RD SHARMA

Solutions

Class 10 Maths

Chapter 1

Ex 1.5

Q.1: Show that the following numbers are irrational.

(i) 7√5 $7\sqrt{5}$

Let us assume that 7√5 $7\sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

7√5 $7\sqrt{5}$ = ab $\frac{a}{b}$

√5 $\sqrt{5}$ = a7b $\frac{a}{7b}$

We know that √5 $\sqrt{5}$ is an irrational number

Here we see that √5 $\sqrt{5}$ is a rational number which is a contradiction.

(ii) 6+√2 $6+\sqrt{2}$

Let us assume that 6+√2 $6+\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

6+√2 $6+\sqrt{2}$ = ab $\frac{a}{b}$

√2 $\sqrt{2}$ = ab−6 $\frac{a}{b}-6$

√2 $\sqrt{2}$ = a−6bb $\frac{a-6b}{b}$

Here we see that √2 $\sqrt{2}$ is a rational number which is a contradiction as we know that √2 $\sqrt{2}$ is an irrational number

Hence 6+√2 $6+\sqrt{2}$ is an irrational number

(iii) 3−√5 $3-\sqrt{5}$

Let us assume that 3−√5 $3-\sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

3−√5 $3-\sqrt{5}$ = ab $\frac{a}{b}$

√5 $\sqrt{5}$ = 3−ab $3-\frac{a}{b}$

√5 $\sqrt{5}$ = 3b−ab $\frac{3b-a}{b}$

Here we see that $\sqrt5\sqrt{5}$ is a rational number which is a contradiction as we know that $\sqrt5\sqrt{5}$ is an irrational number

Hence $3-\sqrt5 3-\sqrt{5}$ is an irrational number.

Q.2: Prove that the following numbers are irrationals.

Sol:

(i) $2\sqrt7 \frac{2}{\sqrt{7}}$

Let us assume that $2\sqrt7 2\sqrt{7}$ is rational. Then, there exist positive co primes a and b such that

$2\sqrt7 2\sqrt{7}$ = ab $\frac{a}{b}$

$\sqrt7\sqrt{7}$ = 2ba $\frac{2b}{a}$

$\sqrt7\sqrt{7}$ is rational number which is a contradiction

Hence $2\sqrt7 2\sqrt{7}$ is an irrational number

(ii) $32\sqrt5 \frac{3}{2\sqrt{5}}$

Let us assume that $32\sqrt5 \frac{3}{2\sqrt{5}}$ is rational. Then, there exist positive co primes a and b such that

$32\sqrt5 \frac{3}{2\sqrt{5}}$ = ab $\frac{a}{b}$

$\sqrt5\sqrt{5}$ = 3b2a $\frac{3b}{2a}$

$\sqrt5\sqrt{5}$ is rational number which is a contradiction

Hence $32\sqrt5 \frac{3}{2\sqrt{5}}$ is irrational.

(iii) $4+\sqrt2 4+\sqrt{2}$

Let us assume that $4+\sqrt2 4+\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

4+√2$4+\sqrt{2}$ = ab $\frac{a}{b}$

√2$\sqrt{2}$ = ab−4$\frac{a}{b}-4$

√2$\sqrt{2}$ = a−4bb $\frac{a-4b}{b}$

√2$\sqrt{2}$ is rational number which is a contradiction

Hence 4+√2$4+\sqrt{2}$ is irrational.

(iv) 5√2$5\sqrt{2}$

Let us assume that 5√2$5\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

5√2$5\sqrt{2}$ = ab $\frac{a}{b}$

√2$\sqrt{2}$ = ab−5$\frac{a}{b}-5$

√2$\sqrt{2}$ = a−5bb $\frac{a-5b}{b}$

√2$\sqrt{2}$ is rational number which is a contradiction

Hence 5√2$5\sqrt{2}$ is irrational

Q.3: Show that 2−√3$2-\sqrt{3}$ **is an irrational number.**

Sol:

Let us assume that 2−√3$2-\sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

2−√3$2-\sqrt{3}$ = ab $\frac{a}{b}$

√3$\sqrt{3}$ = 2−ab$2-\frac{a}{b}$

Here we see that √3$\sqrt{3}$ is a rational number which is a contradiction

Hence 2−√3$2-\sqrt{3}$ is irrational

Q.4: Show that 3+√2$3+\sqrt{2}$ **is an irrational number**.

