Very Short Answer Type Questions

[1 mark]

Que 1. Find the length of each side of an equilateral triangle having an area of $9\sqrt{3}$ cm².

Sol. Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$$\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}\ cm^2$$

⇒ $a^2 = 36\ cm^2$ ⇒ $a = 6$ cm.

Que 2. If the sides of an equilateral triangle is x unit, then find the area of the tringle.

Sol. $\frac{\sqrt{3}}{4}\chi^2$ sq. unit.

Que 3. Find the area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm.

Sol. Area of isosceles triangle $= \frac{a}{4}\sqrt{4b^2 - a^2} = \frac{2}{4}\sqrt{4.4^2 - 2^2}$

$$= \frac{2}{4} \times \sqrt{60} = \frac{2}{4} \times 2\sqrt{15} = \sqrt{15}cm^2.$$

Que 4. One side of an equilateral triangle is 4 cm. Find its area.

Sol. Area of equilateral triangle$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}4^2 = 4\sqrt{3}cm^2.$

Que 5. Find the area of an isosceles triangle having base x cm and equal side y cm.

Sol. $\frac{x}{4}\sqrt{4y^2 - x^2}cm^2.$

Que 6. The base and the corresponding altitude of a parallelogram are 10 cm and 7 cm, respectively. Find its area.

Sol. Area of parallelogram = Base × Corresponding altitude

= $10 \times 7 = 70$ cm².

Que 7. How many times area is changed, when sides of a triangle are doubled.

Sol. Four times.

Short Answer Type Questions – I

[2 marks]

Que 1. The base of a right-angled triangle measures 4 cm and its hypotenuse measures 5 cm. Find the area of the triangle.

Sol.

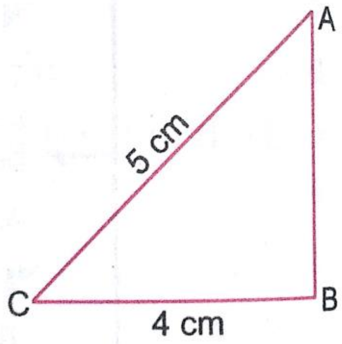

Fig. 12.1

In right-angled triangle ABC

$AB^2 + BC^2 = AC^2$ (By Pythagoras Theorem)

⇒ $AB^2 + 4^2 = 5^2$

⇒ $AB^2 + = 25 - 16 = 9$

⇒ AB = 3 cm

∴ Area of ΔABC = $\frac{1}{2} BC \times AB = \frac{1}{2} \times 4 \times 3 = 6 cm^2$

Que 2. If the area of an equilateral triangle is $36\sqrt{3}$ cm², find its height.

Sol. Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$$\frac{\sqrt{3}}{4} a^2 = 36\sqrt{3} \quad \Rightarrow \quad a^2 = 4 \times 36$$

⇒ a = $\sqrt{4 \times 36} = 12 cm$

Height of equilateral triangle $= \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3} cm$

Que 3. If the area of an equilateral triangle is $81\sqrt{3}$ cm², find its perimeter.

Sol. Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$\frac{\sqrt{3}}{4} a^2 = 81\sqrt{3} \quad \Rightarrow \quad a^2 = 81 \times 4$ ⇒ a = 18 cm

∴ Perimeter of equilateral triangle = 3a = 3×18 = 54 cm

Que 4. The sides of a triangle are 8 cm, 15 cm and 17 cm. Find its area.

Sol. Let a = 8cm, b = 15cm, c = 17 cm

$$s = \frac{a+b+c}{2} = \frac{8+15+18}{2} = \frac{40}{2} = 20cm$$

∴ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{20(20-8)(20-15)(20-17)}$$

$$= \sqrt{20 \times 12 \times 5 \times 3} = 60\ cm^2$$

Que 5. Find the area of a trapezium whose parallel sides are 25 cm and 13 cm long and the distance between them is 8 cm.

Sol. Area of trapezium $= \frac{1}{2}$ (Sum of parallel sides) × (Perpendicular distance between them)

$$= \frac{1}{2}(25 + 13) \times 8 = 152\ cm^2$$

Que 6. Find the area of an isosceles triangle, whose equal sides are of length 15 cm each and third side is 12 cm.

Sol. Area of isosceles triangle $= \frac{12}{4}\sqrt{4 \times 15^2 - 12^2} = \frac{12}{4}\sqrt{900 - 144}$

$$= 3\sqrt{756} = 3 \times 6\sqrt{21} = 18\sqrt{21}\ cm^2$$

Que 7. If the perimeter of an isosceles triangle is 11 cm and its base is 5 cm, its area is $\frac{5}{4}\sqrt{11}\ cm^2$. State true or false and give reason.

Sol. True, 2b + a = 11 ⇒ 2b + 5 = 11

⇒ 2b = 6 ⇒ b = 3

∴ Area of isosceles triangle $= \frac{a}{4}\sqrt{4b^2 - a^2}$

$$= \frac{5}{4}\sqrt{4 \times 3^2 - 5^2} = \frac{5}{4}\sqrt{11} cm^2$$

Que 8. An isosceles right triangle has area 8 cm^2. Find the length of its hypotenuse

Sol.

