Very Short Answer Type Questions

[1 mark]

Que 1. State which of the following formulae is/are incorrect and justify your statement.
(i) V = 6a^2 b. **(ii) V = ab + c** **(iii) A = a (b + c)**

Sol. (ii) V = ab + c, is incorrect
As volume is three-dimensional, so each term in the formula must have a product of three letter terms.

Que 2. Find the volume of the sphere in term of π whose diameter is 6 cm.

Sol. Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (3)^2 = 36\pi$ cubic cm.

Que 3. What is the number of surfaces of a right circular cylinder?

Sol. Three.

Que 4. Find the ratio of surface area and volume of the sphere of unit radius.

Sol. Required ratio $= \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3\times4\times\pi\times(1)^2}{4\times\pi\times(1)^3} = \frac{3}{1}$ $(\because r = 1)$ i.e., 3: 1

Que 5. A cube and a sphere are of the same height. Find the ratio of their volume.

Sol. $\frac{Volume\ of\ cube}{Volumeof\ the\ sphere} = \frac{a^3}{\frac{3}{4}\pi\left(\frac{a}{2}\right)^3} = \frac{6}{\pi}$

(Let edge of cube be a then radius of sphere= $\frac{a}{2}$)

∴ Required ratio = 6: π

Que 6. The radius of sphere is 2r, then find its volume.

Sol. Volume of the sphere $= \frac{4}{3}m(2r)^3 = \frac{32}{3}\pi r^3$

Que 7. Find the total surface area of a cone whose radius is $\frac{r}{2}$ and slant height 2l.

Sol. Total surface area of the cone = $\pi\left(\frac{r}{2}\right)\left(2l + \frac{r}{2}\right) = \pi r\left(l + \frac{r}{4}\right)$.

Que 8. Find the volume in terms of π of a conical vessel with radius 7 cm and slant height 25 cm.

Sol. ∵ $l^2 = r^2 + h^2$
$\Rightarrow (25)^2 = 7^2 + h^2$
$\Rightarrow h^2 = 625 - 49 \Rightarrow h = \sqrt{576} = 24\ cm$

Volume of the vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 7^2 \times 24$

$= \frac{1}{3}\pi \times 49 \times 24$

$= \pi \times 49 \times 8 = 392\pi\ cm^3.$

Que 9. Nidhi has to find the area of a sphere whose diameter was 14 cm. She wrote down area $= \boldsymbol{4\pi r\ cm^2 = 4 \times \frac{22}{7} \times \frac{7}{2}\ cm^2 = 44\ cm^2}$

Kartika knew nothing about sphere formula but was able to tell Nidhi that she was wrong. How did he know?

Sol. Area is two-dimensional while $4\pi r$ represents a length.

Que 10. Find the ratio of the lateral surface area and total surface area of a cube.

Sol. Lateral surface area of cube: Total surface area of cube = $4a^2$: $6a^2$ = 2: 3.

Que 11. Find the volume of the right circular cone with radius 3.5 cm and height 12 cm.

Sol. Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 12\ cm^{3 = 154\ cm^2}$

Que 12. If the radius of a sphere is doubled what will happen to its surface area?

Sol. Surface area of sphere $= 4\pi r^2$

When radius is doubled then new surface area $= 4\pi(2r)^2 = 4\pi \times 4r^2$

$= 4(4\pi r^2)$

$= 4 \times$ Original surface area.

∴ Surface area becomes 4 times.

Short Answer Type Questions – I

[2 marks]

Que 1. The surface area of cuboid is 1792 sq cm. If its length, breadth and height are in the ratio 4: 2: 1, then find the length of the cuboid.

Sol. Let the height = x cm, then breadth = 2x cm
length = 4x cm
According to formula, 2(lb + bh+ lh) = 1792
$2(8x^2 + 2x^2 + 4x^2) = 1792$
$28x^2 = 1792$

$$\Rightarrow\ x^2 = \frac{1792}{28} = 64 \quad \Rightarrow x = 8$$

Length = 8 × 4 = 32 cm

Que 2. If the radius of a cylinder is doubled and height is halved, then show that the volume will be doubled.

Sol. Volume of cylinder $= \pi r^2 h$

New volume $= \pi R^2 H = \pi(2r)^2 \left(\frac{h}{2}\right) = 2(\pi r^2 h)$

∴ New volume = 2 × original volume.

Que 3. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volume.

Sol. v^1 (volume of cone) $= \frac{1}{3\pi r^2 r}$

v^2 (volume of hemisphere) $= \frac{2}{3}\pi r^2$

v^3 (volume of cylinder) $= \pi r^2 . r$

$$v^1 : v^2 : v^3 = \frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1$$

$$v^1 : v^2 : v^3 = 1 : 2 : 3$$

Que 4. The areas of three adjacent faces of a cuboid are A_1, A_2 and A_3. If its volume is V, prove that $V^2 = A_1 . A_2 . A_3$

Sol. Let, length, breadth and height of the cuboid be a, b and c respectively.
∴ Volume = abc
Also, A_1 = ab, A_2 = bc and A_3 = ca

∴ $A_1 . A_2 . A_3 . = (ab)(bc)(ca) = a^2 b^2 c^2 = (abc)^2 = V^2$

⇒ $V^2 = A_1 . A_2 . A_3 .$

Que 5. Prove that the volume of the largest right circular cone that can be fitted in a cube having edge 2r equals to the volume of a hemisphere of radius r.

Sol. Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi r^2 (2r) \ = \frac{2}{3}\pi r^3$$

= volume of the hemisphere.

Que 6. Find the length of the longest rod that can be placed in a room 12 m × 9 m × 8 m.

Sol. Length of the longest rod = Diagonal of the room

$$= \sqrt{a^2 + b^2 + c^2} = \sqrt{12^2 + 9^2 + 8^2}$$
$$= \sqrt{144 + 81 + 64} = \sqrt{289} = 17\ m$$

Que 7. The curved surface area of a right circular cylinder of height 14 cm is 88cm². Find the diameter of the base of the cylinder.

Sol. Curved surface area of cylinder = $2\pi rh$

$$\Rightarrow \ 88 \ = \ 2 \ \times \ \frac{22}{7} \times r \times 14 \quad \Rightarrow \ r = \frac{88 \times 7}{2 \times 22 \times 14} = 1$$

∴ Diameter of the base of cylinder = 2r = 2 × 1 = 2 cm

Que 8. Mukta had to make a modal of a cylinder kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. What would be the area of chart paper requires by her, if she wanted to make a kaleidoscope of length 25 cm with 3.5 cm radius?

Sol. Radius of the base of the cylindrical kaleidoscope = r = 3.5 cm
Height of kaleidoscope = h = 25 cm
Chart paper required = curved surface area of kaleidoscope

$$= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 25 = 550\ cm^2$$

Que 9. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet is required for the same?

Sol. Radius of the closed cylindrical tank $= \frac{140}{2}$ cm = 70 cm = 0.7 m

Height of the closed cylindrical tank = 1 m

Area of metal sheet required = $2\pi r(r + h) = 2 \times \frac{22}{7} \times 0.7(1 + 0.7) = 7.48\ m^2$

Que 10. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Sol. Radius of cone (r) = $\frac{105}{2}$ cm

Slant height of cone (l) = 10 cm

Curved surface area of cone = $\pi rl = \frac{22}{7} \times \frac{10.5}{2} \times 10 = 165\ cm^2$.

Que 11. Find the radius of a sphere whose surface area is 154 cm^2.

Sol. Let 'r' be the radius of sphere

Surface area of sphere = $4\pi r^2$

⇒ $154 = 4\pi r^2$ ⇒ $154 = 4 \times \frac{22}{7} \times r^2$

⇒ $r^2 = \frac{154 \times 7}{4 \times 22} = \frac{49}{4}$

⇒ $r = \frac{7}{2} cm = 3.5\ cm$

Que 12. A hemispherical bowl is made of steel 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Sol. Outer radius of hemispherical bowl = R = r + 0.25

= 5 + 0.25 = 5.25 cm

Outer curved surface area of bowl = $2\pi R^2 = 2 \times \frac{22}{7} \times (5.25)^2$

= 2 × 22 × 5.25 × 0.75 = 173.25 cm^2

Que 13. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many liters of water can it hold? (1 m^3 = 1000 L).

Sol. Volume of cuboidal tank = l × b × h

= 6 m × 5 m × 4.5 m = 135 m^3

= 135 × 1000 L = 135000 L

Que 14. A storage tank is in the form of a cube. Where it is full of water, the volume of water is 15,625 m^3. If the present depth of water is 1.3 m, find the volume of water already used from the tank.