Sol:

Let us assume that $3+\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$3+\sqrt{2} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - 3$

$\sqrt{2} = \frac{a-3b}{b}$

Here we see that $\sqrt{2}$ is a irrational number which is a contradiction

Hence $3+\sqrt{2}$ is irrational

Q.5: Prove that $4-5\sqrt{2}$ is an irrational number.

Sol:

Let us assume that $4-5\sqrt{2}$ is rational. Then, there exist positive co primes a and b such that

$4-5\sqrt{2} = \frac{a}{b}$

$5\sqrt{2} = \frac{a}{b} - 4$

$\sqrt{2} = \frac{\frac{a}{b}-4}{5}$

$\sqrt{2} = \frac{a-4b}{5b}$

This contradicts the fact that $\sqrt{2}$ is an irrational number

Hence $4-5\sqrt{2}$ is irrational

Q.6: Show that $5-2\sqrt{3}$ is an irrational number.

Sol.

Let us assume that $5-2\sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

$5-2\sqrt{3} = \frac{a}{b}$

$2\sqrt{3}2\sqrt{3}$ = $ab-5\frac{a}{b}-5$

$\sqrt{3}=$ $ab-52\sqrt{3}=\frac{\frac{a}{b}-5}{2}$ $\sqrt{3}=$ $a-5b2b\sqrt{3}=\frac{a-5b}{2b}$

This contradicts the fact that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $5-2\sqrt{3}5-2\sqrt{3}$ is irrational

Q.7: Prove that $2\sqrt{3}-12\sqrt{3}-1$ is an irrational number.

Sol:

Let us assume that $2\sqrt{3}-12\sqrt{3}-1$ is rational. Then, there exist positive co primes a and b such that

$2\sqrt{3}-12\sqrt{3}-1$ = $ab\frac{a}{b}$

$2\sqrt{3}2\sqrt{3}$ = $ab+1\frac{a}{b}+1$

$\sqrt{3}=$ $ab+12\sqrt{3}=\frac{\frac{a}{b}+1}{2}$ $\sqrt{3}=$ $a+b2b\sqrt{3}=\frac{a+b}{2b}$

This contradicts the fact that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $5-2\sqrt{3}5-2\sqrt{3}$ is irrational

Q.8: Prove that $2-3\sqrt{5}2-3\sqrt{5}$ is an irrational number.

Sol:

Let us assume that $2-3\sqrt{5}2-3\sqrt{5}$ is rational. Then, there exist positive co primes a and b such that

$2-3\sqrt{5}2-3\sqrt{5}$ = $ab\frac{a}{b}$

$3\sqrt{5}3\sqrt{5}$ = $ab-2\frac{a}{b}-2$

$3\sqrt{5}=$ $ab-233\sqrt{5}=\frac{\frac{a}{b}-2}{3}$ $\sqrt{5}=$ $a-3b3b\sqrt{5}=\frac{a-3b}{3b}$

This contradicts the fact that $\sqrt{5}\sqrt{5}$ is an irrational number

Hence $2-3\sqrt{5}2-3\sqrt{5}$ is irrational

Q.9: Prove that $\sqrt{5}+\sqrt{3}\sqrt{5}+\sqrt{3}$ is irrational.

Sol:

Let us assume that $\sqrt{5}+\sqrt{3}\sqrt{5}+\sqrt{3}$ is rational. Then, there exist positive co primes a and b such that

$\sqrt{5}+\sqrt{3}\sqrt{5}+\sqrt{3}$ = ab $\frac{a}{b}$

$\sqrt{5}=ab-\sqrt{3}\sqrt{5}=\frac{a}{b}-\sqrt{3}$ $(\sqrt{5})^2=(ab-\sqrt{3})^3(\sqrt{5})^2=(\frac{a}{b}-\sqrt{3})^3$ $5=(ab)^2-2a\sqrt{3}b+3$

$5=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}+3$ $\Rightarrow 5-3=(ab)^2-2a\sqrt{3}b \Rightarrow 5-3=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}$ $\Rightarrow 2=(ab)^2-2a\sqrt{3}b$

$\Rightarrow 2=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}$ $\Rightarrow (ab)^2-2=2a\sqrt{3}b \Rightarrow (\frac{a}{b})^2-2=\frac{2a\sqrt{3}}{b}$ $\Rightarrow a^2-2b^2b^2=2a\sqrt{3}b$

$\Rightarrow \frac{a^2-2b^2}{b^2}=\frac{2a\sqrt{3}}{b}$ $\Rightarrow (a^2-2b^2b^2)(b2a)=\sqrt{3} \Rightarrow (\frac{a^2-2b^2}{b^2})(\frac{b}{2a})=\sqrt{3}$ $\Rightarrow (a^2-2b^22ab)=\sqrt{3}$

$\Rightarrow (\frac{a^2-2b^2}{2ab})=\sqrt{3}$

Here we see that $\sqrt{3}\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}\sqrt{3}$ is an irrational number

Hence $\sqrt{5}+\sqrt{3}\sqrt{5}+\sqrt{3}$ is an irrational number

Q.10: Prove that $\sqrt{3}+\sqrt{4}\sqrt{3}+\sqrt{4}$ is irrational.