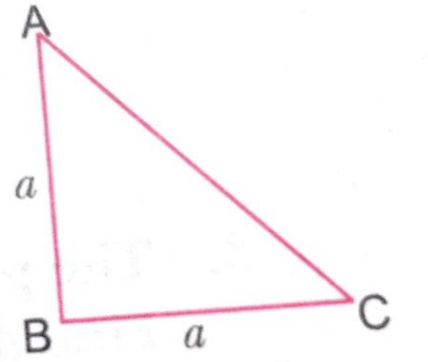

Fig. 12.2

Area $=\frac{1}{2}a^2 \Rightarrow \frac{1}{2}a^2 = 8$

$\Rightarrow a^2 = 16 \Rightarrow a = 4\,cm$

Hypotenuse= $\sqrt{2}a = \sqrt{2.4} = 4\sqrt{4}cm.$

Que 9. The altitude of an equilateral triangle is $3\sqrt{5}$ cm. Find its area.

Sol. Altitude $= \frac{\sqrt{3a}}{2} \Rightarrow \frac{\sqrt{3}}{2}a = 3\sqrt{3} \Rightarrow a = 6\,cm$

Area of equilateral triangle= $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times 6^2 = 9\sqrt{3}\,cm^2.$

Que 10. If the area of an equilateral triangle is $16\sqrt{3}$ cm^2, then find the perimeter of the triangle.

Sol. Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 16\sqrt{3} \quad \Rightarrow a^2 = 64 \quad \Rightarrow a = 8\,cm$

Perimeter of the equilateral triangle = 3a = 3 × 8 = 24 cm

Short Answer Type Questions – II

[3 marks]

Que 1. The cost of levelling a ground in the form of a triangle having the sides 51m, 37m and 20m at the rate of ₹3 per m² is ₹918. State whether the statement is true or false and justify your answer.

Sol. True, Let a= 51m, b=37m, c = 20m

$$s = \frac{a+b+c}{2} = \frac{51+37+20}{2} = \frac{108}{2} = 54m$$

∴ Area of triangle ground = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{54(54-51)(54-37)(54-20)}$$

$$= \sqrt{54 \times 3 \times 17 \times 34} = 306m^2$$

Cost of levelling the ground = ₹3 × 306 = ₹918

Que 2. There is a slide in a park, one of its side walls has been painted in some colour with a message "KEEP THE PARK GREEN AND CLEAN". If the sides of the walls are 15 m, 11 m and 6 m, find the area painted in colour.

Sol. Let the dimensions of triangular shape wall be

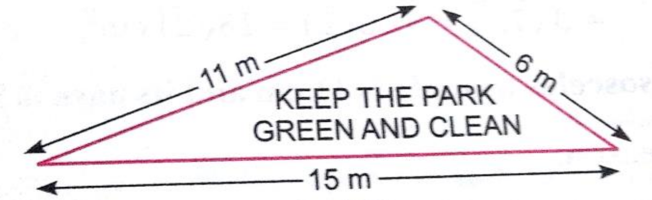

Fig. 12.3

a = 15 m, b = 6 m, c = 11 m

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{15+6+11}{2} = \frac{32}{2} = 16m$$

Area Painted = Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-15)(16-6)(16-11)}$$
$$= \sqrt{16 \times 1 \times 10 \times 5} = 20\sqrt{2}m^2$$

Que 3. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3: 2. Find the area of the triangle.

Sol. Let each of the equal side of isosceles triangle = 3x cm
and base of isosceles triangle = 2x cm
∴ Perimeter = 3x + 3x + 2x
32 = 8x ⇒ x = 4

∴ Sides are 3 ×4, 3× 4, 2 × 4 i.e., 12 cm, 12 cm, 8 cm

Now, $s = \frac{a+b+c}{2} = \frac{12+12+8}{2} = 16\ cm$

∴ Area of triangle = $\sqrt{s(-a)(s-b)(s-c)}$

$= \sqrt{16(16-12)(16-12)(16-8)}$

$= \sqrt{16 \times 4 \times 4 \times 8} = 32\sqrt{2} cm^2$

Que 4. In a rectangular field of diameters 60 m x 50 m, a triangular park is constructed. If the dimensions of the park is 50 m, 45 m and 35 m, find the area of the remaining field.

