

RD SHARMA
Solutions
Class 10 Maths
Chapter 9
Ex 9.4

Question 1. If 12th of an A.P is 82 and 18th term is 124. Then find out the 24th term.

Solution: Given: $a_{12} = 82$ and $a_{18} = 124$

we know : $a_n = a + (n-1) c.d$

$$\Rightarrow a_{12} = a + (12-1) c.d$$

$$\Rightarrow 82 = a + 11c.d \quad \text{---(1)}$$

$$\Rightarrow 124 = a + (18-1) c.d$$

$$\Rightarrow 124 = a + 17c.d \quad \text{---(2)}$$

Subtracting (2) from (1)

$$\Rightarrow (a + 17c.d) - (a + 11c.d) = 124 - 82$$

$$\Rightarrow a + 17c.d - a - 11c.d = 42$$

$$\Rightarrow 6c.d = 42$$

$$\Rightarrow c.d = 7$$

Here we have, Common Difference (c.d) = 7

putting c.d = 7 in equation (1), we get

$$\Rightarrow a + 11 \times 7 = 82$$

$$\Rightarrow a = 82 - 77$$

$$\Rightarrow a = 5$$

Now, we have First Term (a) = 5

we have to find 24th term

$$a_{24} = a + (24 - 1) c.d$$

$$= 5 + 23 \times 7$$

$$= 5 + 161$$

$$= \mathbf{166}$$

Question 2. In an A.P. the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term.

Solution: Given

24th term is twice the 10th term

$$a_{24} = 2 \times a_{10} \quad \dots \dots (1)$$

let, first term be a

and common difference be d

we know, nth term is $a_n = a + (n - 1)d$

from equation (1), we have

$$a + (24-1)d = 2(a + (10-1)d)$$

$$\Rightarrow a + 23d = 2(a + 9d)$$

$$\Rightarrow a + 23d = 2a + 18d$$

$$\Rightarrow (23 - 18)d = a$$

$$\Rightarrow a = 5d$$

we have to prove that,

72nd term is twice the 34th term

$$\Rightarrow a_{72} = 2 \times a_{34}$$

$$\Rightarrow a + (72-1)d = 2[a + (34-1)d]$$

$$\Rightarrow a + 71d = 2(a + 33d)$$

$$\Rightarrow a + 71d = 2a + 66d$$

putting the value of a = 5d in the above equation,

$$\Rightarrow 5d + 71d = 2(5d) + 66d$$

$$\Rightarrow 76d = 76d$$

Hence it is proved...

Question 3. If the $(m+1)^{\text{th}}$ term of an A.P. is twice the $(n+1)^{\text{th}}$ term of the A.P. Then prove that: $(3m+1)^{\text{th}}$ is twice the $(m+n+1)^{\text{th}}$ term.

Solution: From the question, we have

$$a_{(m+1)} = 2 a_{(n+1)}$$

Let, First term = a

and Common Difference = d

$$\Rightarrow a + (m + 1 - 1)d = 2[a + (n + 1 - 1)d]$$

$$\Rightarrow a + md = 2a + 2nd$$

$$\Rightarrow a = md - 2nd$$

$$\Rightarrow a = (m-2n)d \quad \text{---(2)}$$

We have to prove, $a_{(3m+1)} = 2 a_{(m+n+1)}$

$$\Rightarrow a + (3m + 1 - 1)d = 2 [a + (m + n + 1 - 1)d]$$

$$\Rightarrow a + 3md = 2a + 2(m + n)d$$

putting the value of $a = (m - 2n)d$, from equation (1)

$$\Rightarrow (m - 2n)d + 3md = 2[(m - 2n)d] + 2(m + n)d$$

$$\Rightarrow m - 2n + 3m = 2m - 4n + 2m + 2n$$

$$\Rightarrow 4m - 2n = 4m - 2n$$

Hence it is proved...

Question 4. If the n^{th} term of the A.P. 9,7,5, ... is same as the n^{th} term of the A.P. 15, 12, 9, ... find n.

Solution: we have here,

First sequence is 9,7,5, ...

First term (a) = 9,

Common Difference (c.d) = $9 - 7 = -2$

$$n^{\text{th}} \text{ term} = a + (n-1)c.d$$

$$\Rightarrow a_n = 9 + (n-1)(-2)$$

$$= 9 - 2n + 2$$

$$= 11 - 2n$$

Second sequence is 15, 12, 9, ...

here, First term (a) = 15

$$\text{Common Difference (c.d)} = 12 - 15 = -3$$

$$n^{\text{th}} \text{ term} = a + (n-1)d$$

$$\Rightarrow a'_n = 15 + (n-1)(-3)$$

$$= 15 - 3n + 3$$

$$= 18 - 3n$$

We are given in the question that the n^{th} term of both the A.P.s are same,

So, we can write it as

$$a_n = a'_n$$

$$\Rightarrow 11 - 2n = 18 - 3n$$

$$\Rightarrow \mathbf{n = 7}$$

So, the 7th term of both the A.P.s will be equal.

Question 5. Find the 13th term from the end in the following A.P.

(i). 4, 9, 14, ... , 254.

Solution: we have,

$$\text{First term (a)} = 4 \text{ and common difference (c.d)} = 9 - 4 = 5$$

$$\text{last term here (l)} = 254$$

$$n^{\text{th}} \text{ term from the end is : } l - (n-1)d$$

$$\text{we have to find 13}^{\text{th}} \text{ term from end then : } l - 12d$$

$$= 254 - 12 \times 5$$

$$= 254 - 60$$

$$= \mathbf{194}$$

(ii). 3, 5, 7, 9, ..., 201.

