

RS = SM = 6 m

Join OS which intersects RM at A.

In ΔROS and ΔMOS

	OR = OM	(Radii of the same circle)
	OS = OS	(Common)
	RS = SM	(Each 6 cm)
∴	ΔROS ≅ ΔMOS	(By SSS congruence criterion)
∴	∠RSO = ∠MSO	(CPCT)

In ΔRAS and ΔMAS

AS = AS (Common)

$\therefore$ $\angle RSA = \angle MSA$ ($\because \angle RSO = \angle MSO$)

$RS = MS$ (Given)

$\therefore$ $\Delta RAS \cong \Delta MAS$ (By SAS congruence criterion)

$\therefore$ $\angle RAS = \angle MAS$ (CPCT)

$\therefore$ $\angle RAS + \angle MAS = 180^\circ$ (Linear pair)

$\Rightarrow$ $\angle RAS = \angle MAS = 90^\circ$

Let $OA = x$ m $\Rightarrow$ $AS = (5 - x)$ m

In right triangle RAS,

$RS^2 = RA^2 + AS^2$

$\Rightarrow$ $6^2 = RA^2 + (5 - x)^2$ ……(i)

$\Rightarrow$ $RA^2 = 6^2 - (5 - x)^2$

In right triangle RAO,

$RO^2 = RA^2 + OA^2$

$\Rightarrow$ $5^2 = RA^2 + x^2$

$\Rightarrow$ $RA^2 = 5^2 - x^2$ ………(ii)

From equation (i) and (ii), we get

$6^2 - (5 - x)^2 = 5^2 - x^2$

$6^2 - 5^2 = (5 - x)^2$

$36 - 25 = 25 + x^2 - 10x - x^2$

$11 = 25 - 10x$ $\Rightarrow$ $10x = 14$ $\Rightarrow = 1.4$ m

From equation (ii), we have

$RA^2 = 5^2 - (1.4)^2 = 25 - 1.96$

$RA^2 = 23.04$ $\Rightarrow$ $RA = \sqrt{23.04}$

As the Perpendicular from the centre of a bisects the chord.

$\therefore$ $RM = 2RA$

$RM = 2 \times 4.8 = 9.6$ m

Hence, distance between Reshma and Mandeep is 9.6 m.

Que 3. The length of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of other chord from the centre?

Sol.

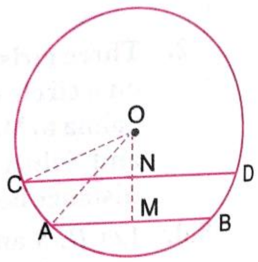

Fig. 10.50

Let, AB and CD be two parallel chords of a circle with centre O such that AB = 6 cm and CD = 8 cm. Draw OM ⊥ AB and ON ⊥ CD.

As AB || CD and OM ⊥ AB, ON ⊥ CD. Therefore, Points O, N and M are collinear.

As the perpendicular from the centre of a circle to the chord bisects the chord.

Therefore,

$$AM = \frac{1}{2}AB = \frac{1}{2}\times 6 = 3\ cm$$

$$CN = \frac{1}{2}CD = \frac{1}{2}\times 8 = 4\ cm$$

In right triangle OAM, we have

$OA^2 = OM^2 + AM^2$

$OA^2 = 4^2 + 3^2$ ⇒ $OA^2 = 25$ ⇒ OA = 5cm

Also, OA = OC (Radii of the same circle)

⇒ OC = 5 cm

In right triangle OCN, we have

$OC^2 = ON^2 + CN^2$

⇒ $5^2 = ON^2 + 4^2$ ⇒ $ON^2 = 5^2 - 4^2$

⇒ $ON^2 = 9$ ⇒ ON = 3 cm

Que 4. AC and BD are chords of a circle that bisect each other. Prove that AC and BD are diameter and ABCD is a rectangle.

