

RD Sharma
Solutions
Class 11 Maths
Chapter 26
Ex 26.1

Ellipse Ex 26.1 Q1

Let $P(x, y)$ be any point on the ellipse whose focus is $S(1, -2)$ and eccentricity $e = \frac{1}{2}$. Let PM be perpendicular from P on the directrix. Then,

$$SP = ePM$$

$$\Rightarrow SP = \frac{1}{2}(PM)$$

$$\Rightarrow SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow 4SP^2 = (PM)^2$$

$$\Rightarrow 4[(x-1)^2 + (y+2)^2] = \left[\frac{3x-2y+5}{\sqrt{(3)^2 + (-2)^2}} \right]^2$$

$$\Rightarrow 4[x^2 + 1 - 2x + y^2 + 4 + 4y] = \frac{(3x-2y+5)^2}{(\sqrt{13})^2}$$

$$\Rightarrow 4[x^2 + y^2 - 2x + 4y + 5] = \frac{(3x-2y+5)^2}{13}$$

$$\Rightarrow 52[x^2 + y^2 - 2x + 4y + 5] = (3x-2y+5)^2$$

$$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = (3x-2y+5)^2$$

$$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = (3x)^2 + (-2y)^2 + (5)^2 + 2 \times 3x \times (-2y) + 2 \times (-2y) \times 5 + 2 \times 5 \times 3x$$

$$\left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \right]$$

$$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 + 25 - 12xy - 20y + 30x$$

$$\Rightarrow 52x^2 - 9x^2 + 52y^2 - 4y^2 + 12xy - 104x - 30x + 208y + 20y + 260 - 25 = 0$$

$$\Rightarrow 43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(i)

Let $P(x, y)$ be a point on the ellipsis. Then, by definition

$$SP = ePM$$

Here $e = \frac{1}{2}$. coordinates of S are $(0, 1)$ and the equation of the directrix is

$$x + y = 0.$$

$$\therefore SP = \frac{1}{2}PM$$

$$\Rightarrow SP^2 = \frac{1}{4}(PM)^2$$

$$\Rightarrow 4SP^2 = (PM)^2$$

$$\Rightarrow 4[(x-0)^2 + (y-1)^2] = \left[\frac{x+y}{\sqrt{1^2+1^2}} \right]^2$$

$$\Rightarrow 4[x^2 + y^2 + 1 - 2y] = \frac{(x+y)^2}{2}$$

$$\Rightarrow 4 \times 2[x^2 + y^2 - 2y + 1] = x^2 + y^2 + 2xy$$

$$\Rightarrow 8x^2 + 8y^2 - 16y + 8 = x^2 + y^2 + 2xy$$

$$\Rightarrow 8x^2 - x^2 + 8y^2 - y^2 - 2xy - 16y + 8 = 0$$

$$\Rightarrow 7x^2 + 7y^2 - 2xy - 16y + 8 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(ii)

Let $P(x, y)$ be a point on the ellipse. Then, by definition

$$SP = e PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(-1, 1)$ and the equation of directrix is

$$x - y + 3 = 0$$

$$\therefore SP = \frac{1}{2} PM$$

$$\Rightarrow SP^2 = \frac{1}{4} (PM)^2$$

$$\Rightarrow 4SP^2 = PM^2$$

$$\Rightarrow 4[(x+1)^2 + (y-1)^2] = \left[\frac{x-y+3}{\sqrt{1^2 + (-1)^2}} \right]^2$$

$$\Rightarrow 4[x^2 + 1 + 2x + y^2 + 1 - 2y] = \frac{(x-y+3)^2}{2}$$

$$\Rightarrow 8[x^2 + y^2 + 2x - 2y + 2] = (x-y+3)^2$$

$$\Rightarrow 8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + (-y)^2 + 3^2 + 2x(-y) \times 3 + 2x(x) \times (-y) + 2 \times 3 \times x$$

$$\left[\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \right]$$

$$\Rightarrow 8x^2 + 8y^2 + 16x - 16y + 16 = x^2 + y^2 + 9 - 6y - 2xy + 6x$$

$$\Rightarrow 8x^2 - x^2 + 8y^2 - y^2 + 2xy + 16x - 6x - 16y + 6y + 16 - 9 = 0$$

$$\Rightarrow 7x^2 + 7y^2 + 2xy + 10x - 10y + 7 = 0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(iii)

Let $P(x, y)$ be a point the ellipse. Then, by definition

$$SP = e PM$$

Here $e = \frac{4}{5}$, coordinates of S are $(-2, 3)$ and the equation of directrix is

$$2x + 3y + 4 = 0$$

$$\therefore SP = \frac{4}{5} PM$$

$$\Rightarrow SP^2 = \frac{16}{25} (PM)^2$$

$$\Rightarrow 25 SP^2 = 16 PM^2$$

$$\Rightarrow 25[(x+2)^2 + (y-3)^2] = 16 \left[\frac{2x+3y+4}{\sqrt{2^2+3^2}} \right]^2$$

$$\Rightarrow 25[x^2 + 4 + 4x + y^2 + 9 - 6y] = \frac{16(2x+3y+4)^2}{13}$$

$$\Rightarrow 325[x^2 + y^2 + 4x - 6y + 13] = 16(2x+3y+4)^2$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q2(iv)

Let $P(x, y)$ be a point on the ellipse. Then, by definition

$$SP = e PM$$

Here $e = \frac{1}{2}$, coordinates of S are $(1, 2)$ and the equation of directrix is

$$3x + 4y - 5 = 0$$

$$\therefore SP = \frac{1}{2} PM$$

$$\Rightarrow SP^2 = \frac{1}{4} (PM)^2$$

$$\Rightarrow 4SP^2 = PM^2$$

$$\Rightarrow 4[(x-1)^2 + (y-2)^2] = \left[\frac{3x+4y-5}{\sqrt{3^2+4^2}} \right]^2$$

$$\Rightarrow 4[x^2+1-2x+y^2+4-4y] = \frac{(3x+4y-5)^2}{25}$$

$$\Rightarrow 100[x^2+y^2-2x-4y+5] = (3x+4y-5)^2$$

$$\Rightarrow 100x^2+100y^2-200x-400y+500 = (3x+4y-5)^2$$

$$\Rightarrow 100x^2+100y^2-200x-400y+500 = (3x)^2+(4y)^2+(-5)^2+2 \times 3x \times 4y+2 \times 4y \times (-5)+2 \times (-5) \times 3x$$

$$\Rightarrow 100x^2+100y^2-200x-400y+500 = 9x^2+16y^2+25+24xy-40y-30x$$

$$\Rightarrow 100x^2-9x^2+100y^2-16y^2-24xy-200x+30x-400y+40y+500-25=0$$

$$\Rightarrow 91x^2+84y^2-24xy-170x-360y+475=0$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q3(i)

$$4x^2+9y^2=1$$

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$

$$\text{eccentricity} = \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}} = \frac{\sqrt{5}}{3}$$

$$\text{Length of latus rectum} = \frac{2 \times \frac{1}{9}}{\frac{1}{2}} = \frac{4}{9}$$

$$\text{Foci are } \left(\frac{\sqrt{5}}{6}, 0\right), \left(-\frac{\sqrt{5}}{6}, 0\right)$$