Solution: we have,

$$\text{First term (a)} = 3 \text{ and common difference (c.d)} = 5 - 3 = 2$$

last term here (l) = 201

n^{th} term from the end is : $l - (n-1)d$

we have to find 13th term from end then : $l - 12d$

$$= 201 - 12 \times 2$$

$$= 201 - 24$$

$$= \mathbf{177}$$

(iii). 1, 4, 7, 10, ... , 88.

Solution: we have,

First term (a) = 1 and common difference (c.d) = $4 - 1 = 3$

last term here (l) = 88

n^{th} term from the end is : $l - (n-1)d$

we have to find 13th term from end then : $l - 12d$

$$= 88 - 12 \times 3 = 88 - 36 = \mathbf{52}$$

Question 6. The 4th term of an A.P. is three times the first term and the 7th term exceeds the twice the

third term by 1. Find the A.P.

Solution: Given, 4th term of the A.P = thrice the first term

$$\Rightarrow a_4 = 3 \text{ first term}$$

Assuming first term to be 'a' and the common difference be 'd'

$$\text{we have, } a + (4 - 1)d = 3a$$

$$\Rightarrow a + 3d = 3a$$

$$\Rightarrow a = 3d \frac{3}{2} \quad \text{---(1)}$$

and also it is given that,

the 7th term exceeds the twice of the 3rd term by 1

$$\Rightarrow a_7 + 1 = 2 \times a_3$$

$$\Rightarrow a + (7-1)d + 1 = 2[a + (3-1)d]$$

$$\Rightarrow a + 6d + 1 = 2a + 4d$$

$$\Rightarrow a = 2d + 1$$

—(2)

putting the value of $a = 32d \frac{3}{2}d$ from equation (1) in equation (2)

$$32d \frac{3}{2}d = 2d + 1$$

$$\Rightarrow 32d \frac{3}{2}d - 2d = 1$$

$$\Rightarrow 3d - 4d^2 = 1 \frac{3d-4d^2}{2} = 1$$

$$\Rightarrow -d = 2$$

$$\Rightarrow d = -2$$

put $d = -2$ in $a = 32d \frac{3}{2}d$

$$\Rightarrow a = 32(-2) \frac{3}{2}(-2)$$

$$\Rightarrow a = -3$$

Now, we have $a = -3$ and $d = -2$, so the A.P. is $-2, -5, -8, -11, \dots$

Question 7. Calculate the third term and the n^{th} term of an A.P. whose 8^{th} term and 13^{th} term are 48

and 78 respectively.

Solution: Given, $a_8 = 48$ and $a_{13} = 78$

n^{th} term of an A.P. is: $a_n = a + (n-1)d$

so,

$$a_8 = a + (8 - 1)d = a + 7d \quad \text{—(1)}$$

$$a_{13} = a + (13 - 1)d = a + 12d \quad \text{—(2)}$$

Equating (1) and (2), we get.

$$\Rightarrow a + 12d - (a + 7d) = 78 - 48$$

$$\Rightarrow a + 12d - a - 7d = 30$$

$$\Rightarrow 5d = 30$$

$$\Rightarrow d = 6$$

Putting the value of $d = 6$ in equation (1),

$$a + 7 \times 6 = 48$$

$$\Rightarrow a + 42 = 48$$

$$\Rightarrow \mathbf{a = 4}$$

Now, we have the first term (a) and the common difference (d) with us,

$$\text{So, } n^{\text{th}} \text{ term will be: } a_n = a + (n-1)d$$

$$= 4 + (n-1)6$$

$$= 4 + 6n - 6$$

$$\mathbf{a_n = 6n - 2}$$

and the 3rd term will be

$$a_3 = 6 \times 3 - 2$$

$$\mathbf{a_n = 16}$$

Question 8. How many three digit numbers are divisible with 3?

Solution: We know the first three digit number which is divisible by 3 is 102 and the last three digit number which is divisible by 3 is 999

So, here we have

$$\text{First term (a) = 102}$$

$$\text{Common Difference (c.d) = 3}$$

$$\text{last term or } n^{\text{th}} \text{ term (l) = 999}$$

$$\Rightarrow a_n = 999$$

$$\Rightarrow a + (n - 1)c.d = 999$$

$$\Rightarrow 102 + (n - 1)3 = 999$$

$$\Rightarrow 102 + 3n - 3 = 999$$

$$\Rightarrow 99 + 3n = 999$$

$$\Rightarrow 3n = 900$$

$$\Rightarrow n = 300$$

Therefore, there are 300 terms in the sequence.

Question 9. An A.P. has 50 terms and the first term is 8 and the last term is 155. Find the 41st term from the A.P.

Solution: Given,

First term (a) = 8

Number of terms (n) = 50

Last term (a_n) = 148

$$\Rightarrow a_n = a + (n - 1)d$$

$$\Rightarrow 155 = 8 + (50 - 1)d$$

$$\Rightarrow 49d = 147$$

$$\Rightarrow d = 3$$

now, 41st term will be: $a + (41-1)d$

$$\Rightarrow 8 + 40 \times 3$$

$$\Rightarrow \mathbf{128}$$

Question 10. The sum of 4th and 8th term of an A.P. is 24 and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P.