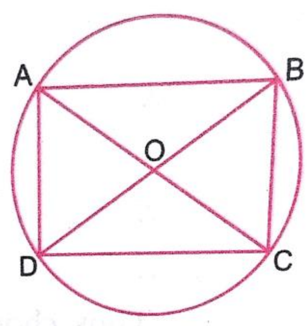

Fig. 10.51

Sol. Let AC and BD bisect each other at point O. Then,

OA = OC and OB = OD ....(i)

In triangles AOB and COD, we have

OA = OC

OB = OD

and ∠AOB = ∠COD (Vertically opposite angles)

∴ ΔAOB ≅ ΔCOD (SAS congruence criterion)

⇒ AB = CD (CPCT)

⇒ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ ......(ii)

Similarly BC = DA

⇒ $\overset{\frown}{BC} \cong \overset{\frown}{DA}$ ……(iii)

From (ii) and (ii), we have

$$\overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{CD} + \overset{\frown}{DA}$$

⇒ $\overset{\frown}{ABC} = \overset{\frown}{CDA}$

⇒ AC divides the circle into two equal parts.

⇒ AC is the diameter of the circle. Similarly, we can prove that BD is also a diameter of the circle.

Since AC and BD are diameter of the circle.

∴ ∠ABC = 90° = ∠ADC

Also, ∠BAD = 90° = ∠BCD

Also, AB = CD and BC = DA (Proved above)

Hence, ABCD is a rectangle.

Que 5. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

Sol.

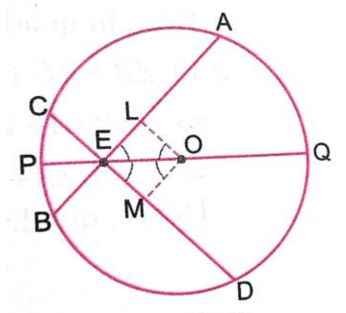

Fig. 10.52

Given: AB and CD are two chords of a circle with centre O, intersecting at point E. PQ is a diameter through E, such that ∠AEQ = ∠DEQ.

To prove: AB = CD

Construction: Draw OL ⊥ AB and OM ⊥ CD

Proof: ∠LOE + ∠LEO + ∠OLE = 180° (Angle sum property of a triangle)

⇒ ∠LOE + ∠LEO + 90° 180°

∠LOE + ∠LEO = 90° …..(i)

Similarly ∠MOE + ∠MEO + ∠OME = 180°

⇒ ∠MOE + ∠MEO + 90° = 180°

∠MOE + ∠MEO = 90° …..(ii)

From (i) and (ii) we get

∠LOE + ∠LEO = ∠MOE + ∠MEO ……(iii)

Also, ∠LEO = ∠MEO (Given) ….(iv)

From (iii) and (iv) we get

$\angle LOE = \angle MOE$

Now in triangle OLE and OME

	$\angle LEO = \angle MEO$	(Given)
∴	$\angle LOE = \angle MOE$	(Proved above)
	$EO = EO$	(Common)
∴	$\Delta OLE \cong \Delta OME$	(ASA congruence criterion)
∴	$OL = OM$	(CPCT)

Thus, chords AB and CD are equidistance from the centre are equal.

∴ $AB = CD$

Que 6. If the non-parallel sides of a trapezium are equal, prove that it is cyclin.

Sol.

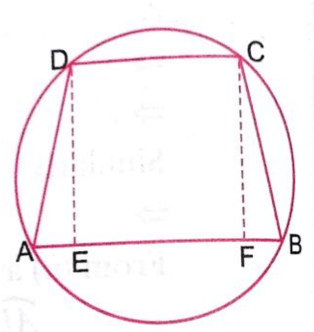

Fig. 10.53

Given: A trapezium ABCD in which AB || CD and AD = BC
To prove: ABCD is a cyclin trapezium.
Construction: Draw $DE \perp AB$ and $CF \perp AB$
In right triangle AED and BFC, We have

	$AD = BC$	(Given)	
	$\angle DEA = \angle CFB$	(Each equal to 90°	
and,	$DE = CF$	(Distance between two parallel lines)	
⇒	$\Delta DEA \cong \Delta CFB$	(RHS congruence criterion)	
⇒	$\angle A = \angle B$	(CPCT)	…….(i)
	$\angle ADE = \angle BCF$	(CPCT)	…….(ii)
⇒	$\angle C = \angle BCF + 90° = \angle ADE + 90° = \angle ADC$		…….(iii)
⇒	$\angle C = \angle D$		

Now, in quadrilateral ABCD, we have

$\angle A + \angle B + \angle C + \angle D = 360°$ (By Angle sum property)

⇒ $2\angle A + 2\angle C = 360°$ (From (i) and (iii)

⇒ $\angle A + \angle C = 180°$

Hence, quadrilateral ACBD is cyclin.

