

**RD Sharma**  
**Solutions**  
**Class 11 Maths**  
**Chapter 29**  
**Ex 29.5**

### Limits Ex 29.5 Q1

$$\begin{aligned} & \lim_{x \rightarrow a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a} \\ &= \lim_{x \rightarrow a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2) - (a+2)} \\ &= \lim_{y \rightarrow b} \frac{y^{\frac{5}{2}} - b^{\frac{5}{2}}}{y-b}, \text{ where } x+2 = y \text{ and } a+2 = b \\ &= \frac{5}{2} b^{\frac{5}{2}-1} \quad \left[ \text{Using formula } \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = n a^{n-1} \right] \\ &= \frac{5}{2} (a+2)^{\frac{5}{2}-1} \\ &= \frac{5}{2} (a+2)^{\frac{3}{2}} \end{aligned}$$

### Limits Ex 29.5 Q2

$$\begin{aligned} & \lim_{x \rightarrow a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{x-a} \\ &= \lim_{x \rightarrow a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{(x+2) - (a+2)} \end{aligned}$$

Let  $x+2 = y$ ,  $a+2 = b$

$$\begin{aligned} \Rightarrow & \lim_{(x+2) \rightarrow (a+2)} \frac{(y)^{\frac{3}{2}} - (b)^{\frac{3}{2}}}{(y) - (b)} \quad \left[ \text{Using formula } \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = n a^{n-1} \right] \\ &= \frac{3}{2} (b)^{\frac{3}{2}-1} \\ &= \frac{3}{2} (a+2)^{\frac{3}{2}-1} \\ &= \frac{3}{2} (a+2)^{\frac{1}{2}} \end{aligned}$$

### Limits Ex 29.5 Q3

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1} \\ &= \lim_{x \rightarrow 0} \frac{(1+x)^6 - 1^6}{(1+x)^2 - 1^2} \\ &= \lim_{x \rightarrow 0} \frac{1+x-1}{1+x-1} \end{aligned}$$

$\Rightarrow$  Let  $1+x = y$ , as  $x \rightarrow 0$ ,  $y \rightarrow 1$

$$\begin{aligned} &= \lim_{y \rightarrow 1} \frac{y^6 - 1^6}{y - 1} \\ &= \lim_{y \rightarrow 1} \frac{y^2 - 1}{y - 1} \end{aligned}$$

$$\begin{aligned} &= \frac{6(1)^{6-1}}{2(1)^{2-1}} \quad \left[ \text{Using formula } \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\ &= \frac{6}{2} \\ &= 3 \end{aligned}$$

### Limits Ex 29.5 Q4

$$\lim_{x \rightarrow a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}$$

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$ , here,  $n = \frac{2}{7}$

$$\Rightarrow \lim_{x \rightarrow a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a} = \frac{2}{7} (a)^{\frac{2}{7}-1}$$

$$= \frac{2}{7} a^{-\frac{5}{7}}$$

## Limits Ex 29.5 Q5

$$\begin{aligned} \lim_{x \rightarrow a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}} &= \lim_{x \rightarrow a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{\frac{x - a}{x^{\frac{5}{7}} - a^{\frac{5}{7}}}} \\ &= \lim_{x \rightarrow a} \frac{x - a}{\frac{x - a}{x^{\frac{5}{7}} - a^{\frac{5}{7}}}} \\ &= \lim_{x \rightarrow a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a} \\ &= \lim_{x \rightarrow a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a} \end{aligned}$$

[Dividing numerator and denominator by  $x - a$ ]

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

Here,  $n = \frac{5}{7}$  is numerator and applying  $\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$  in denominator, where  $m = \frac{2}{7}$

$$\begin{aligned} \Rightarrow \frac{\lim_{x \rightarrow a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \rightarrow a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} &= \frac{\frac{5}{7} (a)^{\frac{5}{7}-1}}{\frac{2}{7} (a)^{\frac{2}{7}-1}} \\ &= \frac{\frac{5}{7} a^{-\frac{2}{7}}}{\frac{2}{7} a^{-\frac{5}{7}}} \\ &= \frac{5}{2} a^{-\frac{2}{7} + \frac{5}{7}} \\ &= \frac{5}{2} a^{\frac{3}{7}} \end{aligned}$$

## Limits Ex 29.5 Q6

$$\lim_{x \rightarrow -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \frac{8}{2} \lim_{x \rightarrow -\frac{1}{2}} \frac{x^3 + \left(\frac{1}{2}\right)^3}{x + \frac{1}{2}}$$

$$= 4 \lim_{x \rightarrow -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)}$$

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here,  $n = 3$ ,  $a = -\frac{1}{2}$

$$\Rightarrow 4 \lim_{x \rightarrow -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)} = 4 \times 3 \left(-\frac{1}{2}\right)^{3-1}$$

$$= 4 \times 3 \times \frac{1}{4}$$

$$= 3$$

## Limits Ex 29.5 Q7

$$\begin{aligned} & \lim_{x \rightarrow 27} \frac{\left(x^{\frac{1}{3}} + 3\right)\left(x^{\frac{1}{3}} - 3\right)}{x - 27} \\ &= \lim_{x \rightarrow 27} \frac{\left(x^{\frac{2}{3}} - 9\right)}{x - 27} \\ &= \lim_{x \rightarrow 27} \frac{x^{\frac{2}{3}} - 27^{\frac{2}{3}}}{x - 27} \end{aligned}$$