Ellipse Ex 26.1 Q3(ii)

$$5x^2 + 4y^2 = 1$$

$$\frac{x^2}{\frac{1}{5}} + \frac{y^2}{\frac{1}{4}} = 1$$

$$\text{eccentricity} = \sqrt{\frac{\frac{1}{4} - \frac{1}{5}}{\frac{1}{4}}} = \frac{1}{\sqrt{5}}$$

$$\text{Length of latus rectum} = \frac{2 \times \frac{1}{5}}{\frac{1}{2}} = \frac{4}{5}$$

$$\text{Foci are } \left(0, \frac{1}{2\sqrt{5}}\right); \left(0, -\frac{1}{2\sqrt{5}}\right)$$

Ellipse Ex 26.1 Q3(iii)

We have,

$$4x^2 + 3y^2 = 1$$

$$\Rightarrow \frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1 \dots\dots\dots (i)$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{3}$ i.e.,

$$a = \frac{1}{2} \text{ and } b = \frac{1}{\sqrt{3}}.$$

Clearly, $b > a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.

Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{\frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{1 - \frac{3}{4}}$$

$$= \sqrt{\frac{1}{4}}$$

$$\therefore e = \frac{1}{2}$$

The coordinates of the foci are $(0, be)$ and $(0, -be)$ i.e., $\left(0, \frac{1}{2\sqrt{3}}\right)$ and $\left(0, -\frac{1}{2\sqrt{3}}\right)$.

Now,

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= 2 \times \frac{\frac{1}{4}}{\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{2}$$

Ellipse Ex 26.1 Q3(iv)

We have,

$$25x^2 + 16y^2 = 1600$$

$$\Rightarrow \frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$

$$\Rightarrow \frac{x^2}{64} + \frac{y^2}{100} = 1 \dots\dots\dots (i)$$

This is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 = 64$ and $b^2 = 100$ i.e.,

$$a = 8 \text{ and } b = 10.$$

Clearly, $b > a$, therefore the major and minor axes of the ellipse (i) are along y and x axes respectively.

Let e be the eccentricity of the ellipse. Then,

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10}$$

$$\therefore = \frac{3}{5}$$

The coordinates of the foci are $(0, be)$ and $(0, -be)$ i.e., $(0, 6)$ and $(0, -6)$.

Now,

$$\text{Length of the latus rectum} = \frac{2a^2}{b}$$

$$= 2 \times \frac{64}{10}$$

$$= \frac{64}{5}$$

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$\Rightarrow \sqrt{\frac{2}{5}} = \sqrt{1 - \frac{b^2}{a^2}} \quad \left[\because \text{eccentricity} = \sqrt{\frac{2}{5}} \right]$$

$$\Rightarrow \frac{2}{5} = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{b^2}{a^2} = 1 - \frac{2}{5}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{3}{5}$$

$$\Rightarrow 5b^2 = 3a^2$$

$$\Rightarrow b^2 = \frac{3a^2}{5} \dots\dots\dots (ii)$$

Putting the value of $b^2 = \frac{3a^2}{5}$ in equation (i), we get

$$\frac{9}{a^2} + \frac{1}{\frac{3a^2}{5}} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{5}{3a^2} = 1$$

$$\Rightarrow \frac{1}{a^2} \left[9 + \frac{5}{3} \right] = 1$$

$$\Rightarrow \frac{1}{a^2} \left[9 + \frac{5}{3} \right] = 1$$

$$\Rightarrow 9 + \frac{5}{3} = a^2$$

$$\Rightarrow a^2 = \frac{32}{3} \dots\dots\dots \text{(iii)}$$

Putting $a^2 = \frac{32}{3}$ in equation (ii), we get

$$b^2 = \frac{3}{5} \times \frac{32}{3} = \frac{32}{5} \dots\dots\dots \text{(iv)}$$

∴ The required equation of ellipse is

$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$$

$$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32} = 1$$

$$\Rightarrow 3x^2 + 5y^2 = 32.$$

This is the required equation of ellipse.

Ellipse Ex 26.1 Q5(i)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of the foci are $(\pm 2, 0)$. This means that the major and minor axes of the ellipse are along x and y axes respectively and the coordinates of foci are $(\pm ae, 0)$

$$\therefore ae = 2$$

$$\Rightarrow a \times \frac{1}{2} = 2 \quad \left[\because e = \frac{1}{2} \right]$$

$$\Rightarrow a = 4$$

$$\Rightarrow a^2 = 16$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = (4)^2 \left[1 - \left(\frac{1}{2} \right)^2 \right]$$

$$\Rightarrow b^2 = 16 \left[1 - \frac{1}{4} \right]$$

$$\Rightarrow b^2 = 16 \times \frac{3}{4} = 12$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\Rightarrow 3x^2 + 4y^2 = 48$$

required equation of ellipse.

Ellipse Ex 26.1 Q5(ii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots (i)$$

The length of latus-rectum = 5

$$\therefore \frac{2b^2}{a} = 5$$

$$\Rightarrow b^2 = \frac{5a}{2} \dots\dots (ii)$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow \frac{5a}{2} = a^2 \left[1 - \left(\frac{2}{3} \right)^2 \right] \quad \left[\because e = \frac{2}{3} \right]$$

$$\Rightarrow \frac{5a}{2} = a^2 \left[1 - \frac{4}{9} \right]$$

$$\Rightarrow \frac{5}{2} = a \left(\frac{5}{9} \right)$$

$$\Rightarrow \frac{5}{2} \times \frac{9}{5} = a$$

$$\Rightarrow a = \frac{9}{2}$$

$$\Rightarrow a^2 = \frac{81}{4}$$

Putting $a = \frac{9}{2}$ in $b^2 = \frac{5a}{2}$, we get

$$b^2 = \frac{5}{2} \times \frac{9}{2}$$

$$\Rightarrow b^2 = \frac{45}{4}$$

Substituting $a^2 = \frac{81}{4}$ and $b^2 = \frac{45}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

$$\Rightarrow \frac{4x^2 \times 5 + 4y^2 \times 9}{405} = 1$$

$$\Rightarrow 20x^2 + 36y^2 = 405$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(iii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

Then, semi-major axis = a

$$\therefore a = 4 \quad [\because \text{semi-major axis} = 4]$$

$$\Rightarrow a^2 = 16$$

Now,

$$b^2 = a^2 (1 - e^2)$$

$$\Rightarrow b^2 = 16 \left[1 - \left(\frac{1}{2} \right)^2 \right] \quad \left[\because e = \frac{1}{2} \right]$$

$$\Rightarrow b^2 = 16 \left[1 - \frac{1}{4} \right]$$

$$\Rightarrow b^2 = 16 \times \frac{3}{4}$$

$$\Rightarrow b^2 = 12$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\Rightarrow 3x^2 + 4y^2 = 48$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q5(iv)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ where major axis} = 2a \dots\dots\dots (i)$$

