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Solutions
Class 11 Maths
Chapter 27
Ex 27.1

Hyperbola Ex 27.1 Q1

Let $S(-1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola. Draw PM perpendicular from P on the directrix. Then, by definition.

$$SP = ePM$$

$$\Rightarrow SP^2 = e^2PM^2$$

$$\Rightarrow (x+1)^2 + (y-1)^2 = (3)^2 \left[\frac{x-y+3}{\sqrt{1^2+(-1)^2}} \right]^2 \quad [\because e=3]$$

$$\Rightarrow x^2 + 1 + 2x + y^2 + 1 - 2y = \frac{9[x-y+3]^2}{2}$$

$$\Rightarrow 2[x^2 + y^2 + 2x - 2y + 2] = 9[x-y+3]^2$$

$$\Rightarrow 2x^2 + 2y^2 + 4x - 4y + 4 = 9[x^2(-y)^2 + 3^2 + 2 \times x \times (-y) + 2 \times (-y) \times 3 + 2 \times 3 \times x]$$

$$\Rightarrow 2x^2 + 2y^2 + 4x - 4y - 4 = 9[x^2 + y^2 + 9 - 2xy - 6y + 6x]$$

$$\Rightarrow 2x^2 + 2y^2 + 4x - 4y + 4 = 9x^2 + 9y^2 + 81 - 18xy - 54y + 4y + 81 - 4 = 0$$

$$\Rightarrow 7x^2 + 7y^2 - 18xy + 50x - 50y + 77 = 0$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(i)

Let $S(0,3)$ be the focus and $P(x,y)$ be a point on the hyperbola.

Draw PM perpendicular from P on the directrix. Then, by definition

$$SP = ePM$$

$$\Rightarrow SP^2 = e^2PM^2$$

$$\Rightarrow (x-0)^2 + (y-3)^2 = 2^2 \left[\frac{x+y-1}{\sqrt{1^2+1^2}} \right]^2 \quad [\because e=2]$$

$$\Rightarrow x^2 + y^2 + 9 - 6y = \frac{4[x+y-1]^2}{2}$$

$$\Rightarrow x^2 + y^2 - 6y + 9 = 2(x+y-1)^2$$

$$\Rightarrow x^2 + y^2 - 6y + 9 = 2[x^2 + y^2 + (-1)^2 + 2xy + 2 \times y \times (-1) + 2 \times (-1) \times x]$$

$$\Rightarrow x^2 + y^2 - 6y + 9 = 2[x^2 + y^2 + 1 + 2xy - 2y - 2x]$$

$$\Rightarrow x^2 + y^2 - 6y + 9 = 2x^2 + 2y^2 + 2 + 4xy - 4y - 4x$$

$$\Rightarrow 2x^2 - x^2 + 2y^2 - y^2 + 4xy - 4x - 4y + 6y + 2 - 9 = 0$$

$$\Rightarrow x^2 + y^2 + 4xy - 4x + 2y - 7 = 0$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(ii)

Let $S(1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola.
 Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 & sP = ePM \\
 \Rightarrow & sP^2 = e^2PM^2 \\
 \Rightarrow & (x-1)^2 + (y-1)^2 = 2^2 \left[\frac{3x+4y+8}{\sqrt{3^2+4^2}} \right]^2 \quad [\because e=2] \\
 \Rightarrow & x^2+1-2x+y^2+1-2y = 4 \left[\frac{3x+4y+8}{\sqrt{25}} \right]^2 \\
 \Rightarrow & x^2+y^2-2x-2y+2 = \frac{4(3x+4y+8)^2}{25} \\
 \Rightarrow & 25x^2+25y^2-50x-50y+50 = 4(3x+4y+8)^2 \\
 \Rightarrow & 25x^2+25y^2-50x-50y+50 = 4[9x^2+16y^2+6y+24xy+64y+48x] \\
 \Rightarrow & 25x^2+25y^2-50x-50y+50 = 36x^2+64y^2+256+96xy+256y+192x \\
 \Rightarrow & 36x^2-25x^2+64y^2-25y^2+96xy+192x+50x+256y+50y+256-50 = 0 \\
 \Rightarrow & 11x^2+39y^2+96xy+242x+306y+206 = 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(iii)

Let $S(1,1)$ be the focus and $P(x,y)$ be a point on the hyperbola.
 Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 & sP = ePM \\
 \Rightarrow & sP^2 = e^2PM^2 \\
 \Rightarrow & (x-1)^2 + (y-1)^2 = (\sqrt{3})^2 \left[\frac{2x+y-1}{\sqrt{2^2+1^2}} \right]^2 \quad [\because e=2] \\
 \Rightarrow & x^2+1-2x+y^2+1-2y = \frac{3[2x+y-1]^2}{5} \\
 \Rightarrow & 5[x^2+y^2-2x-2y+2] = 3(2x+y-1)^2 \\
 \Rightarrow & 5x^2+5y^2-10x-10y+10 = 3[(2x)^2+y^2+(-1)^2+2 \times 2x \times y+2 \times y \times (-1)+2 \times (-1) \times 2x] \\
 \Rightarrow & 5x^2+5y^2-10x-10y+10 = 3[4x^2+y^2+1+4xy-2y-4x] \\
 \Rightarrow & 5x^2+5y^2-10x-10y+10 = 12x^2+3y^2+3+12xy-6y-12x \\
 \Rightarrow & 12x^2-5x^2+3y^2-5y^2+12xy-12x+10x-6y+10y+3-10 = 0 \\
 \Rightarrow & 7x^2-2y^2+12xy-2x+4y-7 = 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(iv)

Let $S(2, -1)$ be the focus and $P(x, y)$ be a point on the hyperbola.
 Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 & SP = ePM \\
 \Rightarrow & SP^2 = e^2 PM^2 \\
 \Rightarrow & (x - 2)^2 + (y + 1)^2 = 2^2 \left[\frac{2x + 3y - 1}{\sqrt{2^2 + 3^2}} \right]^2 \quad [\because e = 2] \\
 \Rightarrow & x^2 + 4 - 4x + y^2 + 1 + 2y = \frac{4[2x + 3y - 1]^2}{13} \\
 \Rightarrow & 13[x^2 + y^2 - 4x + 2y + 5] = 4(2x + 3y - 1)^2 \\
 \Rightarrow & 13x^2 + 13y^2 - 52x + 26y + 65 = 4[2x + 3y - 1]^2 \\
 \Rightarrow & 13x^2 + 13y^2 - 52x + 26y + 65 = 4[(2x)^2 + (3y)^2 + (-1)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-1) + 2 \times (-1) \times 2x] \\
 \Rightarrow & 13x^2 + 13y^2 - 52x + 26y + 65 = 4[4x^2 + 9y^2 + 1 + 12xy - 6y - 4x] \\
 \Rightarrow & 13x^2 + 13y^2 - 52x + 26y + 65 = 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x \\
 \Rightarrow & 16x^2 - 13x^2 + 36y^2 - 13y^2 + 48xy - 16x + 52x - 24y - 26y + 4 - 65 = 0 \\
 \Rightarrow & 3x^2 + 23y^2 + 48xy + 36x - 50y - 61 = 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(v)