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

$$\begin{aligned} &= \frac{2}{3} (27)^{\frac{2}{3}-1} \\ &= \frac{2}{3} (27)^{-\frac{1}{3}} \\ &= \frac{2}{3} \times \frac{1}{(27)^{\frac{1}{3}}} \\ &= \frac{2}{3} \times \frac{1}{3} \\ &= \frac{2}{9} \end{aligned}$$

## Limits Ex 29.5 Q8

$$\begin{aligned} & \lim_{x \rightarrow 4} \frac{x^3 - 64}{x^2 - 16} \\ &= \lim_{x \rightarrow 4} \frac{x^3 - 4^3}{x^2 - 4^2} \\ &= \lim_{x \rightarrow 4} \frac{x^3 - 4^3}{\frac{x^2 - 4^2}{x - 4}} \\ &= \frac{\lim_{x \rightarrow 4} \frac{x^3 - 4^3}{x - 4}}{\lim_{x \rightarrow 4} \frac{x^2 - 4}{x - 4}} \end{aligned}$$

[Dividing numerator and denominator by  $x - 4$ ]

Applying  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$  in numerator and  $\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = m a^{m-1}$  in denominator

$$\Rightarrow n = 3, m = 2$$

$$\begin{aligned} \Rightarrow & \frac{\lim_{x \rightarrow 4} \frac{x^3 - 4^3}{x - 4}}{\lim_{x \rightarrow 4} \frac{x^2 - 4}{x - 4}} = \frac{3(4)^{3-1}}{2(4)^{2-1}} = \frac{3(4)^2}{2(4)} \\ &= 6 \end{aligned}$$

### Limits Ex 29.5 Q9

$$\begin{aligned} & \lim_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^{15} - 1^{15}}{x^{10} - 1^{10}} \\ &= \lim_{x \rightarrow 1} \frac{x - 1}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^{15} - 1^{15}}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^{10} - 1^{10}}{x - 1} \end{aligned}$$

[Dividing numerator and denominator by  $(x - 1)$ ]

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$  in numerator and  $\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = ma^{m-1}$  in denominator

Here,  $n = 15$ ,  $m = 10$

$$\begin{aligned} \Rightarrow & \frac{\lim_{x \rightarrow 1} \frac{x^{15} - 1^{15}}{x - 1}}{\lim_{x \rightarrow 1} \frac{x^{10} - 1^{10}}{x - 1}} = \frac{15(1)^{15-1}}{10(1)^{10-1}} \\ &= \frac{15}{10} \\ &= \frac{3}{2} \end{aligned}$$

### Limits Ex 29.5 Q10

$$\begin{aligned} & \lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1} \\ &= \lim_{x \rightarrow -1} \frac{x^3 - (-1)^3}{x - (-1)} \end{aligned}$$

Applying formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here,  $n = 3$ ,  $a = -1$

$$\begin{aligned} \Rightarrow & \lim_{x \rightarrow -1} \frac{x^3 - (-1)^3}{x - (-1)} = na^{n-1} \\ &= 3(-1)^{3-1} \\ &= 3(-1)^2 \\ &= 3 \end{aligned}$$

### Limits Ex 29.5 Q11

$$\begin{aligned} & \lim_{x \rightarrow a} \frac{\frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{3} - \frac{a^{\frac{2}{3}}}{3}}{\frac{x^{\frac{4}{3}} - a^{\frac{4}{3}}}{3}} \\ &= \lim_{x \rightarrow a} \frac{\frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{3}}{\frac{x^{\frac{4}{3}} - a^{\frac{4}{3}}}{3}} \\ &= \lim_{x \rightarrow a} \frac{\frac{x - a}{3}}{\frac{x - a}{3}} \\ &= \lim_{x \rightarrow a} \frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{3} \\ &= \lim_{x \rightarrow a} \frac{x^{\frac{4}{3}} - a^{\frac{4}{3}}}{3} \end{aligned}$$

[Dividing numerator and denominator by  $x - a$ ]

Applying the formula  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$  in numerator and  $\lim_{x \rightarrow a} \frac{x^m - a^m}{x - a} = ma^{m-1}$  in denominator respectively

$$\text{Here, } n = \frac{2}{3}, m = \frac{3}{4}$$

$$\begin{aligned} \Rightarrow \frac{\lim_{x \rightarrow a} \frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim_{x \rightarrow a} \frac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}} &= \frac{\frac{2}{3}(a)^{\frac{2}{3}-1}}{\frac{3}{4}(a)^{\frac{3}{4}-1}} \\ &= \frac{8}{9} a^{\frac{-1}{3} + \frac{1}{4}} \\ &= \frac{8}{9} a^{\frac{-1}{12}} \end{aligned}$$