Now,

$$2a = 12$$

[\because Major axis = 12]

$$\Rightarrow a = 6$$

$$\Rightarrow a^2 = 36$$

Now,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 36 \left(1 - \frac{1}{4}\right)$$

[$\because e = \frac{1}{2}$]

$$\Rightarrow b^2 = 36 \times \frac{3}{4}$$

$$\Rightarrow b^2 = 27$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{36} + \frac{y^2}{27} = 1$$

$$\Rightarrow \frac{1}{9} \left[\frac{x^2}{4} + \frac{y^2}{3} \right] = 1$$

$$\Rightarrow \frac{3x^2 + 4y^2}{12} = 9$$

$$\Rightarrow 3x^2 + 4y^2 = 108$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(v)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

Since the ellipse passes through

$(1, 4)$ and $(-6, 1)$.

$$\therefore \frac{(1)^2}{a^2} + \frac{(4)^2}{b^2} = 1$$

$$\Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow b^2 + 16a^2 = a^2b^2 \dots\dots\dots (ii)$$

and $\frac{(-6)^2}{a^2} + \frac{(1)^2}{b^2} = 1$

$$\Rightarrow \frac{36}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow \frac{36}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow 36b^2 + a^2 = a^2b^2 \dots\dots\dots (iii)$$

Multiplying equation (iii) by 16, we get

$$576b^2 + 16a^2 = 16a^2b^2 \dots\dots\dots (iv)$$

Multiplying equation (iii) by 16, we get

$$576b^2 + 16a^2 = 16a^2b^2 \dots\dots\dots (iv)$$

Substituting equation (ii) from equation (iv), we get

$$576b^2 - b^2 = 16a^2b^2 - a^2b^2$$

$$\Rightarrow 575b^2 = 15a^2b^2$$

$$\Rightarrow 575 = 15a^2$$

$$\Rightarrow a^2 = \frac{575}{15} = \frac{115}{3}$$

Putting $a^2 = \frac{115}{3}$ in equation (ii), we get

$$b^2 + 16 \times \frac{115}{3} = \frac{115}{3} \times b^2$$

$$\Rightarrow b^2 - \frac{115}{3}b^2 = -16 \times \frac{115}{3}$$

$$\Rightarrow \frac{3b^2 - 115b^2}{3} = -\frac{16 \times 115}{3}$$

$$\Rightarrow -112b^2 = -16 \times 115$$

$$\Rightarrow b^2 = \frac{16 \times 115}{112}$$

Substituting the value of a^2 and b^2 in (i), we get

$$\frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$$

$$\Rightarrow \frac{3x^2 + 7y^2}{115} = 1$$

$$\Rightarrow 3x^2 + 7y^2 = 115$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q5(vi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$$\therefore a = 5 \text{ and } ae = 4 \Rightarrow e = \frac{4}{5}$$

$$\text{Now, } b^2 = a^2(1 - e^2) \Rightarrow b^2 = 25 \left(1 - \frac{16}{25}\right) = 9.$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{25} + \frac{y^2}{9} = 1, \text{ which is the equation of the required ellipse.}$$

Ellipse Ex 26.1 Q5(vii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 13 \quad [\because \text{vertices: } (0, \pm 13)]$$

$$\Rightarrow b^2 = 169$$

$$\text{and } be = 5 \quad [\because \text{foci: } (0, \pm 5)]$$

$$\Rightarrow 13 \times e = 5$$

$$\Rightarrow e = \frac{5}{13}$$

$$\text{Now, } a^2 = b^2(1 - e^2)$$

$$\Rightarrow a^2 = (13)^2 \left[1 - \left(\frac{5}{13} \right)^2 \right]$$

$$\Rightarrow a^2 = 169 \left[1 - \frac{25}{169} \right]$$

$$\Rightarrow a^2 = 169 \left[\frac{144}{169} \right]$$

$$\Rightarrow a^2 = 144$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{144} + \frac{y^2}{169} = 1$$

This is the required equation of ellipse.

Ellipse Ex 26.1 Q5(viii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$$\therefore a = 6 \quad [\because \text{vertices: } (\pm 6, 0)]$$

$$\Rightarrow a^2 = 36$$

$$\text{and } ae = 4 \quad [\because \text{foci: } (\pm 4, 0)]$$

$$\Rightarrow 6 \times e = 4$$

$$\Rightarrow e = \frac{4}{6} = \frac{2}{3}$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 36 \left[1 - \left(\frac{2}{3} \right)^2 \right]$$

$$= 36 \times \left[1 - \frac{4}{9} \right]$$

$$= 36 \times \frac{5}{9}$$

$$= 4 \times 5$$

$$= 20$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(ix)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of its ends of major axis and minor axis are $(\pm a, 0)$ and $(0, \pm b)$ respectively.

$$\therefore a = 3 \quad [\because \text{Ends of major axis} = (\pm 3, 0)]$$

$$\Rightarrow a^2 = 9$$

$$\text{and } b = 2 \quad [\because \text{Ends of major axis} = (0, \pm 2)]$$

$$\Rightarrow b^2 = 4$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(x)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of its ends of major axis and minor axis are $(0, \pm b)$ and $(\pm a, 0)$ respectively.

$$\therefore b = \sqrt{5} \quad [\because \text{ends of major axis} = (0, \pm\sqrt{5})]$$

$$\Rightarrow b^2 = 5$$

$$\text{and } a = 1 \quad [\because \text{ends of major axis} = (\pm 1, 0)]$$

$$\Rightarrow a^2 = 1$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{1} + \frac{y^2}{5} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xi)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

we have,

Length of major axis = 26

$$\Rightarrow 2a = 26$$

$$\Rightarrow a = \frac{26}{2} = 13$$

$$\Rightarrow a^2 = 169$$

The coordinates of foci are $(\pm ae, 0)$.

$$\therefore ae = 5$$

$$\Rightarrow 13 \times e = 5$$

$$\Rightarrow e = \frac{5}{13}$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 169 \left[1 - \left(\frac{5}{13} \right)^2 \right]$$

$$\Rightarrow b^2 = 169 \left[1 - \frac{25}{169} \right]$$

$$\Rightarrow b^2 = 169 \left[\frac{144}{169} \right]$$

$$\Rightarrow b^2 = 144$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{169} + \frac{y^2}{144} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

we have,

Length of major axis = 16

$$\Rightarrow 2a = 16$$

$$\Rightarrow a = \frac{16}{2} = 8$$

$$\Rightarrow a^2 = 64$$

The coordinates of foci are $(0, \pm be)$.

$$\therefore be = 6$$

$$[\because \text{foci: } (0, \pm 6)]$$

$$\Rightarrow (be)^2 = 36$$

$$\text{Now, } a^2 = b^2(1 - e^2)$$

$$\Rightarrow a^2 = b^2 - b^2e^2$$

$$\Rightarrow 64 = b^2 - 36 \quad \left[\because (be)^2 = 36 \text{ and } a^2 = 64 \right]$$

$$\Rightarrow 64 + 36 = b^2$$

$$\Rightarrow b^2 = 100$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q5(xiii)

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

we have,

$$a = 4$$

$$\Rightarrow a^2 = 16$$

and, the coordinates of foci are $(\pm 3, 0)$

$$\therefore ae = 3$$

$$\Rightarrow 4 \times e = 3$$

$$\Rightarrow e = \frac{3}{4}$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$= 4^2 \left[1 - \left(\frac{3}{4} \right)^2 \right]$$

$$= 16 \times \left(1 - \frac{9}{16} \right)$$

$$= 16 \times \frac{7}{16}$$

$$= 7$$

Substituting the values of a^2 and b^2 in (i), we get

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

This is the equation of the required ellipse.

