RD SHARMA
Solutions
Class 10 Maths
Chapter 8
Ex 8.6

Question 1: Determine the nature of the roots of the following quadratic equations.

Solution: (i) $2x^2$ -3x + 5 =0

The given quadratic equation is in the form of $ax^2 +bx +c =0$

So a= 2, b= -3, c= 5

We know, determinant (D) = $b^2 – 4ac$

= $(-3)^2 -4(2)(5)$

= 9 – 40

= -31<0

Since D<0, the determinant of the equation is negative, so the expression does not having any real roots.

(ii) $2x^2$ -6x + 3=0

The given quadratic equation is in the form of $ax^2 +bx +c =0$

So a= 2, b= -6, c= 3

We know, determinant (D) = $b^2 – 4ac$

= $(-6)^2 -4(2)(3)$

= 36 – 24

= 12<0

Since D>0, the determinant of the equation is positive, so the expression does having any real and distinct roots.

(iii) For what value of k $(4-k)x^2 + (2k+4)x +(8k+1) =0$ is a perfect square.

The given equation is $(4-k)x^2 + (2k+4)x +(8k+1) =0$

Here, a= 4-k, b= 2k+4, c= 8k+1

The discriminate (D) = $b^2 - 4ac$

= $(2k+4)^2 – 4(4-k)(8k+1)$

= $(4k^2+16+ 16k) -4(32k+4-8k^2-k)$

= $4(k^2 +8k^2+4k-31k+4-4)$

=$4(9k^2-27k)$

D = $4(9k^2-27k)$

The given equation is a perfect square

D= 0

$4(9k^2-27k) = 0$

$9k^2-27k=0$

Taking out common of of 3 from both sides and cross multiplying

= $k^2 -3k =0$

= K (k-3) =0

Either k=0

Or k =3

The value of k is to be 0 or 3 in order to be a perfect square.

(iv) Find the least positive value of k for which the equation $x^2+kx+4=0$ has real roots.

The given equation is $x^2+kx+4=0$ has real roots

Here, a= 1, b= k, c= 4

The discriminate (D) = $b^2 - 4ac \geq 0$

= $k^2 – 16 \geq 0$

= $k\leq 4$,$k\geq-4$

The least positive value of k =4 for the given equation to have real roots.

(v) Find the value of k for which the given quadratic equation has real roots and distinct roots.

$Kx^2 + 2x + 1 = 0$

The given equation is $Kx^2 + 2x + 1 = 0$

Here, a= k, b= 2, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

$= 4 - 4k \geq 0 = 4k \leq 4$

$K \leq 1$

The value of $k \leq 1$ for which the quadratic equation is having real and equal roots.

(vi) $Kx^2 + 6x + 1 = 0$

The given equation is $Kx^2 + 6x + 1 = 0$

Here, a= k, b= 6, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

$= 36 - 4k \geq 0$

$= 4k \leq 36$

$K \leq 9$

The value of $k \leq 9$ for which the quadratic equation is having real and equal roots.

(vii) $x^2 - kx + 9 = 0$

The given equation is $X^2 - kx + 9 = 0$

Here, a= 1, b= -k, c= 9

Given that the equation is having real and distinct roots.

Hence, the discriminate (D) = $b^2 - 4ac \geq 0$

$= k^2 - 4(1)(9) \geq 0$

$= k^2 - 36 \geq 0$

$= k \geq -6$ and $k \leq 6$

The value of k lies between -6 and 6 respectively to have the real and distinct roots.

Question 2: Find the value of k.

(i) $Kx^2+4x+1=0$

The given equation $Kx^2+4x+1=0$ is in the form of $ax^2+bx+c=0$ where a= k, b= 4, c= 1