Solution:

Let's assume first term be a and common difference be d

Given 4th term + 8th term = 24

$$\Rightarrow a_4 + a_8 = 24$$

$$\Rightarrow (a + (4 - 1)d) + (a + (8 - 1)d) = 24$$

$$\Rightarrow a + 3d + a + 7d = 24$$

$$\Rightarrow 2a + 10d = 24$$

—(1)

And 6th term + 10th term = 34

$$\Rightarrow a_6 + a_{10} = 34$$

$$\Rightarrow (a + 5d) + (a + 9d) = 34$$

$$\Rightarrow 2a + 14d = 34$$

—(2)

Subtracting equation (1) from (2), we get

$$\Rightarrow (2a + 14d) - (2a + 10d) = 34 - 24$$

$$\Rightarrow 2a + 14d - 2a - 10d = 10$$

$$\Rightarrow 4d = 10$$

$$\Rightarrow d = 52 \frac{5}{2}$$

Put $d = 52 \frac{5}{2}$ in equation (1)

$$\Rightarrow 2a + 10 \times 52 \frac{5}{2} = 24$$

$$\Rightarrow 2a + 25 = 24$$

$$\Rightarrow 2a = -1$$

$$\Rightarrow a = -12 - \frac{1}{2}$$

Therefore, we have $a = -12 - \frac{1}{2}$ and $d = 52 \frac{5}{2}$

Question 11. The first term of an A.P. is 7 and its 100th term is -488, Find the 50th term of the same A.P.

Solution: Given,

First term (a) = 7

100th term (a_{100}) = -488

we know, $a_n = a + (n - 1)d$

$$\Rightarrow (a_{100}) = a + (100 - 1)d$$

$$\Rightarrow 7 + 99d = -488$$

$$\Rightarrow 99d = -495$$

$$\Rightarrow \mathbf{d = -5}$$

Now, we have the common difference (d) = 5

We have to find out the 50th term of the A.P.

Then, $a_{50} = a + 49d$

$$= 7 + 49 \times (-5)$$

$$= 7 - 245$$

$$= -238$$

So, the 50th term of the A.P. is -238

Question 12. Find $a_{40} - a_{30}$ of the following A.P.

(i). 3, 5, 7, 9, . . .

Solution: Provided A.P. is 3, 5, 7, 9, . . .

So, we have first term (a) = 3 and the common difference (d) is $5 - 3 = 2$

we have to find $a_{40} - a_{30} = (a + 39d) - (a + 29d)$

$$= a + 39d - a - 29d$$

$$= 10d$$

$$= 10 \times 2$$

$$= 20$$

(ii). 4, 9, 14, 19, . . .

Solution: Given A.P. is 4, 9, 14, 19, . . .

Common difference (d) = $9 - 4 = 5$

we have to find $a_{40} - a_{30} = 10d$

$$= 10 \times 5 = 50$$

Question 13. Write the expression $a_m - a_n$ for the A.P. $a, a + d, a + 2d, . . .$

Solution: General Arithmetic Progression

$a, a + d, a + 2d, . . .$

$$a_m - a_n = (a + (m - 1)d) - (a + (n - 1)d)$$

$$\Rightarrow a + md - d - a - nd + d$$

$$\Rightarrow md - kd$$

$$\Rightarrow (m - n) d \quad \text{---(1)}$$

Hence find the common difference of the A.P. for which

(i). 11th term is 5 and 13th term is 79

Solution: Given,

$$11^{\text{th}} \text{ term } (a_{11}) = 5$$

$$\text{and } 13^{\text{th}} \text{ term } (a_{13}) = 79$$

from equation (1),

$$\text{taking } m = 11 \text{ and } n = 13$$

$$\Rightarrow a_m - a_n = (13 - 11) d$$

$$\Rightarrow 79 - 5 = 2d$$

$$\Rightarrow 74 = 2d$$

$$\Rightarrow \mathbf{d = 37}$$

(ii). $a_{10} - a_5 = 200$

Solution: Given,

here we have the difference between the 10th term and 5th term

Putting the value of m and n in equation (1) as 10 and 5, we have

$$\Rightarrow a_{10} - a_5 = (10 - 5) d$$

$$\Rightarrow 200 = 5 d$$

$$\Rightarrow \mathbf{d = 40}$$

(iii). 20th term is 10 more than the 18th term

Solution: Given,

$$a_{20} + 10 = a_{18}$$

$$\Rightarrow a_{20} - a_{18} = 10$$

from equation (1), we have

$$a_m - a_n = (m - n) d$$

$$\Rightarrow a_{20} - a_{18} = (20 - 18) d$$

$$\Rightarrow 10 = 2d$$

$$\Rightarrow d = 5$$

Question 15. Find n if the given value of x is the n term if the given A.P.

