RD SHARMA
Solutions
Class 10 Maths
Chapter 14
Ex *14.2*

1. Find the distance between the following pair of points:

(i) $(-6, 7)$ and $(-1, -5)$

(ii) $(a+b, b+c)$ and $(a-b, c-b)$

(iii) $(a\sin\alpha, -b\cos\alpha)$ and $(-a\cos\alpha, b\sin\alpha)$

(iv) $(a, 0)$ and $(0, b)$

Sol:

(i) We have $P(-6,7)$ and $Q(-1,-5)$

Here,

$x_1 = -6, y_1 = 7$ and

$x_2 = -1, y_2 = -5$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[-1-(-6)]^2 + (-5-7)^2}$$

$$PQ = \sqrt{(-1+6)^2 + (-5-7)^2}$$

$$PQ = \sqrt{(5)^2 + (-12)^2}$$

$$PQ = \sqrt{25+144}$$

$$PQ = \sqrt{169}$$

$$PQ = 13$$

(ii) we have $P(a+b, b+c)$ and $Q(a-b, c-b)$ here,

$x_1 = a+b, y_1 = b+c$ and $x_2 = a-b, y_2 = c-b$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[a-b-(a+b)]^2 + (c-b-(b+c))^2}$$

$$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$$

$$PQ=\sqrt{(-2b)^2+(-2b)^2}$$

$$PQ=\sqrt{4b^2+4b^2}$$

$$PQ=\sqrt{8b^2}$$

$$PQ=\sqrt{4\times 2b^2}$$

$$PQ=2\sqrt{2}b$$

(iii) we have $P(a\sin\alpha,-b\cos\alpha)$ and $Q(-a\cos\alpha,b\sin\alpha)$ here

$x_1=a\sin\alpha, y_1=-b\cos\alpha$ and

$x_2=-a\cos\alpha, y_2=b\sin\alpha$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(-a\cos\alpha-a\sin\alpha)^2+\left[-b\sin\alpha-(-b\cos\alpha)\right]^2}$$

$$PQ=\sqrt{(-a\cos\alpha)^2+(-a\sin\alpha)^2+2(-a\cos\alpha)(-a\sin\alpha)+(b\sin\alpha)^2+(-b\cos\alpha)^2-2(b\sin\alpha)(-b\cos\alpha)}$$

$$PQ=\sqrt{a^2\cos^2\alpha+a^2\sin^2\alpha+2a^2\cos\alpha\sin\alpha+b^2\sin^2\alpha+b^2\cos^2\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\left(\cos^2\alpha+\sin^2\alpha\right)+2a^2\cos\alpha\sin\alpha+b^2\left(\sin^2\alpha+\cos^2\alpha\right)+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\times 1+2a^2\cos\alpha\sin\alpha+b^2\times 1+2b^2\sin\alpha\cos\alpha} \qquad \left[\because \sin^2\alpha+\cos^2\alpha=1\right]$$

$$PQ=\sqrt{a^2+b^2+2a^2\cos\alpha\sin\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{\left(a^2+b^2\right)+2\cos\alpha\sin\alpha\left(a^2+b^2\right)}$$

$$PQ=\sqrt{\left(a^2+b^2\right)(1+2\cos\alpha\sin\alpha)}$$

(iv) We have $P(a,0)$ and $Q(0,b)$

Here,

$x_1=a, y_1=0, x_2=0, y_2=b,$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(0-a)^2+(b-0)^2}$$

$$PQ=\sqrt{(-a)^2+(b)^2}$$

$$PQ=\sqrt{a^2+b^2}$$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Sol:

We have $P(3,a)$ and $Q(4,1)$

Here,

$x_1 = 3, y_1 = a$

$x_2 = 4, y_2 = 1$

$PQ = \sqrt{10}$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{(1)^2 + (1-a)^2}$

$\Rightarrow \sqrt{10} = \sqrt{1+1+a^2-2a}$ $\left[\because (a-b)^2 = a^2 + b^2 - 2ab\right]$

$\Rightarrow \sqrt{10} = \sqrt{2+a^2-2a}$

Squaring both sides

$\Rightarrow \left(\sqrt{10}\right)^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow 10 = 2 + a^2 - 2a$

$\Rightarrow a^2 - 2a + 2 - 10 = 0$

$\Rightarrow a^2 - 2a - 8 = 0$

Splitting the middle team.