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of foci are $(+ae, 0)$ and $(-ae, 0)$.

$$\therefore ae = 4$$

$$[\because \text{foci: } (\pm 4, 0)]$$

$$\Rightarrow a \times \frac{1}{3} = 4$$

$$[\because e = \frac{1}{3}]$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

$$\text{Now, } b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 144 \left[1 - \left(\frac{1}{3} \right)^2 \right]$$

$$\Rightarrow b^2 = 144 \left[1 - \frac{1}{9} \right]$$

$$\Rightarrow b^2 = 144 \times \frac{8}{9}$$

$$\Rightarrow b^2 = 16 \times 8 = 128$$

Substituting $a^2 = 144$ and $b^2 = 128$ in equation (i), we get

$$= \frac{x^2}{144} + \frac{y^2}{128} = 1$$

$$\Rightarrow \frac{1}{16} \left[\frac{x^2}{9} + \frac{y^2}{8} \right] = 1$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{8} = 16$$

This is the equation of the required ellipse.

The coordinates of foci are $(\pm ae, 0)$.

$$\therefore 2ae = 2b$$

[given]

$$\Rightarrow ae = b$$

$$\Rightarrow (ae)^2 = b^2 \dots\dots\dots (i)$$

The length of latus-rectum is 10.

$$\Rightarrow \frac{2b^2}{a} = 10$$

[\because latus-rectum = $\frac{2b^2}{a}$]

$$\Rightarrow b^2 = \frac{10a}{2}$$

$$\Rightarrow b^2 = 5a \dots\dots\dots (ii)$$

Now,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = a^2 - a^2e^2$$

$$\Rightarrow b^2 = a^2 - b^2$$

$$\Rightarrow 2b^2 = a^2$$

$$\Rightarrow b^2 = \frac{a^2}{2}$$

Substituting $b^2 = \frac{a^2}{2}$ in equation (ii), we get

$$\frac{a^2}{2} = 5a$$

$$\Rightarrow a^2 = 10a$$

$$\Rightarrow a = 10$$

$$\Rightarrow a^2 = 100$$

Putting $a^2 = 100$ in $b^2 = \frac{a^2}{2}$, we get

$$b^2 = \frac{100}{2} = 50$$

\therefore The required equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{100} + \frac{y^2}{50} = 1$$

$$\Rightarrow \frac{x^2 + 2y^2}{100} = 1$$

$$\Rightarrow x^2 + 2y^2 = 100$$

This is the required equation of the ellipse.

Ellipse Ex 26.1 Q8(i)

Let $2a$ and $2b$ the major and minor axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1 \quad [\because \text{centre: } (-2, 3), \dots \dots \text{(i)}]$$

We have,

$$\text{semi-major axis} = a = 3$$

$$\Rightarrow a^2 = 9$$

and semi-minor axis = $b = 2$

$$\Rightarrow b^2 = 4$$

Putting $a^2 = 9$ and $b^2 = 4$ in equation (i), we get

$$\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$$

$$\Rightarrow \frac{4(x+2)^2 + 9(y-3)^2}{36} = 1$$

$$\Rightarrow 4(x+2)^2 + 9(y-3)^2 = 36$$

$$\Rightarrow 4[x^2 + 4 + 4x] + 9[y^2 + 9 - 6y] = 36$$

$$\Rightarrow 4x^2 + 16 + 16x + 9y^2 + 81 - 54y = 36$$

$$\Rightarrow 4x^2 + 9y^2 + 16x - 54y + 16 + 81 - 36 = 0$$

$$\Rightarrow 4x^2 + 9y^2 + 16x - 54y + 61 = 0$$

Ellipse Ex 26.1 Q8(ii)

Let $2a$ and $2b$ the minor and major axes of the ellipse. Then, its equation is

$$\frac{(x+2)^2}{a^2} + \frac{(y-3)^2}{b^2} = 1 \quad [\because \text{centre: } (-2, 3) \dots \dots \dots (i)]$$

We have,

$$\text{semi-major axis} = a = 2$$

$$\Rightarrow a^2 = 4$$

and semi-minor axis = $b = 3$

$$\Rightarrow b^2 = 9$$

Putting $a^2 = 4$ and $b^2 = 9$ in equation (i), we get

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

$$\Rightarrow \frac{9(x+2)^2 + 4(y-3)^2}{36} = 1$$

$$\Rightarrow 9(x+2)^2 + 4(y-3)^2 = 36$$

$$\Rightarrow 9[x^2 + 4 + 4x] + 4[y^2 + 9 - 6y] = 36$$

$$\Rightarrow 9x^2 + 36 + 36x + 4y^2 + 36 - 24y = 36$$

$$\Rightarrow 9x^2 + 4y^2 + 36x - 24y + 36 + 36 - 36 = 0$$

$$\Rightarrow 9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

Ellipse Ex 26.1 Q9(i)

Let $2a$ and $2b$ be the major and minor axes of the ellipse.

(i) when latus-rectum is half of minor axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2b$$

$$\Rightarrow 2b^2 = ab$$

$$\Rightarrow \frac{b^2}{b} = \frac{a}{2}$$

$$\Rightarrow b = \frac{a}{2}$$

$$\Rightarrow b^2 = \frac{a^2}{4}$$

Now, $b^2 = a^2(1 - e^2)$

$$\Rightarrow \frac{a^2}{4} = a^2(1 - e^2) \quad \left[\because b^2 = \frac{a^2}{4} \right]$$

$$\Rightarrow \frac{1}{4} = 1 - e^2$$

$$\Rightarrow e^2 = 1 - \frac{1}{4}$$

$$\Rightarrow e^2 = \frac{3}{4}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

Ellipse Ex 26.1 Q9(ii)

When latus-rectum is half of major-axis.

$$\frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow 2b^2 = a^2$$

$$\Rightarrow a^2 = 2b^2$$

Now,

$$\Rightarrow b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = 2b^2(1 - e^2) \quad \left[\because a^2 = 2b^2 \right]$$

$$\Rightarrow 1 = 2(1 - e^2)$$

$$\Rightarrow 1 = 2 - 2e^2$$

$$\Rightarrow 2e^2 = 2 - 1$$

$$\Rightarrow e^2 = \frac{1}{2}$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

Ellipse Ex 26.1 Q10(i)