Let $S(a, 0)$ be the focus and $P(x, y)$ be a point on the hyperbola.
 Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 & SP = ePM \\
 \Rightarrow & SP^2 = e^2 PM^2 \\
 \Rightarrow & (x - a)^2 + (y - 0)^2 = \left(\frac{4}{3}\right)^2 \left[\frac{2x - y + a}{\sqrt{2^2 + (-1)^2}} \right]^2 \quad \left[\because e = \frac{4}{3}\right] \\
 \Rightarrow & x^2 + a^2 - 2ax + y^2 = \frac{16}{9} \times \frac{[2x - y + a]^2}{5} \\
 \Rightarrow & 45[x^2 + y^2 - 2ax + a^2] = 16[2x - y + a]^2 \\
 \Rightarrow & 45x^2 + 45y^2 - 90ax + 45a^2 = 16[(2x)^2 + (-y)^2 + a^2 + 2 \times 2x(-y) + 2 \times (-y) \times a + 2 \times a \times 2x] \\
 \Rightarrow & 45x^2 + 45y^2 - 90ax + 45a^2 = 16[4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax] \\
 \Rightarrow & 45x^2 + 45y^2 - 90ax + 45a^2 = 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax \\
 \Rightarrow & 64x^2 - 45x^2 + 16y^2 - 45y^2 - 64xy + 64ax + 90ax - 32ay + 16a^2 - 45a^2 = 0 \\
 \Rightarrow & 19x^2 - 29y^2 - 64xy + 154ax - 32ay - 29a^2 = 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q2(vi)

Let $S(2, 2)$ be the focus and $P(x, y)$ be a point on the hyperbola.
 Draw PM perpendicular from P on the directrix. Then, by definition

$$\begin{aligned}
 & SP = ePM \\
 \Rightarrow & SP^2 = e^2 PM^2 \\
 \Rightarrow & (x-2)^2 + (y-2)^2 = 2^2 \left[\frac{x+y-9}{\sqrt{1^2+1^2}} \right]^2 \quad \left[\because e = \frac{4}{3} \right] \\
 \Rightarrow & x^2 + 4 - 4x + y^2 + 4 - 4y = \frac{4[x+y-9]^2}{2} \\
 \Rightarrow & x^2 + y^2 - 4x - 4y + 8 = 2[x+y-9]^2 \\
 \Rightarrow & x^2 + y^2 - 4x - 4y + 8 = 2[x^2 + y^2 + (-9)^2 + 2 \times x \times y + 2 \times y \times (-9) + 2 \times (-9) \times x] \\
 \Rightarrow & x^2 + y^2 - 4x - 4y + 8 = 2[x^2 + y^2 + 81 + 2xy - 18y + 18x] \\
 \Rightarrow & x^2 + y^2 - 4x - 4y + 8 = [2x^2 + 2y^2 + 162 + 4xy - 36y - 36x] \\
 \Rightarrow & 2x^2 - x^2 + 2y^2 - y^2 + 4xy - 36x + 4x - 36y + 4y + 162 - 8 = 0 \\
 \Rightarrow & x^2 + y^2 + 4xy - 32x - 32y + 154 = 0
 \end{aligned}$$

This is the required equation of the hyperbola.

Hyperbola Ex 27.1 Q3(i)

We have,

$$\begin{aligned}
 & 9x^2 - 16y^2 = 144 \\
 \Rightarrow & \frac{9x^2}{144} - \frac{16y^2}{144} = 1 \\
 \Rightarrow & \frac{x^2}{16} - \frac{y^2}{9} = 1
 \end{aligned}$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = 16$ and $b^2 = 9$

Eccentricity: The eccentricity e is given by

$$\begin{aligned}
 e &= \sqrt{1 + \frac{b^2}{a^2}} \\
 &= \sqrt{1 + \frac{9}{16}} \\
 &= \sqrt{\frac{25}{16}} \\
 &= \frac{5}{4}
 \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$ i.e., $(\pm 5, 0)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned}
 & x = \pm \frac{a}{e} \text{ i.e., } x = \pm \frac{16}{5} \\
 \therefore & 5x = \pm 16 \\
 \Rightarrow & 5x \mp 16 = 0
 \end{aligned}$$

Length of latus-rectum: The length of the latus-rectum

$$= \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$$

Hyperbola Ex 27.1 Q3(ii)

We have,

$$16x^2 - 9y^2 = -144$$

$$\Rightarrow \frac{16x^2}{144} - \frac{9y^2}{144} = -1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = -1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, where $a^2 = 9$ and $b^2 = 16$

$$\therefore a = 3 \text{ and } b = 4$$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{a^2}{b^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Foci: The coordinates of the foci are $(0, \pm be)$.

$$\begin{aligned} \therefore (0, \pm be) &= \left(0, \pm 4 \times \frac{5}{4}\right) \\ &= (0, \pm 5) \end{aligned}$$

\therefore the coordinates of the foci are $(0, \pm 5)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned} y &= \frac{\pm b}{e} \\ \Rightarrow y &= \pm \frac{4}{\frac{5}{4}} = \pm \frac{16}{5} \\ \Rightarrow 5y \mp 16 &= 0 \end{aligned}$$

Latus-rectum: The length of the latus-rectum

$$\begin{aligned} &= \frac{2a^2}{b} \\ &= \frac{2 \times 9}{4} = \frac{9}{2} \end{aligned}$$

Hyperbola Ex 27.1 Q3(iii)

We have,

$$4x^2 - 3y^2 = 36$$

$$\Rightarrow \frac{4x^2}{36} - \frac{3y^2}{36} = 1$$

$$\Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a^2 = 9$ and $b^2 = 12$

$$\therefore a = 3 \text{ and } b = \sqrt{12} = 2\sqrt{3}$$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{12}{9}} \\ &= \sqrt{1 + \frac{4}{3}} \\ &= \sqrt{\frac{7}{3}} \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$.

$$\begin{aligned} \therefore \pm ae &= \pm 3 \times \sqrt{\frac{7}{3}} \\ &= \pm 3 \times \frac{\sqrt{7}}{\sqrt{3}} \\ &= \pm \sqrt{3} \times \sqrt{7} \\ &= \pm \sqrt{21} \end{aligned}$$

$$\therefore (\pm ae, 0) = (\pm \sqrt{21}, 0)$$

\therefore the coordinates of the foci are $(\pm \sqrt{21}, 0)$

Equations of the directrices: The equations of the directrices are

$$x = \frac{\pm a}{e}$$

$$\begin{aligned} \therefore x &= \pm 3 \times \frac{1}{\frac{\sqrt{7}}{\sqrt{3}}} \\ &= \pm \frac{3\sqrt{3}}{\sqrt{7}} \end{aligned}$$

$$\Rightarrow \sqrt{7}x \mp 3\sqrt{3} = 0$$

\therefore The equations of the directrices are $\sqrt{7}x \mp 3\sqrt{3} = 0$

Latus-rectum: The length of the latus-rectum

$$= \frac{2b^2}{a} = \frac{2 \times 12}{3} = 8$$

Hyperbola Ex 27.1 Q3(iv)