Sol. Volume of water in cubical storage tank = 15, 625 m^3 = $(2.5\ m)^3 = a^3$

⇒ a = 2.5 m

Volume of water in cubical tank when depth of water is 1.3 m

$= 2.5\times 2.5 \times 1.3 = 8.125$ m^3

Volume of water already used from the tank = 15.625 - 8.125 = 7.5 m^3

Que 15. How many planks each of which is 2 m long, 3 cm broad and 4 cm thick can be cut-off from a wooden block 6 m long, 18 cm broad and 44 cm thick?

Sol. 2 m = 2 × 100 cm = 200 cm, 6 m = 6 × 100 = 600 cm

Number of planks = $\frac{Volume\ of\ wooden\ block}{Volume\ of\ each\ plank} = \frac{600\times18\times44}{200\times3\times4}$ = 198.

Que 16. Find the amount of water displaced by a solid spherical ball of diameter 0.21 m.

Sol. Radius of spherical ball (r) = $\frac{0.21}{2}$ m

Water displaced by spherical ball = Volume of spherical ball

$$= \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times\frac{0.21}{2}\times\frac{0.21}{2}\times\frac{0.21}{2}$$

$= 0.004851$ m^3

Que 17. Find the volume of a sphere whose surface area is 154 cm^2.

Sol. Let r cm be the radius of sphere.

Surface area of the sphere = $4\pi r^2$

$\Rightarrow$ $154 = 4\pi r^2$ $\qquad\Rightarrow 4\times\frac{22}{7}\times r^2 = 154$

$r^2 = \frac{154\times7}{4\times22} = \frac{7^2}{2^2}$ $\qquad\Rightarrow r = \frac{7}{2}$

Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{72}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}cm^3$$

$$= \frac{539}{3}\ cm^3 = 179\frac{2}{3}cm^3.$$

Short Answer Type Questions – II

[3 marks]

Surface Area of a Cube Cuboid

Que 1. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white-washing the walls of the room and ceiling at the rate of 7.50 per m^2.

Sol. Area of four walls = 2h (l + b)
Here, l = 5 m, b = 4 m and h = 3 m
Area of four walls = 2 ×(5 + 4) = 54 m^2
Area of ceiling = l × b = 5× 4 = 20 m^2
Total area to be white-washed = 54 + 20 = 74 m^2
Cost of white-washing of 1 square metre = ₹7.50
∴ Cost of white-washing = ₹74 × 7.50 = ₹555.

Que 2. Shahid has built a cubical water tank with lid for his house, with each edge 2 m. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if one dozen of tiles costs him ₹480.

Sol. Edge of tank = 2 m = 2 × 100 cm = 200 cm
Area of five faces of the tank $5a^2 = 5\ (200\ cm)^2 = 2,00,000\ cm^2$
Area of a square tile = 25 cm × 25 cm = 625 cm^2

Number of tiles required $= \frac{Area\ of\ five\ walls}{Area\ of\ a\ tile} = \frac{200000}{625} = 320 = \frac{320}{12} dozen$

Cost of one dozen of tiles = ₹480

∴ Cost of $\frac{320}{12}$ dozen tiles $=$ ₹$480 \times \frac{320}{12} =$ ₹12,800

Que 3. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held with tape. It is 30 cm long, 25 cm wide and 25 cm high.
(i) Which is the area of the glass?
(ii) How much of tape is needed for all the 12 edges?

Sol. Here, l = 30 cm, b = 25 cm, h = 25 cm
(i) Area of the glass = 2 (lb + bh + hl)
= 2 (30×25+25×25+25×30) = 2(750+625+750)
= 4250 cm^2
(ii) Tape needed for all the 12 edges = 4(30 + 25 + 25) = 320 cm.

Que 4. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.
(i) Which box has greater lateral surface area and by how much?
(ii) Which box has smaller total surface area and by how much?

Sol. (i) Lateral surface area of cubical box = $4a^2 = 4 \times 10^2 = 400$ cm^2
Lateral surface area of cuboidal box = 2h (l + b)
= $2\times8(12.5 + 10) = 16 \times 22.5 = 360$ cm^2
Thus, lateral surface area of cubical box is greater by
(400 cm^2 – 360 cm^2) = 40 cm^2
(ii) Total surface area of cubical box = $6a^2 = 6 \times 10^2$ cm^2 = 600 cm^2
Total surface area of cuboidal box = 2 (lb + bh + hl)
= $2(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)$
= $2(125 + 80 + 100) = 2 \times 305$ cm^2 = 610 cm^2
Thus, total surface area of cuboidal box is greater by (610 – 600) cm^2
= 10 cm^2

Surface Area of Right circular Cylinder

Que 1. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find
(i) its inner curved surface area,
(ii) the cost of plastering this curved surface at the rate of ₹40 per m^2.

Sol. Radius of well = (r) = $\frac{3.5}{2}$ m
Depth of well = (h) 10 m
(i) Inner curved surface area of well = $2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10 = 110$ m^2

(ii) Cost of plastering 1 m^2 = ₹40
∴ Cost of plastering 110 m^2 = ₹110 × 40 = ₹4400

Que 2. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolution to move once over the level of a playground. Find the area of the playground in m^2.

Sol. Radius of roller = $\frac{84}{2}$ cm = 42 cm

Length of roller = 120 cm

∴ Curved surface area of roller = $2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680$ cm^2

As the roller takes 500 complete revolutions to level the playground
∴ Area of playground = 500 × 31680 cm^2

$$= 15840000\, cm^2 = \frac{15840000}{10000} m^2 = 1584\, m^2$$

Que 3. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholder in the shapes of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply competitions with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Sol. Radius of penholder = r = 3 cm
Height of penholder = h = 10.5 cm
Total surface area of penholder = $\pi r^2 + 2\pi rh = \pi r\,(r + 2h)$

$$= \frac{22}{7} \times 3\,(3 + 2 \times 10.5) = \frac{66}{7} \times 24\ cm^2$$

Cardboard required for 35 competitors = $35 \times \frac{66}{7} \times 24 = 7920\ cm^2$

Surface Area of Cone

Que 1. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white-washing its curved surface at the rate of ₹210 per 100 m^2.

Sol. Radius of the base of the conical tomb (r) = $\frac{14}{2} m = 7\ m$

Slant height of conical tomb (l) = 25 m

Curved surface area of conical tomb = $\pi rl = \frac{22}{7} \times 7 \times 25 = 550\ m^2$

Cost of white-washing 1 m^2 = ₹ $\frac{210}{100}$ = ₹2.1

∴ Cost of white-washing 550 m^2 = ₹550 × 2.1 = ₹1155

Que 2. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Sol. Radius of the base of the conical cap (r) = 7 cm
Height of the conical cap (h) = 24 cm
Let 'l' be the slant height of the conical cap. Then

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$$

$$l = \sqrt{625} = 25$$

Area of the sheet required for a cap = curved surface area of a conical cap

$$= \frac{22}{7} \times 7 \times 25\ cm^2 = 550\ cm^2$$

∴ Sheet required for 10 caps = 550 × 10 cm^2 = 5500 cm^2

Que 3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find
(i) radius of the base (ii) total surface area of the cone.

Sol. Slant height of the cone (l) = 14 cm
Curved surface area of the cone = 308 cm^2
Let 'r' be the radius of the base of cone
(i) Curved surface area of cone = πrl

∴ $\frac{22}{7} \times r \times 14 = 308$

⇒ $r = \frac{308 \times 7}{22 \times 14}$ = 7 cm

(ii) Total surface area of cone = πr (r + l) = $\frac{22}{7} \times 7\ (7 + 14) = \frac{22}{7} \times 7 \times 21 = 462\ cm^2$

Que 4. The radius and slant height of a cone are in the ratio 4: 7. If its curved surface area is 792 cm^2, find its radius.

Sol. Let the radius of cone (r) = 4x cm
and the slant height of the cone (l) = 7x cm
Curved surface area of cone = πrl

∴ πrl = 792 cm^2 ⇒ $\frac{22}{7} \times 4x \times 7x = 792$

⇒ $x^2 = \frac{792}{22 \times 4} = 9$ ⇒ x = 3 cm

∴ Radius of the cone = 4 × 3 = 12 cm

Que 5. What length of 5 m wide cloth will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m?