We have,

$$x^2 + 2y^2 - 2x + 12y + 10 = 0$$

$$\Rightarrow x^2 - 2x + 2y^2 + 12y + 10 = 0$$

$$\Rightarrow (x^2 - 2x + 1 - 1) + 2(y^2 + 6y) + 10 = 0$$

$$\Rightarrow [(x - 1)^2 - 1] + 2[(y^2 + 2 \times y \times 3 + 9) - 9] + 10 = 0$$

$$\Rightarrow (x - 1)^2 - 1 + 2[(y + 3)^2 - 9] + 10 = 0$$

$$\Rightarrow (x - 1)^2 + 2(y + 3)^2 - 18 - 1 + 10 = 0$$

$$\Rightarrow (x - 1)^2 + 2(y + 3)^2 - 19 + 10 = 0$$

$$\Rightarrow (x - 1)^2 + 2(y + 3)^2 - 9 = 0$$

$$\Rightarrow (x - 1)^2 + 2(y + 3)^2 = 9$$

$$\Rightarrow \frac{(x - 1)^2}{9} + 2 \frac{(y + 3)^2}{9} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{9} + \frac{(y + 3)^2}{\frac{9}{2}} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{(3)^2} + \frac{(y + 3)^2}{\left(\frac{3}{\sqrt{2}}\right)^2} = 1 \dots\dots\dots (i)$$

\therefore The coordinates of centre of the ellipse are $(1, -3)$.

Shifting the origin at $(1, -3)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y , we have

$$x = X + 1 \quad \text{and} \quad y = Y - 3 \dots\dots\dots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2} + \frac{Y^2}{\left(\frac{3}{\sqrt{2}}\right)^2} = 1 \dots\dots\dots (iii)$$

This is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a = 3,$$

$$\text{and } b = \frac{3}{\sqrt{2}}.$$

Clearly, $a > b$. so, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

$$\text{major-axis} = 2a = 2 \times 3 = 6$$

$$\text{and, minor-axis} = 2b = \frac{2 \times 3}{\sqrt{2}} = 3\sqrt{2}$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{\left(\frac{3}{\sqrt{2}}\right)^2}{3^2}}$$

$$= \sqrt{1 - \frac{9}{2 \times 9}}$$

$$= \sqrt{1 - \frac{1}{2}}$$

$$= \sqrt{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X = \pm a e, Y = 0)$ i.e.,

$$\left(X = \pm \frac{3}{\sqrt{2}}, Y = 0\right)$$

Putting $X = \pm \frac{3}{\sqrt{2}}$ and $Y = 0$ in equation (ii), we get

$$x = \pm \frac{3}{\sqrt{2}} + 1 \text{ and } y = 0 - 3$$

$$\Rightarrow x = 1 \pm \frac{3}{\sqrt{2}} \text{ and } y = -3$$

We have,

$$x^2 + 4y^2 - 4x + 24y + 31 = 0$$

$$\Rightarrow x^2 - 4x + 4(y^2 + 6y) + 31 = 0$$

$$\Rightarrow [x^2 - 2 \times x \times 2 + 2^2 - 2^2] + 4[y^2 + 2 \times 3 \times y + 3^2 - 3^2] + 31 = 0$$

$$\Rightarrow [(x-2)^2 - 2^2] + 4[(y+3)^2 - 9] + 31 = 0$$

$$\Rightarrow (x-2)^2 - 4 + 4(y+3)^2 - 36 + 31 = 0$$

$$\Rightarrow (x-2)^2 + 4(y+3)^2 - 5 - 4 = 0$$

$$\Rightarrow (x-2)^2 + 4(y+3)^2 = 9$$

$$\Rightarrow \frac{(x-2)^2}{9} + \frac{4(y+3)^2}{9} = 1$$

$$\Rightarrow \frac{(x-2)^2}{9} + \frac{(y+3)^2}{\frac{9}{4}} = 1$$

$$\Rightarrow \frac{(x-2)^2}{3^2} + \frac{(y+3)^2}{\left(\frac{3}{2}\right)^2} = 1 \dots\dots\dots(i)$$

\(\therefore\) The coordinates of centre of the ellipse are (2, -3).

Shifting the origin at (2, -3) without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y, we have

$$x = X + 2 \quad \text{and} \quad y = Y - 3 \dots\dots\dots(ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{3^2} + \frac{Y^2}{\left(\frac{3}{2}\right)^2} = 1 \dots\dots\dots(iii)$$

This is of the form

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1, \text{ where}$$

$$a = 3 \quad \text{and} \quad b = \frac{3}{2}$$

Clearly, $a > b$. so, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

Length of the axes:

$$\text{Major-axis} = 2a = 2 \times 3 = 6$$

$$\text{and, Minor-axis} = 2b = 2 \times \frac{3}{2} = 3$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{9}{9}}$$

$$= \sqrt{1 - \frac{9}{4 \times 9}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{1 - \frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X = \pm ae, Y = 0)$

$$\text{i.e., } \left(X = \pm \frac{3\sqrt{3}}{2}, Y = 0 \right)$$

Putting $X = \pm \frac{3\sqrt{3}}{2}$ and $Y = 0$ in equation (ii), we get

$$x = +2 \text{ and } y = 0 - 3$$

$$\Rightarrow x = 2 \pm \frac{3\sqrt{3}}{2} \text{ and } y = -3.$$

so, the coordinates of foci with respect to old axes are given by $\left(2 \pm \frac{3\sqrt{3}}{2}, -3 \right)$

We have,

$$4x^2 + y^2 - 8x + 2y + 1 = 0$$

$$\Rightarrow 4(x^2 - 2x) + (y^2 + 2y) + 1 = 0$$

$$\Rightarrow 4[(x^2 - 2x + 1) - 1] + [(y^2 + 2y + 1) - 1] + 1 = 0$$

$$\Rightarrow 4[(x - 1)^2 - 1] + [(y + 1)^2 - 1] + 1 = 0$$

$$\Rightarrow 4(x - 1)^2 - 4 + (y + 1)^2 - 1 + 1 = 0$$

$$\Rightarrow 4(x - 1)^2 + (y + 1)^2 - 4 = 0$$

$$\Rightarrow 4(x - 1)^2 + (y + 1)^2 = 4$$

$$\Rightarrow \frac{(x - 1)^2}{1} + \frac{(y + 1)^2}{4} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{1^2} + \frac{(y + 1)^2}{2^2} = 1 \dots\dots\dots (i)$$

\therefore The coordinates of centre of the ellipse are $(1, -1)$.