Que 7. Prove that quadrilateral formed by angle bisectors of a cyclin quadrilateral is also cyclin.

Sol.

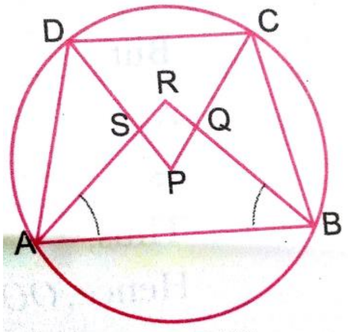

Fig. 10.54

Given: A cyclin quadrilateral ABCD in which the angle bisectors AR, CP and DP of internal angles A, B, C and D respectively form a quadrilateral PQRS.

To prove: PQRS is a cyclin quadrilateral.
Proof: In ΔARB, we have

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle R = 180°$ ….(i) (∵ AR, BR are bisectors of ∠A ∠B)

In ΔDPC, We have

$\frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle P = 180°$ ...(ii)

(∵ DP, CP are bisectors of ∠D and ∠C respectively)

Adding (i) and (ii), we get

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle R + \frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle P = 180° + 180°$$

$$\angle P + \angle R = 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$\angle P + \angle R = 360° - \frac{1}{2} \times 360° = 360° - 180°$$

⇒ ∠P + ∠R = 180°

As the sum of a pair of opposite angles of quadrilateral PQRS is 180°. Therefore, quadrilateral PQRS is cyclin.

Que 8. If two circles intersects at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Sol.

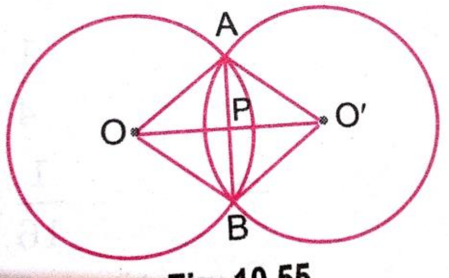

Fig. 10.55

Given: Two circles, with centres O and O' intersect at two points A and B. AB is the common chord of the two circles and OO' is the line segment joining the centres of the two circles. Let OO' intersect AB at P.

To prove: OO' is the perpendicular bisector of AB.
Construction: Join OA, OB, O' A and O' B
Proof: In triangles OAO' and OBO', we have

	OO' = OO'	(Common)
	OA = OB	(Radii of the same circle)
	O'A = O' B	(Radii of the same circle)
⇒	ΔOAO' ≅ ΔOBO'	(SSS congruence criterion)
⇒	∠AOO' = ∠BOO'	(CPCT)
I.e.,	∠AOP = ∠BOP	

In triangle AOP and BOP, we have

	OP = OP	(Common)
	∠AOP = ∠BOP	(Proved above)
	OA = OB	(Radio of the same circle)
∴	ΔAOR ≅ ΔBOP	(By SAS congruence criterion)
⇒	AP = BP	(CPCT)
And	∠APO = ∠BPO	(CPCT)

But ∠APO + ∠BPO = 180° (Linear)

∴ ∠APO + ∠APO = 180° ⇒ 2∠APO = 180°

⇒ ∠APO = 90°

Thus, AP = BP and ∠APO = ∠BPO = 90°

Hence, OO' is the perpendicular bisectors of AB.

HOTS (Higher Order Thinking Skills)

Que 1. AB and AC are two chords of a circle of radius r such that AB = 2AC. If P and q are the distances of AB and AC from the centre. Prove that
$4q^2 = p^2 + 3r^2$.

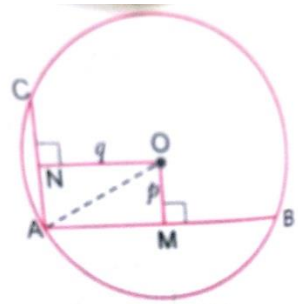

Fig. 10.56

Sol. Draw OM ⊥ AB and ON ⊥ AC

Join OA.