Given that, the equation has real and equal roots

D= $b^2-4ac=0$

= $4^2-4(k)(1)=0$

= 16-4k=0

= k = 4

The value of k is 4

(ii) $kx^2-2\sqrt{5}x+4=0$ $kx^2-2\sqrt{5}x+4=0$

The given equation $kx^2-2\sqrt{5}x+4=0$ $kx^2-2\sqrt{5}x+4=0$ is in the form of $ax^2+bx+c=0$ where

a= k , b= $-2\sqrt{5}$ $-2\sqrt{5}$, c= 4

Given that, the equation has real and equal roots

D= $b^2-4ac=0$

= $-2\sqrt{5}^2-4\times k\times 4=0$ $-2\sqrt{5}^2-4\times k\times 4=0$

= 20-16k =0

= k= 54 $k=\frac{5}{4}$

The value of k is k= 54 $k=\frac{5}{4}$

(iii) $3x^2-5x+2k=0$

The given equation $3x^2-5x+2k=0$ is in the form of $ax^2+bx+c=0$ where a= 3, b= -5, c= 2k

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $(-5)^2$-4(3)(2k)=0

= 25-24k =0

K = k=2524 $k = \frac{25}{24}$

The value of the k is k=2524 $k = \frac{25}{24}$

(iv) $4x^2$+kx+9=0

The given equation $4x^2$+kx+9=0 is in the form of ax^2+bx+c=0 where a= 4, b= k, c= 9

Given that, the equation has real and equal roots

D= b^2-4ac=0

= k^2-4(4)(9)=0

= k^2 -144=0

= k = 12

The value of k is 12

(v) $2kx^2$-40x+25=0

The given equation $2kx^2$-40x+25=0 is in the form of ax^2+bx+c=0 where a= 2k, b= -40, c= 25

Given that, the equation has real and equal roots

D= b^2-4ac=0

= $(-40)^2$-4(2k)(25)=0

= 1600-200k=0

= k = 8

The value of k is 8

(vi) $9x^2$-24x+k = 0

The given equation $9x^2-24x+k=0$ is in the form of $ax^2+bx+c=0$ where a= 9, b= -24, c= k

Given that, the equation has real and equal roots

$D= b^2-4ac=0$

$= (-24)^2-4(9)(k)=0$

= 576-36k = 0 = k = 16

The value of k is 16

(vii) $4x^2-3kx+1 =0$

The given equation $4x^2-3kx+1=0$ is in the form of $ax^2+bx+c=0$ where a= 4, b= -3k, c= 1

Given that, the equation has real and equal roots $D= b^2-4ac=0$

$= (-3k)^2-4(4)(1)=0$

$= 9k^2-16=0$

K = $43\frac{4}{3}$

The value of k is $43\frac{4}{3}$

(viii) $x^2-2(5+2k)x+3(7+10k) =0$

The given equation $X^2-2(5+2k)x+3(7+10k) =0$ is in the form of $ax^2+bx+c=0$ where a= 1, b=+2(52k) , c= 3(7+10k)

Given that, the nature of the roots of the equation are real and equal roots

$D= b^2-4ac=0$

$= (+2(52k))^2-4(1)(3(7+10k))=0$

$= 4(5+2k)^2 -12(7+10k)=0$

$= 25+4k^2+20k-21-30k=0$

$= 4k^2-10k+4=0$

Simplifying the above equation. We get,

= $2k^2-5k+2=0$

= $2k^2-4k-k+2=0$

=2k(k-2)-1(k-2) =0

= (k-2)(2k-1) =0K=2 and k = 12 $\frac{1}{2}$

The value of k can either be 2 or 12 $\frac{1}{2}$

(ix) $(3k+1)x^2+2(k+1)x+k=0$

The given equation $(3k+1)x^2+2(k+1)x+k=0$is in the form of $ax^2+bx+c=0$ where a= 3k+1, b=+2(k+1) , c=(k)

Given that, the nature of the roots of the equation are real and equal roots

D= $b^2-4ac=0$

= $[2(k+1)]^2 -4(3k+1)(k) =0$

= $(k+1)^2-k(3k+1)=0$

= $-2k^2+k+1=0$

This equation can also be written as $2k^2-k-1=0$

The value of k can be obtained by

K = k= 1+√94 $k = \frac{1+\sqrt{9}}{4}$ = 1

Or , k= 1−√94 $k = \frac{1-\sqrt{9}}{4}$ = −12 $\frac{-1}{2}$

The value of k are 1 and −12 $\frac{-1}{2}$ respectively.

