RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.8

Limits Ex 29.8 Q1

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$Let\ y = \frac{\pi}{2} - x$$

$$as\ x \to \pi/2, \quad y \to 0$$

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$= \lim_{y \to 0} y\tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y\frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$

$$= \lim_{y \to 0} y\frac{\cos y}{\sin y}$$

$$= \lim_{y \to 0} \cos y = \lim_{y \to 0} \frac{y}{\sin y}$$

$$= 1$$

Limits Ex 29.8 Q2

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2\sin x \cos x}{\cos x}$$

$$= 2\lim_{x \to \frac{\pi}{2}} \sin x$$

$$= 2 \times \sin\frac{\pi}{2}$$

$$= 2 \times 1$$

$$= 2$$

Limits Ex 29.8 Q3

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(1+\sin x)}{(1-\sin x)}$$

$$= \lim_{x\to\frac{\pi}{2}} (1+\sin x)$$

$$= 1+\sin\frac{\pi}{2}$$

$$= 1+1$$

$$= 2$$

Limits Ex 29.8 Q4

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{1-\sin^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{1+\sin x}$$

$$= \frac{1}{1+\sin\frac{\pi}{2}}$$

$$= \frac{1}{1+1}$$

$$= \frac{1}{2}$$

Limits Ex 29.8 Q5

$$\lim_{x \to a} \frac{\cos x - \cos a}{x - a}$$

$$= \lim_{x \to a} \frac{\left(-2 \sin\left(\frac{x+a}{2}\right) \sin\left(\frac{x-a}{2}\right)\right)}{x - a}$$

$$= -2 \lim_{x \to a} \sin\left(\frac{x+a}{2}\right) \lim_{x \to a} \frac{\sin\left(\frac{x-a}{2}\right)}{x - a}$$

$$= -2 \times \sin\left(\frac{a+a}{2}\right) \times \left(\lim_{x-a \to 0} \frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right) \times \frac{1}{2}$$

$$= -2 \sin a \times 1 \times \frac{1}{2} \qquad \left[\because \lim_{x \to a} \frac{\sin x}{x} = 1\right]$$

$$= -\sin a$$

Limits Ex 29.8 Q6

$$\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{x - \frac{\pi}{4}}$$

If $x \to \frac{\pi}{4}$, then $x - \frac{\pi}{4} \to 0$

Let $x - \frac{\pi}{4} = y \Rightarrow y \to 0$

$$= \lim_{y \to 0} \frac{1 - \tan\left(y + \frac{\pi}{4}\right)}{y}$$

$$= \lim_{y \to 0} \frac{1 - \left(\dfrac{\tan y + \tan\frac{\pi}{4}}{1 - \tan y \tan\frac{\pi}{4}}\right)}{y}$$

$$= \lim_{y \to 0} \frac{(1 - \tan y - \tan y - 1)}{y\,(1 - \tan y)} \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \lim_{y \to 0} \frac{(-2\tan y)}{y\,(1 - \tan y)}$$

$$= -2 \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}(1 - \tan y)}$$

$$= -2 \times 1 \frac{1}{(1 - 0)} \qquad \left[\because \lim_{\theta \to 0} = \frac{\tan\theta}{\theta} = 1\right]$$

$$= -2$$

Limits Ex 29.8 Q7

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}$$

If $x \to \frac{\pi}{2}, \frac{\pi}{2} - x \to 0$

Let $\frac{\pi}{2} - x = y$ they $y \to 0$

$$= \lim_{y \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - y\right)}{y^2}$$

$$= \lim_{y \to 0} \frac{1 - \cos y}{y^2}$$

$$= \lim_{y \to 0} \frac{2\sin^2\frac{y}{2}}{y^2}$$

$$= 2\left(\lim_{y \to 0} \frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{1}{4} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Limits Ex 29.8 Q8

$$\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

If $x \to \frac{\pi}{3}, \frac{\pi}{3} - x \to 0, \pi - 3x \to 0$

Let $\frac{\pi}{3} - x = y$ then $y \to 0$

$$= \lim_{y \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} - y\right)}{3\left(\frac{\pi}{3} - x\right)}$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \frac{\tan\frac{\pi}{3} - \tan y}{1 + \tan\frac{\pi}{3} . \tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \frac{\sqrt{3} - \tan y}{1 + \sqrt{3} \tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} + 3\tan y - \sqrt{3} + \tan y \right)}{3\left(1 + \sqrt{3}\tan y\right) y}$$

$$= \lim_{y \to 0} \frac{4 \tan y}{3\left(1 + \sqrt{3} \tan y\right) y}$$

$$= \frac{4}{3} \times \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0} \left(1 + \sqrt{3} \frac{\tan y}{y} \times y\right)}$$

$$= \frac{4 \times 1}{3} \times \frac{1}{1 + 0}$$

$$= \frac{4}{3}$$

Limits Ex 29.8 Q9

$$\lim_{x\to a}\frac{a\sin x - x\sin a}{ax^2 - xa^2} = \lim_{x\to a}\frac{(a\sin x - x\sin a)}{ax(x-a)}$$

Let $t = x - a$

Then, as $x \to a$, $t \to 0$

$$\therefore \lim_{x\to a}\frac{(a\sin x - x\sin a)}{ax(x-a)} = \lim_{t\to 0}\frac{(a\sin(t+a) - (t+a)\sin a)}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a\cos t - t\sin a - a\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a(\cos t - 1) - t\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a + a\sin a(2\sin^2(t/2)) - t\sin a}{a(t+a)t}$$

$$= \lim_{t\to 0}\frac{a\sin t\cos a}{a(t+a)t} + \lim_{t\to 0}\frac{a\sin a(2\sin^2(t/2))}{a(t+a)t} - \lim_{t\to 0}\frac{t\sin a}{a(t+a)t}$$

$$= \frac{a\cos a}{a^2} + 0 - \frac{\sin a}{a^2}$$

$$= \frac{a\cos a - \sin a}{a^2}$$

Limits Ex 29.8 Q10

$$\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}\,\frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{2-1-\sin x}{\cos^2 x\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(1-\sin^2 x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{1}{(1+\sin x)\left(\sqrt{2}+\sqrt{1+\sin x}\right)}$$

$$= \frac{1}{(1+1)\left(\sqrt{2}+\sqrt{2}\right)}$$

$$= \frac{1}{\left(4\sqrt{2}\right)}$$