Sol. Area of rectangular field = length × breadth = 60 × 50 = 3,000 m^2

Now, a = 50 m, b = 45 m and c = 35 m

$$s = \frac{a+b+c}{2} = \frac{50+45+35}{2} = \frac{130}{2} = 65\ m$$

By Heron's formula:
∴ Area of triangle = $\sqrt{s(-a)(s-b)(-c)}$

$= \sqrt{65(65-50)(65-45)(65-35)} = \sqrt{65 \times 15 \times 20 \times 30}$

$= \sqrt{13 \times 5 \times 5 \times 3 \times 5 \times 2 \times 2 \times 5 \times 2 \times 3}$

$= 5 \times 5 \times 3 \times 2\sqrt{13 \times 2}$

$= 150\sqrt{26} = 764.85\ m^2$ (approximately)

Hence, the remaining area
= Area of rectangle – Area of triangle
3,000 – 764.85 = 2,235.15 m^2

Que 5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, then the area of the rhombus is 96 cm^2. State whether the statement is True or False and justify your answer.

Sol.

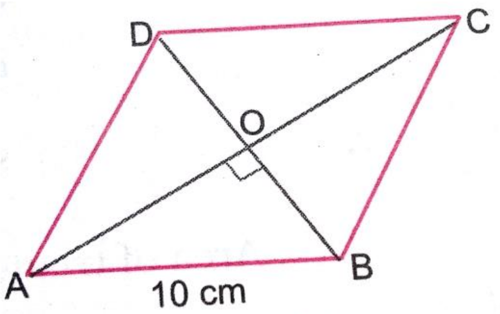

Fig. 12.4

True. AC = 16 cm

BD =? And AB = 10 cm

As the diagonals of a rhombus bisect each other at 90°

∴ OA = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 16 = 8 cm

OB = $\frac{1}{2}$ BD

∴ OA^2 + OB^2 **=** AB^2

$8^2 + OB^2 = 10^2$ ⇒ $OB^2 = 100 – 64$

$OB^2 = 36$ ⇒ OB = 6 cm

∴ BD = 2 × OB = 2×6 = 12 cm

Area of rhombus = $\frac{1}{2}$ AC × BD= $\frac{1}{2}$ ×16× 12 = 96cm^2

Que 6. An umbrella is made by stitching 10 triangular pieces of cloth of two different designs, each piece measuring 20 cm, 50 cm. How much cloth of each design is required for the umbrella?

Sol. The sides of triangular pieces are 20 cm, 50 cm and 50 cm.

Let, a = 20 cm, b = 50 cm, c = 50 cm

∴ Semi-Perimeter, $s = \frac{a+b+c}{2} = \frac{20+50+50}{2}$

$$s = 60\ cm$$

∴ Area Of one triangular Piece $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-)(60-50)(60-50)}$

$= \sqrt{60 \times 40 \times 10 \times 10} = 200\sqrt{6}cm^2$

Cloth of each design required = Area of 5 triangular pieces

$= 5 \times 200\sqrt{6} = 1000\sqrt{6}cm^2$

Que 7. The sides of a triangular field are 41 m, 40 m and 9 m. Find the number of rose beds that can be prepared in the field, If each rose bed on an average needs 900cm^2 space.

Sol. Let a = 41 m, b = 40 m, c = 9 m.

$$s = \frac{a+b+c}{2} = \frac{41+40+9}{2} = \frac{90}{2} \qquad \Rightarrow\ s = 45\ m$$

Area of the triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{45(45-41)(45-40)(45-9)}$$

$$= \sqrt{45 \times 4 \times 5 \times 36} = 180\ m^2\ = 1800000\ cm^2$$

Number of rose beds $= \dfrac{Total\ area}{Area\ needed\ for\ one\ rose\ bed} = \dfrac{1800000}{900}$ = 2000

Long Answer Type Questions

[4 Marks]

Que 1. Sides of a triangle are in the ratio of 12:17:25 and its perimeter is 540 cm. Find its area.

Sol. Let the sides of the triangle be 12x, 17x and 25x
Perimeter of the triangle = 540 cm
∴ 12x + 17x + 25x = 540
⇒ 54x = 540 ⇒ x = 10
Let a = 12x = 12 × 10 = 120 cm
b = 17x = 17 × 10 = 170 cm
c = 25x = 25 × 10 = 250 cm

∴ $s = \frac{a+b+c}{2} = \frac{120+170+250}{2} = \frac{540}{2}$ = 270 cm

∴ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{270(270-120)(270-170)(270-250)}$$
$$= \sqrt{270 \times 150 \times 100 \times 20} = 100\sqrt{27 \times 15 \times 20}$$
$$= 100 \times 9 \times 5 \times 2 = 9000\ cm^2$$

Que 2. A traffic signal board indicating 'SCHOOL AHEAD' is an equilateral triangle with side a. Find the area of the signal board, using Heron's formula. If its perimeter is 180 cm, what will be the area of the signal board?

Sol.