(i) **1, 2111, 3111, 4111, ... : x = 14111** $1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \dots : x = \frac{141}{11}$

(ii) **5 12, 11, 16 12, 22, ... : x = 550** $5\frac{1}{2}, 11, 16\frac{1}{2}, 22, \dots : x = 550$

(iii) **-1, -3, -5, -7, ... : x = -151**

(iv) **25, 50, 70, 100, ... : x = 1000**

Solution:

(i) Given sequence is

$$1, 2111, 3111, 4111, \dots : x = 14111 \quad 1, \frac{21}{11}, \frac{31}{11}, \frac{41}{11}, \dots : x = \frac{141}{11}$$

first term (a) = 1

$$\text{Common difference (d)} = 2111 - 1 = \frac{21}{11} - 1$$

$$= 21 - 1111 = \frac{21 - 11}{11}$$

$$= 1011 = \frac{10}{11}$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \times d$$

$$\Rightarrow 17111 = 1 + (n-1) \cdot 1011 = 1 + (n-1) \cdot \frac{10}{11}$$

$$\Rightarrow 17111 - 1 = (n-1) \cdot 1011 = (n-1) \cdot \frac{10}{11}$$

$$\Rightarrow 171 - 1111 = (n-1) \cdot 1011 = (n-1) \cdot \frac{10}{11}$$

$$\Rightarrow 16011 = (n-1) \cdot 1011 = (n-1) \cdot \frac{10}{11}$$

$$\Rightarrow (n-1) = 16011 \times \frac{11}{10} = \frac{160}{11} \times \frac{11}{10}$$

$$\Rightarrow n = 17$$

(ii) Given sequence is

$$5, 11, 16, 22, \dots : x = 550$$

$$\text{first term } (a) = 5$$

$$\text{Common Difference } (d) = 11 - 5 = 6$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \times d$$

$$\Rightarrow 550 = 5 + (n-1) \times 6$$

$$\Rightarrow 550 = 6n - 1$$

$$\Rightarrow n = \frac{550 + 1}{6}$$

$$\Rightarrow n = 92$$

(iii) Given sequence is,

$$-1, -3, -5, -7, \dots : x = -151$$

$$\text{first term } (a) = -1$$

$$\text{Common Difference } (d) = -3 - (-1)$$

$$= -3 + 1$$

$$= -2$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \times d$$

$$\Rightarrow -151 = -1 + (n-1) \times (-2)$$

$$\Rightarrow -151 = -1 - 2n + 2$$

$$\Rightarrow -151 = 1 - 2n$$

$$\Rightarrow 2n = 152$$

$$\Rightarrow n = 76$$

(iv) Given sequence is,

$$25, 50, 75, 100, \dots : x = 1000$$

$$\text{First term } (a) = 25$$

$$\text{Common Difference } (d) = 50 - 25 = 25$$

$$n^{\text{th}} \text{ term } a_n = a + (n-1) \times d$$

$$\text{we have } a_n = 1000$$

$$\Rightarrow 1000 = 25 + (n - 1) 25$$

$$\Rightarrow 975 = (n - 1)25$$

$$\Rightarrow n - 1 = 39$$

$$\Rightarrow n = 40$$

Question 16. If an A.P. consists of n terms with the first term a and n^{th} term l . Show that the sum of the m^{th} term from the beginning and the m^{th} term from the end is $(a + l)$.

Solution: First term of the sequence is a

$$\text{Last term } (l) = a + (n - 1) d$$

Total no. of terms = n

Common Difference = d

$$m^{\text{th}} \text{ term from the beginning } a_m = a + (n - 1) d$$

$$m^{\text{th}} \text{ term from the end } = l + (n - 1)(-d)$$

$$\Rightarrow a_{(n - m + 1)} = l - (n - 1)d$$

$$\Rightarrow a_m + a_{(n - m + 1)} = a + (n - 1)d + (l - (n - 1)d)$$

$$= a + (n-1)d + l - (n-1)d$$

$$= a + l$$

Question 17. Find the A.P. whose third term is 16 and seventh term exceeds its fifth term by 12.

Solution: Given, $a_3 = 16$

$$\Rightarrow a + (3 - 1)d = 16$$

$$\Rightarrow a + 2d = 16$$

... (i)

$$\text{and } a_7 - 12 = a_5$$

$$\Rightarrow a + 6d - 12 = a + 4d$$

$$\Rightarrow 2d = 12$$

$$\Rightarrow d = 6$$

Put $d = 6$ in equation (1)

$$a + 2 \times 6 = 16$$

$$\Rightarrow a + 12 = 16$$

$\Rightarrow a = 4$. So, the sequence is 4, 10, 16, . . .

Question 18. The 7th term of an A.P is 32 and its 13th term is 62. Find the A.P.

Solution: Given

$$a_7 = 32$$

$$\Rightarrow a + (7 - 1)d = 32$$

$$\Rightarrow a + 6d = 32 \quad \dots (i)$$

$$\text{and } a_{13} = 62$$

$$\Rightarrow a + (13 - 1)d = 62$$

$$\Rightarrow a + 12d = 62 \quad \dots (ii)$$

equation (ii) – (i), we have

$$(a + 12d) - (a + 6d) = 62 - 32$$

$$\Rightarrow 6d = 30$$

$$\Rightarrow d = 5$$

Putting $d = 5$ in equation (i)

$$a + 6 \times 5 = 32$$

$$\Rightarrow a = 32 - 30$$

$$\Rightarrow a = 2$$

So, the obtained A.P. is

2, 7, 12, 17, . . .

Question 19. Which term of the A.P. 3, 10, 17, . . . will be 84 more than its 13th term?

Solution:

Given A.P. is 3, 10, 17, . . .

First term (a) = 3

Common Difference (d) = 10 - 3 = 7

Let n^{th} term of the A.P. will be 84 more than its 13^{th} term, then

$$a_n = 84 + a_{13}$$

$$\Rightarrow a + (n - 1)d = a + (13 - 1)d + 84$$

$$\Rightarrow (n - 1) \times 7 = 12 \times 7 + 84$$

$$\Rightarrow n - 1 = 24$$

$$\Rightarrow n = 25$$

Hence, 25^{th} ter, of the given A.P. is 84 more than the 13^{th} term.