In right ΔOAM,

$$OA^2 = OM^2 + AM^2$$

⇒ $r^2 = p^2 + \left(\frac{1}{2}AB\right)^2$ (∵ OM ⊥ AB, ∴ OM bisects AB)

⇒ $\frac{1}{4}AB^2 = r^2 - p^2$ or $AB^2 = 4r^2 - 4p^2$ …(i)

In right ΔOAN,

$$OA^2 = ON^2 + AN^2$$

⇒ $r^2 = q^2 + \left(\frac{1}{2}AC\right)^2$ (∵ ON ⊥ AC, ∴ ON bisects AC)

⇒ $\frac{1}{4}AC^2 = r^2 - q^2$ or $\frac{1}{4}\left(\frac{1}{2}AB\right)^2 = r^2 - q^2$ (∵ AB = 2AC)

⇒ $\frac{1}{16}AB^2 = r^2 - q^2$ or $AB^2 = 16r^2 - 16p^2$ …(ii)

From (i) and (ii), we have

$$4r^2 - 4p^2 = 16r^2 - 16q^2$$

Or $r^2 - p^2 = 4r^2 - 4q^2$

Or $4q^2 = 3r^2 + p^2$

Que 2. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is

equal to half the difference of the angles subtended by the chords AC and DE at the center.

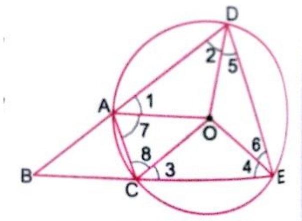

Fig. 10.57

Sol. Given: AD = CE

To prove: $\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$

In ΔAOD and ΔCOE

AD = CE (Given)

AO = OC and DO = OE (Radii of same circle)

∴ ΔAOD ≅ ΔCOE (By SSS congruence criterion)

⇒ $\angle 1 = \angle 3$, $\angle 2 = \angle 4$ (CPCT) …(i)

But OA = OD and OC = OE ⇒ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ …(ii)

From (i) and (ii), we have

$\angle 1 = \angle 2 = \angle 3 = \angle 4$ (= x say)

Also, OA = OC and OD = OE

⇒ $\angle 7 = \angle 8$ (= z say) and $\angle 5 = \angle 6$ (= y say)

Now, ADEC is a cyclic quadrilateral

⇒ $\angle DAC + \angle DEC = 180^0$

⇒ $x + z + x + y = 180^0$ ⇒ $y = 180^0 - 2x - z$ …(iii)

In ΔDOE, $\angle DOE = 180^0 - 2y$

And in ΔAOC, $\angle AOC = 180^0 - 2z$

∴ $\angle DOE - \angle AOC = (180^0 - 2y) - (180^0 - 2z) = 2z - 2y$

$= 2z - 2(180^0 - 2x - z)$ (Using (iii))

$= 4z + 4x - 360^0$ …(iv)

Again, $\angle BAC + \angle CAD = 180^0$ ⇒ $\angle BAC = 180^0 - (z + x)$ …(v)

Similarly, $\angle BAC = 180^0 - (z + x)$ …(vi)

In ΔABC, $\angle ABC = 180^0 - \angle BAC - \angle BCA$

$= 180^0 - 2[180^0 - (z + x)]$ (Using (v) and (vi))

$= 2z + 2x - 180^0 = \frac{1}{2}(4z + 4x - 360^0)$ …(vii)

From (iv) and (vii), we have

$\angle BAC = \frac{1}{2}(\angle DOE - \angle AOC)$

Que 3. Bisectors of angles A, B and C of a triangle ABC intersects its circumcircle at D, E and F respectively. Prove that angles of triangle DEF are $90^0 - \frac{A}{2}$, $90^0 - \frac{B}{2}$ and $90^0 - \frac{C}{2}$.

