

RD Sharma
Solutions
Class 11 Maths
Chapter 29
Ex 29.4

Limits Ex 29.4 Q1

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2}-1}{x} &= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x+x^2}-1)(\sqrt{1+x+x^2}+1)}{x(\sqrt{1+x+x^2}+1)} \\ &= \lim_{x \rightarrow 0} \frac{((1+x+x^2)-1)}{x(\sqrt{1+x+x^2}+1)} \\ &= \lim_{x \rightarrow 0} \frac{x(1+x)}{x(\sqrt{1+x+x^2}+1)} \\ &= \lim_{x \rightarrow 0} \frac{1+x}{\sqrt{1+x+x^2}+1} \\ &= \frac{1+0}{\sqrt{1+0+0}+1} \\ &= \frac{1}{1+1} \\ &= \frac{1}{2}\end{aligned}$$

Limits Ex 29.4 Q2

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} \\ &= \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} \times \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} + \sqrt{a-x}} \\ &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{((a+x) - (a-x))} \\ &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{2x} \\ &= \lim_{x \rightarrow 0} (\sqrt{a+x} + \sqrt{a-x}) \\ &= \sqrt{a} + \sqrt{a} \\ &= 2\sqrt{a} \end{aligned}$$

Limits Ex 29.4 Q3

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{a^2 + x^2} - a)}{x^2} \times \frac{(\sqrt{a^2 + x^2} + a)}{(\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{(a^2 + x^2 - a^2)}{x^2 \sqrt{a^2 + x^2} + a} \\ &= \lim_{x \rightarrow 0} \frac{x^2}{x^2 (\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{a^2 + x^2} + a} \\ &= \frac{1}{a+a} \\ &= \frac{1}{2a} \end{aligned}$$

Limits Ex 29.4 Q4

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - \sqrt{1-x})}{2x} \times \frac{(\sqrt{1+x} + \sqrt{1-x})}{(\sqrt{1+x} + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{(1+x) - (1-x)}{2x(\sqrt{1+x} + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{2x}{2x(\sqrt{1+x} + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1+x} + \sqrt{1-x})} \\ &= \frac{1}{\sqrt{1} + \sqrt{1}} \\ &= \frac{1}{2} \end{aligned}$$

Limits Ex 29.4 Q5

$$\begin{aligned} & \lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x} \\ &= \lim_{x \rightarrow 2} \frac{(\sqrt{3-x} - 1)}{2-x} \times \frac{(\sqrt{3-x} + 1)}{(\sqrt{3-x} + 1)} \\ &= \lim_{x \rightarrow 2} \frac{(3-x) - 1}{(2-x)(\sqrt{3-x} + 1)} \\ &= \lim_{x \rightarrow 2} \frac{(2-x)}{(2-x)(\sqrt{3-x} + 1)} \\ &= \lim_{x \rightarrow 2} \frac{1}{(\sqrt{3-x} + 1)} \\ &= \frac{1}{\sqrt{3-2} + 1} = \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

Limits Ex 29.4 Q6

$$\begin{aligned} & \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}} \\ &= \lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}} \times \frac{(\sqrt{x-2}+\sqrt{4-x})}{(\sqrt{x-2}+\sqrt{4-x})} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{(x-2)-(4-x)} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{x-2-4+x} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(\sqrt{x-2}+\sqrt{4-x})}{2(x-3)} \\ &= \frac{1}{2} \lim_{x \rightarrow 3} (\sqrt{x-2}+\sqrt{4-x}) \\ &= \frac{1}{2} (\sqrt{3-2}+\sqrt{4-3}) \\ &= \frac{1}{2} (\sqrt{1}+\sqrt{1}) \\ &= \frac{1}{2} (1+1) = \frac{2}{2} \\ &= 1 \end{aligned}$$

Limits Ex 29.4 Q7

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}} \\ &= \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x}-\sqrt{1-x}} \times \frac{(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}+\sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{1+x-1+x} \\ &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x}+\sqrt{1-x})}{2x} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sqrt{1+x}+\sqrt{1-x}}{x} \right) x \\ &= \frac{1}{2} \lim_{x \rightarrow 0} (\sqrt{1+x}+\sqrt{1-x}) \\ &= \frac{1}{2} (\sqrt{1}+\sqrt{1}) \\ &= \frac{2}{2} = 1 \end{aligned}$$

Limits Ex 29.4 Q8

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{(\sqrt{5x-4} - \sqrt{x})}{x-1} &= \lim_{x \rightarrow 1} \frac{(\sqrt{5x-4} - \sqrt{x})}{x-1} \times \frac{(\sqrt{5x-4} + \sqrt{x})}{(\sqrt{5x-4} + \sqrt{x})} \\ &= \lim_{x \rightarrow 1} \frac{((5x-4) - x)}{(x-1)(\sqrt{5x-4} + \sqrt{x})} \\ &= 4 \lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(\sqrt{5x-4} + \sqrt{x})} \\ &= 4 \lim_{x \rightarrow 1} \frac{1}{\sqrt{5x-4} + \sqrt{x}} \\ &= 4 \times \frac{1}{\sqrt{5-4} + \sqrt{1}} \\ &= 4 \times \frac{1}{\sqrt{1} + \sqrt{1}} \\ &= \frac{4}{2} = 2\end{aligned}$$