We have,

$$3x^2 - y^2 = 4$$

$$\Rightarrow \frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\Rightarrow \frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{2^2} = 1$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = \frac{2}{\sqrt{3}}$ and $b = 2$

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{4}{\frac{4}{3}}} \\ &= \sqrt{1 + 3} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$\therefore \pm ae = \pm \frac{2}{\sqrt{3}} \times 2 = \pm \frac{4}{\sqrt{3}}$$

The coordinates of the foci are $\left(\pm \frac{4}{\sqrt{3}}, 0\right)$

Equations of the directrices: The equations of the directrices are

$$\begin{aligned}x &= \pm \frac{a}{e} \\&= \pm \frac{2}{\frac{2}{\sqrt{3}}} \\&= \pm \frac{1}{\sqrt{3}}\end{aligned}$$

$$\Rightarrow \sqrt{3}x \mp 1 = 0$$

Latus-rectum: The length of the latus-rectum = $\frac{2b^2}{a}$.

$$\begin{aligned}\therefore \frac{2b^2}{a} &= 2 \times \frac{4}{\sqrt{3}} \\&= 4\sqrt{3}\end{aligned}$$

Hyperbola Ex 27.1 Q3(v)

Hyperbola Ex 27.1 Q4

We have,

$$\begin{aligned}25x^2 - 36y^2 &= 225 \\ \Rightarrow \frac{25x^2}{225} - \frac{36y^2}{225} &= 1 \\ \Rightarrow \frac{x^2}{9} - \frac{4y^2}{25} &= 1 \\ \Rightarrow \frac{x^2}{9} - \frac{y^2}{\frac{25}{4}} &= 1 \\ \Rightarrow \frac{x^2}{(3)^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} &= 1\end{aligned}$$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a = 3$ and $b = \frac{5}{2}$

Length of the transverse axis: The length of the transverse axis

$$\begin{aligned}&= 2a \\&= 2 \times 3 = 6\end{aligned}$$

Length of the conjugate axis: The length of the conjugate axis is

$$2b = 2 \times \frac{5}{2} = 5$$

Eccentricity: The eccentricity e is given by

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$
$$= \sqrt{1 + \frac{25}{9}}$$

$$= \sqrt{1 + \frac{25}{36}}$$

$$= \sqrt{\frac{61}{36}}$$

$$= \frac{\sqrt{61}}{6}$$

$$\text{Length of LR} = \frac{2b^2}{a} = \frac{25}{6}$$

$$\text{Foci } \left(\pm \frac{\sqrt{61}}{2}, 0 \right)$$

Hyperbola Ex 27.1 Q5(i)

We have,

$$\begin{aligned} & 16x^2 - 9y^2 + 32x + 36y - 164 = 0 \\ \Rightarrow & 16x^2 + 32x - 9y^2 + 36y - 14 = 0 \\ \Rightarrow & 16(x^2 + 2x) - 9(y^2 + 4y) - 164 = 0 \\ \Rightarrow & 16[x^2 + 2x + 1 - 1] - 9[y^2 - 4y + 4 - 4] - 164 = 0 \\ \Rightarrow & 16[(x + 1)^2 - 1] - 9[(y - 2)^2 - 4] - 164 = 0 \\ \Rightarrow & 16(x + 1)^2 - 16 - 9(y - 2)^2 + 36 - 164 = 0 \\ \Rightarrow & 16(x + 1)^2 - 9(y - 2)^2 + 20 - 164 = 0 \\ \Rightarrow & 16(x + 1)^2 - 9(y - 2)^2 - 144 = 0 \\ \Rightarrow & 16(x + 1)^2 - 9(y - 2)^2 = 144 \\ \Rightarrow & \frac{16(x + 1)^2}{144} - \frac{9(y - 2)^2}{144} = 1 \\ \Rightarrow & \frac{(x + 1)^2}{9} - \frac{(y - 2)^2}{16} = 1 \quad \text{---(i)} \end{aligned}$$

Shifting the origin at $(-1, 2)$ without rotating the coordinate axes and denoting the new coordinates with respect to the new axes by X and y ,

We have,

$$x = X - 1 \text{ and } y = Y + 2 \quad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 9$ and $b^2 = 16$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

$\therefore x = -1$ and $y = 2$ [Using equation (ii)]

So, the coordinates of the centre w.r.t the old axes are $(-1, 2)$.

Eccentricity: The eccentricity e is given by

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{16}{9}} \\ &= \sqrt{\frac{25}{9}} \\ &= \frac{5}{3} \end{aligned}$$

Foci: The coordinates of the foci with respect to the new axes are given by $(X = \pm ae, Y = 0)$
i.e., $(X = \pm 5, Y = 0)$.

Putting $X = \pm 5$ and $Y = 0$ in equation (ii), we get

$$\begin{aligned} x &= \pm 5 - 1 \text{ and } y = 0 + 2 \\ \Rightarrow x &= 4, -6 \text{ and } y = 2 \end{aligned}$$

Equation of the directrix: The equations of the directrix are

$$\begin{aligned}X &= \pm \frac{a}{e} \\ &= \pm \frac{3}{\frac{5}{3}} \\ X &= \pm \frac{9}{5}\end{aligned}$$

Putting $X = \pm \frac{9}{5}$ in equation (ii), we get

$$\begin{aligned}x &= \pm \frac{9}{5} - 1 \\ \Rightarrow x &= \frac{\pm 9 - 5}{5} \\ \Rightarrow x &= \frac{4}{5} \text{ and } x = \frac{-14}{5} \\ \Rightarrow 5x - 4 &= 0 \text{ and } 5x + 14 = 0\end{aligned}$$

So, the equations of the directrices w.r.t the old axes are

$$5x - 4 = 0 \text{ and } 5x + 14 = 0.$$

We have,

$$x^2 - y^2 + 4x = 0$$

$$\Rightarrow x^2 + 4x - y^2 = 0$$

$$\Rightarrow x^2 + 4x + 4 - 4 - y^2 = 0$$

$$\Rightarrow (x + 2)^2 - y^2 = 4$$

$$\Rightarrow \frac{(x + 2)^2}{4} - \frac{y^2}{4} = 1 \quad \text{---(i)}$$

Shifting the origin at $(-2, 0)$ without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y ,

We have,

$$x = X - 2 \text{ and } y = Y \quad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{4} - \frac{Y^2}{4} = 1 \quad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 4$ and $b^2 = 4$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$$x = -2 \text{ and } y = 0.$$

So, the coordinates of the centre w.r.t the old axes are $(-2, 0)$.

Eccentricity: The eccentricity e is given by

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\&= \sqrt{1 + \frac{4}{4}} \\&= \sqrt{1 + 1} \\&= \sqrt{2}\end{aligned}$$

Foci: The coordinates of the foci w.r.t the new axes are $(X = \pm ae, Y = 0)$ i.e., $(X = \pm 2\sqrt{2}, Y = 0)$.

