RD SHARMA

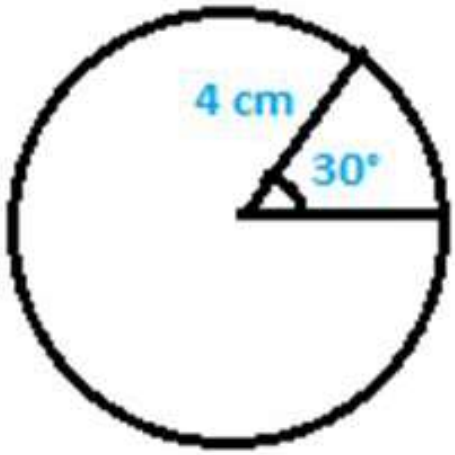
Solutions

Class 10 Maths

Chapter 15

Ex15.2

Q1. Find in terms of π, the length of the arc that subtends an angle of 30 degrees, at the center of 'O' of the circle with a radius of 4 cm.



Soln:

Given Data :

Radius = 4 cm

Angle subtended at the centre 'O' = 30°

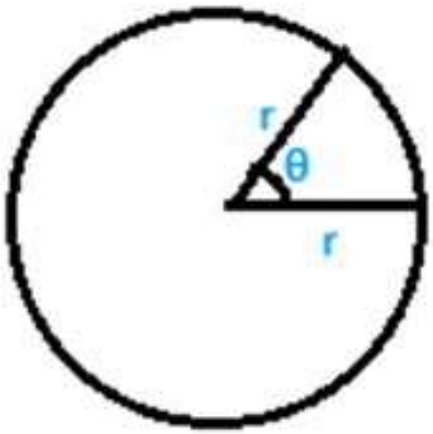
Formula to be used :

Lengthofarc=θ360∗2πrcmLengthofarc = $\frac{\theta}{360} * 2\pi rcm$ Lengthofarc=30360∗2π∗4cm
Lengthofarc = $\frac{30}{360} * 2\pi * 4cm$

= 2π3 cm $\frac{2\pi}{3}$cm

Therefore, the Length of arc the length of the arc that subtends an angle of 60 degrees is 2π3 cm
$\frac{2\pi}{3}$cm

Q2. Find the angle subtended at the centre of circle of radius 5 cm by an arc of length 5π3 cm
$\frac{5\pi}{3}$cm **.**



Soln:

Given data:

Radius = 5 cm

Length of arc = $5\pi 3\,\text{cm}\frac{5\pi}{3}\text{cm}$

Formula to be used:

Length of arc = $\theta 360 * 2\pi r\text{cm}\frac{\theta}{360} * 2\pi r\text{cm}$

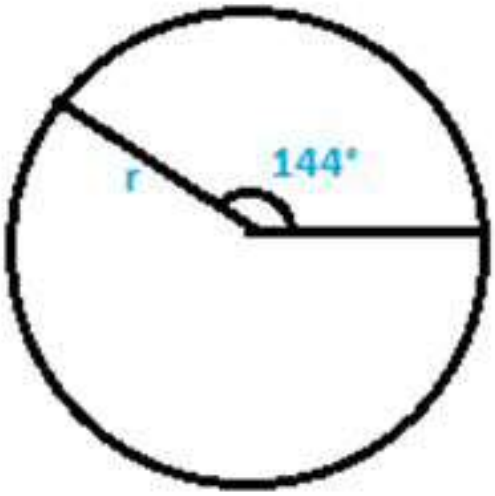
$5\pi 3\,\text{cm}\frac{5\pi}{3}\text{cm} = \theta 360 * 2\pi r\text{cm}\frac{\theta}{360} * 2\pi r\text{cm}$

Solving the above equation, we have:

θ = 60°

Therefore, angle subtended at the centre of circle is60°

Q3. An arc of length cm subtends an angle of 144° at the center of the circle.