(x) $Kx^2+kx+1 = -4x^2-x$

Bringing all the x components on one side we get ,

$x^2(4+k)+x(k+1)+1=0$

The given equation $Kx^2+kx+1 = -4x^2-x$ is in the form of $ax^2+bx+c=0$ where a=4+k,b=+k+1 , c= 1

Given that, the nature of the roots of the equation are real and equal roots

D= b^2-4ac=0

= $(k+1)^2$-4(4+k)(1) =0

= k^2-2k-10 =0

The equation is also in the form $ax^2+bx+c=0$

The value of k is obtained by a=1 , b= -2 , c = -15

$$k = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Putting the respective values in the above formula we will obtain the value of k

The value of k are 5 and -3 for different given quadratic equation.

(xi) $(k+1)x^2+2(k+3)x+k+8 =0$

The given equation (k+1) x^2+2(k+3)x+k+8 =0 is in the form of $ax^2+bx+c=0$ where a=k+1,b=2(k+3), c= k+8

Given the nature of the roots of the equation are real and equal .

D= b^2-4ac =0

= $[2(k+30]^2$-4(k+1)(k+8)=0

= $4(k+3)^2$-4(k+1)(k+8) =0

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $(k+3)^2$-(k+1)(k+8) =0

= k^2+9+6k – (k^2+9k+18) =0

Cancelling out the like terms on the LHS side

= 9+6k-9k-8 =0

= -3k+1 =0

= 3k = 1

$$k = \frac{1}{3}$$

The value of k of the given equation is $k = \frac{1}{3}$

(xii) x^2-2kx+7k-12=0

The given equation is X^2-2kx+7k-12=0

The given equation is in the form of $ax^2+bx+c=0$ where a=1,b=-2k, c= 7k-12

Given the nature of the roots of the equation are real and equal .

D= b^2-4ac =0

= $(2k)^2-4(1)(7k-12)$ =0

= $4k^2-28k+48$ =0

= $k^2-7k+12$ =0

The value of k can be obtained by

k= $-b\pm\sqrt{b^2-4ac}2a$ $k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a = 1 , b = -7k , c = 12

By calculating the value of k is $7\pm\sqrt{1}2$ $\frac{7\pm\sqrt{1}}{2}$ = 4 , 3

The value of k for the given equation is 4 and 3 respectively.

(xiii) $(k+1)x^2-2(3k+1)x+8k+1$ =0

The given equation is $(k+1)x^2-2(3k+1)x+8k+1$ =0

The given equation is in the form of $ax^2+bx+c=0$ where a=k+1,b=-2(k+1), c= 8k+1

Given the nature of the roots of the equation are real and equal.

D= b^2-4ac =0

= $(-2(k+1))^2-4(k+1)(8k+1)$ =0

= $4(3k+1)^2-4(k+1)(8k+1)$ =0

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $(3k+1)^2-(k+1)(8k+1)$ =0

$= 9k^2+6k+1 – (8k^2+9k+1) =0$

$= 9k^2+6k+1 – 8k^2-9k-1 =0$

$= k^2-3k =0$

$= k(k-3) =0$

Either k =0

Or, k-3 =0 = k=3

The value of k for the given equation is 0 and 3 respectively.

(xiv) $5x^2-4x+2 +k(4x^2-2x+1) =0$

The given equation $5x^2-4x+2 +k(4x^2-2x+1) =0$ can be written as $x^2(5+4k)-x(4+2k)+2-k =0$

The given equation is in the form of $ax^2+bx+c=0$ where a=5+4k, b=-(4+2k), c= 2-k

Given the nature of the roots of the equation are real and equal.

$D= b^2-4ac =0$

$= [-(4+2k)]^2-4(5+4k)(2-k)=0$

$= 16+4k^2+16- 4(10-5k+8k-4k^2] =0$

$= 16+4k^2+16- 40+20k-32k+16k^2 =0$

$= 20k^2 -4k-24 =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

$= 5k^2-k-6 =0$

The value of k can be obtained by equation

k=−b±√b2−4ac2a $k=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Here a = 5 , b= -1 , c =-6

k= 1±√−12−4(5)(−6)2(5) $k=\frac{1\pm\sqrt{-1^2-4(5)(-6)}}{2(5)}$ k=65 and−1 $k=\frac{6}{5}\text{and}-1$

The value of k for the given equation are k=65 and−1 $k=\frac{6}{5}\text{and}-1$ respectively.