Sol. Radius of the base of the conical tent (r) = 7 m
Height of the conical tent (h) = 24 m
Let 'l' be the slant height of the cone then

$$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25\ m$$

Curved surface area of the conical tent = πrl = $\frac{22}{7} \times 7 \times 25 = 550\ m^2$

∴ Area of the cloth used = 550 m^2

Length of 5 m wide cloth used = $\frac{Area}{Width} = \frac{550\ m^2}{5\ m}$ = 110 m

Que 6. A cylinder and a cone have equal height and equal radii of their bases. If their curved surface areas are in the ratio 8: 5. Show that the ratio of radius to height of each is 3: 4.

Sol. $\frac{Curved\ surface\ area\ of\ cylinder}{Curved\ surface\ area\ of\ cone} = \frac{2\pi rh}{\pi rl} = \frac{2\pi rh}{\pi r\sqrt{r^2+h^2}}$

$\frac{8}{5} = \frac{2h}{\sqrt{r^2+h^2}}$

$\Rightarrow$ $\frac{64}{25} = \frac{4h^2}{r^2+h^2}$ $\Rightarrow$ $64r^2 + 64h^2 = 100h^2$

$\Rightarrow$ $64r^2 = 100h^2 - 64h^2 \Rightarrow$ $64r^2 = 36h^2$

$\Rightarrow$ $\frac{r^2}{h^2} = \frac{36}{64} = \frac{9}{16}$ $\Rightarrow$ $\frac{r}{h} = \frac{3}{4}$

$\therefore$ r: h = 3: 4

Surface Area of Sphere

Que 1. The diameter of the moon is approximately one-fourth of the diameter of the earth. Find the ratio of their surface areas.

Sol. $D_{moon} = \frac{1}{4} D_{earth}$

$\Rightarrow$ $2. R_{moon} = \frac{1}{4} . 2\ R_{earth}$

$\Rightarrow$ $R_{earth} = 4\ R_{moon}$

$\frac{Surface\ area\ of\ moon\ (S_{moon})}{Surface\ area\ earth\ (S_{earth})} = \frac{4\pi R^2{}_{moon}}{4\pi R^2{}_{earth}}$

$\Rightarrow \frac{S_{moon}}{S_{earth}} = \frac{R^2{}_{moon}}{(4R_{moon})^2} = \frac{1}{16} \frac{R^2{}_{moon}}{R^2{}_{moon}} = \frac{1}{16}$

$\therefore$ $S_{moon} : S_{earth} = 1 : 16$

Que 2. In Fig. 13.8, a right circular cylinder just encloses a sphere of radius r. Find
(i) Surface area of the sphere:
(ii) Curved surface area of the cylinder:
(iii) Ratio of the areas obtained in (i) and (ii).

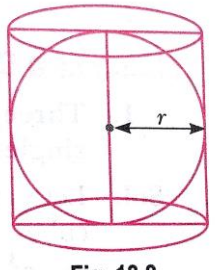

Fig. 13.8

Sol. (i) Surface area S^1 of the sphere = $4\pi r^2$
(ii) We have
Radius of the cylinder = r
Height of the cylinder = h = 2^r
$\therefore$ Curved surface area S_2 of the cylinder

$$2\pi rh = 2\pi r \times 2r = 4\pi r^2$$

(iii) $\frac{S^1}{S^2} = \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1}$

$\therefore$ $S_1 : S_2 = 1 : 1$

Que 3. Two hemispherical domes are to be painted as shown in the given figure. If the circumference of the bases of the domes are 17.6 cm and 70.4 cm respectively, then find the cost of painting at the rate of ₹10 per cm².

Sol.

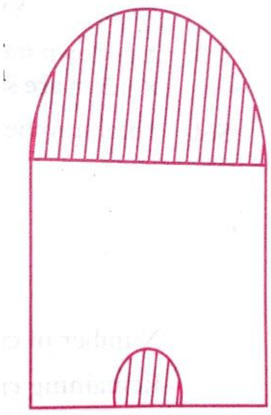

Fig. 13.9

Let radii of the bases of two domes be r and R.

$$\therefore\ 2\pi r = 17.6 \quad \Rightarrow\ 2 \times \frac{22}{7} \times r = 17.6$$

$$\Rightarrow\ r = \frac{17.6 \times 7}{2 \times 22} = 2.8\ cm$$

And $2\pi R = 70.4 \Rightarrow\ 2 \times \frac{22}{7} \times R = 70.4$

$$\Rightarrow\ R = \frac{70.4 \times 7}{2 \times 22} = 11.2\ cm$$

Now, area of two hemispherical domes = $2\pi r^2 + 2\pi R^2$

$$= 2 \times \frac{22}{7} \times 2.8 \times 2.8 \times 2 \times \frac{22}{7} \times 112 \times 112$$

$$= 49.28 + 788.48\ cm^2 = 837.76\ cm^2$$

Cost of painting at the rate of ₹10 per cm^2 = 837.76 × 10

= ₹8377.6

Que 4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in two cases.

Sol. Radius of the spherical balloon = r^1 = 7 cm

Surface area S_1 of the balloon = $4\pi {r_1}^2$

$$= 4 \times \frac{22}{7} \times 7^2 \ = \ 616\ cm^2$$

Radius of the spherical balloon when air is pumped into it = r^2 = 1 cm

Surface area S^2 of the balloon = $4\pi {r_1}^2 \ = 4 \times \frac{22}{7} \times 14^2$

$$\frac{S_1}{S_2} = \frac{4\times\frac{22}{7}\times 7^2}{4\times\frac{22}{7}\times 14^2} = \frac{1}{4}$$

$S_1 : S^2 = 1 : 4$

Volume of a Cuboid

Que 1. Three copper cubes whose edges measure 5 cm, 4 cm and 3 cm respectively are melted to form a single cube. Find the surface area of the new cube.

Sol. Let a cm be the edge of new cube. Then volume of the new cube = Sum of the volumes of three cubes.

⇒ $a^3 = 5^3 + 4^3 + 3^3 = 125 + 64 + 27$

⇒ $a^3 = 216$ ⇒ $a^3 = 6^3$ ⇒ a = 6 cm

∴ Surface area of the new cube = $6a^2 = 6\times6^2 = 216\ cm^2$

Que 2. If V is the volume of a cuboid of dimensions l, b, h and S is its surface area, then prove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{l} + \frac{1}{b} + \frac{1}{h}\right)$.

Sol. V = lbh and S = 2(lb + bh + hl)

Now RHS $= \frac{2}{S}\left(\frac{1}{l} + \frac{1}{b} + \frac{1}{h}\right) = \frac{2}{S}\left(\frac{bh + hl + lb}{lbh}\right) = \frac{2}{S} \times \frac{S}{2} \times \frac{1}{V} = \frac{1}{V} = \ LHS.$

Que 3. A godown measurers 45 m × 25 m × 10m. If 8000 wooden crates each measuring 1.5 m × 1.25 m × 0.5 m are stored in the godown, find how many more such crates can be stored in the godown.

Sol. Total number of crates that can be accommodated in the godown

$$= \frac{Volume\ of\ godown}{Volume\ of\ one\ crate}$$

$$= \frac{45 \times 25 \times 10}{1.5 \times 1.25 \times 0.25} = 12000$$

Number of crates already stored = 8000
Remaining crates that can be accommodated = 12000 – 8000 = 4000

Que 4. The lateral surface area of a cube is 576 cm^2. Find its volume and the total surface area.

Sol. Let each side of the cube be a cm.
Then, the lateral surface area of the cube = $4a^2$

$$\therefore \quad 4a^2 = 576 \quad \Rightarrow \quad a^2 = \frac{576}{4} cm^2 = 144 cm^2 \quad \Rightarrow \quad a = 12\ cm$$

Volume of the cube = a^3 = $(12\text{ cm})^3$ = 1728 cm^3
Total surface area of the cube = $6a^2 = 6 \times 12^2$ = 864 cm^2

Que 5. A village, having a population of 4000, required 150litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Sol. Volume of water = Volume of tank = l × b × h
= 20 × 15 × 6 m^3 = 1800 m^3
= 1800 × 1000 L = 1800000 L
Water used in a day = 4000 × 150 = 600000 L

Number of days the water will last = $\frac{Capacity\ of\ \text{tank}}{Water\ used\ in\ a\ day}$

$$= \frac{1800000\ L}{600000\ L} = 3$$

Que 6. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water (in Litre) will fall into the sea in a minute?

Sol. Length of water canal in one minute $= \frac{2 \times 1000}{60} = \frac{100}{3} m$

Volume of water flowing into the sea in one minute

= l × b × h = $\frac{100 \times 40 \times 3}{3} = 4000\ m^3$

= 4000 × 1000 L
= 4000000 L (∵ 1 m^3 = 1000 L)

Volume of a Cylinder

Que 1. If the lateral surface of a cylinder is 94.2 cm^2 and its height is 5 cm, then find
(i) radius of its base,
(ii) its volume (Use, π = 3.14).