Soln:

Given Data : length of arc = cm

θ = angle subtended at the centre of circle = 144°

Formula to be used :

Length of arc = $\theta 360 * 2\pi rcm \frac{\theta}{360} * 2\pi rcm$

$\theta 360 * 2\pi rcm \frac{\theta}{360} * 2\pi rcm$ = 144360 $ast2\pi rcm \frac{144}{360}$ $ast2\pi rcm$

= $4\pi 5 * rcm \frac{4\pi}{5} * rcm$

As given in the question, length of arc = cm ,

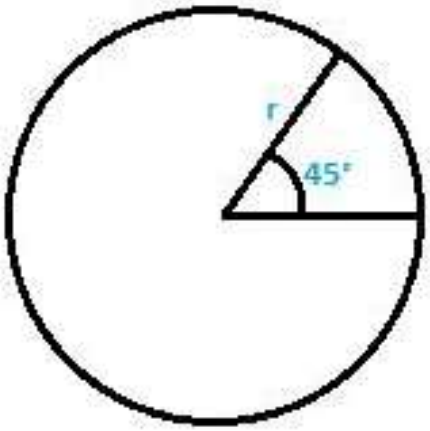
Therefore, cm = $4\pi 5 * rcm \frac{4\pi}{5} * rcm$

Solving the above equation, we have

r = 25 cm.

Therefore the radius of the circle is found to be 25 cm.

Q4. An arc of length 25 cm subtends an angle of 55° at the center of a circle. Find in terms of radius of the circle.



Soln:

Given Data :

length of arc =25 cm

θ = angle subtended at the centre of circle = 55°

Formula to be used :

Length of arc = θ360∗2πrcm$\frac{\theta}{360} * 2\pi rcm$

=5536∗2πrcm$\frac{55}{36} * 2\pi rcm$

As given in the question length of arc =25 cm ,hence,

25 cm =55360∗2π∗rcm$\frac{55}{360} * 2\pi * rcm$

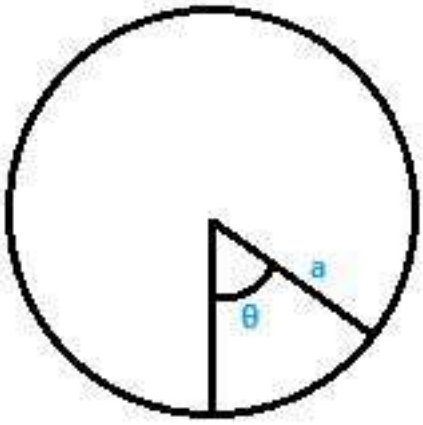
25 = 11πr36 $\frac{11\pi r}{36}$

radius=25∗3611∗π radius = $\frac{25*36}{11*\pi}$

= 90011π $\frac{900}{11\pi}$

Therefore, the radius of the circle is 90011π $\frac{900}{11\pi}$

Q5. Find the angle subtended at the center of the circle of radius 'a' cm by an arc of fraca pi4 fraca pi4 **length cm .**



Soln:

Given data :

Radius = a cm

Length of arc = aπ4 $\frac{a\pi}{4}$ cm

θ = angle subtended at the centre of circle

Formula to be used:

Length of arc = θ360 ∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

Length of arc = θ360 ∗2πacm $\frac{\theta}{360} * 2\pi acm$

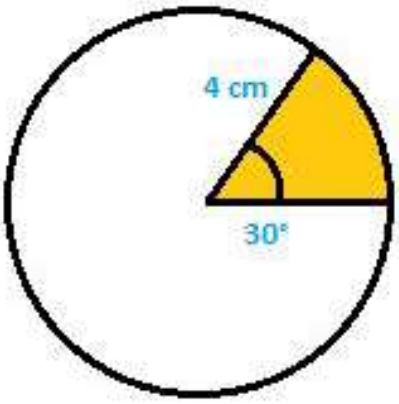
θ360 ∗2πacm $\frac{\theta}{360} * 2\pi acm$ = aπ4 $\frac{a\pi}{4}$ cm

Solving the above equation, we have

θ = 45°

Therefore, the angle subtended at the centre of circle is 45°

Q6. A sector of the circle of radius 4 cm subtends an angle of 30°. Find the area of the sector.