Question 20. Two arithmetic progressions have the same common difference. The difference between their 100^{th} terms is 100. What is the difference between their 1000^{th} terms?

Solution:

Let the two A.P. be a_1, a_2, a_3, \dots and b_1, b_2, b_3, \dots

$$a_n = a_1 + (n - 1)d \text{ and } b_n = a_1 + (n - 1)d$$

Since common difference of two equation is same and given difference between 100^{th} terms is 100

$$\Rightarrow a_{100} - b_{100} = 100$$

$$\Rightarrow a + (100 - 1)d - [b + (100 - 1)d] = 100$$

$$\Rightarrow a + 99d - b - 99d = 100$$

$$\Rightarrow a + b = 100 \quad \dots (1)$$

Difference between 1000^{th} term is

$$\Rightarrow a_{1000} - b_{1000}$$

$$= a + (1000 - 1)d - [b + (1000 - 1)d]$$

$$= a + 999d - b - 999d = a - b = 100 \quad (\text{from equation 1})$$

Therefore, Difference between 1000th terms of two A.P. is 100.

Question 21. For what value of n , the n^{th} terms of the Arithmetic Progression 63, 65, 67, . . . and , 3, 10, 17, . . . are equal?

Solution:

Given two A.P.s are:

63, 65, 67, . . . and 3, 10, 17, . . .

First term for first A.P. is $(a) = 63$

Common difference (d) is $65 - 63 = 2$

n^{th} term $(a_n) = a + (n - 1)d$

$$= 63 + (n - 1) 2$$

First term for second A.P. is $(a') = 3$

Common Difference $(d') = 10 - 3$

$$= 7$$

n^{th} term $(a'_n) = a' + (n - 1)d$

$$= 3 + (n - 1) 7$$

Let n^{th} term of the two sequence be equal then,

$$\Rightarrow 63 + (n - 1)2 = 3 + (n - 1)7$$

$$\Rightarrow 60 = (n - 1).7 - (n - 1).2$$

$$\Rightarrow 60 = 5(n - 1)$$

$$\Rightarrow n - 1 = 12$$

$$\Rightarrow n = 13$$

Hence, the 13th term of both the A.P.s are same.

Question 22. How many multiple of 4 lie between 10 and 250?

Solution: Multiple of 4 after 10 is 12 and multiple of 4 before 250 is $120/4$, remainder is 2, so,

$$250 - 2 = 248$$

248 is the last multiple of 4 before 250

the sequence is

$$12, \dots, 248$$

with first term (a) = 12

Last term (l) = 258

Common Difference (d) = 4

n^{th} term (a_n) = $a + (n - 1)d$

Here n^{th} term $a_n = 248$

$$\Rightarrow 248 = a + (n - 1)d$$

$$\Rightarrow 12 + (n - 1)4 = 248$$

$$\Rightarrow (n - 1)4 = 236$$

$$\Rightarrow n - 1 = 59$$

$$\Rightarrow n = 59 + 1$$

$$\Rightarrow N = 60$$

Therefore, there are 60 terms between 10 and 250 which are multiples of 4

Question 23. How many three digit numbers are divisible by 7 ?

Solution: The three digit numbers are 100, , 999

105 is the first 3 digit number which is divisible by 7

and when we divide 999 with 7 remainder is 5, so, $999 - 5 = 994$

994 is the last three digit number which is divisible by 7 .

The sequence here is

$$105, \dots, 994$$

First term (a) = 105

Last term (l) = 994

Common Difference (d) = 7

Let there are n numbers in the sequence then,

$$\Rightarrow a_n = 994$$

$$\Rightarrow a + (n - 1)d = 994$$

$$\Rightarrow 105 + (n - 1)7 = 994$$

$$\Rightarrow (n - 1) \times 7 = 889$$

$$\Rightarrow n - 1 = 127$$

$$\Rightarrow n = 128$$

Therefore, there are 128 three digit numbers which are divisible by 7.

Question 24. Which term of the A.P. 8, 14, 20, 26, . . . will be 72 more than its 41st term?

Solution: Given sequence

8, 14, 20, 26, . . .

Let its n term be 72 more than its 41st term

$$\Rightarrow a_n = a_{41} + 72 \quad \dots (1)$$

For the given sequence,

first term (a) = 8,

Common Difference (d) = $14 - 8 = 6$

from equation (1), we have

$$a_n = a_{41} + 72$$

$$\Rightarrow a + (n - 1)d = a + (41 - 1)d + 72$$

$$\Rightarrow 8 + (n - 1)6 = 8 + 40 \times 6 + 72$$

$$\Rightarrow (n - 1)6 = 312$$

$$\Rightarrow n - 1 = 52$$

$$\Rightarrow n = 53$$

Therefore, 53rd term is 72 more than its 41st term.

Question 25. Find the term of the Arithmetic Progression 9, 12, 15, 18, . . . which is 39 more than its 36th term.

Solution: Given A.P. is

9, 12, 15, 18, . . .

