

RD Sharma
Solutions
Class 11 Maths
Chapter 29
Ex 29.9

Limits Ex 29.9 Q1

$$= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{\tan^2 x}$$

As $x \rightarrow \pi$, $x - \pi \rightarrow 0$, let $x - \pi = y$

$$= \lim_{y \rightarrow 0} \frac{1 + \cos(\pi + y)}{\tan^2(\pi + y)}$$

$$= \lim_{y \rightarrow 0} \frac{1 - \cos y}{\tan^2 y}$$

$$= \lim_{y \rightarrow 0} \frac{2 \sin^2 \frac{y}{2}}{\tan^2 y}$$

$$= \frac{\lim_{y \rightarrow 0} 2 \sin^2 \frac{y}{2}}{\lim_{y \rightarrow 0} \tan^2 y}$$

$$= \frac{2 \left(\lim_{y \rightarrow 0} \frac{\sin y}{\frac{y}{2}} \right)^2 \times \frac{y^2}{4}}{\left(\lim_{y \rightarrow 0} \frac{\tan y}{y} \right) \times y^2}$$

$$= \frac{2 \times 1 \times \frac{y^2}{4}}{1 \times y^2}$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

$$\left[\begin{array}{l} \because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \end{array} \right]$$

Limits Ex 29.9 Q2

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot^2 x + 1 - 2}{\cot x - 1} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot^2 x - 1}{\cot x - 1} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} (\cot x + 1) \\ &= \cot \frac{\pi}{4} + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Limits Ex 29.9 Q3

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2} \\ &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} \\ &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)} \\ &= \lim_{x \rightarrow \frac{\pi}{6}} (\operatorname{cosec} x + 2) \\ &= \operatorname{cosec} \frac{\pi}{6} + 2 \\ &= 2 + 2 \\ &= 4 \end{aligned}$$

Limits Ex 29.9 Q4

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - (1 + \cot^2 x)}{1 - \cot x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - 1 - \cot^2 x}{1 - \cot x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \cot^2 x}{1 - \cot x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 - \cot x)(1 + \cot x)}{(1 - \cot x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} (1 + \cot x) \\ &= 1 + \cot \frac{\pi}{4} \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Limits Ex 29.9 Q5

$$\begin{aligned} & \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \\ &= \lim_{x \rightarrow \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1} \\ &= \lim_{x \rightarrow \pi} \frac{(2 + \cos x) - 1}{(\pi - x)^2 (\sqrt{2 + \cos x} + 1)} \\ &= \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2 (\sqrt{2 + \cos x} + 1)} \end{aligned}$$

Let $\pi - x = y, x \rightarrow \pi, y \rightarrow 0$

$$\Rightarrow \lim_{x \rightarrow \pi} \frac{1 + \cos x}{(\pi - x)^2 (\sqrt{2 + \cos x} + 1)} = \lim_{y \rightarrow 0} \frac{1 + \cos(\pi - y)}{y^2 (\sqrt{2 + \cos(\pi - y)} + 1)}$$

$$\begin{aligned}
&= \lim_{y \rightarrow 0} \frac{1 - \cos y}{y^2 \sqrt{2 - \cos y} + 1} \\
&= \lim_{y \rightarrow 0} \frac{2 \sin^2 y}{2} \\
&= \lim_{y \rightarrow 0} \left(\frac{\sin y}{\frac{y}{2}} \right)^2 \times \frac{1}{4 \sqrt{2 - \cos y} + 1} \\
&= 2 \times \left(\lim_{y \rightarrow 0} \frac{\sin y}{2} \right)^2 \times \frac{1}{4 \lim_{y \rightarrow 0} \sqrt{2 - \cos y} + 1} \\
&= 2 \times 1 \times \frac{1}{4} \times \frac{1}{\sqrt{2 - \cos 0} + 1} \\
&= 2 \times 1 \times \frac{1}{4} \times \frac{1}{\sqrt{2 - 1} + 1} \\
&= 2 \times 1 \times \frac{1}{4} \times \frac{1}{1 + 1} \\
&= 2 \times 1 \times \frac{1}{4} \times \frac{1}{2} \\
&= \frac{1}{4}
\end{aligned}$$

Limits Ex 29.9 Q6

$$\begin{aligned}
&\lim_{x \rightarrow \frac{3\pi}{2}} \frac{1 + \operatorname{cosec}^3 x}{\cot^2 x} \\
&= \lim_{x \rightarrow \frac{3\pi}{2}} \frac{(1 + \operatorname{cosec} x)(1 + \operatorname{cosec}^2 x - \operatorname{cosec} x)}{(\operatorname{cosec}^2 x - 1)} \\
&= \lim_{x \rightarrow \frac{3\pi}{2}} \frac{(\operatorname{cosec} x + 1)(1 + \operatorname{cosec}^2 x - \operatorname{cosec} x)}{(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)} \\
&= \lim_{x \rightarrow \frac{3\pi}{2}} \frac{(1 + \operatorname{cosec}^2 x - \operatorname{cosec} x)}{\operatorname{cosec} x - 1}
\end{aligned}$$

$$= \frac{1 + \operatorname{cosec}^2 \frac{3\pi}{2} - \operatorname{cosec} \frac{3\pi}{2}}{\operatorname{cosec} \frac{3\pi}{2} - 1}$$

$$= \frac{1 + (-1)^2 - (-1)}{(-1) - 1}$$

$$= \frac{1 + 1 + 1}{-2}$$

$$= \frac{-3}{2}$$

$$\left[\because \operatorname{cosec} \frac{3\pi}{2} = -1 \right]$$