Soln:

Given Data:

Radius = 4 cm

Angle subtended at the centre 'O' = 30°

Formula to be used :

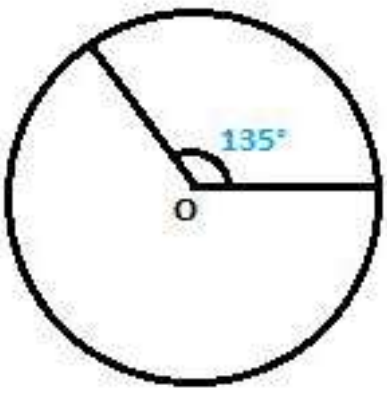
$\text{Areaofthesector}=\frac{\Theta}{360}*\pi r^2 \quad \text{Areaofthesector}=\frac{30}{360}*\pi 4^2$

Solving the above equation, we have:

Area of the sector = 4.9 cm^2

Therefore, Area of the sector is found to be 4.9 cm^2

Q7. A sector of a circle of radius 8 cm subtends an angle of 135°. Find the area of sector.



Soln:

Given Data:

Radius = 8 cm

Angle subtended at the centre ‘O’ = 135°

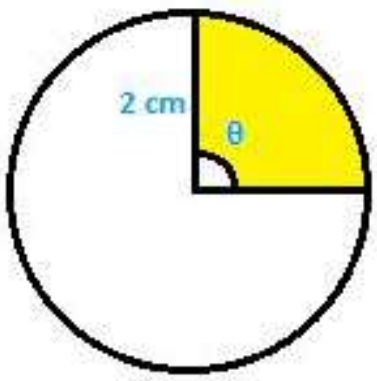
Formula to be used:

$\text{Areaofthesector}=\Theta 360*\pi r^2 \text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ $\text{Areaofthesector}=135360*\pi 8^2$
$\text{Areaofthesector} = \frac{135}{360} * \pi 8^2$

$= 5287\frac{528}{7}\ cm^2$

Therefore, Area of the sector calculated is $5287\frac{528}{7}\ cm^2$

Q8. The area of sector of circle of radius 2 cm is cm^2. Find the angle subtended by the sector.



Soln:

Given Data:

Radius = 2 cm

Angle subtended at the centre ‘O’ =?

Area of sector of circle = cm^2

Formula to be used:

$\text{Area of the sector} = \frac{\Theta}{360} * \pi r^2$ $\text{Area of the sector} = \frac{\Theta}{360} * \pi 3^2$

$= \pi\Theta 90 \frac{\pi\Theta}{90}$

As given in the question area of sector of circle = cm^2

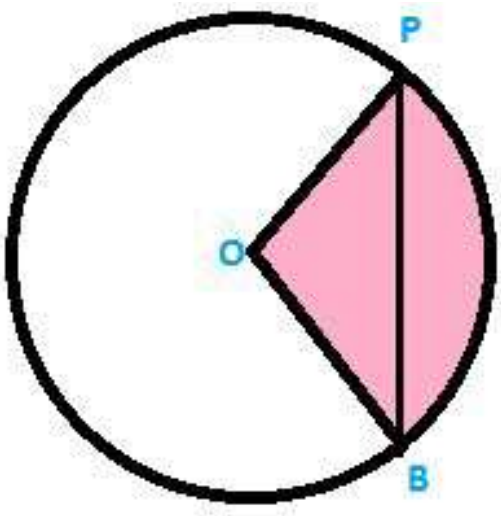
$cm^2 = \pi\Theta 90 \frac{\pi\Theta}{90}$

Solving the above equation, we have

θ = 90°

Therefore, the angle subtended at the centre of circle is 90°

Q10. PQ is a chord of circle with centre 'O' and radius 4 cm. PQ is of the length 4 cm. Find the area of sector of the circle formed by chord PQ.



Soln:

Given Data: PQ is chord of length 4 cm.

Also, PO = QO= 4 cm

OPQ is an equilateral triangle.