Limits Ex 29.4 Q9

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{(x-1)}{(\sqrt{x^2+3}-2)} &= \lim_{x \rightarrow 1} \frac{(x-1) \times (\sqrt{x^2+3}+2)}{(\sqrt{x^2+3}-2)(\sqrt{x^2+3}+2)} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x^2+3}+2)}{(x^2+3-4)} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x^2+3}+2)}{(x^2-1)} \\ &= \lim_{x \rightarrow 1} \frac{\sqrt{x^2+3}+2}{x+1}\end{aligned}$$

Putting the value $x = 1$

$$\begin{aligned}\Rightarrow & \frac{\sqrt{1+3}+2}{1+1} \\ &= \frac{2+2}{2} \\ &= \frac{4}{2} = 2\end{aligned}$$

Limits Ex 29.4 Q10

$$\begin{aligned}
& \lim_{x \rightarrow 3} \frac{(\sqrt{x+3} - \sqrt{6})}{(x^2 - 9)} \\
&= \lim_{x \rightarrow 3} \frac{(\sqrt{x+3} - \sqrt{6})(\sqrt{x+3} + \sqrt{6})}{(x-3)(x+3)(\sqrt{x+3} + \sqrt{6})} \\
&= \lim_{x \rightarrow 3} \frac{((x+3) - 6)}{(x-3)(x+3)(\sqrt{x+3} + \sqrt{6})} \\
&= \lim_{x \rightarrow 3} \frac{(x-3)}{(x-3)(x+3)(\sqrt{x+3} + \sqrt{6})} \\
&= \lim_{x \rightarrow 3} \frac{1}{(x+3)(\sqrt{x+3} + \sqrt{6})} \\
&= \frac{1}{(3+3)\sqrt{3+3} + \sqrt{6}} \\
&= \frac{1}{6(\sqrt{6} + \sqrt{6})} = \frac{1}{6 \times 2\sqrt{6}} \\
&= \frac{1}{12\sqrt{6}}
\end{aligned}$$

Limits Ex 29.4 Q11

$$\begin{aligned}
& \lim_{x \rightarrow 1} \frac{(\sqrt{5x-4} - \sqrt{x})}{(x^2 - 1)} \\
&= \lim_{x \rightarrow 1} \frac{(\sqrt{5x-4} - \sqrt{x})(\sqrt{5x-4} + \sqrt{x})}{(x-1)(x+1)(\sqrt{5x-4} + \sqrt{x})} \\
&= \lim_{x \rightarrow 1} \frac{((5x-4) - x)}{(x-1)(x+1)(\sqrt{5x-4} + \sqrt{x})} \\
&= \lim_{x \rightarrow 1} \frac{4(x-1)}{(x-1)(x+1)(\sqrt{5x-4} + \sqrt{x})} \\
&= \lim_{x \rightarrow 1} \frac{4}{(x+1)(\sqrt{5x-4} + \sqrt{x})} \\
&= \frac{4}{(1+1)(\sqrt{5-4} + \sqrt{1})} \\
&= \frac{4}{2(1+1)} \\
&= \frac{4}{4} = 1
\end{aligned}$$

Limits Ex 29.4 Q12

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - 1)}{x} &= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - 1)}{x} \times \frac{(\sqrt{1+x} + 1)}{(\sqrt{1+x} + 1)} \\ &= \lim_{x \rightarrow 0} \frac{(1+x-1)}{x(\sqrt{1+x} + 1)} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{1+x} + 1)} \\ &= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1+x} + 1)} \\ &= \frac{1}{\sqrt{1} + 1} = \frac{1}{2}\end{aligned}$$

Limits Ex 29.4 Q13

$$\begin{aligned}\lim_{x \rightarrow 2} \frac{\sqrt{x^2+1} - \sqrt{5}}{(x-2)} &= \lim_{x \rightarrow 2} \frac{\sqrt{x^2+1} - \sqrt{5}}{(x-2)} \times \frac{(\sqrt{x^2+1} + \sqrt{5})}{(\sqrt{x^2+1} + \sqrt{5})} \\ &= \lim_{x \rightarrow 2} \frac{(x^2+1-5)}{(x-2)(\sqrt{x^2+1} + \sqrt{5})} \\ &= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)}{(x-2)(\sqrt{x^2+1} + \sqrt{5})} \\ &= \lim_{x \rightarrow 2} \frac{(x+2)}{(\sqrt{x^2+1} + \sqrt{5})}\end{aligned}$$

$$\begin{aligned}&= \frac{(2+2)}{\sqrt{4+1} + \sqrt{5}} \\ &= \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}\end{aligned}$$