Putting $X = \pm 2\sqrt{2}$ and $Y = 0$ in equation (ii), we get

$$\begin{aligned}x &= \pm 2\sqrt{2} - 2 \text{ and } y = 0 \\ \Rightarrow x &= -2 \pm 2\sqrt{2} \text{ and } y = 0\end{aligned}$$

So, the coordinates of foci w.r.t the old axes are $(-2 \pm 2\sqrt{2}, 0)$

Directrices: The equations of the directrices w.r.t the new axes are

$$X = \pm \frac{a}{e} \text{ i.e., } X = \pm \frac{2}{\sqrt{2}}$$

Putting $X = \pm \frac{2}{\sqrt{2}}$ in equation (ii), we get

$$\begin{aligned}x &= \pm \frac{2}{\sqrt{2}} - 2 \\ \Rightarrow x + 2 &= \pm \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} \\ \Rightarrow x + 2 &= \pm \sqrt{2}\end{aligned}$$

So, the equations of the directrices w.r.t to the old axes are $x + 2 = \pm \sqrt{2}$.

We have,

$$x^2 - 3y^2 - 2x = 8$$

$$\Rightarrow x^2 - 2x - 3y^2 = 8$$

$$\Rightarrow x^2 - 2x + 1 - 1 - 3y^2 = 8$$

$$\Rightarrow (x - 1)^2 - 1 - 3y^2 = 8$$

$$\Rightarrow (x - 1)^2 - 3y^2 = 9$$

$$\Rightarrow \frac{(x - 1)^2}{9} - \frac{3y^2}{9} = 1$$

$$\Rightarrow \frac{(x - 1)^2}{9} - \frac{y^2}{3} = 1 \quad \text{---(i)}$$

Shifting the origin at (1,0) without rotating the axes and denoting the new coordinates w.r.t these axes by X and Y , We have,

$$x = X + 1 \text{ and } y = Y \quad \text{---(ii)}$$

Using these relations, equation (i) reduces to

$$\frac{X^2}{9} - \frac{Y^2}{3} = 1 \quad \text{---(ii)}$$

This is of the form $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1$, where $a^2 = 9$ and $b^2 = 3$. so,

We have,

Centre: The coordinates of the centre w.r.t the new axes are $(X = 0, Y = 0)$

Putting $X = 0$ and $Y = 0$ in equation (ii), we get

$$x = 1 \text{ and } y = 0.$$

So, the coordinates of the centre w.r.t the old axes are (1,0).

Eccentricity: The eccentricity e is given by

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\&= \sqrt{1 + \frac{3}{9}} \\&= \sqrt{1 + \frac{1}{3}} \\&= \sqrt{\frac{4}{3}} \\&= \frac{2}{\sqrt{3}} \\&= \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\&= \frac{2\sqrt{3}}{3}\end{aligned}$$

Foci: The coordinates of the foci w.r.t the new axes are $(X = \pm ae, Y = 0)$ i.e., $(X = \pm 2\sqrt{3}, Y = 0)$.

Putting $X = \pm 2\sqrt{3}$ and $Y = 0$ in equation (ii), we get

$$\begin{aligned}x &= \pm 2\sqrt{3} + 1 \text{ and } y = 0 \\ \Rightarrow x &= 1 \pm 2\sqrt{3} \text{ and } y = 0\end{aligned}$$

So, the coordinates of foci w.r.t the old axes are $(1 \pm 2\sqrt{3}, 0)$

Directrices: The equations of the directrices w.r.t the new axes are

$$X = \pm \frac{a}{e} \text{ i.e., } X = \pm \frac{3}{\frac{2\sqrt{3}}{3}} = \pm \frac{9}{2\sqrt{3}}$$

Putting $X = \pm \frac{9}{2\sqrt{3}}$ in equation (ii), we get

$$\begin{aligned}x &= \pm \frac{9}{2\sqrt{3}} + 1 \\ \Rightarrow x &= \pm \frac{9}{2\sqrt{3}}\end{aligned}$$

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

Then,

Distance between the foci = 16

$$\Rightarrow 2ae = 16 \quad [\because \text{Distance between foci} = 2ae]$$

$$\Rightarrow ae = 8$$

$$\Rightarrow a \times \sqrt{2} = 8 \quad [\because e = \sqrt{2}]$$

$$\Rightarrow a = \frac{8}{\sqrt{2}}$$

$$\Rightarrow a^2 = \frac{64}{2} = 32$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ &= 32 \left((\sqrt{2})^2 - 1 \right) \\ &= 32 \times (2 - 1) \\ &= 32 \end{aligned}$$

Putting $a^2 = 32$ and $b^2 = 32$ in equation (i), we get

$$\begin{aligned} \frac{x^2}{32} - \frac{y^2}{32} &= 1 \\ \Rightarrow x^2 - y^2 &= 32 \end{aligned}$$

Hence, the equation of the required hyperbola is $x^2 - y^2 = 32$.

Hyperbola Ex 27.1 Q6(ii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

Then,

The length of the conjugate axis = $2b$

$$\therefore 2b = 5 \quad [\because \text{Conjugate axis} = 5]$$

$$\Rightarrow b = \frac{5}{2}$$

$$\Rightarrow b^2 = \frac{25}{4}$$

And, the distance between foci = $2ae$

$$\therefore 2ae = 13 \quad [\because \text{The distance between foci is } 13]$$

$$\Rightarrow a^2e^2 = \frac{169}{4}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow \frac{25}{4} = a^2e^2 - a^2$$

$$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2$$

$$\Rightarrow a^2 = \frac{169}{4} - \frac{25}{4}$$

$$\Rightarrow a^2 = \frac{169 - 25}{4}$$

$$\Rightarrow a^2 = \frac{144}{4} = 36$$

Putting $a^2 = 36$ and $b^2 = \frac{25}{4}$ in equation (i), we get

$$\frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{x^2}{36} - \frac{4y^2}{25} = 1$$

$$\Rightarrow 25x^2 - 144y^2 = 900$$

Hence, the equation of the required hyperbola is $25x^2 - 144y^2 = 900$.

Hyperbola Ex 27.1 Q6(iii)

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

Then,

The length of the conjugate axis = $2b$

$$\therefore 2b = 7 \quad [\because \text{Conjugate axis is } = 5]$$

$$\Rightarrow b = \frac{7}{2}$$

$$\Rightarrow b^2 = \frac{49}{4} \quad \text{---(ii)}$$

The required hyperbola passes through the point $(3, -2)$.

$$\therefore \frac{(3)^2}{a^2} - \frac{(-2)^2}{b^2} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{16}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 + \frac{16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{65}{49}$$

$$\Rightarrow a^2 = \frac{49 \times 9}{65}$$

$$\Rightarrow a^2 = \frac{441}{65}$$

Putting $a^2 = \frac{441}{65}$ and $b^2 = \frac{49}{4}$ in equation (i), we get

$$\frac{x^2}{\frac{441}{65}} - \frac{y^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{65x^2}{441} - \frac{4y^2}{49} = 1$$

$$\Rightarrow \frac{65x^2 - 36y^2}{441} = 1$$

$$\Rightarrow 65x^2 - 36y^2 = 441$$

Hence, the equation of the required hyperbola is $65x^2 - 36y^2 = 441$.