$\Rightarrow a^2 - 4a + 2a - 8 = 0$

$\Rightarrow a(a-4) + 2(a-4) = 0$

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4, a = -2$

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Sol:

We have $P(2,1)$ and $Q(1,-2)$ and $R(X,Y)$

Also, $PR = QR$

$PR = \sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow PR = \sqrt{x^2 + (2)^2 - 2xx \times 2 + y^2 + (1)^2 - 2 \times y \times 1}$

$\Rightarrow PR = \sqrt{x^2 + 4 - 4x + y^2 + 1 - 2y}$

$$\Rightarrow PR=\sqrt{x^2+5-4x+y^2-2y}$$

$$QR=\sqrt{(x-1)^2+(y+2)^2}$$

$$\Rightarrow PR=\sqrt{x^2+1-2x+y^2+4+4y}$$

$$\Rightarrow PR=\sqrt{x^2+5-2x+y^2+4y}$$

$$\because PR=QR$$

$$\Rightarrow \sqrt{x^2+5-4x+y^2-2y}=\sqrt{x^2+5-2x+y^2+4y}$$

$$\Rightarrow x^2+5-4x+y^2-2y=x^2+5-2x+y^2+4y$$

$$\Rightarrow x^2+5-4x+y^2-2y=x^2+5-2x+y^2+4y$$

$$\Rightarrow -4x+2x-2y-4y=0$$

$$\Rightarrow -2x-6y=0$$

$$\Rightarrow -2(x+3y)=0$$

$$\Rightarrow x+3y=\frac{0}{-2}$$

$$\Rightarrow x+3y=0$$

Hence proved.

4. Find the values of x, y if the distances of the point (x, y) from (−3, 0) as well as from (3,0) are 4.

Sol:

We have $P(x,y), Q(-3,0)$ and $R(3,0)$

$$PQ=\sqrt{(x+3)^2+(y-0)^2}$$

$$\Rightarrow 4=\sqrt{x^2+9+6x+y^2}$$

Squaring both sides

$$\Rightarrow (4)^2=\left(\sqrt{x^2+9+6x+y^2}\right)^2$$

$$\Rightarrow 16=x^2+9+6x+y^2$$

$$\Rightarrow x^2+y^2=16-9-6x$$

$$\Rightarrow x^2+y^2=7-6x \qquad \text{.............(1)}$$

$$PR=\left(\sqrt{(x-3)^2+(y-0)^2}\right)$$

$$\Rightarrow 4=\sqrt{x^2+9-6x+y^2}$$

Squaring both sides

$$(4)^2 = \left(\sqrt{x^2 + 9 - 6x + y^2}\right)^2$$

$$\Rightarrow 16 = x^2 + 9 - 6x + y^2$$

$$\Rightarrow x^2 + y^2 = 16 - 9 + 6x$$

$$\Rightarrow x^2 + y^2 = 7 + 6x \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

Equating (1) and (2)

$$7 - 6x = 7 + 6x$$

$$\Rightarrow 7 - 7 = 6x + 6x$$

$$\Rightarrow 0 = 12x$$

$$\Rightarrow x = 12$$

Equating (1) and (2)

$$7 - 6x = 7 + 6x$$

$$\Rightarrow 7 - 7 = 6x + 6x$$

$$\Rightarrow 0 = 12x$$

$$\Rightarrow x = 12$$

Substituting the value of $x = 0$ in (2)

$$x^2 + y^2 = 7 + 6x$$

$$0 + y^2 = 7 + 6 \times 0$$

$$y^2 = 7$$

$$y = \pm\sqrt{7}$$