(xv) $(4-k)x^2+(2k+4)x+(8k+1) = 0$

The given equation is $(4-k)x^2+(2k+4)x+(8k+1) = 0$

The given equation is in the form of $ax^2+bx+c=0$ where $a=4-k$, $b=(2k+4)$, $c= 8k+1$

Given the nature of the roots of the equation are real and equal.

$D= b^2-4ac =0$

$= (2k+4)^2-4(4-k)(8k+1)=0$

$= 4k^2+16k+16 -4(-8k^2+32k+4-k)=0$

$= 4k^2+16k+16 + 32k^2-124k-16 =0$

Cancelling out the like and opposite terms. We get,

$= 36k^2-108k =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

$= 9k^2-27k =0$

$= 9k(k-3) =0$

Either $9k =0$

$K =0$

Or, $k-3 =0$

$K= 3$

The value of k for the given equation is 0 and 3 respectively.

(xvi) $(2k+1)x^2+2(k+3)x+(k+5) = 0$

The given equation is $(2k+1)x^2+2(k+3)x+(k+5) = 0$

The given equation is in the form of $ax^2+bx+c=0$ where $a=2k+1$, $b=2(k+3)$, $c= k+5$

Given the nature of the roots of the equation are real and equal.

$D= b^2-4ac =0$

$= [2(k+3)]^2-4(2k+1)(k+5) =0$

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $[(k+3)]^2$-(2k+1)(k+5) =0

= K^2+9+6k-($2k^2$+11k+5) =0

= $-k^2$– 5k+4 =0

= k^2+5k-4 =0

The value of k can be obtained by k=65and−1 $k = \frac{6}{5} \text{and} - 1$ respectively.

Here a= 1 , b = 5 , c= -4

Now k=−5±√5²−4(1)(−4)2×1 $k = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2 \times 1}$

K = k=−5±√412 $k = \frac{-5 \pm \sqrt{41}}{2}$

The value of k for the given equation is k=−5±√412 $k = \frac{-5 \pm \sqrt{41}}{2}$

(xvii) $4x^2$-2(k+1)x+(k+4) =0

The given equation is $4x^2$-2(k+1)x+(k+4) = 0

The given equation is in the form of ax^2+bx+c=0 where a=4, b=-2(k+1), c= k+4

Given the nature of the roots of the equation are real and equal.

D= b^2-4ac =0

= $[-2(k+1)]^2$-4(4)(k+4)=0

Taking out 4 as common from the LHS of the equation and dividing the same on the RHS

= $(k+1)^2$-4(k+4)=0

= k^2+1+2k-4k-16 =0

= k^2-2k-15 =0

The value of k can be obtained by k=65and−1 $k = \frac{6}{5} \text{and} - 1$ respectively.

Here a= 1 , b = -2 , c= -15

K = k=2±√692 $k = \frac{2 \pm \sqrt{69}}{2}$

The value of k for the given equation is k=2±√692 $k = \frac{2\pm\sqrt{69}}{2}$

Question 3: In the following, determine the set of values of k for which the given quadratic equation has real roots:

Solution:

(i) $2x^2+3x+k = 0$

The given equation is $2x^2+3x+k = 0$

The given quadratic equation has equal and real roots

$D = b^2-4ac \geq 0$

The given equation is in the form of $ax^2+bx+c =0$ so , a= 2 , b= 3 , c =k

$= 9 – 4(2)(k) \geq 0$

$= 9-8k \geq 0$

= k≤98 $k \leq \frac{9}{8}$

The value of k does not exceed k≤98 $k \leq \frac{9}{8}$ to have a real root.

(ii) $2x^2+kx+3 = 0$

The given equation is $2x^2+kx+3 = 0$

The given quadratic equation has equal and real roots

$D = b^2-4ac \geq 0$

The given equation is in the form of $ax^2+bx+c =0$ so, a= 2 , b= k , c =3

$= k^2-4(2)(3) \geq 0$

$= k^2-24 \geq 0$

$= k^2 \geq 24$

k≥√24 $k \geq \sqrt{24}$ k≥√24 $k \geq \sqrt{24}$

The value of k should not exceed k≥√24$k \geq \sqrt{24}$ in order to obtain real roots .