Hyperbola Ex 27.1 Q7(i)

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{6-4}{2}, \frac{4+4}{2}\right)$ i.e., $(1, 4)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$$\frac{(x-1)^2}{a^2} - \frac{(y-4)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between two foci = $2ae$

$$\Rightarrow \sqrt{(6+4)^2 + (4-4)^2} = 2ae \quad [\because \text{Foci} = (6, 4) \text{ and } (-4, 4)]$$

$$\Rightarrow \sqrt{(10)^2} = 2ae$$

$$\Rightarrow 10 = 2ae$$

$$\Rightarrow 2ae = 10$$

$$\Rightarrow 2a \times 2 = 10 \quad [\because e = 2]$$

$$\Rightarrow a = \frac{10}{4}$$

$$\Rightarrow a = \frac{5}{2}$$

$$\Rightarrow a^2 = \frac{25}{4}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = \frac{25}{4}(2^2 - 1)$$

$$= \frac{25}{4}(4 - 1)$$

$$= \frac{25}{4} \times 3 = \frac{75}{4}$$

Putting $a^2 = \frac{25}{4}$ and $b^2 = \frac{75}{4}$ in equation (i), we get

$$\frac{(x-1)^2}{\frac{25}{4}} - \frac{(y-4)^2}{\frac{75}{4}} = 1$$

$$\Rightarrow \frac{4(x-1)^2}{25} - \frac{4(y-4)^2}{75} = 1$$

$$\Rightarrow \frac{4 \times 3(x-1)^2 - 4(y-4)^2}{75} = 1$$

$$\Rightarrow 12(x-1)^2 - 4(y-4)^2 = 75$$

$$\Rightarrow 12[x^2 + 1 - 2x] - 4[y^2 + 16 - 8y] = 75$$

$$\Rightarrow 12x^2 + 12 - 24x - 4y^2 - 64 + 32y = 75$$

$$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 52 - 75 = 0$$

$$\Rightarrow 12x^2 - 4y^2 - 24x + 32y - 127 = 0$$

This is the equation of the required hyperbola.

The centre of the hyperbola is the mid-point of the line joining the two vertices.

So, the coordinates of the centre are $\left(\frac{16-8}{2}, \frac{-1-1}{2}\right)$ i.e., $(4, -1)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$$\frac{(x-4)^2}{a^2} - \frac{(y+1)^2}{b^2} = 1 \quad \text{---(i)}$$

Now,

The distance between two vertices = $2a$

$$\therefore \sqrt{(16+8)^2 + (-1+1)^2} = 2ae \quad [\because \text{vertices} = (-8, -1) \text{ and } (16, -1)]$$

$$\Rightarrow 24 = 2a$$

$$\Rightarrow a = 12$$

$$\Rightarrow a^2 = 144$$

and, the distance between the focus and vertex is = $ae - a$

and, the distance between the focus and vertex is = $ae - a$

$$\therefore \sqrt{(17-16)^2 + (-1+1)^2} = ae - a \quad [\because \text{Focus} = (17, -1) \text{ and vertex} = (16, -1)]$$

$$\Rightarrow \sqrt{1^2} = ae - a$$

$$\Rightarrow ae - a = 1$$

$$\Rightarrow 12 \times e - 12 = 1$$

$$[\because a = 12]$$

$$\Rightarrow 12e = 1 + 12$$

$$\Rightarrow e = \frac{13}{12}$$

$$\Rightarrow e^2 = \frac{169}{144}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$= (12)^2 \left(\frac{169}{144} - 1 \right)$$

$$[\because a = 12 \text{ and } e = \frac{13}{12}]$$

$$= 144 \times \left(\frac{169 - 144}{144} \right)$$

$$= 144 \times \frac{25}{144}$$

$$= 25$$

Putting $a^2 = 144$ and $b^2 = 25$ in equation (i), we get

$$\frac{(x-4)^2}{144} - \frac{(y+1)^2}{25} = 1$$

$$\Rightarrow \frac{25(x-4)^2 - 144(y+1)^2}{3600} = 1$$

$$\Rightarrow 25[x^2 + 16 - 8x] - 144[y^2 + 1 + 2y] = 3600$$

$$\Rightarrow 25x^2 + 400 - 200x - 144y^2 - 144 - 288y = 3600$$

$$\Rightarrow 25x^2 - 144y^2 - 200x - 288y + 256 = 3600$$

$$\Rightarrow 25x^2 - 144y^2 - 200x - 288y - 3344 = 0$$

This is the equation of the required hyperbola.

Hyperbola Ex 27.1 Q7(iii)

The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{4+8}{2}, \frac{2+2}{2}\right)$ i.e., $(6, 2)$.

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of the hyperbola is

$$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between two foci = $2ae$

$$\Rightarrow \sqrt{(8-4)^2 + (2-2)^2} = 2ae \quad [\because \text{Foci} = (4, 2) \text{ and } (8, 2)]$$

$$\Rightarrow \sqrt{(4)^2} = 2ae$$

$$\Rightarrow 2ae = 4$$

$$\Rightarrow 2 \times a \times 2 = 4 \quad [\because e = 2]$$

$$\Rightarrow a = \frac{4}{4} = 1$$

$$\Rightarrow a^2 = 1$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1(2^2 - 1) \quad [\because e = 2]$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1(2^2 - 1) \quad [\because e = 2]$$

$$\Rightarrow b^2 = 4 - 1$$

$$\Rightarrow b^2 = 3$$

Putting $a^2 = 1$ and $b^2 = 3$ in equation (i), we get

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x-6)^2 - (y-2)^2}{3} = 1$$

$$\Rightarrow 3(x-6)^2 - (y-2)^2 = 3$$

$$\Rightarrow 3[x^2 + 36 - 12x] - [y^2 + 4 - 4y] = 3$$

$$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y = 3$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$$

This is the equation of the required hyperbola.

Hyperbola Ex 27.1 Q7(iv)

Since, the vertices are on y -axis, so let the equation of the required hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 7 \quad [\because \text{vertices} = (0, \pm 7)]$$

$$\Rightarrow b^2 = 49$$

and,

$$be = \frac{28}{3} \quad [\because \text{Foci} = (0, \pm \frac{28}{3})]$$

$$\Rightarrow 7 \times e = \frac{28}{3}$$

$$\Rightarrow e = \frac{4}{3}$$

$$\Rightarrow e^2 = \frac{16}{9}$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = 49 \left(\frac{16}{9} - 1 \right)$$

$$\Rightarrow a^2 = 49 \times \frac{7}{9}$$

$$\Rightarrow a^2 = \frac{343}{9}$$

Putting $a^2 = \frac{343}{9}$ and $b^2 = 49$ in equation (i), we get

$$\frac{x^2}{\frac{343}{9}} - \frac{y^2}{49} = -1$$

This is the equation of the required hyperbola.