Sol. (i) Height of the cylinder (h) = 5 cm
Let r cm be the radius of the base
Lateral surface area of cylinder = 94.2 cm^2

$\Rightarrow \quad 2\pi rh = 94.2\ cm^2$

$2 \times 3.14 \times r \times 5 = 94.2$

$$\Rightarrow \quad r = \frac{94.2}{2\times3.14\times5} = \frac{94.2}{31.4} = 3\ cm$$

Thus, radius of the base of cylinder = 3 cm.

(ii) Volume of cylinder = $\pi r^2 h$

$= 3.14 \times 3^2 \times 5 = 141.3$ cm^2

Que 2. 30 circular plates, each of radius 14 cm and thickness 3 cm are placed one above the another to form a cylindrical solid. Find:
(i) the total surface area. (ii) Volume of the cylinder so formed.

Sol. Height of the cylinder formed (h) = 30 × 3 = 90 cm
Radius of the base of the cylinder formed (r) = 14 cm
(i) Total surface area of the cylinder = $2\pi r$ (r + h)

$$= 2 \times \frac{22}{7} \times 14(14 + 90)$$

$$= 2 \times \frac{22}{7} \times 14 \times 104 = 9152\ cm^2$$

(ii) Volume of the cylinder formed = $\pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 90 = 55440\ cm^2$$

Que 3. The capacity of a closed cylindrical vessel of height 1 m is 15.4 liters. How many square metres of metal sheet would be needed to make it?

Sol. Height of the cylindrical vessel (h) = 1 m

Capacity of the cylindrical vessel = 15.4 L = $\frac{15.4}{1000} m^3 = 0.0154\ m^3$

Let ‘r’ m be the radius of the base of the cylindrical vessel
Volume of the cylindrical vessel = $\pi r^2 h$

$\Rightarrow \quad \pi r^2 h = 0.0154$

$\Rightarrow \quad \frac{22}{7} \times r^2 \times 1 = 0.0154$

$\Rightarrow \qquad r^2 = \frac{0.0154 \times 7}{22} = 0.0049 \quad \Rightarrow \qquad$ r = 0.07 m

Metal sheet needed to make the cyclindrical vessel

= Total surface area of the cylindrical vessel

= 2πr (r + h) = 2 × $\frac{22}{7} \times 0.07(0.07 + 1)\, m^2$

$= \frac{1}{7} \times 44 \times 0.07 \times 1.07\, m^2 = 0.4708\, m^2$

Que 4. A soft drink is available in two packs:
(i) A tin can with rectangular base of length 5 cm and width 4 cm, having a height of 15 cm.
(ii) A plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Sol. Capacity of tin can = l × b × h = 5 × 4 × 15 cm^3 = 300 cm^3

Radius of the base of the plastic cylinder = $\frac{7}{2}$ cm

Capacity of plastic cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10\, cm = 385\, cm^3$$

Thus, the plastic cylinder has greater capacity by (385 cm^3 – 300 cm^3**)** = 85 cm^3

Que 5. A patient in a hospital is given soup daily in cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height 4 cm. how much soup needs to be prepared daily to serve 250 patients?

Sol. Radius of cylindrical bowl = $\frac{7}{2} cm = 3.5\, cm$

Height of the bowl filled with soap (h) =4 cm

Volume of soup for 1 patient = $\pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 4 = 154\, cm^3$

∴ Volume of soup for 250 patients = 250 × 154 cm^3 = 38500 cm^3

$$= \frac{38500\, L}{1000} \quad (\because \;\; 1\, L = 1000\, cm^3)$$

= 38.5 L

Que 6. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? (1000 cm^3 = 1 L)

Sol. Height of the cylindrical vessel (h) = 25 cm
Let r cm be the radius of the base of the cylindrical vessel.
Circumference of the base = $2\pi r$ = 132 cm

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$$

$$\Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Volume of the cylindrical vessel = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25 = 34650\ cm^3$$

∴ Volume of water which vessel can hold = $\frac{34650\ L}{1000} = 34{,}65\ L$

Que 7. Rainwater which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if the rainfall is 1 cm? (Take π = 3.14)

Sol. 6 m = 600 cm, 4 m = 4 × 100 cm = 400 cm
Volume of the rainwater = 600 × 400 × 1 cm^3
Let the water level in the cylindrical vessel be h
Volume of the water in cylindrical vessel = $\pi r^2 h = 3.14 \times (20)^2 \times h$
According to statement, 600 × 400 × 1 cm^3 = 3.14 × $(20)^2$ × h

$$\Rightarrow \quad h = \frac{600 \times 400}{314 \times 20 \times 20} cm = 191.08\ cm$$

Que 8. A cylindrical tube opened at both the ends is made of iron sheet which is 2 cm thick. If the outer diameter is 16 cm and its length is 100 cm, find how many cubic centimeters of iron has been used in making the tube.

Sol. Outer radius of the cylindrical tube (R) $= \frac{16}{2} cm = 8\ cm$

Inner radius of the cylindrical tube (r) = (8 – 2) cm = 6 cm
Length of the tube (h) = 100 cm
Volume of iron used in making the cylindrical tube = $\pi R^2 h - \pi r^2 h$

$$= \pi h(R^2 - r^2)$$

$$= \frac{22}{7} \times 100(8^2 - 6^2) = \frac{22}{7} \times 100 \times 14 \times 2$$

8800 cm^3

Volume of Cone

Que 1. The radius and height of a cone are in the ratio 3: 4 and its volume is 301.44 cm^3. Find the radius and slant height of the cone.

Sol. Let the radius of the cone (r) = 3x cm
Height of the cone (h) = 4x cm

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow$ $301.44 = \frac{1}{3} \times 3.14 \times (3x)^2 . 4x$

$\Rightarrow$ $x^3 = \frac{301.44}{3.14 \times 12} = 8$

$\Rightarrow$ $X^3 = 2^3 \Rightarrow x = 2$ cm

Radius of the cone = 3x = 3 × 2 = 6 cm
Height of the cone = 4x = 4 × 2 = 8 cm

Slant height of the cone (l) = $\sqrt{r^2 + h^2} = \sqrt{6^2 + 8^2} = \sqrt{100} = 10\ cm$

Que 2. The height of a cone is 15 cm. If its volume is 1570 cm^3, find the diameter of the base. (Use =$\pi = 3.14$)

Sol. Let, the radius of the base of cone be r cm
Height of the cone = 15 cm
Volume of the cone = 1570 cm^3

$\Rightarrow$ $\frac{1}{3}\pi r^2 h = 1570$

$\Rightarrow$ $\frac{1}{3} \times 3.14 \times r^2 \times 15 = 1570$

$\Rightarrow$ $r^2 = \frac{1570 \times 3}{314 \times 15} = 100$ cm

$\Rightarrow$ $r = \sqrt{100} = 10\ cm$

Thus, the diameter of the base of cone = 2r = 2 × 10 cm = 20 cm

Que 3. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Sol.

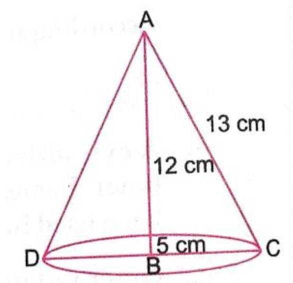

Fig. 13.10

Let ABC be a right triangle with AB = 12 cm, BC = 5 cm and AC = 13 cm.
When ΔABC is revolved about AB, it forms a right circular cone of radius BC = 5 cm and height AB = 12 cm.

Thus, volume of cone formed $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5^2 \times 12 = 100\pi\ cm^3$$

Volume of Sphere and Hemisphere

Que 1. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of metal is 8.9 g/cm^3?

Sol. Radius of the metallic ball $= \frac{4.2}{2} cm = 2.1\ cm$

Volume of the ball $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

38.808 cm^3

Mass of the ball = density × volume
= 8.9 g/cm^3 × 38.808 cm^3 = 345.39 g

Que 2. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Sol. Let, the diameter of the moon = $2R_M$
Diameter of the earth = $2R_E$

According to statement = $2R_M = \frac{1}{4}(2R_E)$

$$R_E = 4R_M$$

$$\frac{Volume\ of\ moon\ (V_M)}{Volume\ of\ Earth\ (V_E)} = \frac{\frac{4}{3}\pi R^3{}_M}{\frac{4}{3}\pi R^3{}_E}$$

$$= \frac{\frac{4}{3}\pi R^3{}_M}{\frac{4}{3}\pi (4\ R_M)^3} = \frac{1}{64}\frac{R^3{}_M}{64\ R^3{}_M}$$

$$\frac{V_M}{V_E} = \frac{1}{64} \Rightarrow V_M = \frac{1}{64}V_E$$

i.e., volume of moon is $\frac{1}{64}$ of the volume of earth.