Shifting the origin at $(1, -1)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and Y , we have

$$x = X + 1 \quad \text{and} \quad y = Y - 1 \dots\dots\dots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{2^2} = 1$$

This is of the form

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1, \text{ where}$$

$$a = 1 \quad \text{and} \quad b = 2$$

Clearly, $b > a$. so, the given equation represents an ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

$$\text{Major-axis} = 2b = 2 \times 2 = 4$$

$$\text{Minor-axis} = 2a = 2 \times 1 = 2$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{a^2}{b^2}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{4-1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X = 0, Y = \pm be)$

i.e., $(X = 0, Y = \pm\sqrt{3})$

Putting $X = 0$ and $Y = \pm\sqrt{3}$ in equation (iii), we get

$$x = 0 + 1 \text{ and } y = \pm\sqrt{3} - 1$$

$$\Rightarrow x = 1 \text{ and } y = -1 \pm \sqrt{3}$$

We have,

$$3x^2 + 4y^2 - 12x - 8y + 4 = 0$$

$$\Rightarrow 3x^2 - 12x + 4y^2 - 8y + 4 = 0$$

$$\Rightarrow 3(x^2 - 4x) + 4(y^2 - 2y) + 4 = 0$$

$$\Rightarrow 3[x^2 - 2 \times x \times 2 + 2^2 - 2^2] + 4[y^2 - 2 \times y \times 1^2 - 1^2] + 4 = 0$$

$$\Rightarrow 3[(x - 2)^2 - 4] + 4[(y + 1)^2 - 1] + 4 = 0$$

$$\Rightarrow 3(x - 2)^2 - 12 + 4(y + 1)^2 - 4 + 4 = 0$$

$$\Rightarrow 3(x - 2)^2 + 4(y - 1)^2 - 12 = 0$$

$$\Rightarrow 3(x - 2)^2 + 4(y - 1)^2 = 12$$

$$\Rightarrow \frac{3(x - 2)^2}{12} + \frac{4(y - 1)^2}{12} = 1$$

$$\Rightarrow \frac{(x - 2)^2}{4} + \frac{(y - 1)^2}{3} = 1$$

$$\Rightarrow \frac{(x - 2)^2}{2^2} + \frac{(y - 1)^2}{(\sqrt{3})^2} = 1 \dots\dots\dots (i)$$

\therefore The coordinates of centre of the ellipse are $(2, 1)$.

Shifting the origin at $(2, 1)$ without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by X and Y , we have

$$x = X + 2 \quad \text{and} \quad y = Y - 1 \dots\dots\dots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{2^2} + \frac{Y^2}{(\sqrt{3})^2} = 1, \text{ where}$$

$$a = 2 \quad \text{and} \quad b = \sqrt{3}$$

Clearly, $a > b$, so, the given equation represents an ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

$$\text{Major-axis} = 2a = 2 \times 2 = 4$$

$$\text{and, Minor-axis} = 2b = 2 \times \sqrt{3} = 2\sqrt{3}$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{3}{4}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $(X = \pm ae, Y = 0)$

$$\text{i.e., } (X = \pm 1, Y = 0)$$

Putting $X = \pm 1$ and $Y = 0$ in equation (ii), we get

$$x = \pm 1 + 2 \text{ and } y = 0 + 1$$

$$\Rightarrow x = 2 \pm 1 \text{ and } y = 1$$

so, the coordinates of foci with respect to old axes are given by $(2 \pm 1, 1)$ i.e., $(3, 1)$ and $(1, 1)$.

Ellipse Ex 26.1 Q10(v)

We have,

$$4x^2 + 16y^2 - 24x - 32y - 12 = 0$$

$$\Rightarrow 4x^2 - 24x + 16y^2 - 32y - 12 = 0$$

$$\Rightarrow 4(x^2 - 6x) + 16(y^2 - 2y) - 12 = 0$$

$$\Rightarrow 4[x^2 - 2 \times x \times 3 + 3^2 - 3^2] + 16[y^2 - 2y + 1^2 - 1^2] - 12 = 0$$

$$\Rightarrow 4[(x - 3)^2 - 9] + 16[(y - 1)^2 - 1] - 12 = 0$$

$$\Rightarrow 4(x - 3)^2 - 36 + 16(y - 1)^2 - 16 - 12 = 0$$

$$\Rightarrow 4(x - 3)^2 + 16(y - 1)^2 - 36 - 28 = 0$$

$$\Rightarrow 4(x - 3)^2 + 16(y - 1)^2 - 64 = 0$$

$$\Rightarrow 4(x - 3)^2 + 16(y - 1)^2 = 64$$

$$\Rightarrow \frac{4(x - 3)^2}{64} + \frac{16(y - 1)^2}{64} = 1$$

$$\Rightarrow \frac{(x - 3)^2}{16} + \frac{(y - 1)^2}{4} = 1$$

$$\Rightarrow \frac{(x - 3)^2}{(4)^2} + \frac{(y - 1)^2}{(2)^2} = 1 \dots\dots\dots (i)$$

\therefore The coordinates of centre of the ellipse are (3, 1).

Shifting the origin at (3, 1) without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by X and Y , we have

$$x = X + 3 \quad \text{and} \quad y = Y + 1 \dots\dots\dots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1, \text{ where}$$

$$a = 4 \quad \text{and} \quad b = 2$$

Clearly, $a > b$. so, the given equation represents an ellipse whose major and minor axes are along Y and X axes respectively.

Length of the axes:

$$\text{Major-axis} = 2a = 2 \times 4 = 8$$

$$\text{and, Minor-axis} = 2b = 2 \times 2 = 4$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{4}{16}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $(X = \pm a e, Y = 0)$

$$\text{i.e., } (X = \pm 2\sqrt{3}, Y = 0)$$

Putting $X = \pm 2\sqrt{3}$ and $Y = 0$ in equation (ii), we get

$$x = \pm 2\sqrt{3} + 3 \text{ and } y = 0 + 1$$

$$\Rightarrow x = 3 \pm 2\sqrt{3} \text{ and } y = 1$$

Ellipse Ex 26.1 Q10(vi)

We have,

$$x^2 + 4y^2 - 2x = 0$$

$$\Rightarrow x^2 - 2x + 4y^2 = 0$$

$$\Rightarrow (x^2 - 2x + 1^2 - 1^2) + 4y^2 = 0$$

$$\Rightarrow (x - 1)^2 - 1 + 4y^2 = 0$$

$$\Rightarrow (x - 1)^2 + 4y^2 = 1$$

$$\Rightarrow \frac{(x - 1)^2}{1} + \frac{y^2}{\frac{1}{4}} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1 \dots\dots\dots (i)$$

∴ The coordinates of centre of the ellipse are (1, 0).

Shifting the origin at (1, 0) without rotating the coordinate axes and denoting the new coordinates w.r.t the new axes by X and Y , we have

$$x = X + 1 \quad \text{and} \quad y = Y \dots\dots\dots (ii)$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{1^2} + \frac{Y^2}{\left(\frac{1}{2}\right)^2} = 1, \text{ where}$$

$$a = 1 \quad \text{and} \quad b = \frac{1}{2}$$

Clearly, $a > b$. so, the given equation represents an ellipse whose major and minor axes are along X and Y axes respectively.

Length of the axes:

$$\text{Major-axis} = 2 \times a = 2 \times 1 = 2$$

$$\text{and, Minor-axis} = 2 \times b = 2 \times \frac{1}{2} = 1$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Foci: The coordinates of the foci w.r.t the new axes are given by $(X = \pm a e, Y = 0)$

$$\text{i.e., } \left(X = \pm \frac{\sqrt{3}}{2}, Y = 0 \right)$$

Putting $X = \pm \frac{\sqrt{3}}{2}$ and $Y = 0$ in equation (ii), we get

$$x = \pm \frac{\sqrt{3}}{2} + 1 \text{ and } y = 0$$

$$\Rightarrow x = 1 \pm \frac{\sqrt{3}}{2} \text{ and } y = 0.$$

so, the coordinates of foci w.r.t the old axes are given by $\left(1 \pm \frac{\sqrt{3}}{2}, 0 \right)$.