Angle POQ = 60°

Area of sector (formed by the chord (shaded region)) = (area of sector)

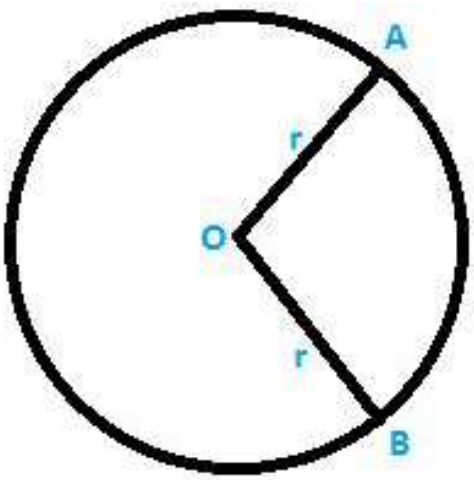
Formula to be used:

$\text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ $\text{Areaofthesector} = \frac{60}{360} * \pi 4^2$

$= 32\pi 3\ \frac{32\pi}{3}$

Therefore, Area of the sector is $32\pi 3\ \frac{32\pi}{3}\ cm^2$

Q11. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of arc and area of sector.



Soln:

Given Data:

Radius = 35 cm

Angle subtended at the centre 'O' = 72°

Area of sector of circle =?

Formula to be used:

$\text{Lengthofarc} = \frac{\theta}{360} * 2\pi rcm$ $\text{Lengthofarc} = \frac{108}{360} * 2\pi x42cm$

Solving the above equation we have,

Length of arc = 44 cm

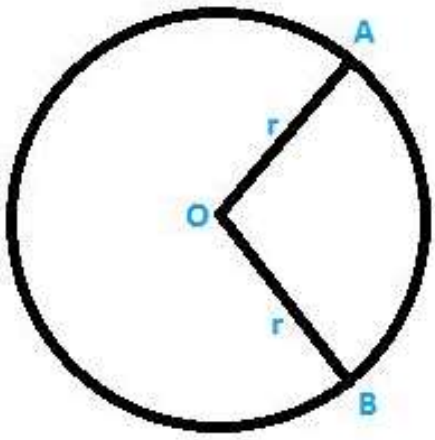
We know that,

$\text{Area of the sector} = \frac{\Theta}{360} * \pi r^2$ $\text{Area of the sector} = \frac{72}{360} * \pi 35^2$

Solving the above equation, we have, Area of the sector = (35 x 22) cm^2

Therefore, Area of the sector is 770 cm^2

Q12. The perimeter of a sector of a circle of radius 5.7 m is 27.2m. find the area of the sector.



Soln:

Given Data:

Radius = 5.7 cm = OA = OB [from the figure shown above]

Perimeter = 27.2 m

Let the angle subtended at the centre be θ

Perimeter = $\frac{\theta}{360} * 2\pi rcm$ $\frac{\theta}{360} * 2\pi rcm$ + OA + OB

= $\frac{\theta}{360} * 2\pi x 5.7cm$ $\frac{\theta}{360} * 2\pi x 5.7cm$ + 5.7 + 5.7

Solving the above equation we have,

θ = 158.8°

We know that,

$\text{Area of the sector} = \frac{\Theta}{360} * \pi r^2$ $\text{Area of the sector} = \frac{\Theta}{360} * \pi r^2$

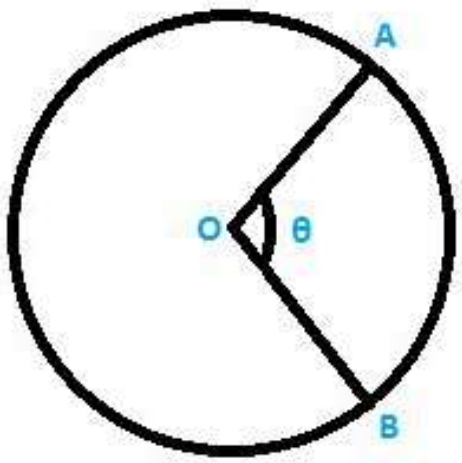
Area of the sector = $158.8360 * \pi 5.7^2 \frac{158.8}{360} * \pi 5.7^2$

Solving the above equation we have,

Area of the sector = 45.048 cm^2

Therefore, Area of the sector is 45.048 cm^2

Q13. The perimeter of a certain sector of a circle of radius is 5.6 m and 27.2 m. find the area of a sector.