Que 3. A hemisphere tank is made up of an iron sheet 1 cm thick. If the inner radius is 1 m, then find the volume of the iron used to make the tank.

Sol. Inner radius of the hemispherical tank (r) = 1 m
Outer radius of the hemispherical tank (R) = 1 + 0.01 = 1.01 m

Volume of iron used to make the hemispherical tank = $\frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7}[(1.01)^3 - 1^3]$$

$$= \frac{44}{21}(1.0303 - 1) = \frac{44}{21} \times 0.0303 = 0.06349\ m^3$$

Que 4. A shopkeeper has one spherical laddoo of radius 5 cm. With the same amount of material. How many laddoos of radius 2.5 cm can be made?

Sol. Radius of larger spherical laddoo (R) = 5 cm
Radius of smaller spherical laddoo (r) = 2.5 cm

Number of laddoo = $\frac{Volume\ of\ larger\ laddoo}{Volume\ of\ smaller\ one}$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{\frac{4}{3}\pi(5)^3}{\frac{4}{3}\pi(2.5)^3} = \frac{5\times5\times5}{2.5\times2.5\times2.5} = 8$$

Que 5. The water for a factory is stored in a hemispherical tank whose internal diameter is 14 m. The tank contains 50 kilolitres of water. Water is pumped into the tank to fill its capacity. Calculate the volume of water pumped into the tank.

Sol. Internal radius of the hemispherical tank (r) = $\frac{14}{2}$ m = 7 m

Capacity of the tank = $\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

= 718.67 m^3 = 718.67 kilolitres

Volume of water pumped into the tank = 718.67 – 50 = 668.67 kilolitres

Que 6. A cub of side 5 cm contain a sphere touching its sides. Find the volume of the gap in between.

Sol. Each side of the cube (a) = 5 cm
Diameter of the sphere (2r) = 5 cm
∴ Radius of the sphere (r) = $\frac{5}{2}$ cm

Volume of the sphere = $a^3 = 5^3\ cm^3 = 125\ cm^3$

Volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2}$$

= 65.476 cm^3

Volume of gap between cube and sphere = 125.000 cm^3 – 65.476 cm^3 = 59.524 cm^3

Que 7. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceeds its height?

Sol. Let the radius of sphere and cylinder be r and be h height of cylinder. Then according to the question.
Volume of sphere = Volume of cylinder

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi^2 h \qquad \Rightarrow r = \frac{3}{4}h$$

Diameter of the cylinder = $\frac{3}{2}$ h

Difference between the diameter and height of the cylinder = $\frac{3}{2}\ h - h\ = \frac{h}{2}$

Percentage by which the diameter exceeds the height of cylinder

$$= \frac{\frac{h}{2}}{h} \times 100 = \frac{h}{2} \times \frac{1}{h} \times 100 = 50\%$$

Thus, the diameter of the cylinder exceeds its height by 50%.

Long Answer Type Questions

[4 Marks]

Que 1. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the sides of new cube? Also, find the ratio between their surface areas.

Sol. Volume of given cube = a3 = 123 = 12 × 12 × 12 cm3
Let the edge of the new cube = x
∴ Volume of new cubes = x3
Volume of 8 new cubes = 8x3
Now, 8x3 = 12 × 12 × 12

$$\Rightarrow \quad x^3 = \frac{12\times12\times12}{8} = 6^3$$

$$\Rightarrow \quad x = 6 \text{ cm}$$

$$\Rightarrow \frac{Surface\ area\ of\ given\ cube}{Surface\ area\ of\ new\ cubes} = \frac{6a^2}{6x^2} = \frac{6\times12^2}{6\times6^2} = \frac{6\times12\times12}{6\times6\times6} = \frac{4}{1} = 4:1$$

Que 2. A field is 70 m long and 40 m broad. In the corner of the field, a pit which is 10 m long, 8 m broad and 5 m deep, has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Sol.

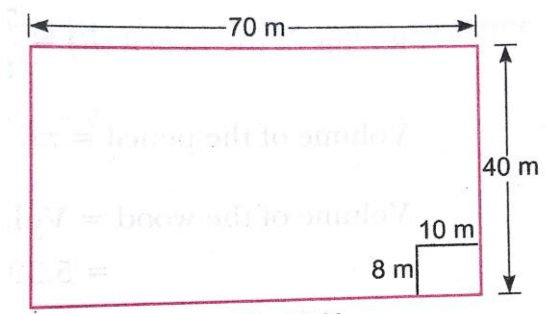

Fig. 13.11

Area of the field on which earth taken out is to be spread
= 70 × 40 m2 – 10 × 8 m2
= 2800 m2 – 80 m2 = 2720 m2
Volume of the earth dug out = 10 × 8 × 5 m3 = 400 m3

$$\text{Rise in level of the field} = \frac{Volume\ of\ the\ earth\ dugout}{Area\ on\ which\ earth\ taken\ out\ is\ to\ be\ spread}$$

$$= \frac{400}{2720} = 0.147\ m = 14.7\ cm$$

Que 3. A solid cylinder has total surface area of 462 cm2. Its curved surface area is one third of its total surface area. Find the volume of the cylinder.

Sol. Let r cm be the radius of the base and h cm be the right of the cylinder.
Then, total surface area of cylinder = 2πr (r + h)
Curved surface area of cylinder = 2πrh
We have,

Curved surface area = $\frac{1}{3}$ (Total surface area) = $\frac{1}{3} \times 462\ cm^2\ = 154\ cm^2$

$\Rightarrow \quad 2\pi rh\ = 154$
Also, 2πrh + 2πr2 = 462 $\quad \Rightarrow \quad 154\ +\ 2\pi r^2\ =\ 462$
$\Rightarrow \quad 2\pi r^2\ =\ 462\ -\ 154\ =\ 308\ cm^2$

$2 \times \frac{22}{7} \times r^2\ =\ 308$

$\Rightarrow \quad r^2 = \frac{308 \times 7}{2 \times 22} = 7^2 \quad \Rightarrow \quad r\ =\ 7\ cm$

Again $\quad 2\pi$rh = 154 $\quad \Rightarrow \quad 2 \times \frac{22}{7} \times 7 \times h\ =\ 154$

$\Rightarrow \quad h\ =\ \frac{154}{2 \times 22} = \frac{7}{2}\ cm$

Volume of the cylinder = πr2h

$$= \frac{22}{7} \times 7 \times 7 \times \frac{7}{2}\ =\ 539\ cm^3$$

Que 4. A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of graphite.

Sol. Radius of solid graphite cylinder (r) = $\frac{1}{2}mm\ = \frac{1}{2} \times \frac{1}{10} = \frac{1}{20}cm$

Length of graphite cylinder (h) = 14 cm

Volume of graphite cylinder (h) = πr2h

$$= \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 14 = 0.11\ cm^3$$

Radius of the pencil (R) = $\frac{7}{2}mm\ =\ \frac{7}{2} \times \frac{1}{10} = \frac{7}{20}cm$

Volume of the pencil = $\pi R^2 h = \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 14 =\ 5.39\ cm^3$

Volume of the wood = Volume of the pencil – Volume of the graphite

= 5.39 cm^2 – 0.11 cm^3

= 5.28 cm^3

Que 5. It costs ₹2200 to paint the inner curved surface of cylinder vessel 10 m deep. If the cost of painting is at the rate of ₹20 per m^2 find
(i) inner curved surface area of the vessel,
(ii) radius of the base,
(iii) capacity of the vessel.

Sol. (i) Inner curved surface area of the vessel = $\frac{Total\ cost\ of\ painting}{Cost\ of\ painting}$

$$= \frac{₹2200}{₹20/m^2} = 110\ m^2$$

(ii) Let the radius of the base of the cylinder vessel be r m.
Depth of the cylindrical vessel (h) = 10 m
Curved surface area of the cylindrical vessel = $2\pi rh$

∴ $2\pi rh = 110 \Rightarrow 2 \times \frac{22}{7} \times r \times 10 = 110$

⇒ $r = \frac{110\times7}{2\times22\times10} = 1.75$

(iii) Capacity of the cylindrical vessel = $\pi r^2h = \frac{22}{7} \times (1.75)^2 \times 10m^3$

= 96.25 m^3

Que 6. The inner diameter of cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if 1 cm^3 of wood has mass of 0.6 g.