Hyperbola Ex 27.1 Q8

Let $2a$ and $2b$ be the transverse and conjugate axes and e be the eccentricity. Then,

The length of conjugate axis = $\frac{3}{4}$ [length of transverse axis]

$$\Rightarrow 2b = \frac{3}{4} \times (2a)$$

$$\Rightarrow \frac{b}{a} = \frac{3}{4}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{9}{16}$$

Now,

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4} \end{aligned}$$

Hence, $e = \frac{5}{4}$

Hyperbola Ex 27.1 Q9(i)

Let (x_2, y_2) be the coordinates of the second vertex.

We know that, the centre of the hyperbola is the mid-point of the line-joining the two vertices.

$$\begin{aligned} \therefore \quad \frac{x_1 + 4}{2} &= 3 \text{ and } \frac{y_1 + 2}{2} = 2 && [\because \text{Centre} = (3, 2) \text{ and vertex} = (4, 2)] \\ \Rightarrow \quad x_1 &= 2 \text{ and } y_2 = 2 \end{aligned}$$

\therefore The coordinates of the second vertex is $(2, 2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be eccentricity. Then, the equation of hyperbola is

$$\frac{(x - 3)^2}{a^2} - \frac{(y - 2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between the two vertices = $2a$

$$\begin{aligned} \Rightarrow \quad \sqrt{(4 - 2)^2 + (2 - 2)^2} &= 2a && [\because \text{Vertices} = (4, 2) \text{ and } (2, 2)] \\ \Rightarrow \quad \sqrt{2^2} &= 2a \\ \Rightarrow \quad 2a &= 2 \\ \Rightarrow \quad a &= 1 && \text{---(ii)} \end{aligned}$$

Now, the distance between the vertex and focus is = $ae - a$

$$\begin{aligned} \Rightarrow \quad \sqrt{(5 - 4)^2 + (2 - 2)^2} &= ae - a && [\because \text{Focus} = (5, 2) \text{ and vertex} = (4, 2)] \\ \Rightarrow \quad \sqrt{1} &= ae - a \\ \Rightarrow \quad ae - a &= 1 \\ \Rightarrow \quad 1 \times e - 1 &= 1 && [\because e = 1] \\ \Rightarrow \quad e &= 1 + 1 = 2 \end{aligned}$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ &= a^2(2^2 - 1) \\ &= 1 \times (4 - 1) \\ &= 1 \times 3 \\ &= 3 \end{aligned}$$

Putting $a^2 = 1$ and $b^2 = 3$ in equation (i), we get

$$\begin{aligned} \Rightarrow \quad \frac{(x - 3)^2}{1} - \frac{(y - 2)^2}{3} &= 1 \\ \Rightarrow \quad \frac{3(x - 3)^2 - (y - 2)^2}{3} &= 1 \\ \Rightarrow \quad 3(x - 3)^2 - (y - 2)^2 &= 3 \end{aligned}$$

This is the equation of the required hyperbola.

Hyperbola Ex 27.1 Q9(ii)

Let (x_1, y_1) be the coordinates of the second focus of the required hyperbola.

We know that, the centre of the hyperbola is the mid-point of the line-joining the two foci.

$$\therefore \frac{x_1 + 4}{2} = 6 \text{ and } \frac{y_1 + 2}{2} = 2 \quad [\because \text{Centre} = (6, 2) \text{ and focus} = (4, 2)]$$

$$\Rightarrow x_1 = 8 \text{ and } y_2 = 2$$

\(\therefore\) The coordinates of the second focus is $(8, 2)$

Let $2a$ and $2b$ be the length of transverse and conjugate axes and let e be the eccentricity. Then, the equation of hyperbola is

$$\frac{(x - 6)^2}{a^2} - \frac{(y - 2)^2}{b^2} = 1 \quad \text{---(i)}$$

Now, distance between the two vertices = $2ae$

$$\Rightarrow \sqrt{(8 - 4)^2 + (2 - 2)^2} = 2ae \quad [\because \text{foci} = (4, 2) \text{ and } (8, 2)]$$

$$\Rightarrow \sqrt{2^2} = 2a$$

$$\Rightarrow 2a = 2$$

$$\Rightarrow a = 1 \quad \text{---(ii)}$$

Now, the distance between the vertex and focus is = $ae - a$

$$\Rightarrow \sqrt{(5 - 4)^2 + (2 - 2)^2} = ae - a \quad [\because \text{Focus} = (5, 2) \text{ and vertex} = (4, 2)]$$

$$\Rightarrow \sqrt{(1)^2} = 2ae$$

$$\Rightarrow 2ae = 4$$

$$\Rightarrow 2 \times a \times 2 = 4 \quad [\because e = 2]$$

$$\Rightarrow a = 1$$

$$\Rightarrow a^2 = 1$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 1(2^2 - 1)$$

$$= 1(4 - 1)$$

$$= 3$$

$$\Rightarrow b^2 = 3$$

Putting $a^2 = 1$ and $b^2 = 3$ in equation (i), we get

$$\Rightarrow \frac{(x - 6)^2}{1} - \frac{(y - 2)^2}{3} = 1$$

$$\Rightarrow \frac{3(x - 6)^2 - (y - 2)^2}{3} = 1$$

$$\Rightarrow 3(x - 6)^2 - (y - 2)^2 = 3$$

This is the equation of the required hyperbola.

For a hyperbole if the length of semi transverse and semi conjugate axes are equal.

Then $a = b$

Equation of the given hyperbole is

$$x^2 - y^2 = a^2 \dots (1)$$

$$\text{Then } e = \sqrt{2}, C = (0, 0), S = (\sqrt{2}a, 0), S' = (-\sqrt{2}a, 0)$$

Let coordinates of any point P on hyperbole be (α, β) . Since P lies on (1)

$$\alpha^2 - \beta^2 = a^2 \dots (2)$$

$$\text{Now } SP^2 = (\sqrt{2}a - \alpha)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 - 2\sqrt{2}a\alpha$$

$$\text{and } S'P^2 = -(\sqrt{2}a - \alpha)^2 + \beta^2 = 2a^2 + \alpha^2 + \beta^2 + 2\sqrt{2}a\alpha$$

$$\begin{aligned} \text{Now } SP^2 \cdot S'P^2 &= (2a^2 + \alpha^2 + \beta^2)^2 - 8a^2\alpha^2 \\ &= 4a^4 + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 - 8a^2\alpha^2 \\ &= 4a^2(\alpha^2 - 2a^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 \\ &= 4a^2(\alpha^2 - \beta^2 - 2a^2) + 4a^2(\alpha^2 + \beta^2) + (\alpha^2 + \beta^2)^2 \\ &= (\alpha^2 + \beta^2)^2 = CP^2 \end{aligned}$$

$$SP \cdot S'P = CP^2$$

Hyperbola Ex 27.1 Q11(i)

Let the equation of hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots (i)$$

The coordinates of its vertices and foci are $(\pm a, 0)$ and $(\pm ae, 0)$ respectively.