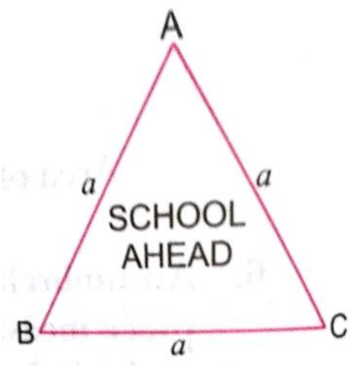

Fig. 12.5

Perimeter of the signal board,

$2s = a + a + a \qquad \Rightarrow \; s = \frac{3}{2}a$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{\frac{3a}{2}\left(\frac{3}{2}a - a\right)\left(\frac{3}{2}a - a\right)\left(\frac{3}{2}a - a\right)}$$

$$= \sqrt{\frac{3a}{3} \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}} = \sqrt{\frac{3a^4}{16}} = \frac{\sqrt{3}}{4}a^2 \ sq.units$$

Now, if perimeter = 180 cm

3a = 180 ⇒ a = 60 cm

∴ Area of signal board $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times (60)^2 = 900\sqrt{3}cm^2$

Que 3. The lengths of the sides of a triangle are 7 cm, 13 cm and 12 cm. Find the length of perpendicular from the opposite vertex to the side whose length is 12 cm.

Sol.

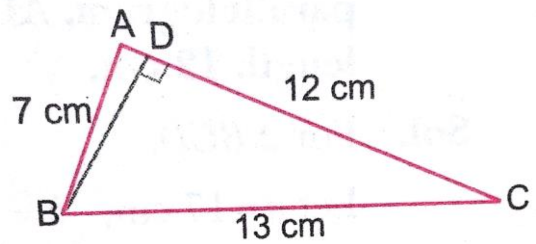

Fig. 12.6

Let, a = 7 cm, b = 13 cm, c = 12 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{7+13+12}{2} = \frac{32}{2} = 16\ cm$$

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-7)(16-13)(16-12)}$$

$$= \sqrt{16 \times 9 \times 3 \times 4} = 24\sqrt{3}cm^2$$

Also, Area of ΔABC = $\frac{1}{2}$ AC.BD

$$24\sqrt{3} = \frac{1}{2} \times 12 \times BD \quad \Rightarrow \quad BD = \frac{24\sqrt{3}\times 2}{12} = 4\sqrt{3}\ cm$$

Que 4. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm. Find the cost of polishing the tiles at the rate of 50 p per cm^2.

Sol. Measures of the sides of the triangular tile are 28 cm, 9 cm and 35 cm.

Let a = 28 cm, b = 9 cm, c = 35 cm

Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{28+9+35}{2}$ = 39 cm

∴ Area of one triangular tile = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{36(36-28)(36-9)(36-35)}$

$= \sqrt{36 \times 8 \times 27 \times 1} = 36\sqrt{6}cm^2$

So, area of 16 triangular tile = $16 \times 36\sqrt{6}cm^2 \;=\; 576\sqrt{6}cm^2$

$= 576 \times 2.45 = 1411.2\ cm^2$

Hence, cost of polishing the tiles at the rate of ₹ $\frac{1}{2}$ per cm^2 = ₹ $\frac{1}{2}$ × 1411.2

= ₹ 705.60

Que 5. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm, and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Sol. Let a = 26cm, b = 28cm, c = 30cm

∴ $s = \frac{a+b+c}{2} = \frac{26+28+30}{2} = \frac{84}{2} = 42$

∴ Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-26(42-28)(42-30)}$

$= \sqrt{42 \times 16 \times 14 \times 12}$

$= \sqrt{2 \times 3 \times 7 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 2 \times 2 \times 3}$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 336\ cm^2$

Now, Area of parallelogram = Area of triangle

⇒ Base × height = 336

⇒ 28 × height = 336

⇒ height $= \frac{336}{28}$ = 12 cm

Que 6. The length of two adjacent sides of a parallelogram are 17 cm and 12 cm. One of its diagonals is 25 cm long. Find the area of the parallelogram. Also find the altitude from vertex on the side of length 12 cm.

Sol.

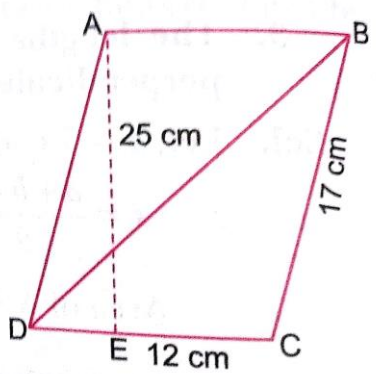

Fig. 12.7

For ΔBCD.

Let a= 17 cm. b = 12 cm, c = 25 cm

So its semi-Perimeter, $s = \frac{a+b+c}{2}$

$$= \frac{17+12+25}{2} = 27\ cm$$

∴ Area of $\Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{27(27-17)(27-12)(27-25)}$$
$$= \sqrt{27 \times 10 \times 15 \times 2} = 90cm^2$$

Now, area of parallelogram ABCD = 2 × Area of ΔBCD

= 2 ×90 = 180 cm^2

Also, area of parallelogram ABCD = DC × AE

∴ 180 = 12 × AE

⇒ AE $= \frac{180}{12} = 15\ cm$

Que 7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and side 6 cm each is to be made of three different shades as shown in Fig. 12.8. How much paper of each shade has been used in it?