Soln:

Given data:

Radius of the circle = 5.6 m = OA = OB

(AB arc length) + OA + OB = 27.2

Let the angle subtended at the centre be θ

We know that,

Length of arc = $\theta 360 * 2\pi rcm \frac{\theta}{360} * 2\pi rcm$

$\theta 360 * 2\pi rcm \frac{\theta}{360} * 2\pi rcm$ + OA + OB = 27.2 m

$\theta 360 * 2\pi rcm \frac{\theta}{360} * 2\pi rcm$ + 5.6 + 5.6 = 27.2 m

Solving the above equation, we have,

θ = 163.64°

We know that,

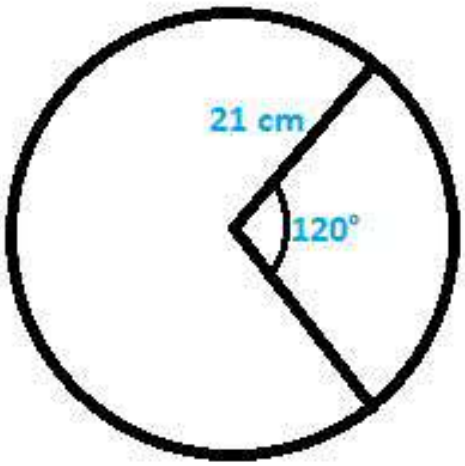
$\text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ $\text{Areaofthesector} = \frac{163.64}{360} * \pi 5.6^2$ $\text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ $\text{Areaofthesector} = \frac{163.64}{360} * \pi 5.6^2$

On solving the above equation, we have,

Area of the sector = 44.8 cm^2

Therefore, Area of the sector is 44.8 cm^2

Q14. A sector was cut from a circle of radius 21 cm. The angle of sector is 120°. Find the length of its arc and its area.



Soln:

Given data:

Radius of circle (r) = 21 cm

θ = angle subtended at the centre of circle = 120°

Formula to be used:

$\text{Lengthofarc} = \frac{\theta}{360} * 2\pi rcm$ $\text{Lengthofarc} = \frac{120}{360} * 2\pi x21cm$ $\text{Lengthofarc} = \frac{\theta}{360} * 2\pi rcm$ $\text{Lengthofarc} = \frac{120}{360} * 2\pi x21cm$

On solving the above equation, we get,

Length of arc = 44 cm

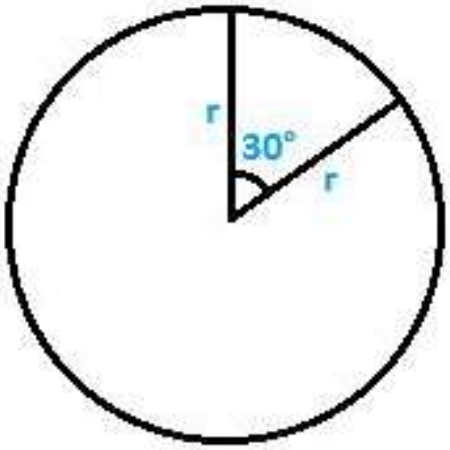
We know that,

$\text{Area of the sector} = \frac{\Theta}{360} * \pi r^2$ $\text{Area of the sector} = \frac{120}{360} * \pi 21^2$

Area of the sector = (22 x 21) cm^2

Therefore, Area of the sector is 462 cm^2

Q15. The minute hand of a circle is $\sqrt{21}\sqrt{21}$ cm long. Find the area described by the minute hand on the face of clock between 7:00 a.m to 7:05 a.m.