(iii)$2x^2$-5x-k =0

The given equation is $2x^2$-5x-k = 0

The given quadratic equation has equal and real roots

D = b^2-4ac ≥ 0

The given equation is in the form of ax^2+bx+c =0 so, a= 2 , b= -5 , c =-k

= 25 -4(2)(-k) ≥ 0

= 25-8k ≥ 0

= k≤258$k \leq \frac{25}{8}$

The value of k should not exceed k≤258$k \leq \frac{25}{8}$

(iv) Kx^2+6x+1 =0

The given equation is Kx^2+6x+1 = 0

The given quadratic equation has equal and real roots

D = b^2-4ac ≥ 0

The given equation is in the form of ax^2+bx+c =0 so, a= k , b= 6 , c =1

= 36 -4(k)(1) ≥ 0

= 36-4k ≥ 0

= k ≤ 9

The value of k for the given equation is k ≤ 9

(v) x^2-kx+9 =0

The given equation is X^2-kx+9 = 0

The given quadratic equation has equal and real roots

$D = b^2 - 4ac \geq 0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= 1 , b= -k , c =9

$= k^2 - 4(1)(-9) \geq 0$

$= k^2 - 36 \geq 0$

$= k^2 \geq 36$

$k \geq \sqrt{36} k \geq \sqrt{36}$

$K \geq 6$ and $k \leq -6$

The value of k should in between $K \geq 6$ and $k \leq -6$ in order to maintain real roots.

Question 4: Determine the nature of the roots of the following quadratic equations.

Solution:

(i) $2x^2 - 3x + 5 = 0$

The given quadratic equation is in the form of $ax^2 + bx + c = 0$

So a= 2, b= -3, c= 5

We know, determinant (D) = $b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$= -31 < 0$

Since D<0, the determinant of the equation is negative, so the expression does not having any real roots.

(ii) $2x^2 - 6x + 3 = 0$

The given quadratic equation is in the form of $ax^2 + bx + c = 0$

So a= 2, b= -6, c= 3

We know, determinant (D) = $b^2 - 4ac$

$= (-6)^2 - 4(2)(3)$

$= 36 – 24$

$= 12<0$

Since D>0, the determinant of the equation is positive, so the expression does having any real and distinct roots.

(iii) For what value of k $(4-k)x^2 + (2k+4)x +(8k+1) =0$ is a perfect square

The given equation is $(4-k)x^2 + (2k+4)x +(8k+1) =0$

Here, a= 4-k, b= 2k+4, c= 8k+1

The discriminate (D) = $b^{2 -} 4ac$

$= (2k+4)^2 – 4(4-k)(8k+1)$

$= (4k^2+16+ 16k) -4(32k+4-8k^2-k)$

$= 4(k^2 +8k^2+4k-31k+4-4)$

$=4(9k^2-27k)$

$D = 4(9k^2-27k)$

The given equation is a perfect square

D= 0

$4(9k^2-27k) = 0$

$9k^2-27k=0$

Taking out common of of 3 from both sides and cross multiplying

$K^2 -3k =0$

K (k-3) =0

Either k=0

Or k =3

The value of k is to be 0 or 3 in order to be a perfect square.

(iv) Find the least positive value of k for which the equation $x^2+kx+4=0$ has real roots.

The given equation is $x^2+kx+4=0$ has real roots

Here, a= 1, b= k, c= 4

The discriminate (D) = $b^2 - 4ac \geq 0$

= $k^2 - 16 \geq 0$

= $k \leq 4$,$k \geq -4$

The least positive value of k =4 for the given equation to have real roots.

(v) Find the value of k for which the given quadratic equation has real roots and distinct roots.

$Kx^2 + 2x+1 =0$

The given equation is $Kx^2 + 2x+1 =0$

Here, a= k, b= 2, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $4 - 4k \geq 0$

= $4k \leq 4$

$K \leq 1$

The value of $k \leq 1$ for which the quadratic equation is having real and equal roots.