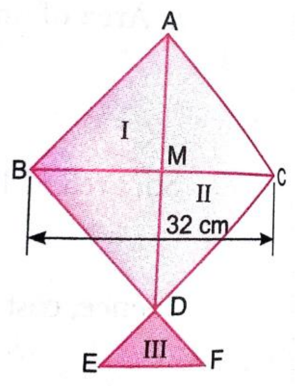

Fig. 12.8

Sol. As the diagonals of a square bisect each other at right angle

∴ AM = DM $= \frac{32}{2} = 16\ cm$

Area of shade I = Area of shade II = Area of ΔABD

$$= \frac{1}{2} \times AD \times BM = \frac{1}{2} \times 32 \times 16 = 256\ cm^2$$

For the area I = Area of shade III

Area of isosceles ΔDEF $= \frac{a}{4}\sqrt{4b^2 - a^2}$

$= \frac{8}{4}\sqrt{4}(6)^2 - \ 8^2$

$= 2\sqrt{144 - 64} = 2\sqrt{80} = 8\sqrt{5}cm^2$

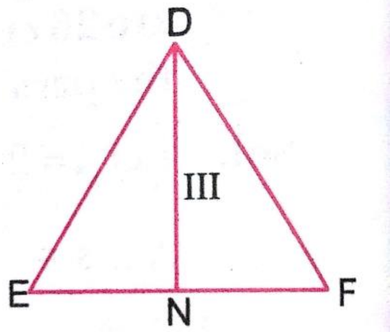

Fig. 12.9

Que 8. The perimeter of triangle is 50cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Sol. Let the length of the smaller side = x
According to the statement, other two sides of the triangle will be x + 4 and 2x -6
Perimeter of triangle = x + x + 4 + 2x -6
⇒ 50 = 4x – 2 ⇒ 4x = 52 ⇒ x = 13
∴ Sides of triangle are = 13, (13 + 4), (2 x 13 – 6) = 13 cm, 17 cm, 20 cm
Let, a = 13 cm, b = 17 cm and c = 20 cm

∴ $s = \frac{a+b+c}{2} = \frac{13+17+20}{2}$ = 25 cm

∴ Area of Δ $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{25(25-13)(25-17)(25-20)}$
$= \sqrt{25 \times 12 \times 8 \times 8} = \sqrt{5 \times 5 \times 3 \times 4 \times 4 \times 2 \times 5}$
$= 20\sqrt{30} = 20 \times 5.48 \ = 109.6\ cm^2$

Que 9. The area of a trapezium is 475 cm^2 and the height is 19 cm. Find the lengths of its parallel sides if one side is 4 cm greater than the other.

Sol.

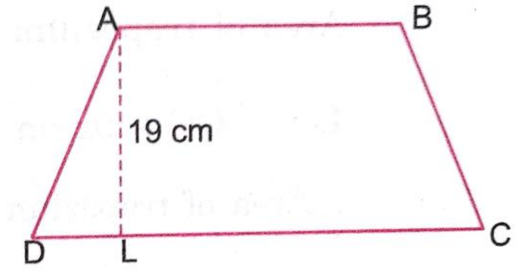

Fig. 12.10

Let AB = x cm $\therefore$ DC = x + 4

Area of trapezium ABCD $= \frac{1}{2}(AB + DC)AL$

$$475 = \frac{1}{2}(x + x + 4) \times 19$$

$950 = (2x + 4)19 \quad \Rightarrow 38x + 76 = 950$

$\Rightarrow$ 38x = 950 – 76 $\quad \Rightarrow x = \frac{874}{38} \quad \Rightarrow x = 23$

$\therefore$ Sides of trapezium are 23, 23 + 4 i.e., 23, 27 cm

Que 10. The triangle side walls of a flyover is used for advertisement. The sides of walls are 122 m, 22 m and 120 m as shown in figure. The advertisement yields an earning of ₹5000 per m^2 per year. A company fixed one of its wall for 3 months. How much rent did it ray?

Sol. Let the sides of triangle wall be

a = 122 m, b = 22 m, c = 120 m

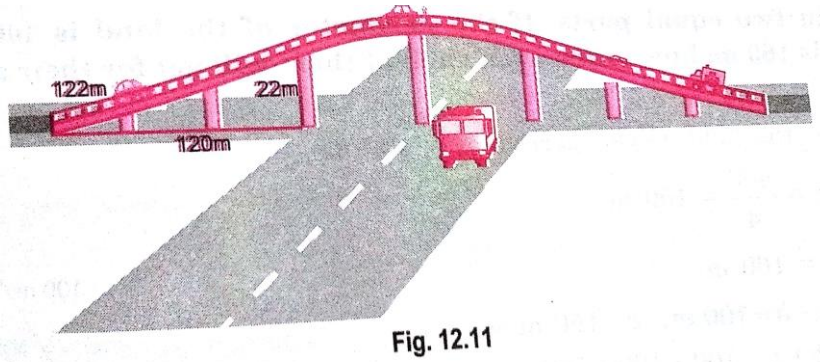

Fig. 12.11

$$\therefore s = \frac{a+b+c}{2} = \frac{122+22+120}{2} = \frac{264}{2} = 132\,m$$