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The coordinates of its foci are $(\pm ae, 0)$ i.e., $(\pm 3, 0)$.

$$\therefore ae = 3$$

$$\Rightarrow (ae)^2 = 9 \dots\dots\dots (ii)$$

The required ellipse passes through $(4, 1)$.

$$\therefore \frac{(4)^2}{a^2} + \frac{(1)^2}{b^2} = 1$$

$$\Rightarrow \frac{16}{a^2} + \frac{1}{b^2} = 1$$

$$\Rightarrow 16b^2 + a^2 = a^2b^2$$

$$\Rightarrow a^2 + 16b^2 = a^2b^2 \dots\dots\dots (iii)$$

Now,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = a^2 - a^2e^2$$

$$\Rightarrow b^2 = a^2 - 9 \quad \text{[Using equation (ii)]} \dots\dots\dots (iv)$$

Substituting $b^2 = a^2 - 9$ in equation (iii), we get

$$a^2 + 16(a^2 - 9) = a^2(a^2 - 9)$$

$$\Rightarrow a^2 + 16a^2 - 144 = a^4 - 9a^2$$

$$\Rightarrow 17a^2 - 144 = a^4 - 9a^2$$

$$\Rightarrow a^4 - 9a^2 - 17a^2 + 144 = 0$$

$$\Rightarrow a^4 - 26a^2 + 144 = 0$$

$$\Rightarrow a^4 - 18a^2 - 8a^2 + 144 = 0$$

$$\Rightarrow a^2(a^2 - 18) - 8(a^2 - 18) = 0$$

$$\Rightarrow (a^2 - 18)(a^2 - 8) = 0$$

$$\Rightarrow a^2 = 18 \quad \text{or, } a^2 \neq 8$$

$$\Rightarrow a^2 = 18$$

Putting $a^2 = 18$ in equation (iv), we get

$$b^2 = 18 - 9 = 9$$

\therefore The required equation of the ellipse is

$$\frac{x^2}{18} + \frac{y^2}{9} = 1$$

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \dots\dots\dots (i)$$

The length of latus-rectum = 5

$$\therefore \frac{2b^2}{a} = 5$$

$$\Rightarrow b^2 = \frac{\sqrt{5}}{2} \dots\dots\dots (ii)$$

Now,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow \frac{5a}{2} = a^2 \left[1 - \left(\frac{2}{3} \right)^2 \right] \quad \left[\because e = \frac{2}{3} \right]$$

$$\Rightarrow \frac{5a}{2} = a^2 \left[1 - \frac{4}{9} \right]$$

$$\Rightarrow \frac{5}{2} = a \left(\frac{5}{9} \right)$$

$$\Rightarrow \frac{5}{2} \times \frac{9}{5} = a$$

$$\Rightarrow a = \frac{9}{2}$$

$$\Rightarrow a^2 = \frac{81}{4}$$

Putting $a = \frac{9}{2}$ in $b^2 = \frac{5a}{2}$, we get

$$b^2 = \frac{5}{2} \times \frac{9}{2}$$

$$\Rightarrow b^2 = \frac{45}{4}$$

Substituting $a^2 = \frac{81}{4}$ and $b^2 = \frac{45}{4}$ in equation(i), we get

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q13

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a < b \dots\dots\dots(i) \quad [\text{foci on y-axis}]$$

Now,

$$a^2 = b^2(1 - e^2)$$

$$\Rightarrow a^2 = b^2 \left[1 - \left(\frac{3}{4} \right)^2 \right]$$

$$\Rightarrow a^2 = b^2 \left[1 - \frac{9}{16} \right]$$

$$\Rightarrow a^2 = b^2 \times \frac{7}{16}$$

$$\Rightarrow a^2 = \frac{7}{16} b^2 \dots\dots\dots(ii)$$

The required ellipse through (6, 4).

$$\therefore \frac{(6)^2}{a^2} + \frac{(4)^2}{b^2} = 1$$

$$\Rightarrow \frac{36}{a^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \frac{36}{7b^2} + \frac{16}{b^2} = 1 \quad \left[\because a^2 = \frac{7}{16} b^2 \right]$$

$$\Rightarrow \frac{36 \times 16}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \frac{576}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \frac{36}{7b^2} + \frac{16}{b^2} = 1 \quad \left[\because a^2 = \frac{7}{16}b^2 \right]$$

$$\Rightarrow \frac{36 \times 16}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \frac{576}{7b^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow \frac{1}{b^2} \left[\frac{576}{7} + \frac{16}{1} \right] = 1$$

$$\Rightarrow \frac{576}{7} + \frac{16}{1} = b^2$$

$$\Rightarrow \frac{576 + 112}{7} = b^2$$

$$\Rightarrow b^2 = \frac{688}{7}$$

Putting $b^2 = \frac{688}{7}$ in equation (i), we get

$$a^2 = \frac{7}{16} \times \frac{688}{7}$$

$$\Rightarrow a^2 = \frac{688}{16} = 43$$

Putting $a^2 = 43$ and $b^2 = \frac{688}{7}$ in equation (i), we get

$$\frac{x^2}{43} + \frac{y^2}{\frac{688}{7}} = 1$$

$$\Rightarrow \frac{x^2}{43} + \frac{7y^2}{688} = 1$$

This is the equation of the required ellipse.

Ellipse Ex 26.1 Q14

Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b \dots\dots\dots(i)$$

The required ellipse passes through (4,3) and (-1, 4).

$$\therefore \frac{(4)^2}{a^2} + \frac{(3)^2}{b^2} = 1$$

$$\Rightarrow \frac{16}{a^2} + \frac{9}{b^2} = 1$$

$$\Rightarrow 16b^2 + 9a^2 = a^2b^2 \dots\dots\dots(ii)$$

$$\text{and } \frac{(-1)^2}{a^2} + \frac{(4)^2}{b^2} = 1$$

$$\Rightarrow \frac{1}{a^2} + \frac{16}{b^2} = 1$$

$$\Rightarrow b^2 + 16a^2 = a^2b^2 \dots\dots\dots(iii)$$

Multiplying equation (iii) by 16, we get

$$16b^2 + 256a^2 = 16a^2b^2 \dots\dots\dots(iv)$$

Subtracting equation (ii) from equation (iv), we get

$$256a^2 - 9a^2 = 16a^2b^2 - a^2b^2$$

$$\Rightarrow 247a^2 = 15a^2b^2$$

$$\Rightarrow \frac{247}{15} = b^2$$

$$\Rightarrow b^2 = \frac{247}{15}$$

Putting $b^2 = \frac{247}{15}$ in equation (iii) we get

$$\frac{247}{15} + 16a^2 = a^2 \times \frac{247}{15}$$

$$\Rightarrow 16a^2 - \frac{247a^2}{15} = \frac{-247}{15}$$

$$\Rightarrow \frac{240a^2 - 247a^2}{15} = \frac{-247}{15}$$

$$\Rightarrow -7a^2 = -247$$

$$\Rightarrow a^2 = \frac{247}{7}$$

Putting $a^2 = \frac{247}{7}$ and $b^2 = \frac{247}{15}$ in equation equation (i), we get

$$\frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1$$

$$\Rightarrow \frac{7x^2}{247} + \frac{15y^2}{247} = 1$$

This is the equation of the required ellipse.