$$\therefore a = 2 \quad [\because \text{vertices} = (\pm 2, 0)]$$

$$\Rightarrow a^2 = 4$$

and,

$$ae = 3 \quad [\because \text{Foci} = (\pm 3, 0)]$$

$$\Rightarrow 2 \times e = 3 \quad [\because a = 2]$$

$$\Rightarrow e = \frac{3}{2}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow b^2 = 2^2 \left[\left(\frac{3}{2} \right)^2 - 1 \right]$$

$$\Rightarrow b^2 = 4 \left[\frac{9}{4} - 1 \right]$$

$$\Rightarrow b^2 = 4 \left[\frac{9 - 4}{4} \right]$$

$$= 4 \times \frac{5}{4}$$

$$= 5$$

Putting $a^2 = 4$ and $b^2 = 5$ in equation (1), we get

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Hence, the equation of the required hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Hyperbola Ex 27.1 Q11(ii)

Since, the vertices lie on y -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 5 \quad [\because \text{vertices} = (0, \pm 5)]$$

$$\Rightarrow b^2 = 25$$

$$\text{and, } be = 8 \quad [\because \text{Foci} = (0, \pm 8)]$$

$$\Rightarrow 5 \times e = 8 \quad [\because b = 5]$$

$$\Rightarrow e = \frac{8}{5}$$

$$\Rightarrow e^2 = \frac{64}{25}$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow a^2 = 25 \left(\frac{64}{25} - 1 \right) \quad \left[\because b^2 = 25 \text{ and } e^2 = \frac{64}{25} \right]$$

$$\Rightarrow a^2 = 25 \times \frac{39}{25}$$

$$\Rightarrow a^2 = 39$$

Putting $a^2 = 39$ and $b^2 = 25$ in equation (i), we get

$$\frac{x^2}{39} - \frac{y^2}{25} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{39} - \frac{y^2}{25} = -1.$$

Since, the vertices line on y -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The coordinates of its vertices and foci are $(0, \pm b)$ and $(0, \pm be)$ respectively.

$$\therefore b = 3 \quad [\because \text{vertices} = (0, \pm 3)]$$

$$\Rightarrow b^2 = 9$$

$$\text{and, } be = 5 \quad [\because \text{Foci} = (0, \pm 5)]$$

$$\Rightarrow e \times 3 = 5$$

$$\Rightarrow e = \frac{5}{3}$$

$$\Rightarrow e^2 = \frac{25}{9}$$

Now,

$$a^2 = b^2 (e^2 - 1)$$

$$\begin{aligned} \Rightarrow a^2 &= 9 \left(\frac{25}{9} - 1 \right) \\ &= 9 \times \left(\frac{25 - 9}{9} \right) \\ &= 9 \times \frac{16}{9} \\ &= 16 \end{aligned}$$

Putting $a^2 = 16$ and $b^2 = 9$ in equatoin (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = -1$$

Hence, the equaton of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = -1.$$

Since, the vertices line on x-axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of transverse axis = 8

$$\therefore 2a = 8 \quad [\because \text{transverse axis is } 2a]$$

$$\Rightarrow a = 4$$

$$\Rightarrow a^2 = 16$$

This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore ae = 5 \quad [\because \text{foci} = (\pm 5, 0)]$$

$$\Rightarrow 4 \times e = 5 \quad [\because a = 4]$$

$$\Rightarrow e = \frac{5}{4}$$

$$\rightarrow e^2 = \frac{25}{16}$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ &= 16\left(\frac{25}{16} - 1\right) \\ &= 16 \times \frac{9}{16} \\ &= 9 \end{aligned}$$

Putting $a^2 = 16$ and $b^2 = 9$ in equation (i), we get

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

Hyperbola Ex 27.1 Q11(v)

Since, the vertices line on x -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

The length of conjugate axis of the required hyperbola is 24.

$$\therefore 2a = 24 \quad [\because \text{conjugate axis is } 2a]$$

$$\Rightarrow a = \frac{24}{2} = 12$$

$$\Rightarrow a^2 = 144$$

This coordinates of foci of the required hyperbola is $(0, \pm be)$

$$\therefore be = 13$$

$$b^2e^2 = 169$$

Now,

$$a^2 = b^2(e^2 - 1)$$

$$\Rightarrow 144 = b^2e^2 - b^2$$

$$\Rightarrow 144 = 169 - b^2$$

$$\Rightarrow b^2 = 169 - 144 = 25$$

Putting $a^2 = 144$ and $b^2 = 25$ in equation (i), we get

$$\frac{x^2}{144} - \frac{y^2}{25} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{144} - \frac{y^2}{25} = -1.$$

Hyperbola Ex 27.1 Q11(vi)

Since, the vertices line on x -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of conjugate axis of the required hyperbola is 8.

$$\therefore \frac{2b^2}{a} = 8$$

$$\Rightarrow b^2 = \frac{8}{2} \times a$$

$$\Rightarrow b^2 = 4a \quad \text{---(ii)}$$

Now,

This coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore ae = 3\sqrt{5} \quad [\because \text{Foci} = (\pm 3\sqrt{5}, 0)]$$

$$\Rightarrow e = \frac{3\sqrt{5}}{a}$$

$$\Rightarrow e^2 = \frac{45}{a^2} \quad \text{---(iii)}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow 4a = a^2e^2 - a^2$$

$$\Rightarrow 4a = a^2 \times \frac{45}{a^2} - a^2$$

$$\Rightarrow 4a = 45 - a^2$$

$$\Rightarrow a^2 + 4a - 45 = 0$$

$$\Rightarrow a^2 + 9a - 5a - 45 = 0$$

$$\Rightarrow a(a+9) - 5(a+9) = 0$$

$$\Rightarrow (a-5)(a+9) = 0$$

$$\Rightarrow a = 5 \quad [\because a+9 \neq 0]$$

$$\Rightarrow a^2 = 25$$

$$\Rightarrow b^2 = 4 \times 5 \quad [\text{Using equation (ii)}]$$

$$\Rightarrow b^2 = 20$$

Putting $a^2 = 25$ and $b^2 = 20$ in equation (i), we get

$$\frac{x^2}{25} - \frac{y^2}{20} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{25} - \frac{y^2}{20} = 1.$$

Hyperbola Ex 27.1 Q11(vii)

Since, the vertices lie on x -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of the latus-rectum of the required hyperbola is 12

$$\therefore \frac{2b^2}{a} = 12$$

$$\Rightarrow b^2 = 6a \quad \text{---(ii)}$$

Now,

The coordinates of foci of the required hyperbola is $(\pm ae, 0)$

$$\therefore ae = 4$$

$$\Rightarrow e = \frac{4}{a}$$

$$\Rightarrow e^2 = \frac{16}{a^2} \quad \text{---(iii)}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow 6a = a^2e^2 - a^2$$

$$\Rightarrow 6a = a^2 \times \frac{16}{a^2} - a^2$$

$$\Rightarrow 6a = 16 - a^2$$

$$\Rightarrow a^2 + 6a - 16 = 0$$

$$\Rightarrow a^2 + 8a - 2a - 16 = 0$$

$$\Rightarrow a(a+8) - 2(a+8) = 0$$

$$\Rightarrow (a+8)(a-2)$$

$$\Rightarrow (a-2) = 0 \quad \left[\begin{array}{l} \because \text{length cannot be negative} \\ \therefore a+8 \neq 0 \end{array} \right]$$

$$\Rightarrow a = 2$$

$$\Rightarrow a^2 = 4$$

$$\Rightarrow b^2 = 6 \times 2 = 12 \quad [\text{Using equation (ii)}]$$

Putting $a^2 = 4$ and $b^2 = 12$ in equation (i), we get

$$\frac{x^2}{4} - \frac{y^2}{12} = 1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{4} - \frac{y^2}{12} = 1.$$