$\therefore$ Area of triangle wall $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{132(132-122)(132-22)(132-120)}$$

$$= \sqrt{132 \times 10 \times 110 \times 12}$$

$$= \sqrt{12 \times 11 \times 10 \times 11 \times 10 \times 12}$$
$$= 10 \times 11 \times 12 = 1320\ m^2$$

Now, yearly rent = ₹5000 per m^2

$$\therefore\ Monthly\ rent\ = ₹\left(\frac{5000}{12}\right) Per\ m^2$$

$$\therefore\ Rent\ paid\ by\ company\ fo\ the\ three\ months\ = ₹\frac{5000}{12} \times 3 \times 1320$$

$$= ₹5000 \times 330\ = ₹16{,}50{,}000$$

HOTS (Higher Order Thinking Skills)

Que 1. The dimension of a rectangle ABCD are 51 cm x 25 cm. A trapezium PBCQ with its parallel sides QC and PB in the ratio 9: 8, is cut off from the rectangle as shown in the Fig. 12.12. If the area of the trapezium PBCQ is $\frac{5}{6}$ th part of the area of the area of the rectangle, Find the lengths of QC and PB.

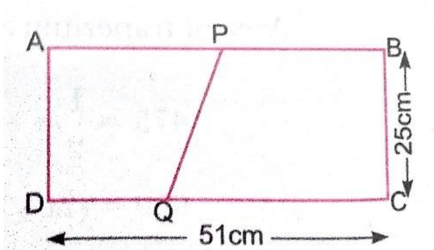

Fig. 12.12

Sol. Area of rectangle ABCD = AB x BC = 51 x 25 = 1275 cm^2

Area of trapezium PBCQ = $\frac{5}{6}$ 1275 = $\frac{6375}{6}$ cm^2

Let QC = $9x$ cm and PB = $8x$ cm

∴ Area of trapezium PBCQ = rrr (QC + PB) x BC

⇒ $\frac{6375}{6} = \frac{1}{2}(9x + 8x) \times 25$

⇒ $\frac{17x \times 25}{2} = \frac{6375}{6}$

⇒ $x = \frac{6375}{6} \times \frac{2}{17 \times 25}$

⇒ $x = 5$

∴ QC = 9 X 5 cm = 45 cm and PB = 8 X 5 cm = 40 cm

Que 2. Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops. She divided the land in two equal parts. If the perimeter of the land is 400 m and one of the diagonals is 160 m, how much area each of them will get for their crops?

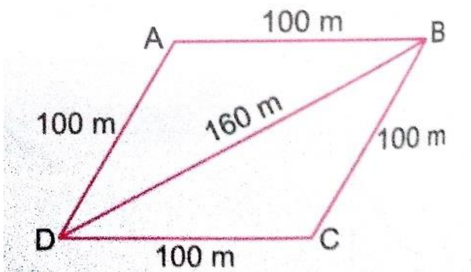

Fig. 12.13

Sol. Let ABCD be the filed. Give perimeter = 400 m

So, each side = $\frac{400}{4}$ = 100 m

Diagonal BD = 160 m

Let a = 100 m, b = 100 m c = 160 m

∴ s = $\frac{a+b+c}{2} = \frac{100+100+160}{2}$ = 180 m

Therefore, Area of ΔABD = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{180(180-100)(180-100)(180-160)}$

$= \sqrt{180 \times 80 \times 80 \times 20}$ = 4800 m^2

Alternative method:

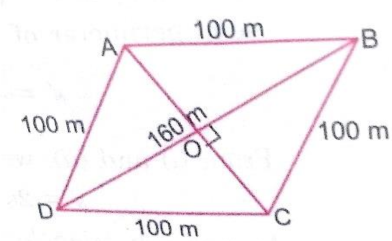

Fig. 12.14

As the diagonals of rhombus bisect each other: Therefore

OD = $\frac{1}{2}$ BD = $\frac{1}{2}$ x 160 = 80 m

OC = $\frac{1}{2}$ AC

In ΔOCD, we have,

$CD^2 = OC^2 + OD^2$

$100^2 = OC^2 + 80^2 \quad \Rightarrow \quad OC^2 = 10000 - 6400$

$\Rightarrow \quad OC^2 = 3600 \quad \Rightarrow \quad OC = 60$ m

Therefore, area of ΔBCD = $\frac{1}{2}$ (BD x OC) = $\frac{1}{2}$ x 160 x 60 = 4800 m^2

∴ Each of them will get 4800 m^2 of area for their crops.