Sol. Inner radius of the cylindrical pipe (r) = $\frac{24}{2}cm = 12\ cm$

Outer radius of the cylindrical pipe (R) = $\frac{28}{2}cm = 14\ cm$

Length of the wood used in making the cylindrical pipe

$$= \pi R^2h - \pi r^2h = \pi h(R^2 - r^2)$$

$$= \frac{22}{7} \times 35(14^2 - 12^2) = \frac{22}{7} \times 35(14 + 12)\ (14 - 12)$$

$$= \frac{22}{7} \times 35 \times 26 \times 2 = 5720\ cm^3$$

Mass of 1 cm^3 of wood = 0.6 g

∴ Mass of 5720 cm^2 of wood = (5720 × 0.6) g = 3432 g

$$\frac{3432}{1000}kg = 3.432\ kg$$

Thus, the mass of pipe = 3.43 kg

Que 7. A metal pipe is 77 cm long. The inner diameter of a cross-section is 4 cm, the outer diameter being 4.4 cm. Find its
(i) inner curved surface its, (ii) outer curved surface area,
(iii) total surface area.

Sol. We have,

Internal radius = r = $\frac{4}{2} cm = 2\ cm$

External radius = R = $\frac{4.4}{2}$ cm = 2.2 cm

Length of the pipe = h = 77 cm

(i) Inner curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 2 \times 77\ cm^2 = 968\ cm^2$

(ii) Outer curved surface area = $2\pi Rh = 2 \times \frac{22}{7} \times 2.2 \times 77\ cm^2 = 1064.8\ cm^2$

(iii) Total surface area of pipe = Inner curved surface area + Outer curved surface area + surface area of the ends

∴ Total surface area of pipe = $2\pi rh + 2\pi Rh + 2\pi(R^2 - r^2)$

$$= 2 \times \frac{22}{77} \times 2 \times 77 + 2 \times \frac{22}{7} \times 2.2 \times 77 + 2 \times \frac{22}{7}(2.2^2 - 2^2)$$

= 968 cm^2 + 1064 cm^2 + 5.28 cm^2 = 2038.08 cm^2

Que 8. The volume of a right circular cone is 9856 cm^2. If the diameter of the base is 28 cm, find
(i) height of the cone, (ii) slant height of the cone,
(iii) curved surface area of the cone.

Sol.(i) Let the height of the cone be h cm.

Radius of the base of the cone (r) = $\frac{28}{2}$ cm = 14 cm

Volume of the cone = 9856 cm^3

⇒ $\frac{1}{3}\pi r^2 h = 9856$

⇒ $\frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856$

⇒ $h = \frac{9856 \times 7 \times 3}{14 \times 14 \times 22} = 48\ cm$

(ii) Let l cm be the slant height of the cone. Then

$$l = \sqrt{r^2 + h^2} = \sqrt{14^2 + 48^2}$$

$$\Rightarrow \quad l = \sqrt{196 + 2304} = \sqrt{2500}$$

$\therefore$ l = 50 cm

(iii) Curved surface area of cone = πrl = $\frac{22}{7} \times 14 \times 50 = 2200\ cm^2$

Que 9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered with canvas, find the area of the canvas required.

Sol. Radius of the conical heap of wheat (r) = $\frac{10.5}{2}$ m

Height of the conical heap of wheat (h) = 3 m

Volume of the conical heap of wheat = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{10.5}{2}\right)^2 \times 3$

$$= \frac{173.25}{2} = 86.625$$

Slant height of the cone $\quad l = \sqrt{r^2 + h^2}$

$$= \sqrt{\left(\frac{10.5}{2}\right)^2 + 3^2} = \sqrt{(5.25)^2 + 3^2}$$

$$= \sqrt{27.5625 + 9} = \sqrt{36.5625}$$

$$l = 6.05\ \text{m}$$

Area of canvas required = curved surface area of cone

$$= \pi r l = \frac{22}{7} \times \frac{10.5}{2} \times 6.05 = 99.825\ \text{m}^2$$

Que 10. What length of tarpaulin 3 m wide will be required to make a conical tent of length 8 m and base radius 6 m? Assume that the extra length of material required for stitching margins and wastage in cutting will be approximately 20 cm.

Sol. Radius of the base of cone (r) = 6 m
Height of the cone (h) = 8 m
Let 'l' be the slant height of the cone. Then

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{6^2 + 8^2} = \sqrt{100} = 10\ m$$

Surface area of the conical tent = πrl

= 3.14 × 6 × 10 = 188.4 m^2

Length of tarpaulin required to make a conical tent of width

3 m = $\frac{118.4}{2} = 62.8\ m$

Wastage = 20 cm = $\frac{20}{100}m\ =\ 0.2\ m$

Total length of tarpaulin required to make conical tent = 62.8 m + 0.2 m = 63 m

Que 11. A conical tent is 10 m high and the radius of its base is 24 m. Find
(i) slant height of the tent, and
(ii) cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is ₹70.

Sol. Radius of conical tent = r = 24 m
Height of conical tent = h = 10 m
(i) Let l be the slant height of the cone. Then l = $\sqrt{r^2 + h^2}$

⇒ $$l = \sqrt{24^2 + 10^2} = \sqrt{576 + 100}$$
$$= \sqrt{676}\quad l = 26\ m$$

(ii) Canvas required to make the conical tent = Curved surface area of cone

$$= \pi rl = \frac{22}{7} \times 24 \times 26\ cm^2$$

Cost of 1 m^2 canvas = ₹70

$$\therefore\ Cost\ of\ \frac{22}{7} \times 24 \times 26\ cm^2\ canvas\ = ₹70 \times \frac{22}{7} \times 24 \times 26 = ₹137280$$

Que 12. A semi-circular sheet of metal of diameter 28 cm is bent to from an open conical cup. Find the capacity of the cup.

Sol.

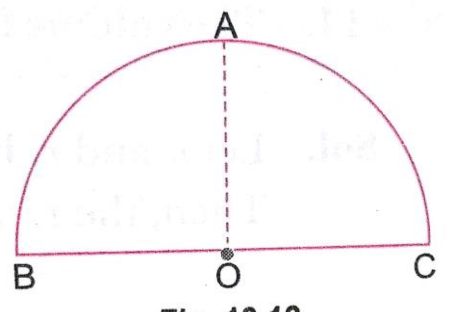

Fig. 13.12

When semi-circular sheet is bent to form an open conical cu, the radius of the sheet becomes slant height of the cup and the semi-circular part of the sheet becomes the circumference of the base of the cone.
∴ Slant height of the conical cup (l) = 14 cm
Let, r cm be the radius and h cm be the height of the conical cup. Then,

circumference of the base of the conical cup = circumference of the semi-circular sheet

$$2\pi r = \frac{1}{2} \times 2\pi \times 14$$

⇒ r = 7 cm

Now, $l^2 = h^2 + r^2$

⇒ $h = \sqrt{l^2 - r^2} = \sqrt{14^2 - 7^2}$

$= \sqrt{147} = 7\sqrt{3}$

Capacity of conical cup $= \frac{1}{3}\pi r^{2h}$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7\sqrt{3} = \frac{1078}{3}\sqrt{3}$$

$= 359.3 \times 1.732 = 622.31\ cm^2$

Que 13. A cloth having an area of 165 m^2 is shaped into the form of a conical tent of radius 5 cm.

(i) How many students can sit in the tent if a student on an average, occupies $\frac{5}{7}m^2$ on the ground?

(ii) Find the volume of the cone.

Sol. Let l m be the height of the conical tent.

Radius of the base of conical tent (r) = 5 m

(i) Area of the circular base of the cone = $\pi r^2 = \frac{22}{7} \times 5^2 m^2$

Number of students = $\frac{Area\ of\ the\ base}{Area\ occupied\ by\ one\ student}$

$$= \frac{\frac{22}{7} \times 5 \times 5\ m^2}{\frac{5}{7}m^2} = \frac{22}{7} \times 5 \times 5 \times \frac{7}{5} = 110$$

(ii) Also, curved surface area of cone = πrl

⇒ $165 = \frac{22}{7} \times 5 \times l$

⇒ $l = \frac{165 \times 7}{22 \times 5}$ ⇒ $l = \frac{21}{2}m = 10.5\ m$

Also, $h^2 = l^2 - r^2$

⇒ $h = \sqrt{(10.5)^2 - 5^2} = \sqrt{15.5 \times 5.5} = \sqrt{85.25} \approx\ 9.23\ cm$

Volume of conical tent $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 9.25\, m^3 \;=\; 24174\, m^3$$

Que 14. The volume of two spheres are in the ratio 64: 27. Find the ratio of their surface areas.