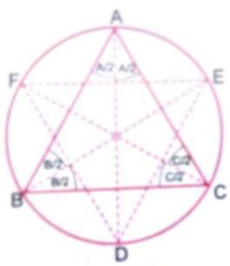

Fig. 10.58

Sol. We have $\angle BED = \angle BAD$

(Angles in the same segment)

$\Rightarrow \quad \angle BED = \frac{1}{2}\angle A$ …(i)

Also, $\angle BEF = \angle BCF$ (Angles in the same segment)

$\Rightarrow \quad \angle BEF = \frac{1}{2}\angle C$ …(ii)

From (i) and (ii) $\angle BED + \angle BEF = \frac{1}{2}\angle A + \frac{1}{2}\angle C$

$\angle DEF = \frac{1}{2}(\angle A + \angle C)$

$\Rightarrow \quad \angle DEF = \frac{1}{2}(180^0 - \angle B)$ $(\because \angle A + \angle B + \angle C = 180^0)$

$\Rightarrow \quad \angle DEF = 90^0 - \frac{1}{2}\angle B$

Value Based Questions

Que 1. Teacher held two sticks AB and CD of equal length in her hands and marked their mid points M and N respectively. She then asked the students whether AM is equal to ND or not. Aprajita answered yes. Is Aprajita correct? State the axiom of Euclid that supports her answer. Which values of Aprajita are depicted here?

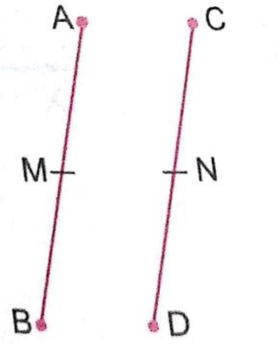

Fig. 5

Sol. Yes, Things which are halves of the same things are equal to one another. Curiosity, knowledge, truthfulness.

Que 2. For her records, a teacher asked the students about their heights. Manav said his height is same as that of Arnav. Raghav also answered the same, way that his height is same as that of Arnav. She then asked the students to relate the height of Manav and Raghav. Arnav answered they both have same height. Is Arnav correct? If yes, state Euclid's axiom which supports his answer.
Which values of Arnav are depicted here?

Sol. Yes, Things which are equal to the same thing are equal to one another. Knowledge, curiosity, truthfulness.

Que 3. The number of members of society A who participated in 'Say No to Crackers' campaign is double the number of members from society B. Also, the number of members from society C is double the number of members from society B. Can you relate the number of participants from society A and C? Justify your answer using Euclid's axiom.
Which values are depicted here?

Sol. The number of participants from society A and C is equal. Things which are double of the same thing are equal to one another. Social service, helpfulness, cooperation, environmental concern.

Que 4. In a society, the number of persons using CNG instead of petrol for their vehicles has increased by 15 and now the number is 25. Form a linear equation to find the original number of persons using CNG and solve it using Euclid's axiom.
Which values are depicted in the question?

Sol. X + 15 = 25
⇒ $x + 15 - 15 = 25 - 15$ (Using Euclid's third axiom)
⇒ x = 10
Environmental care, responsible citizens, futuristic.

Que 5. Teacher asked the students to find the value of x in the following figure if l|| m.
Shalini answered 35°. Is she correct? Which values are depicted here?

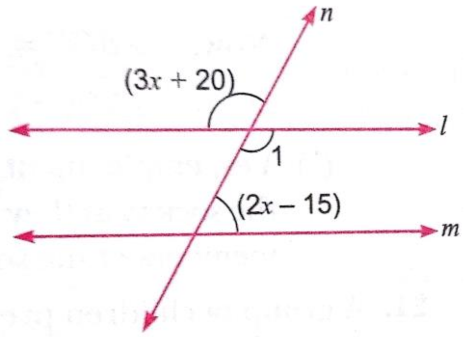

Fig. 6

Sol. ∠1 = 3x + 20 (Vertically opposite angles)
∴ 3x + 20 2x – 15 = 180 ° (Co-interior angles are supplementary)
⇒ $5x + 5 = 180°$ ⇒ $5x = 180° - 5°$
⇒ 5x = 175 ° ⇒ $x = \frac{175}{5} = 35°$

Yes, Knowledge, truthfulness.

Que 6. For spreading the message 'Save Environment Save Future' a rally was organised by some students of a school. They were given triangular cardboard pieces which they divided into two parts by drawing bisectors of base angles (say ∠B and ∠C) intersecting at O in the given figure. Prove that ∠BOC = $90 + \frac{1}{2}\angle A$
Which values are depicted by these students?