(vi) $Kx^2 + 6x+1 = 0$

The given equation is $Kx^2 + 6x+1 =0$

Here, a= k, b= 6, c= 1

The discriminate (D) = $b^2 - 4ac \geq 0$

= $36 - 4k \geq 0$

= $4k \leq 36$

= $K \leq 9$

The value of $k \leq 9$ for which the quadratic equation is having real and equal roots.

(vii) $x^2 - kx+9 =0$

The given equation is $X^2 - kx+9 =0$

Here, a= 1, b= -k, c= 9

Given that the equation is having real and distinct roots.

Hence, the discriminate (D) = $b^2 - 4ac \geq 0$

$= k^2 – 4(1)(9) \geq 0$

$= k^2 -36 \geq 0$

$= k \geq -6$ and $k \leq 6$

The value of k lies between -6 and 6 respectively to have the real and distinct roots.

Question 5: Find the values of k for which the given quadratic equation has real and distinct roots.

Solution:

(i) $Kx^2+2x+1 =0$

The given equation is $Kx^2+2x+1 =0$

The given equation is in the form of $ax^2+bx+c =0$ so, a= k, b= 2 , c =1

$D = b^2-4ac \geq 0$

$= 4-4(1)(k) \geq 0$

$= 4k \leq 4$

$= k \leq 1$

The value of k for the given equation is $k \leq 1$

(ii) $Kx^2+6x+1 =0$

The given equation is $Kx^2+6x+1 =0$

The given equation is in the form of $ax^2+bx+c =0$ so, a= k, b= 6 , c =1

$D = b^2-4ac \geq 0$

$= 36-4(1)(k) \geq 0$

$= 4k \leq 36$

$= k \leq 9$

The value of k for the given equation is $k \leq 9$

Question 6: For what value of k , $(4-k)x^2+(2k+4)x+(8k+1)=0$, is a perfect square.

Solution:

The given equation is $(4-k)x^2+(2k+4)x+(8k+1)=0$

The given equation is in the form of $ax^2+bx+c=0$ so, a= 4-k , b= 2k+4 , c =8k+1

$D = b^2-4ac$

$= (2k+4)^2-4(4-k)(8k+1)$

$= 4k^2 +16+4k-4(32+4-8k^2-k)$

$= 4(k^2 +4+k-32-4+8k^2+k)$

$=4(9k^2-27k)$

Since the given equation is a perfect square

Therefore D =0

$=4(9k^2-27k) =0$

$= (9k^2-27k) =0$

= 3k (k-3) =0

Therefore 3k =0

K = 0

Or, k-3 =0

K =3

The value of k should be 0 or 3 to be perfect square.

Question 7: If the roots of the equation $(b-c)x^2 +(c-a)x+(a-b)=0$ are equal , then prove that 2b = a + c.

Solution:

The given equation is $(b-c)x^2 +(c-a)x+(a-b)=0$.

The given equation is the form of $ax^2 + bx + c =0$. So,

a= (b-c), b= (c-a), c= (a-b)

According to question the equation is having real and equal roots.

Hence discriminant(D) = b^2 -2ac =0

= $(c-a)^2$ -4(b-c)(a-b) =0

= c^2+a^2 -2ac – 4(ab-b^2-ac+cb) =0

= c^2+a^2 -2ac – 4ab+4b^2+4ac-4cb =0

= c^2+a^2 +2ac – 4ab+4b^2-4cb =0

= $(a+c)^2$ -4ab +4b^2 -4cb =0

= $(c+a-2b)^2$ = 0

= (c+a-2b) = 0

= c+a = 2b

Hence it is proved that c+a = 2b.

Question 8: If the roots of the equation $(a^2 +b^2) x^2$– 2(ac+bd)x+$(c^2+ d^2)$= 0 are equal. Prove that a÷b = c÷ d.

Solution:

The given equation is $(a^2 +b^2)x^2$ – 2(ac+bd)x+$(c^2+ d^2)$= 0 .

The equation is in the form of ax^2 + bx= c =0

Hence, a= $(a^2 +b^2)$,b= – 2(ac+bd) , c=$(c^2+ d^2)$.

The given equation is having real and equal roots.