Que 3. A field is in the shape of a trapezium, its parallel sides are 25 m and 10 m and non-parallel sides are 14 m and 13 m. Find the area of the field.

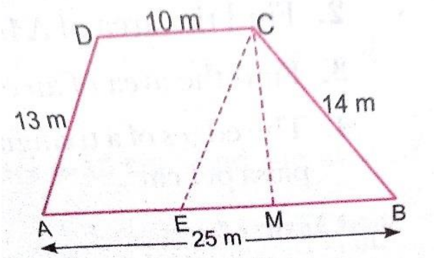

Fig. 12.15

Sol. Let ABCD be a trapezium, with parallel sides AB = 25 m, CD = 10 cm and non-parallel sides

BC = 14 m and AD = 13 m.

Draw CM ⊥ AB and CE || AD.

For ΔBCE

BC = 14 m

CE = AD = 13 m

BE = AB – AE

= 25 – 10 = 15 m (∵ AE = CD = 10 m)

Now, Let a = 14 m, b = 13 m and c = 15 m

∴ Semi-perimeter (s) = $\frac{a+b+c}{2} = \frac{14+13+15}{2}$ = 21 m

∴ Area of Δ BCE = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-13)(21-15)}$

$= \sqrt{21 \times 7 \times 8 \times 6} = \sqrt{3 \times 7 \times 7 \times 2 \times 2 \times 2 \times 2 \times 3}$

= 2 x 2 x 3 x 7 = 84 m^2

Also, area (Δ BCE) = $\frac{1}{2}$ x BE x CM   ⇒   84 = $\frac{1}{2}$ x 15 x CM

⇒   CM = $\frac{2 \times 84}{15}$ ⇒   CM = $\frac{56}{5}$ m

Now, area of parallelogram AECD = base x altitude

= AE x CM = 10 x $\frac{56}{5}$ m^2 = 112 m^2

∴   Area of trapezium = area of parallelogram AECD + area of ΔBCE

= 112 + 84 = 196 m^2.

Que 4. If each side of a triangle is doubled, then find the ratio of area of new triangle thus formed and the given triangle.

Sol. Let a, b, c be the side of the given triangle and s be its semi-perimeter.

Then,   s = $\frac{a+b+c}{2}$ …(i)

∴   Area of the given triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ = Δ . Say

According to the question, the sides of the new triangle will be 2a, 2b and 2c. Let s' be the semi-perimeter of the new triangle.

S' = $\frac{2a+2b+2c}{2}$ = a + b + c   …(ii)

From (i) and (ii), we get

S' = 2s

Area of new triangle = $\sqrt{s'(s'-2a)(s'-2b)(s'-2c)}$

= $\sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$

= $\sqrt{16s(s-a)(s-b)(s-c)}$

= $\sqrt{s(s-a)(s-b)(s-c)}$ = 4Δ

Therefore, the required ratio is 4: 1

Value Based Questions

Que 1. The perimeter of an isosceles triangle is 25 cm and its base is 7 cm. The teacher asked the students to find its area. Sapna answered $\frac{7}{4}\sqrt{15}\ cm^2$. Is she correct? Justify. Which values are depicted here?

Sol. Yes, 2b + a = 15
⇒ 2b + 7 = 15 ⇒ b = 4

∴ Area of isosceles triangle = $\frac{7}{4}\sqrt{4b^2 - a^2} = \frac{7}{4}\sqrt{4 \times 4^2 - 7^2}$

$$=\frac{7}{4}\sqrt{64 - 49} = \frac{7}{4}\sqrt{15}cm^2$$

Curiosity, knowledge, truthfulness.

Que 2. An umbrella is made by stitching 10 triangular pieces of cloth of two different designs, each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each design is required by Mr. Amit if he wants to donate 20 such umbrellas to the children of slum areas?

Sol. The sides of triangular pieces are 20 cm, 50 cm and 50 cm.
Let, a = 20 cm, b = 50 cm, c = 50 cm

∴ Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{20+50+50}{2}$

S = 60 cm

∴ Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{60(60-20)(60-50)(60-50)}$

$= \sqrt{60 \times 40 \times 10 \times 10} = 200\sqrt{6}cm^2$Cloth of each design

required for one umbrella = Area of 5 triangular pieces

= $5 \times 200\sqrt{6} = 1000\sqrt{6}cm^2$

Cloth of each design required for 20 umbrella = $20 \times 1000\sqrt{6} = 20{,}000\sqrt{6}cm^2$
Helpful, caring, loving.

Que 3. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and side 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?
How much paper of each shade is required by Arushi if she wants to donate 40 kites to the children of an orphanage? Which values does Arushi possess?