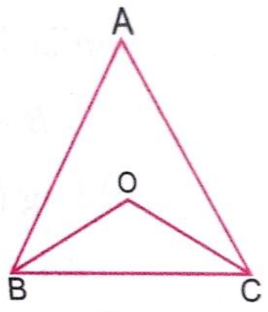

Fig. 7

Sol. In ΔABC, we have
∠A + ∠B + ∠C = 180 ° (∵ sum of the angles of a Δ is 180 °)

$\Rightarrow \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180°}{2}$

$\Rightarrow \quad \frac{1}{2}\angle A + \angle 1 + \angle 2 = 90°$

$\therefore \quad \angle 1 + \angle 2 = 90° - \frac{1}{2}\angle A$ ....(i)

Now, in ΔOBC, we have:

∠1 + ∠2 + ∠BOC = 180° [∵ sum of the angles of Δ is 180 °]

$\Rightarrow$ ∠BOC = 180° $- (\angle 1 + \angle 2)$

$\Rightarrow$ ∠BOC = 180° $- (90° - \frac{1}{2}\angle A)$ [using (i)]

$\Rightarrow$ ∠BOC = 180° $- 90° + \frac{1}{2}\angle A$

$\therefore$ ∠BOC = 90° $+ \frac{1}{2}\angle A$

Environmental care, social, futuristic.

Que 7. Three bus stops situated at A, B and C in the figure are operated by handicapped persons. These 3 bus stops are equidistant from each other. OB is the bisector of ∠ABC and OC is the bisector of ∠ACB.
(a) Find ∠BOC.
(b) Do you think employment provided to handicapped persons is important for the development of the society? Express your views with relevant points.

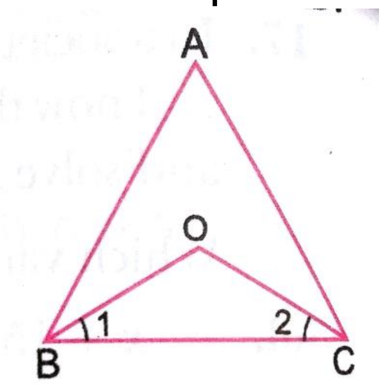

Fig. 9

Sol. (a) Since, A, B, C are equidistant from each other.

∴ ∠ABC is an equilateral triangle.

⇒ ∠ABC = ∠ABC = 60°

⇒ ∠OBC = ∠OCB = $\frac{1}{2} \times 60° = 30°$ (∵ OB and OC are angle bisectors)

Now, ∠BOC = 180° - ∠OBC - ∠OCB (Using angle sum property of triangle)

⇒ ∠BOC = 180°- 30°-30° = 120°

(b) Yes, employment provided to the handicapped persons is important for the development of the society as they would become independent, self-reliant, confident, social, helpful and useful members of the society.

Que 8. A group of children prepared some decorative pieces in the shape of a star for the orphans in an orphanage. Show that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Which values of the children are depicted here?

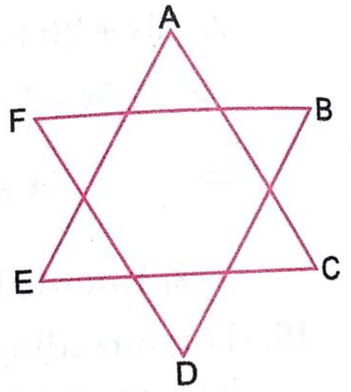

Fig. 10

Sol. In ΔAEC,
$\angle A + \angle E + \angle C = 180°$ … (i) (Angle sum property of a triangle)

Similarly, in ΔBDF,
$\angle B + \angle D \angle F = 180°$ …. (ii)

Adding (i) and (ii), we get
$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360°$
Social, caring, cooperative, hardworking.

Que 9. For annual day, Sakshi and Nidhi were asked to make one rangoli each on two different places. They started it with triangles (say ABC and ΔPQR) and their medians (AM and PN). If two sides (AB and BC) and a median (AM) of one triangle are respectively equal to two sides (PQ and QR) and a median (PN) of other triangle, prove that the two triangles (ΔABC and ΔPQR) are congruent. Which values of the girls are depicted here?