Discriminant(D) = b^2 -4ac = 0

= $[-2(ac+bd)]^2$ -4 $(a^2 +b^2)(c^2 + d^2)$ =0

= $(ac+bd)^2$ – $(a^2 +b^2)(c^2 + d^2)$ =0

= $a^2c^2 + b^2d^2$ +2abcd – $(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$ =0

Cancelling out the equal and opposite terms. We get,

= 2abcd – a^2d^2 – b^2c^2 =0

= abcd +abcd – a^2d^2 – b^2c^2 =0

= ad(bc-ad) +bc(ad-bc) =0

= ad(bc-ad) -bc(bc-ad) =0

= (ad-bc)(bc-ad) =0

= ad –bc =0

= (a÷b) = (c÷ d)

Hence, it is proved.

Question 9: If the roots of the equation $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ca}x+b=0$ $bx^2-2\sqrt{ca}\,x+b=0$ are simultaneously real , then prove that $b^2-ac=0$.

Solution:

The given equations are $ax^2+2bx+c=0$ and $bx^2-2\sqrt{ca}x+b=0$$bx^2-2\sqrt{ca}\,x+b=0$

These two equations are of the form $ax^2+bx+c=0$.

Given that the roots of the two equations are real. Hence, D ≥ 0 that is $b^2-4ac \ge 0$

Let us assume that $ax^2+2bx+c=0$ be equation (i)

And $bx^2-2\sqrt{ca}x+b=0$$bx^2-2\sqrt{ca}\,x+b=0$ be (ii)

From equation (i) $b^2-4ac \ge 0$

= $4b^2-4ac \ge 0$ ……………………………………. (iii)

From equation (ii) $b^2-4ac \ge 0$

= $(2\sqrt{ca})^2-4b^2 \ge 0$$(2\sqrt{ca})^2-4b^2 \ge 0$ ……………………………. (iv)

Given, that the roots of equation (i) and (ii) are simultaneously real and hence equation (iii) = equation (iv).

= $4b^2-4ac = 4ac-4b^2$

= $8ac = 8b^2$

= $b^2-ac=0$.

Hence it is proved that $b^2-ac=0$.

Question 10: If p, q are the real roots and p ≠q . Then show that the roots of the equation $(p-q)x^2+5(p+q)x-2(p-q)=0$ are real and equal.

Solution:

The given equation is $(p-q)x^2 + 5(p+q)x - 2(p-q) = 0$

Given, p , q are real and $p \neq q$.

Then, Discriminant (D) = b^2 –ac

$= [5(p+q)]^2 - 4(p-q)(-2(p-q))$

$= 25(p+q)^2 + (p-q)^2$

We know that the square of any integer is always positive that is , greater than zero.

Hence, (D) = b^2 –ac ≥ 0

As given, p , q are real and $p \neq q$.

Therefore,

$= 25(p+q)^2 + (p-q)^2 > 0 = D > 0$

Therefore, the roots of this equation are real and unequal.

Question 11: If the roots of the equation $(c^2-ab)x^2 - 2(a^2-bc)x + b^2-ac = 0$ are equal , then prove that either a=0 or $a^3+b^3+c^3 = 3abc$.

Solution:

The given equation is $(c^2-ab)x^2 - 2(a^2-bc)x + b^2-ac = 0$

This equation is in the form of $ax^2 + bx + c = 0$

So, $a = (c^2-ab)$, $b = -2(a^2-bc)$, $c = b^2-ac$.

According to the question, the roots of the given question are equal.

Hence, D= 0 , $b^2-4ac = 0$

$= [-2(a^2-bc)]^2 - 4(c^2-ab)(b^2-ac) = 0$

$= 4(a^2-bc)^2 - 4(c^2-ab)(b^2-ac) = 0$

$= 4a(a^3+b^3+c^3 - 3abc) = 0$

Either 4a =0 therefore, a =0

Or, $(a^3+b^3+c^3 - 3abc) = 0$

$= (a^3+b^3+c^3) = 3abc$

Hence its is proved.

Question 12: Show that the equation $2(a^2+b^2)x^2+2(a+b)x+1 =0$ has no real roots , when $a \neq b$.