Here we have,

$$\text{First term (} a \text{)} = 9$$

$$\text{Common Difference (} d \text{)} = 12 - 9 = 3$$

Let its n th term is 39 more than its 36th term

$$\text{So, } a_n = 39 + a_{36}$$

$$\Rightarrow a + (n - 1)d = 39 + a + (36 - 1)d$$

$$\Rightarrow (n - 1)3 = 39 + 35 \times 3$$

$$\Rightarrow (n - 1)3 = (13 + 35) \times 3$$

$$\Rightarrow n - 1 = 48$$

$$\Rightarrow n = 49$$

Therefore, 49th term of the A.P. 39 more than its 36th term.

Question 26. Find the 8th term from the end of the A.P. 7, 10, 13, . . . , 184.

Solution:

Given A.P. is 7, 10, 13, . . . , 184

$$\text{First term (} a \text{)} = 7$$

$$\text{Common Difference (} d \text{)} = 10 - 7 = 3$$

$$\text{last term (} l \text{)} = 184$$

$$n^{\text{th}} \text{ term from end} = l - (n - 1)d$$

$$8^{\text{th}} \text{ term from end} = 184 - (8 - 1)3$$

$$= 184 - 7 \times 3$$

$$= 184 - 21$$

$$= 183$$

Therefore, 8th term from the end is 183

Question 27. Find the 10th term from the end of the A.P. 8, 10, 12, . . . , 126

Solution: Given A.P. is 8, 10, 12, . . . , 126

First term (a) = 8

Common Difference (d) = 10 – 8 = 2

Last term (l) = 126

n^{th} term from end is : $l - (n - 1)d$

So, 10^{th} term from end is : $l - (10 - 1)d$

$$= 126 - 9 \times 2$$

$$= 126 - 18$$

$$= 108$$

Therefore, 109 is the 10^{th} term from the last in the A.P. 8, 10, 12, . . . 126.

Question 28. The sum of 4^{th} and 8^{th} term of an A.P. is 24 and the sum of 6^{th} and 10^{th} term is 44. Find the Arithmetic Progression.

Solution: Given $a_4 + a_8 = 24$

$$\Rightarrow a + (4 - 1)d + a + (8 - 1)d = 24$$

$$\Rightarrow 2a + 3d + 7d = 24$$

$$\Rightarrow 2a + 10d = 24 \quad \dots(1)$$

and $a_6 + a_{10} = 44$

$$\Rightarrow a + (6 - 1)d + a + (10 - 1)d = 44$$

$$\Rightarrow 2a + 5d + 9d = 44$$

$$\Rightarrow 2a + 14d = 44 \quad \dots(2)$$

equation (2) – equation (1), we get

$$2a + 14d - (2a + 10d) = 44 - 24$$

$$\Rightarrow 4d = 20$$

$$\Rightarrow d = 5$$

Put $d = 5$ in equation (1), we get

$$2a + 10 \times 5 = 24$$

$$\Rightarrow 2a = 24 - 50$$

$$\Rightarrow 2a = -26$$

$$\Rightarrow a = -13$$

The A.P is $-13, -7, -2, \dots$

Question 29: Which term of the A.P. is $3, 15, 27, 39, \dots$ will be 120 more than its 21st term?

Solution: Given A.P. is $3, 15, 27, 39, \dots$

First term (a) = 3

Common Difference (d) = $15 - 3 = 12$

Let n^{th} term is 120 more than 21st term

$$\Rightarrow a_n = 120 + a_{21}$$

$$\Rightarrow a + (n - 1)d = 120 + a + (21 - 1)d$$

$$\Rightarrow (n - 1)d = 120 + 20d$$

$$\Rightarrow (n - 1)12 = 120 + 20 \times 12$$

$$\Rightarrow n - 1 = 10 + 20$$

$$\Rightarrow n = 31$$

Therefore, 31st term of the A.P. is 120 more than the 21st term.

Question 30. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43. Find the n^{th} term.

Solution: Given

17th term of an A.P is 5 more than twice its 8th term

$$\Rightarrow a_{17} = 5 + 2a_8$$

$$\Rightarrow a + (17 - 1)d = 5 + 2 [a + (8 - 1)d]$$

$$\Rightarrow a + 16d = 5 + 2a + 14d$$

$$\Rightarrow a + 5 = 2d$$

... (1)

and 11th term of the A.P. is 43

$$a_{11} = 43$$

$$\Rightarrow a + (11 - 1)d = 43$$

$$\Rightarrow a + 10d = 43$$

$$\Rightarrow a + 5 \times 2d = 43$$

from equation (1)

$$\Rightarrow a + 5 \times (a + 5) = 43$$

$$\Rightarrow a + 5a + 25 = 43$$

$$\Rightarrow 6a = 18$$

$$\Rightarrow a = 3$$

Putting the value of $a = 3$, in equation (1), we get

$$3 + 5 = 2d$$

$$\Rightarrow 2d = 8$$

$$\Rightarrow d = 4$$

We have to find the n^{th} term $(a_n) = a + (n - 1)d$

$$= 3 + (n - 1)4$$

$$= 3 + 4n - 4$$

$$= 4n - 1$$

Therefore, n^{th} term is $4n - 1$

Question 31. Find the number of all three digit natural number which are divisible by 9.

Solution: First three-digit number that is divisible by 9 is 108.

Next number is $108 + 9 = 117$.

And the last three-digit number that is divisible by 9 is 999.

Thus, the progression will be 108, 117,, 999.