Sol. In ΔABC and ΔPQR

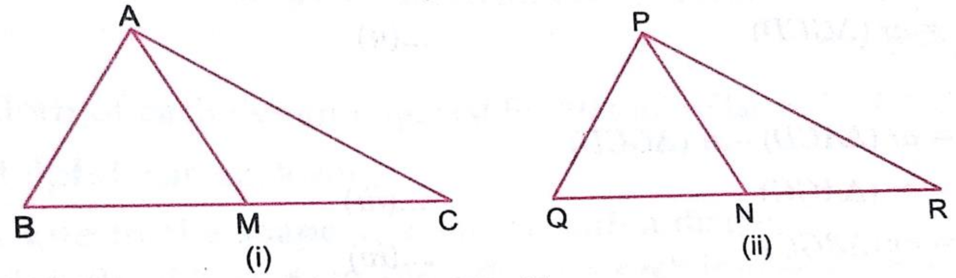

Fig. 11

BC = QR

⇒ $\frac{1}{2}BC = \frac{1}{2}QR$

⇒ BM = QN

In triangle ABM and PQN, we have

AB = PQ	(Given)
BM = QN	(Proved above)
AM = PN	(Given)

∴ $\Delta ABM \cong \Delta PQN$ (SSS congruence criterion)

⇒ $\angle B = \angle Q$ (CPCT)

Now, in triangles ABC and PQR, we have

AB = PQ	(Given)
$\angle B$ = ∠Q	(Proved above)
BC = QR	(Given)

∴ ΔABC ≅ ΔPQR (SSS congruence criterion)

Participation, beauty, hardworking.

Que 10. Triangular pieces of cardboards were cut out by some people who were organising 'No Pollution' campaign in their area. If the three angles of one cutout are respectively equal to the three angles of the other cutout, can we say the two cutouts are congruent? Justify your answer.
Which values of these people are depicted here?

Sol. The two cutouts may not be congruent. For example all equilateral triangles have equal angles but may have different sides.
Environmental concern, cooperative, caring, social.

Que 11. Anya wants to prepare a poster on education of girlchild for a campaign. She takes a triangular sheet and divides it into three equal parts by drawing its medians which intersect at the point G (see Fig. 12).

Show that ar (ΔAGC) = ar (ΔAGC) = ar (ΔAGB) = (ΔBGC) = $\frac{1}{3}ar\ (\Delta ABC)$

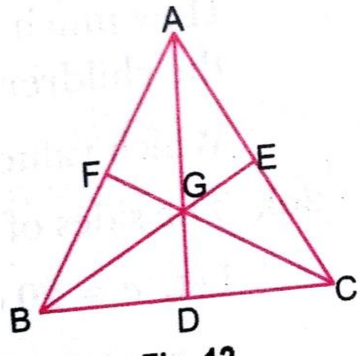

Fig. 12

Do you think education of a girl child is important for the development of a society? Justify your answer.

Sol. Given: A ΔABC in which medians AD, BE and CF intersects at G.

Proof: (ΔAGB) = ar (ΔBGC) = ar (ΔCGA) = $\frac{1}{3}$ ar (ΔABC)

Proof: In ΔABC, AD is the median. As a median of a triangle divides it into two triangles of equal area.

∴ ar (ΔABD) = ar (ΔACD) … (i)

In ΔGBC, GD is the median

∴ aq (ΔGBD) = ar (ΔGCD) …. (ii)

Subtracting (ii) from (i), we get

ar (ΔABD) – ar (ΔGBD) = ar (ACD) – ar (ΔGCD)

ar (ΔAGB) = ar (ΔAGC) … (iii)

Similarly, ar (ΔAGB) = ar (ΔBGC) … (iv)

From (iii) and (iv), we get

ar (ΔAGB) = ar (ΔBGC) = ar (ΔAGC)…. (v)

But, ar (ΔAGB) + ar (ΔBGC) + ar (ΔAGC) = ar (ΔABC) …. (vi)

From (v) and (vi), we get

3 ar (ΔAGB) = ar (ΔABC)

⇒ ar (ΔAGB) = $\frac{1}{3} ar(\Delta ABC)$

Hence, ar (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

Yes, for the development of a society, education of each girl child is essential. An educated society always progresses.