Soln:

Given data:

Radius of the minute hand (r) = $\sqrt{21}\sqrt{21}$ cm

Time between 7:00 a.m to 7:05a.m = 5 min

We know that, 1 hr = 60 min, minute hand completes

One revolution = 360°

60 min = 360°

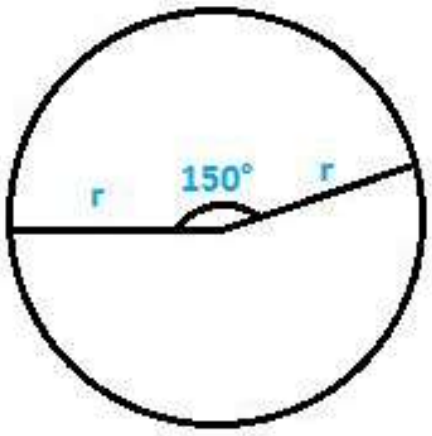
θ = angle subtended at the centre of circle = 5 x 6° = 30°

Areaofthesector=Θ360∗πr2Areaofthesector = $\frac{\Theta}{360} * \pi r^2$ Areaofthesector=30360∗ pi35^2
Areaofthesector = $\frac{30}{360} *$ pi35^2

Area of the sector = 5.5 cm^2

Therefore, Area of the sector is 5.5 cm^2

Q 16. The minute hand of clock is 10 cm long. Find the area of the face of the clock described by the minute hand between 8 a.m to 8:25 a.m.



Soln:

Given data:

Radius of the circle = radius of the clock = length of the minute hand = 10 cm

We know that, 1 hr = 60 min

60 min = 360°

1 min = 6°

Time between 8:00 a.m to 8:25 a.m = 25 min

Therefore, the subtended = 6° x 25 = 150°

Formula to be used :

Areaofthesector=Θ360∗πr2Areaofthesector = $\frac{\Theta}{360} * \pi r^2$ Areaofthesector=150360∗π10^2
Areaofthesector = $\frac{150}{360} * \pi 10^2$

Area of the sector = 916.6 cm^2= 917 cm^2

Therefore, Area of the sector is 917 cm^2

Q17. A sector of 56° cut out from a circle subtends area of 4.4 cm^2. Find the radius of the circle.

Soln:

Given data:

Angle subtended by the sector at the centreof the circle, θ = 56°

Let the radius of the circle be = 'r' cm

Formula to be used:

$Areaofthesector=56360*\pi r^2 Areaofthesector = \frac{56}{360} * \pi r^2$

On solving the above equation, we get,

$r^2 = \sqrt{91}\sqrt{\frac{9}{1}}$ cm

r = 3 cm

Therefore, radius of the circle is r = 3 cm

Q18. In circle of radius 6 cm. Chord of length 10 cm makes an angle of 110° at the centre of circle. Find:

(i) Circumference of the circle

(ii) Area of the circle

(iii) Length of arc

(iv) The area of sector

Soln:

Given data:

Radius of the circle = 6 cm

Chord of length = 10 cm

Angle subtended by chord with the centre of the circle = 110°

Formulae to be used:

Circumference of a circle = 2

Area of a Circle =

Areaofthesector=Θ360∗πr2$\text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ Lengthofarc=90360∗2πx28cm $\text{Lengthofarc} = \frac{90}{360} * 2\pi x28\text{cm}$

Circumference of a circle = 2 = 2 x 3.14 x 8 = 37.7 cm

Area of a Circle = = 3.14 x 6 x 6 = 113.14 cm^2

Areaofthesector=Θ360∗πr2$\text{Areaofthesector} = \frac{\Theta}{360} * \pi r^2$ Areaofthesector=110360∗π62 $\text{Areaofthesector} = \frac{110}{360} * \pi 6^2$

On solving the above equation we get,

Area of the sector = 33.1 cm^2

Lengthofarc=θ360∗2πrcm$\text{Lengthofarc} = \frac{\theta}{360} * 2\pi rcm$ Lengthofarc=110360∗2π6cm $\text{Lengthofarc} = \frac{110}{360} * 2\pi 6\text{cm}$

On solving the above equation we get,

Length of arc = 22.34 cm.