All are three digit numbers which are divisible by 9, and thus forms an A.P.

having first term (a) : 108

last term (l) = 999

and the common difference (d) as 9

We know that, n^{th} term (a_n) = $a + (n - 1)d$

According to the question,

$$999 = 108 + (n - 1)9$$

$$\Rightarrow 999 = 108 + 9n - 9$$

$$\Rightarrow 999 = 99 + 9n$$

$$\Rightarrow 999 = 9n$$

$$\Rightarrow 999 - 99$$

$$\Rightarrow 9n = 900$$

$$\Rightarrow n = 100$$

Therefore, There are 100 three digit terms which are divisible by 9.

Question 32. The 19^{th} term of an A.P. is equal to three times its 6^{th} term. if its 9^{th} term is 19, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, n^{th} term = $a_n = a + (n - 1)d$

According to the question,

$$a_{19} = 3a_6$$

$$\Rightarrow a + (19 - 1)d = 3(a + (6 - 1)d)$$

$$\Rightarrow a + 18d = 3a + 15d$$

$$\Rightarrow 18d - 15d = 3a - a$$

$$\Rightarrow 3d = 2a$$

$$\Rightarrow a = 32d \quad \dots (1)$$

Also, $a_9 = 19$

$$\Rightarrow a + (9 - 1)d = 19$$

$$\Rightarrow a + 8d = 19 \quad \dots (2)$$

On substituting the values of (1) in (2), we get

$$\Rightarrow 32d + 8d = 19$$

$$\Rightarrow 3d + 16d = 19 \times 2$$

$$\Rightarrow 19d = 38$$

$$\Rightarrow d = 2$$

$$\text{Now, } a = 32 \times 2 \quad [\text{From (1)}]$$

$$a = 3$$

Therefore, The A.P. is : 3, 5, 7, 9, . . .

Question 33. The 9th term of an A.P. is equal to 6 times its second term. If its 5th term is 22, find the A.P.

Solution: Let a be the first term

and d be the common difference.

We know that, n th term (a_n) = $a + (n - 1)d$

According to the question,

$$a_9 = 6a_2$$

$$\Rightarrow a + (9 - 1)d = 6(a + (2 - 1)d)$$

$$\Rightarrow a + 8d = 6a + 6d$$

$$\Rightarrow 8d - 6d = 6a - a$$

$$\Rightarrow 2d = 5a$$

$$\Rightarrow a = 25 \frac{2}{5} \quad \dots (1)$$

$$\text{Also, } a_5 = 22$$

$$\Rightarrow a + (5 - 1)d = 22$$

$$\Rightarrow a + 4d = 22 \quad \dots (2)$$

On substituting the values of (1) in (2), we get

$$25 \frac{2}{5} d + 4d = 22$$

$$\Rightarrow 2d + 20d = 22 \times 5$$

$$\Rightarrow 22d = 110$$

$$\Rightarrow d = 5$$

$$\text{Now, } a = 25 \frac{2}{5} \times 5 \quad [\text{From (1)}]$$

$$\Rightarrow a = 2$$

Thus, the A.P. is : 2, 7, 12, 17, . . .

Question 34. The 24th term of an A.P. is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

Solution: Let a be the first term
and d be the common difference.

We know that,

$$n^{\text{th}} \text{ term } (a_n) = a + (n - 1)d$$

According to the question,

$$a_{24} = 2 a_{10}$$

$$\Rightarrow a + (24 - 1)d = 2(a + (10 - 1)d)$$

$$\Rightarrow a + 23d = 2a + 18d$$

$$\Rightarrow 23d - 18d = 2a - a$$

$$\Rightarrow 5d = a$$

$$\Rightarrow a = 5d \quad \dots (1)$$

Also,

$$a_{72} = a + (72 - 1) d$$

$$= 5d + 71d \quad \text{[From (1)]}$$

$$= 76d \quad \dots (2)$$

and

$$a_{15} = a + (15 - 1) d$$

$$= 5d + 14d \quad \text{[From (1)]}$$

$$= 19d \quad \dots (3)$$

On comparing (2) and (3), we get

$$\Rightarrow 76 d = 4 \times 19 d$$

$$\Rightarrow a_{72} = 4 \times a_{15}$$

Thus, 72nd term of the given A.P. is 4 times its 15th term.

Question 35. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution: Since, the number is divisible by both 2 and 5, means it must be divisible by 10.

In the given numbers, first number that is divisible by 10 is 110.

Next number is $110 + 10 = 120$.

The last number that is divisible by 10 is 990.

Thus, the progression will be 110, 120, ..., 990.

All the terms are divisible by 10,

and thus forms an A.P. having first term as 110

and the common difference as 10.

We know that,

$$n\text{th term} = a_n = a + (n - 1)d$$

According to the question,

$$990 = 110 + (n - 1)10$$

$$\Rightarrow 990 = 110 + 10n - 10$$

$$\Rightarrow 10n = 990 - 100$$

$$\Rightarrow n = 89$$

Thus, the number of natural numbers between 101 and 999 which are divisible by both 2 and 5 is 89.

Question 36. If the seventh term of an A.P. is $\frac{1}{9}$ and its 9th term is $\frac{1}{7}$, find the 63rd term.

Solution: Let a be the first term and d be the common difference.