Since, the vertices line on x -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

The length of the vertices of the required hyperbola are $(\pm a, 0)$.

$$\begin{aligned} \therefore a &= 7 && [\because \text{vertices} = (\pm 7, 0)] \\ \Rightarrow a^2 &= 49 && \text{---(ii)} \end{aligned}$$

Now,

$$\begin{aligned} b^2 &= a^2(e^2 - 1) \\ \Rightarrow b^2 &= 49 \left[\left(\frac{4}{3} \right)^2 - 1 \right] && [\because e = \frac{4}{3}] \\ \Rightarrow b^2 &= 49 \left[\frac{16}{9} - 1 \right] \\ \Rightarrow b^2 &= 49 \left[\frac{7}{9} \right] \\ \Rightarrow b^2 &= \frac{343}{9} \end{aligned}$$

Putting $a^2 = 49$ and $b^2 = \frac{343}{9}$ in equation (i), we get

$$\begin{aligned} \frac{x^2}{49} - \frac{y^2}{\frac{343}{9}} &= 1 \\ \Rightarrow \frac{x^2}{49} - \frac{9y^2}{343} &= 1 \end{aligned}$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{49} - \frac{9y^2}{343} = 1.$$

Since, the vertices lie on y -axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{---(i)}$$

It passes through $(2,3)$

$$\therefore \frac{(2)^2}{a^2} - \frac{(3)^2}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{b^2} = -1$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{a^2(e^2 - 1)} = -1 \quad [\because b^2 = a^2(e^2 - 1)]$$

$$\Rightarrow \frac{4}{a^2} - \frac{9}{a^2 e^2 - a^2} = -1 \quad \text{---(ii)}$$

The coordinates of foci of the required hyperbola are $(0, \pm ae)$.

$$\therefore ae = \sqrt{10}$$

$$\Rightarrow a^2 e^2 = 10 \quad \text{---(iii)}$$

Putting $a^2 e^2 = 10$ in equation (ii), we get

$$\frac{4}{a^2} - \frac{9}{10 - a^2} = -1$$

$$\Rightarrow \frac{4(10 - a^2) - 9(a^2)}{a^2(10 - a^2)} = -1$$

$$\Rightarrow \frac{40 - 4a^2 - 9a^2}{10a^2 - a^4} = -1$$

$$\Rightarrow 40 - 13a^2 = -10a^2 + a^4$$

$$\Rightarrow a^4 + 3a^2 - 40 = 0$$

$$\Rightarrow a^4 + 8a^2 - 5a^2 - 40 = 0$$

$$\Rightarrow a^2(a^2 + 8) - 5(a^2 + 8) = 0$$

$$\Rightarrow (a^2 + 8)(a^2 - 5) = 0$$

$$\Rightarrow a^2 - 5 = 0 \quad [\because a^2 \neq -8]$$

$$\Rightarrow a^2 = 5 \quad \text{---(iv)}$$

Now,

$$b^2 = a^2(e^2 - 1)$$

$$= a^2 e^2 - a^2$$

$$= 10 - 5$$

$$= 5$$

[Using equation (iii) and (iv)]

Putting $a^2 = 5$ and $b^2 = 5$ in equation (i), we get

$$\frac{x^2}{5} - \frac{y^2}{5} = -1$$

Hence, the equation of the required hyperbola is

$$\frac{x^2}{5} - \frac{y^2}{5} = -1.$$

Hyperbola Ex 27.1 Q11(x)

Since, the vertices lie on x - axis, so let the equation of the required hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1 \quad \text{----- (i)}$$

The length of the latus-rectum of the required hyperbola is 36.

$$\frac{2a^2}{b} = 36$$

$$a^2 = 18b \quad \text{----- (ii)}$$

Now,

The coordinates of foci of the required hyperbola is $(0, \pm be)$.

$$be = 12$$

$$e = \frac{12}{b}$$

$$e^2 = \frac{144}{b^2}$$

Now,

$$a^2 = b^2 (e^2 - 1)$$

$$18b = b^2 \left(\frac{144}{b^2} - 1 \right)$$

$$18b = 144 - b^2$$

$$b^2 + 18b - 144 = 0$$

$$(b - 6)(b + 24) = 0$$

$$b_{1,2} = 6, -24$$

Consider the positive value of $b = 6$.

On putting $b^2 = 36$, $a^2 = 18(6) = 108$ in equation (i), we get

$$\frac{x^2}{108} - \frac{y^2}{36} = -1$$

$$\frac{x^2 - 3y^2}{108} = -1$$

$$x^2 - 3y^2 = -108$$

$$3y^2 - x^2 = 108$$

Therefore, the equation of the hyperbola is $3y^2 - x^2 = 108$.

Hyperbola Ex 27.1 Q12

$$\text{Eccentricity } = e = \sqrt{2}$$

Distance between foci is

$$2ae = 16$$

$$2a\sqrt{2} = 16$$

$$a = \frac{16}{2\sqrt{2}} = 4\sqrt{2}$$

$$e = \frac{\sqrt{a^2 + b^2}}{a}$$

$$\sqrt{2} = \frac{\sqrt{32 + b^2}}{4\sqrt{2}}$$

$$8 = \sqrt{32 + b^2}$$

$$64 = 32 + b^2$$

$$b^2 = 32$$

$$\text{Equation of hyperbola is } \frac{x^2}{32} - \frac{y^2}{32} = 1$$

Rewriting we get, $x^2 - y^2 = 32$

Hyperbola Ex 27.1 Q13

Let P (x,y) be a point of the set.

$$\text{Distance of P(x,y) from (4,0)} = \sqrt{(x-4)^2 + y^2}$$

$$\text{Distance of P(x,y) from (-4,0)} = \sqrt{(x+4)^2 + y^2}$$

Difference between distance = 2

$$\sqrt{(x-4)^2 + y^2} - \sqrt{(x+4)^2 + y^2} = 2$$

$$\sqrt{(x-4)^2 + y^2} = 2 + \sqrt{(x+4)^2 + y^2}$$

Squaring both sides, we get,

$$(x-4)^2 + y^2 = 4 + 4\sqrt{(x+4)^2 + y^2} + (x+4)^2 + y^2$$

$$(x-4)^2 + y^2 - (x+4)^2 - y^2 = 4 + 4\sqrt{(x+4)^2 + y^2}$$

$$(x-4-x-4)(x-4+x+4) = 4 + 4\sqrt{(x+4)^2 + y^2}$$

$$-16x - 4 = 4\sqrt{(x+4)^2 + y^2}$$

$$-4x - 1 = \sqrt{(x+4)^2 + y^2}$$

Squaring both sides, we get,

$$16x^2 + 8x + 1 = x^2 + 8x + 16 + y^2$$

$$15x^2 - y^2 = 15$$

This is a hyperbola.