Sol. Let r^1 and r^2 be the radii of two spheres.
Then, the ratio of their volumes is given by

$$\frac{\frac{4}{3}\pi {r_1}^3}{\frac{4}{3}\pi {r_2}^3} = \frac{64}{27}$$

$$\left(\frac{r^1}{r^2}\right)^3 = \left(\frac{4}{3}\right)^3 \qquad \Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

Now, ratios of surface areas of two spheres = $\frac{4\pi {r_1}^2}{4\pi {r_2}^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$

∴ Required ratio = 16: 9

Que 15. Twenty-seven solid iron spheres, each of radius and surface area S are melted to form a sphere with surface area S'. Find the
(i) radius r' of the new sphere, and (ii) ratio of S and s'.

Sol. (i) Volume of 27 solid sphere, each of radius, r = 27 $\times \frac{4}{3}\pi r^3 = 36\pi r^3$

According to statement,
Volume of sphere of radius r' = Volume of 27 solid spheres

$$\Rightarrow \quad \frac{4}{3}\pi(r')^3 = 36\pi r^3$$

$$\Rightarrow \quad (r') \;=\; 27r^3 \;= (3r)^3 \qquad \Rightarrow \;\; r' \;=\; 3r$$

(ii) We have,

S' = $4\pi r'^2 \;=\; 4\pi(3r)^2 \;=\; 36\pi r^2$

$$\therefore \qquad \frac{S}{S\prime} = \frac{4\pi r^2}{36\pi r^2} = \frac{1}{9}$$

⇒ S: S = 1: 9

Que 16. A dome of a building is in the form of a hemisphere. From inside, it was white-washed at the cost of ₹498.96. If the cost of white-washing is ₹2.00 per square metre, find the
(i) inside surface area of the dome, and
(ii) volume of the air inside the dome.

Sol. Let, r m be inner radius of the hemispherical dome. Then,

(i) Inside surface area of the hemispherical dome = $\frac{Total\text{ cost}}{Cost\ per\ square\ metre}$

$= \frac{496.96}{2} m^2 = 249.48\ m^2$

Now, $2\pi r^2$= 249.48

$$\Rightarrow \quad r^2 = \frac{249.48 \times 7}{2 \times 22} = 39.69 \quad \Rightarrow \quad r = \sqrt{39.69} = 6.3\ m$$

(ii) Volume of the air inside the dome
= Volume of the hemispherical dome

$= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (6.3)^3 m^3 = 523.908\ m^3$

Que 17. A cylinder, a cone and a sphere are of the same radius and same height. Find the ratio of their curved surface.

Sol. Let r be the common radius of a cylinder, cone and a sphere.
There, height of the cylinder = Height of the cone = Height of the sphere
= 2r

Let 'l' be the slant height of the cone. Then

$$l = \sqrt{r^2 + h^2} = \sqrt{r^2 + (2r)^2} = \sqrt{5}r$$

Also, S_1 = Curved surface area of cylinder = $2\pi rh = 2\pi r.\ 2\pi = 4\pi r^2$
S_2 = Curved surface area of cone = $\pi rl = \pi r \sqrt{5}r = \sqrt{5}\ \pi r^2$
S_3 = Curved surface area of sphere = $4\ \pi r^2$

Now, $S_1: S_2: S_3 = 4\pi r^2 : \sqrt{5}\pi r^2 : 4\pi r^2$
∴ $S_1: S_2: S_3 = 4 : \sqrt{5} : 4$

HOTS (Higher Order Thinking Skills)

Que 1. A wooden bookshelf has external dimensions as follows: height = 110 cm, depth = 25 cm, breadth = 85 cm. The thickness of the planks is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2, find the total expenses required for polishing and painting the surface of the bookshelf.

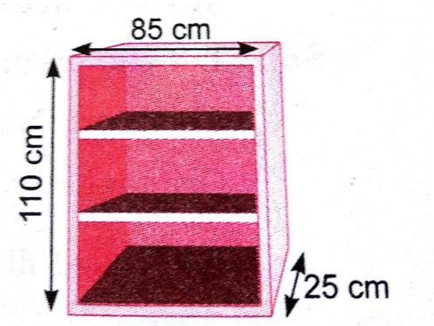

Fig. 13.13

Sol. Area of the bookshelf to be polished = Area of the five complete surfaces

+ Area of 2 rectangles of dimension 110 cm x 5 cm in the front

+ Area of 4 rectangles of dimension 75 cm x 5 cm in the front

= (110 x 85 + 2 x 85 x 25 + 2 x 110 x 25 + 2 x 110 x 5 + 4 x 75 x 5)

= 9350 + 4250 + 5500 + 1100 + 1500 = 21700 cm^2

∴ Total cost of polishing = ₹ $\frac{20}{100}$ x 21700 = ₹4340

Now, Height of the bookshelf excluding the thickness of the plank = 110 – 4 x 5 = 90 cm

∴ Area to be painted = Area of 3 open cuboids of dimensions

75 cm x 30 cm x 20 cm

= 3 x [75 x 30 + 2 x 30 x 20 + 2 x 20 x 75]

= 3(2250 + 1200 + 3000) = 3 x 6450 = 19350 cm^2

∴ Total cost of painting = ₹ $\frac{10}{100}$ x 19350 = ₹1935

Hence, total expenses required = ₹(4340 + 1935) = ₹6275

Que 2. A cone of height 24 cm has a curved surface area 550 cm^2. Find its volume.

Sol. Height of the cone (h) = 24 cm

Let r cm be the radius of the base and l cm be the slant height of the cone. Then,

$$L = \sqrt{r^2 + h^2} = \sqrt{r^2 + 24^2} = \sqrt{r^2 + 576}$$

Now, Curved surface area = πrl $\Rightarrow$ $\frac{22}{7}$ x r x $\sqrt{r^2 + 576}$ = 550

$\Rightarrow$ $\sqrt{r^2 + 576} = 550 \times \frac{7}{22}$ $\Rightarrow$ $r\sqrt{r^2 + 576} = 175$

Squaring both the sides we get

$$r^2(r^2 + 576) = 30625$$

$$(r^2)^2 + 576r^2 - 30625 = 0$$

Let $r^2 = x$

$\therefore$ $x^2 + 576x - 30625 = 0$

$$x^2 + 625x - 49x - 30625 = 0$$

$$x(x + 625) - 49(x + 625) = 0$$

$$(x + 625)(x - 49) = 0$$

$$x + 625 = 0,\ x - 49 = 0$$

$$x = -625,\ x = 49$$

$$x \neq -625,\ x = 49$$

$\therefore$ $r^2 = 49$ $\Rightarrow$ $r = \sqrt{49}$ $\Rightarrow$ r = 7 cm

$\therefore$ Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24 = 1232$ cm^3

Que 3. The diameter of a sphere is decreased by 25%. By what per cent does its curved surface area decrease?

Sol. Let the original diameter of the sphere be $2x$.

Then, original radius of the sphere = x

Original curved surface area = $4\pi r^2$

Decreased diameter of the sphere = $2x$ – 25% of $2x$ = $2x - \frac{x}{2} = \frac{3}{2}x$

Decreased radius of the sphere = $\frac{3}{4}x$

$\therefore$ Decreased curved surface area = $4\pi\left(\frac{3}{4}x\right)^2 = \frac{9}{4}\pi x^2$

Decreased in area = $4\pi x^2 - \frac{9}{4}\pi x^2 = \frac{7}{4}\pi x^2$

Hence, percentage decrease in area = $\frac{\frac{7}{4}\pi x^2}{4\pi x^2}$ x 100% = $\frac{7}{16}$ x 100% = $\frac{175}{4}$% = 43.75%.

Que 4. The surface area of a sphere of radius 5 cm is five times the curved surface area of a cone of radius 4 cm. Find the height and volume of the cone.

Sol. Radius of the sphere (r_1) = 5 cm

Radius of the base of cone (r_2) = 4 cm

Let r cm be the height of the cone.

Surface area of sphere = 4 πx^2 $\Rightarrow$ $4\pi(5)^2 = 100\pi$ cm^2

Curved surface area of cone = πrl = 4 πrl cm^2

Where l is the slant height of the cone.