Therefore,Circumference = 37.7 cm

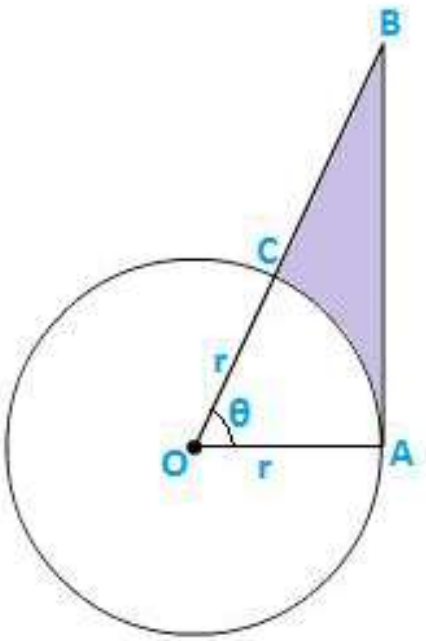
Area of a Circle = 113.14 cm^2

Area of the sector = 33.1 cm^2

Q19. The given figure shows a sector of a circle with centre 'O' subtending an angle θ°. Prove that:

1. Perimeter of shaded region is r(tanθ+secθ+(π theta180)–1) $r(\tan\theta + \sec\theta + (\frac{\pi\ \text{theta}}{180}) - 1)$

2. Area of the shaded region is r²2(tanθ−πθ180) $\frac{r^2}{2}(\tan\theta - \frac{\pi\theta}{180})$



Soln:

Given Data: Angle subtended at the centre of the circle = θ°

Angle OAB = 90° [at point of contact, tangent is perpendicular to radius]

OAB is a right angle triangle

Cos θ = adjsidehypotenuse=rOB=OB=rsecθ $\frac{\text{adjside}}{\text{hypotenuse}} = \frac{r}{OB} = OB = r\sec\theta$

sec θ = oppsideadjside=ABr=AB=rtanθ $\frac{\text{oppside}}{\text{adjside}} = \frac{AB}{r} = AB = r\tan\theta$

Perimeter of the shaded region = AB + BC + CA (arc)

= r tanθ + (OB – OC) + θ360∗2πrcm $\frac{\theta}{360} * 2\pi rcm$

= r(tanθ+secθ+πθ180–1) $r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$

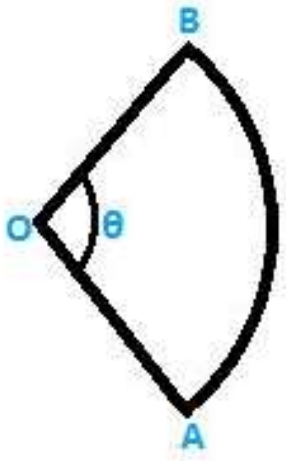
Area of the shaded region = (area of triangle AOB) – (area of sector)

(12∗OA∗AB)–θ360∗πr2 $\left(\frac{1}{2} * OA * AB\right) - \frac{\theta}{360} * \pi r^2$

On solving the above equation we get,

r22[tanθ–πθ180] $\frac{r^2}{2}\left[\tan\theta - \frac{\pi\theta}{180}\right]$

Q 20. The diagram shows a sector of circle of radius 'r' cm subtends an angle θ. The area of sector is A cm^2 and perimeter of sector is 50 cm. Prove that θ=360π(25r−1)
$\theta = \frac{360}{\pi}(\frac{25}{r} - 1)$ **and A = 25 r – r^2**



Soln:

Given Data:

Radius of circle = 'r' cm

Angle subtended at centre of the circle = θ

Perimeter = OA + OB + (AB arc)

r+r+θ360∗2πr=2r+2r[πθ360]$r + r + \frac{\theta}{360} * 2\pi r = 2r + 2r[\frac{\pi\theta}{360}]$

As given in the question, perimeter = 50

θ=360π[25r−1]$\theta = \frac{360}{\pi}[\frac{25}{r} - 1]$

Therefore, θ=360π[25r−1]$\theta = \frac{360}{\pi}[\frac{25}{r} - 1]$

Areaofthesector=Θ360∗πr2$Areaofthesector = \frac{\Theta}{360} * \pi r^2$

On solving the above equation, we have

A = 25r – r^2

Hence, proved.