We know that, $n\text{th term} = a_n = a + (n - 1)d$

According to the question,

$$a_7 = \frac{1}{9}$$

$$\Rightarrow a + (7 - 1)d = \frac{1}{9}$$

$$\Rightarrow a + 6d = 19 \frac{1}{9} \quad \dots(1)$$

$$\text{Also, } a_9 = 17 \frac{1}{7}$$

$$\Rightarrow a + (9 - 1)d = 17 \frac{1}{7}$$

$$\Rightarrow a + 8d = 17 \frac{1}{7} \quad \dots(2)$$

On Subtracting (1) from (2), we get

$$\Rightarrow 8d - 6d = 17 \frac{1}{7} - 19 \frac{1}{9}$$

$$\Rightarrow 2d = 9 - 763 \frac{9-7}{63}$$

$$\Rightarrow 2d = 263 \frac{2}{63}$$

$$\Rightarrow d = 163 \frac{1}{63}$$

Put value of $d = 163 \frac{1}{63}$ in equation (1), we get

$$\Rightarrow a + 6 \times 163 \frac{1}{63} = 19 \frac{1}{9}$$

$$\Rightarrow a = 19 \frac{1}{9} - 663 \frac{6}{63}$$

$$\Rightarrow a = 7 - 663 \frac{7-6}{63}$$

$$\Rightarrow a = 163 \frac{1}{63}$$

Therefore, $a_{63} = a + (63 - 1)d$

$$= 163 \frac{1}{63} + 6263 \frac{62}{63}$$

$$= 6363 \frac{63}{63}$$

$$= 1$$

Thus, 63rd term of the given A.P. is 1.

Question 37. The sum of 5th and 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, Find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term (a_n) = $a + (n - 1)d$

According to the question,

$$a_5 + a_9 = 30$$

$$\Rightarrow a + (5 - 1)d + a + (9 - 1)d = 30$$

$$\Rightarrow a + 4d + a + 8d = 30$$

$$\Rightarrow 2a + 12d = 30$$

$$\Rightarrow a + 6d = 15$$

.... (1)

Also, $a_{25} = 3(a_8)$

$$\Rightarrow a + (25 - 1)d = 3[a + (8 - 1)d]$$

$$\Rightarrow a + 24d = 3a + 21d$$

$$\Rightarrow 3a - a = 24d - 21d$$

$$\Rightarrow 2a = 3d$$

$$\Rightarrow a = 32d \frac{3}{2}d$$

....(2)

Substituting the value of (2) in (1), we get $32d+6d= 15$

$$\Rightarrow 32d \frac{3}{2}d + 6d = 15$$

$$\Rightarrow 3d + 12d = 15 \times 2$$

$$\Rightarrow 15d = 30$$

$$\Rightarrow d = 2$$

now, $a = 32d \frac{3}{2}d \times 2$

[From (1)]

$$\Rightarrow a = 3$$

Therefore, the A.P. is 3, 5, 7, 9, . . .

Question 38. Find whether 0 (zero) is a term of the A.P. 40, 37, 34, 31, . . .

Solution: Let a be the first term and d be the common difference.

We know that, nth term = $a_n = a + (n - 1)d$

It is given that $a = 40$, $d = -3$

and $a_n = 0$

According to the question,

$$\Rightarrow 0 = 40 + (n - 1)(-3)$$

$$\Rightarrow 0 = 40 - 3n + 3$$

$$\Rightarrow 3n = 43$$

$$\Rightarrow n = 43 \frac{43}{3} \quad \dots (1)$$

Here, n is the number of terms, so must be an integer.

Thus, there is no term where 0 (zero) is a term of the A.P. 40, 37, 34, 31, . . .

Question 39. Find the middle term of the A.P. 213, 205, 197, . . . 37.

Solution: Let a be the first term and d be the common difference.

We know that, n th term (a_n) = $a + (n - 1)d$

It is given that $a = 213$,

$$d = -8$$

$$\text{and } a_n = 37$$

According to the question,

$$\Rightarrow 37 = 213 + (n - 1)(-8)$$

$$\Rightarrow 37 = 213 - 8n + 8$$

$$\Rightarrow 8n = 221 - 37$$

$$\Rightarrow 8n = 184$$

$$\Rightarrow n = 23$$

....(1)

Therefore, total number of terms is 23.

Since, there is odd number of terms.

So, Middle term will be 23 + 12th term, i.e., the 12th term.

$$a_{12} = 213 + (12 - 1)(-8)$$

$$a_{12} = 213 - 88$$

$$= 125$$

Thus, the middle term of the A.P. 213, 205, 197, . . . , 37 is 125.

Question 40. If the 5th term of the A.P. is 31 and 25th term is 140 more than the 5th term, find the A.P.

Solution: Let a be the first term and d be the common difference.

We know that, nth term $(a_n) = a + (n - 1)d$

According to question,

$$a_6 = 31$$

$$\Rightarrow a + (5 - 1) = 31$$

$$\Rightarrow a + 4d = 31$$

$$\Rightarrow a = 31 - 4d$$

$$a_5$$

$$\dots (1) \text{ Also, } a_{25} = 140 +$$

$$\Rightarrow a + (25 - 1) = 140 + 31$$

$$\Rightarrow a + 24d = 171$$

$$\dots (3)$$

On substituting the values of (1) in (2), we get

$$31 - 4d + 24d = 171$$

$$\Rightarrow 20d = 171 - 31$$

$$\Rightarrow 20d = 140$$

$$\Rightarrow d = 7$$

$$\Rightarrow a = 31 - 4 \times 7 \quad \text{[From (1)]}$$

$$\Rightarrow a = 3$$

Thus, the A.P. obtained is 3, 10, 17, 24, . .