According to the statement,

$100\pi = 5(4\pi l)$ $\Rightarrow$ l = 5 cm

Now, $h^2 = l^2 = 5^2 - 4^2 = 3^2$ $\Rightarrow$ h = 3 cm

∴ Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 4^2 \times 3$

= $\frac{352}{7}$ cm^3 = 50.29 cm^3 (Approximately)

Que 5. Two solid sphere made of the same metal have masses 5920 g and 740 g, respectively. Determine the radius of the larger sphere, if the diameter of the smaller sphere is 5 cm.

Sol. Let r and R be the radii of the smaller and larger spheres respectively.

We have, r = $\frac{5}{2}$ cm

Volume of the smaller sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{5}{2}\right)^3$ cm^3 = $\frac{4}{3} \times \pi \times \frac{125}{8}$ cm^3

Density of metal = $\frac{\text{Mass}}{\text{Volume}} = \frac{740}{\frac{4}{3}\times\frac{125}{8}\pi}$ gcm^{-3} …(i)

Volume of larger sphere = $\frac{4}{3}\pi R^3$

Density of metal = $\frac{\text{Mass}}{\text{Volume}} = \frac{5920}{\frac{4}{3}\pi R^3}$ …(ii)

From (i) and (ii), we have

$$\frac{740}{\frac{4}{3} \times \frac{125}{8}\pi} = \frac{5920}{\frac{4}{3}\pi R^3}$$

$\Rightarrow$ $R^3 = \frac{5920 \times 125}{740 \times 8} = 125$

$\Rightarrow$ $R^3 = 5^3$ $\Rightarrow$ $R = 5$ cm

Value Based Questions

Que 1. The perimeter of an isosceles triangle is 25 cm and its base is 7 cm. The teacher asked the students to find its area. Sapna answered $\frac{7}{4}\sqrt{15}\ cm^2$. Is she correct? Justify. Which values are depicted here?

Sol. Yes, 2b + a = 15
⇒ 2b + 7 = 15 ⇒ b = 4

∴ Area of isosceles triangle = $\frac{7}{4}\sqrt{4b^2 - a^2} = \frac{7}{4}\sqrt{4 \times 4^2 - 7^2}$

$$=\frac{7}{4}\sqrt{64 - 49} = \frac{7}{4}\sqrt{15}cm^2$$

Curiosity, knowledge, truthfulness.

Que 2. An umbrella is made by stitching 10 triangular pieces of cloth of two different designs, each piece measuring 20 cm, 50 cm and 50 cm.
How much cloth of each design is required by Mr. Amit if he wants to donate 20 such umbrellas to the children of slum areas?

Sol. The sides of triangular pieces are 20 cm, 50 cm and 50 cm.
Let, a = 20 cm, b = 50 cm, c = 50 cm

∴ Semi-perimeter, s = $\frac{a+b+c}{2} = \frac{20+50+50}{2}$

S = 60 cm

∴ Area of one triangular piece = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{60(60-20)(60-50)(60-50)}$$
$$= \sqrt{60 \times 40 \times 10 \times 10} = 200\sqrt{6}cm^2$$

Cloth of each design required for one umbrella = Area of 5 triangular pieces

$$= 5 \times 200\sqrt{6} = 1000\sqrt{6}cm^2$$

Cloth of each design required for 20 umbrella = $20 \times 1000\sqrt{6} = 20{,}000\sqrt{6}cm^2$
Helpful, caring, loving.

Que 3. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and side 6 cm each is to be made of three different shades as shown in figure. How much paper of each shade has been used in it?
How much paper of each shade is required by Arushi if she wants to donate 40 kites to the children of an orphanage? Which values does Arushi possess?

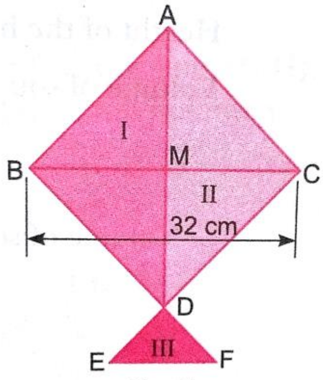

Fig. 13

Sol. As the diagonals of a square are equal bisect each other at right angle

∴ AD = BC =32 cm and AM = DM = $\frac{32}{2} = 16\ cm$

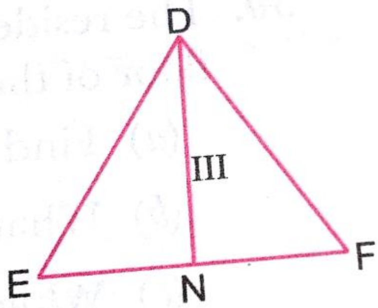

Fig. 14

Area of shade I = Area of shade II

$= \text{Area of } \Delta ABD = \frac{1}{2} \times AD \times BM$

$= \frac{1}{2} \times 32 \times 16 = 256\ Cm^2$

For the area of shade III

Area of isosceles $\Delta DEF = \frac{a}{4}\sqrt{4b^2 - a^2}$

$= \frac{8}{4}\sqrt{4(6)^2 - 8^2} = 2\sqrt{144 - 64}$

$= 2\sqrt{80} = 8\sqrt{5}cm^2$

Area of shade I = Area of shade II = 256 cm^2

∴ Area of sheet of shade I required for making 40 kites

= Area of sheet of shade II required for making 40 kites

= 40 × 256 = 10240 cm^2

Area of sheet of shade III required for making 40 kites = $40 \times 8\sqrt{5} = 320\sqrt{5}cm^2$

Social, loving, caring.

Que 4. A craft mela is organised by welfare Association to promote the art and culture of tribal people. The pandal is to be decorated by using triangular flags around the field. Each flag has dimension 25 cm, 25 cm and 22 cm. Find the

area of cloth required for making 200 such flags. Which values are depicted here?

Sol. Area of cloth required for one flag

$$= \sqrt{s(s-25)(s-25)(s-22)}, \text{ where } s = \frac{25+25+22}{2} = 36\ cm$$

Area of cloth required = $\sqrt{36(36-25)(36-25)(36-22)}$

$= \sqrt{36 \times 11 \times 11 \times 14}$

$= 6 \times 11\sqrt{14}\ cm^2 = 66\sqrt{14}$

Area of cloth required for 200 such flags = $66\sqrt{14} \times 200 = 13200\sqrt{14}cm^2$
Helpfulness, cooperation, beauty.

Que 29. A person donates cylindrical bowls of diameter 7 cm to a charitable hospital in which soup is served to patients. If the bowl is filled with soup to a height of 4 cm, how much soup needs to be prepared daily to serve 250 patients? Which values of the person are depicted here?

Sol. Radius of cylindrical bowl = $\frac{7}{2}cm = 3.5\ cm$

Height of the filled with soup (h) = 4 cm

Volume of soup for 1 patient = $\pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 = 154\ cm^3$$

∴ Volume of soup for 250 patients = 250 ×154 cm^3 = 38500 cm^3

$$= \frac{38500L}{1000} \qquad (\because 1L = 1000\ cm^3)$$

= 38.5 L

The person is kind heated, caring and contributing for the welfare of society.

Que 30. The resident of society decided to paint the hall of cancer detective centre in their premises. If the floor of the cuboidal hall has a perimeter equal to 260 m and height 6 m then
(a) Find the cost of painting of its four walls (including doors etc.) at the rate of ₹9 per m^2.
(b) What is the amount contributed by 50 people?
(c) Which value is depicted by the residents?

Sol. Perimeter = 2(l + b) = 260
= l + b = 130
(a) Surface area of four walls = 2h (l + b) = 2 × 6 × 130 = 1560 m^2

Cost of painting = 9 × 1560 = ₹14,040

(b) Amount contributed = $\frac{14040}{50}$ = ₹280.8

(c) Cooperation, social cohesion.

Que 31. A person pays ₹2200 to children to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate ₹20 per m^2, find
(i) inner curved surface area of the vessel, (ii) radius of the base,
(iii) capacity of the vessel.
Which social is the person violating?

Sol. (i) Inner curved surface area of the vessel = $\frac{Total\ cost\ of\ painting}{Cost\ of\ painting\ per\ m^2}$

$$= \frac{₹2200}{20₹/m^2} = 110\ m^2$$

(ii) Let the radius of the base of the cylindrical vessel be r m.
Depth of the cylindrical vessel (h) = 10 m
Curved surface area of the cylindrical vessel = 2πrh

∴ 2πrh = 110⇒ $2 \times \frac{22}{7} \times r \times 10 = 110$

⇒ r = $\frac{110\times7}{2\times22\times10} = 1.75$

(iii) Capacity of the cylindrical vessel = $\pi r^2 h$

$$= \frac{22}{7} \times (1.75)^2 \times 10\ m^3 = 96.25\ m^3$$

Child labour is abolished under the law. So, the person is violating this law.