Sol:

Let us assume that $\sqrt{3}+\sqrt{4}\sqrt{3}+\sqrt{4}$ is rational. Then, there exist positive co primes a and b such that

$\sqrt{3}+\sqrt{4}\sqrt{3}+\sqrt{4}$ = ab $\frac{a}{b}$

$\sqrt{4}=ab-\sqrt{3}\sqrt{4}=\frac{a}{b}-\sqrt{3}$ $(\sqrt{4})^2=(ab-\sqrt{3})^3(\sqrt{4})^2=(\frac{a}{b}-\sqrt{3})^3$ $4=(ab)^2-2a\sqrt{3}b+3$

$4=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}+3$ $\Rightarrow 4-3=(ab)^2-2a\sqrt{3}b \Rightarrow 4-3=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}$ $\Rightarrow 1=(ab)^2-2a\sqrt{3}b$

$\Rightarrow 1=(\frac{a}{b})^2-\frac{2a\sqrt{3}}{b}$ $\Rightarrow (ab)^2-1=2a\sqrt{3}b \Rightarrow (\frac{a}{b})^2-1=\frac{2a\sqrt{3}}{b}$ $\Rightarrow a^2-b^2b^2=2a\sqrt{3}b$

$\Rightarrow \frac{a^2-b^2}{b^2}=\frac{2a\sqrt{3}}{b}$ $\Rightarrow (a^2-b^2b^2)(b2a)=\sqrt{3} \Rightarrow (\frac{a^2-b^2}{b^2})(\frac{b}{2a})=\sqrt{3}$ $\Rightarrow (a^2-b^22ab)=\sqrt{3}$

$\Rightarrow (\frac{a^2-b^2}{2ab})=\sqrt{3}$

Here we see that $\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}$ is an irrational number

Hence $\sqrt{3}+\sqrt{4}$ is an irrational number

Q.11: Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number.

Sol:

Let us assume that $\sqrt{p}$ is rational. Then, there exist positive co primes a and b such that

$\sqrt{p} = \frac{a}{b}$

$p = \left(\frac{a}{b}\right)^2$

$\Rightarrow p = \frac{a^2}{b^2}$

$\Rightarrow pb^2 = a^2 \Rightarrow p|a^2 \Rightarrow p|a \Rightarrow a = pc$ for some positive integer c

$\Rightarrow b^2p = a^2$

$\Rightarrow b^2p = p^2c^2$ ($\because$ a=pc)

$\Rightarrow p|b^2$ (since $p|c^2p$) $\Rightarrow p|b \Rightarrow p|a$ and $p|b$

This contradicts the fact that a and b are co primes

Hence $\sqrt{p}$ is irrational

Q.12: If p, q are prime positive integers, prove that $\sqrt{p}+\sqrt{q}$ is an irrational number.

Sol:

Let us assume that $\sqrt{p}+\sqrt{q}$ is rational. Then, there exist positive co primes a and b such that

$\sqrt{p}+\sqrt{q} = \frac{a}{b}$

$$\sqrt{p} = \frac{a}{b} - \sqrt{q}$$

$$(\sqrt{p})^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2$$

$$p = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} + q$$

$$p - q = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b}$$

$$\left(\frac{a}{b}\right)^2 - (p - q) = \frac{2a\sqrt{q}}{b}$$

$$\frac{a^2 - b^2(p-q)}{b^2} = \frac{2a\sqrt{q}}{b}$$

$$\left(\frac{a^2 - b^2(p-q)}{b^2}\right)\left(\frac{b}{2a}\right) = \sqrt{q}$$

$$\sqrt{q} = \frac{a^2 - b^2(p-q)}{2ab}$$

Here we see that $\sqrt{q}$ is a rational number which is a contradiction as we know that $\sqrt{q}$ is an irrational number

Hence $\sqrt{p} + \sqrt{q}$ is an irrational number