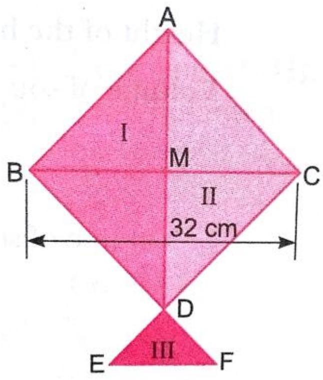

Fig. 13

Sol. As the diagonals of a square are equal bisect each other at right angle

∴ AD = BC =32 cm and AM = DM = $\frac{32}{2} = 16\ cm$

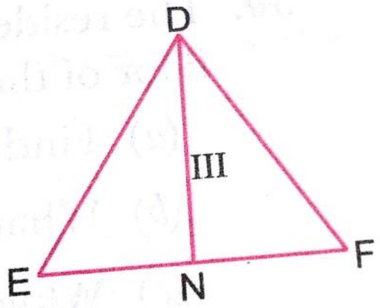

Fig. 14

Area of shade I = Area of shade II

= Area of ΔABD = $\frac{1}{2} \times AD \times BM$

$= \frac{1}{2} \times 32 \times 16 = 256\ Cm^2$

For the area of shade III

Area of isosceles ΔDEF = $\frac{a}{4}\sqrt{4b^2 - a^2}$

$= \frac{8}{4}\sqrt{4(6)^2 - 8^2} = 2\sqrt{144 - 64}$

$= 2\sqrt{80} = 8\sqrt{5}cm^2$

Area of shade I = Area of shade II = 256 cm^2

∴ Area of sheet of shade I required for making 40 kites

= Area of sheet of shade II required for making 40 kites

= 40 × 256 = 10240 cm^2

Area of sheet of shade III required for making 40 kites = $40 \times 8\sqrt{5} = 320\sqrt{5}cm^2$

Social, loving, caring.

Que 4. A craft mela is organised by welfare Association to promote the art and culture of tribal people. The pandal is to be decorated by using triangular flags around the field. Each flag has dimension 25 cm, 25 cm and 22 cm. Find the

area of cloth required for making 200 such flags. Which values are depicted here?

Sol. Area of cloth required for one flag

$$= \sqrt{s(s-25)(s-25)(s-22)},\text{ where } s = \frac{25+25+22}{2} = 36 \text{ cm}$$

Area of cloth required $= \sqrt{36(36-25)(36-25)(36-22)}$

$$= \sqrt{36 \times 11 \times 11 \times 14}$$

$$= 6 \times 11\sqrt{14}\ cm^2 = 66\sqrt{14}$$

Area of cloth required for 200 such flags = $66\sqrt{14} \times 200 = 13200\sqrt{14}cm^2$

Helpfulness, cooperation, beauty.

Que 5. A person donates cylindrical bowls of diameter 7 cm to a charitable hospital in which soup is served to patients. If the bowl is filled with soup to a height of 4 cm, how much soup needs to be prepared daily to serve 250 patients? Which values of the person are depicted here?

Sol. Radius of cylindrical bowl = $\frac{7}{2}cm = 3.5\ cm$

Height of the filled with soup (h) = 4 cm

Volume of soup for 1 patient = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 = 154\ cm^3$$

∴ Volume of soup for 250 patients = 250 ×154 cm^3 = 38500 cm^3

$$= \frac{38500L}{1000} \qquad (\therefore 1L = 1000\ cm^3)$$

= 38.5 L

The person is kind heated, caring and contributing for the welfare of society.

Que 6. The resident of society decided to paint the hall of cancer detective centre in their premises. If the floor of the cuboidal hall has a perimeter equal to 260 m and height 6 m then

(a) Find the cost of painting of its four walls (including doors etc.) at the rate of ₹9 per m^2.

(b) What is the amount contributed by 50 people?

(c) Which value is depicted by the residents?

Sol. Perimeter = 2(l + b) = 260

= l + b = 130

(a) Surface area of four walls = 2h (l + b) = 2 × 6 × 130 = 1560 m^2

Cost of painting = 9 × 1560 = ₹14,040

(b) Amount contributed = $\frac{14040}{50}$ = ₹280.8

(c) Cooperation, social cohesion.

Que 7. A person pays ₹2200 to children to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate ₹20 per m^2, find
(i) inner curved surface area of the vessel, (ii) radius of the base,
(iii) capacity of the vessel.
Which social is the person violating?

Sol. (i) Inner curved surface area of the vessel = $\frac{Total\ \text{cost}\ of\ painting}{Cost\ of\ painting\ per\ m^2}$

$$= \frac{₹2200}{20₹/m^2} = 110\ m^2$$

(ii) Let the radius of the base of the cylindrical vessel be r m.
Depth of the cylindrical vessel (h) = 10 m
Curved surface area of the cylindrical vessel = 2πrh

∴ 2πrh = 110⇒ $2 \times \frac{22}{7} \times r \times 10 = 110$

⇒ r = $\frac{110\times7}{2\times22\times10}$ = 1.75 cm

(iii) Capacity of the cylindrical vessel = $\pi r^2 h$

$$= \frac{22}{7} \times (1.75)^2 \times 10\ m^3 = 96.25\ m^3$$

Child labour is abolished under the law. So, the person is violating this law.