Solution:

The given equation is $2(a^2+b^2)x^2+2(a+b)x-1 =0$

This equation is in the form of $ax^2+bx+c =0$

Here, $a= 2(a^2+b^2)$, $b= 2(a+b)$, $c = +1$.

Given, $a \neq b$

The discriminant(D) = $b^2 -4ac$

$=[2(a+b)]^2 -4 (2(a^2+b^2))(1)$

$= 4(a+b)^2 -8(a^2+b^2)$

$= 4(a^2+b^2+2ab) –8a^2-8b^2$

$= =+2ab –4a^2-4b^2$

According to the question $a \neq b$, as the discriminant D has negative squares so the value of D will be less than zero.

Hence, $D < 0$, when $a \neq b$.

Question 13: Prove that both of the roots of the equation $(x-a)(x-b) +(x-c)(x-b)+(x-c)(x-a) =0$ are real but they are equal only when $a=b=c$.

Solution:

The given equation is $(x-a)(x-b) +(x-c)(x-b)+(x-c)(x-a) =0$

By solving the equation, we get it as,

$3x^2-2x(a+b+c)+(ab+bc+ca) =0$

This equation is in the form of $ax^2+bx+c = 0$

Here , $a =3$, $b = 2(a+b+c)$, $c = (ab+bc+ca)$

The discriminat (D) = b^2-4ac

$= [-2(a+b+c)]^2 - 4(3)(ab+bc+ca)$

$= 4(a+b+c)^2 - 12(ab+bc+ca)$

$= 4[(a+b+c)^2 - 3(ab+bc+ca)]$

$= 4[a^2+b^2+c^2 - ab-bc-ca]$

$= 2[2a^2+2b^2+2c^2 - 2ab-2bc-2ca]$

$= 2[(a-b)^2+(b-c)^2+(c-a)^2]$

Here clearly $D \geq 0$, if $D = 0$ then ,

$[(a-b)^2+(b-c)^2+(c-a)^2] = 0$

$a - b = 0$

$b - c = 0$

$c - a = 0$

Hence, a=b=c=0

Hence, it is proved.

Question 14: If a, b, c are real numbers such that $ac \neq 0$, then, show that at least one of the equations $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ has real roots.

Solution:

The given equation are $ax^2 + bx + c = 0$ ………………………… (i)

And- $ax^2 + bx + c = 0$ …………………………(ii)

Given, equations are in the form of $ax^2 + bx + c = 0$ also given that a ,b, c are real numbers and $ac \neq 0$.

The Discriminant(D) = $b^2 - 4ac$

For equation (i) = $b^2 - 4ac$ ……………………..(iii)

For equation (ii) = $b^2 - 4(-a)(c)$

$= b^2 + 4ac$ ……………………(iv)

As a , b , c are real and given that $ac \neq 0$, hence $b^2 - 4ac > 0$ and $b^2 + 4ac > 0$

Therefore, $D > 0$

Hence proved.

Question 15: If the equation $(1+m^2)x+2mcx+(c^2-a^2)=0$ has real and equal roots , prove that $c^2= a^2(1+m^2)$.

Solution:

The given equation is $(1+m^2)x^2+2mcx+(c^2-a^2)=0$

The above equation is in the form of $ax^2+bx +c =0$.

Here $a = (1+m^2)$, $b= 2mc$, $c= +(c^2-a^2)$

Given, that the nature of the roots of this equation is equal and hence D=0 , $b^2-4ac =0$

$= (2mc)^2 -4(1+m^2) (c^2-a^2) =0$

$= 4m^2c^2 – 4(c^2+m^2c^2-a^2 –a^2m^2) =0$

$= 4(m^2c^2– c^2+m^2c^2+a^2 +a^2m^2) =0$

$= m^2c^2– c^2+m^2c^2+a^2 +a^2m^2 =0$

Now cancelling out the equal and opposite terms ,

$= a^2 +a^2m^2 – c^2 =0$

$= a^2 (1+ m^2) – c^2 =0$

Therefore, $c^2 = a^2 (1+ m^2)$

Hence it is proved that as D= 0 , then the roots are equal of $c^2 = a^2(1+ m^2)$.