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b$$

[∵ axes lie along the coordinates axes]

Now,

$$b^2 = a^2(1 - e^2)$$

$$\Rightarrow b^2 = a^2 \left[1 - \left(\frac{2}{\sqrt{5}} \right)^2 \right] \quad \left[\because e = \frac{2}{\sqrt{5}} \right]$$

$$\Rightarrow b^2 = a^2 \left[1 - \frac{2}{5} \right]$$

$$b^2 = a^2 \times \frac{3}{5}$$

$$\Rightarrow b^2 = \frac{3a^2}{5} \dots\dots\dots (i)$$

The required ellipse passes through (-3, 1)

$$\therefore \frac{(-3)^2}{a^2} + \frac{1^2}{b^2} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{1}{b^2} = 1 \dots\dots\dots (ii)$$

Putting $b^2 = \frac{3a^2}{5}$ in equation (ii), we get

$$\frac{9}{a^2} + \frac{1}{\frac{3a^2}{5}} = 1$$

$$\Rightarrow \frac{9}{a^2} + \frac{5}{3a^2} = 1$$

$$\Rightarrow \frac{1}{a^2} \left[\frac{9}{1} + \frac{5}{3} \right] = 1$$

$$\Rightarrow \frac{27 + 5}{3} = a^2$$

Putting $a^2 = \frac{32}{3}$ in equation (ii), we get

$$b^2 = \frac{3}{5} \times \frac{32}{3} = \frac{32}{5}$$

Substituting $a^2 = \frac{32}{3}$ and $b^2 = \frac{32}{5}$ in equation (i), we get

$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$$

$$\Rightarrow \frac{3x^2}{32} + \frac{5y^2}{32} = 1$$

$$\Rightarrow 3x^2 + 5y^2 = 32$$

This is the equation of the required ellipse.

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \dots\dots\dots(i)$$

We have,

$$2ae = 8 \quad \text{[given]}$$

$$\Rightarrow e = \frac{8}{2a}$$

$$\Rightarrow e = \frac{4}{a} \dots\dots\dots(ii)$$

Now,

$$\frac{2a}{e} = 18 \quad \text{[given]}$$

$$\Rightarrow a = \frac{18e}{2}$$

$$\Rightarrow a = 9e \dots\dots\dots(iii)$$

Using equation (ii) and equation (iii), we get

$$a = \frac{9 \times 4}{a}$$

$$\Rightarrow a^2 = 36$$

Now,

$$b^2 = a^2 (1 - e^2)$$

$$\Rightarrow b^2 = 36 - (ae)^2$$

$$\Rightarrow b^2 = 36 - 16 \quad \text{[Using equation (iii)]}$$

$$\Rightarrow b^2 = 20$$

Putting $a^2 = 36$ and $b^2 = 20$ in equation (i), we get

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

This is the equation of the required ellipse.

Let the equation of the ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \dots\dots\dots (i)$$

The coordinates of vertices are $(0, \pm b)$ i.e., $(0, \pm 10)$.

$$\therefore b = 10$$

$$\Rightarrow b^2 = 100$$

Now,

$$a^2 = b^2 (1 - e^2)$$

$$\Rightarrow a^2 = 100 \left[1 - \left(\frac{4}{5} \right)^2 \right]$$

$$\Rightarrow a^2 = 100 \left[1 - \frac{16}{25} \right]$$

$$\Rightarrow a^2 = 100 \left[\frac{9}{25} \right]$$

$$\Rightarrow a^2 = 4 \times 9 = 36$$

Putting $a^2 = 36$ and $b^2 = 100$ in equation (i), we get

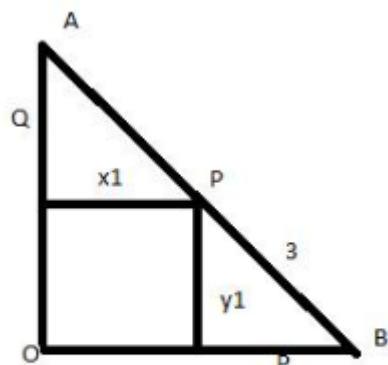
$$\frac{x^2}{36} + \frac{y^2}{100} = 1$$

$$\Rightarrow \frac{100x^2 + 36y^2}{3600} = 1$$

$$\Rightarrow 100x^2 + 36y^2 = 3600$$

This is the equation of the required ellipsis.

Ellipse Ex 26.1 Q18



Using similar triangles principle, we can write

$$\frac{Q}{9} = \frac{y_1}{3}$$

$$Q = 3y_1$$

$$\text{Similarly, } p = \frac{x}{3}$$

Point P(x,y)

$$\text{So } OB = x + \frac{x}{3}$$

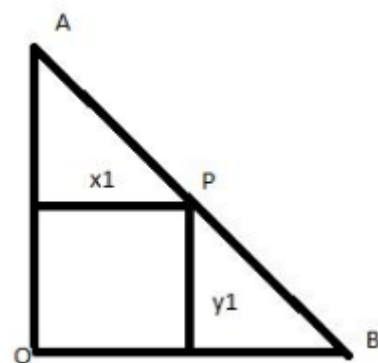
$$OA = y + 3y = 4y$$

using pythagoreus theorem, we get

$$(4y)^2 + \left(\frac{4x}{3}\right)^2 = 12^2$$

$$\frac{y^2}{9} + \frac{x^2}{81} = 1 \text{ is the equation of ellipse}$$

Ellipse Ex 26.1 Q19



From above figure,

Assume length $AB = l$

$$AP = a, PB = b$$

Assume $\widehat{ABO} = \theta$

$$\text{so } x_1 = a \cos \theta, y_1 = b \sin \theta$$

$$\Rightarrow \left(\frac{x_1}{a}\right)^2 + \left(\frac{y_1}{b}\right)^2 = 1$$

Ellipse Ex 26.1 Q20

Let point be (x,y)

Given distances of point from $(0, 4)$ are

$2/3$ of their distances from the line $y = 9$

$$\sqrt{(x-0)^2 + (y-4)^2} = \frac{2}{3} \left(\sqrt{(y-9)^2} \right)$$

Squaring on both sides, we get

$$9 \left[(x-0)^2 + (y-4)^2 \right] = 4 \left[(y-9)^2 \right]$$

$$9x^2 + 9y^2 + 144 - 72y = 4y^2 + 324 - 72y$$

$$9x^2 + 5y^2 = 180$$