

**RD SHARMA**

**Solutions**

**Class 9 Maths**

**Chapter 2**

**Ex 2.2**

1. Assuming that  $x, y, z$  are positive real numbers, simplify each of the following

(i)

$$(\sqrt{x^{-3}})^5$$

$$(\sqrt{x^{-3}})^5 = (\sqrt{\frac{1}{x^3}})^5$$

$$\left(\frac{1}{x^{\frac{3}{2}}}\right)^5 = \frac{1}{x^{\frac{15}{2}}}$$

$$(\sqrt{x^{-3}})^5 = \frac{1}{x^{\frac{15}{2}}}$$

(ii)

$$\sqrt{x^3 y^{-2}}$$

$$\sqrt{x^3 y^{-2}} = \sqrt{\frac{x^3}{y^2}}$$

$$= \left(\frac{x^3}{y^2}\right)^{\frac{1}{2}}$$

$$= \frac{x^{3 \times \frac{1}{2}}}{y^{2 \times \frac{1}{2}}}$$

$$\frac{x^{\frac{3}{2}}}{y}$$

$$\sqrt{x^3 y^{-2}} = \frac{x^{\frac{3}{2}}}{y}$$

(iii)

$$(x^{-\frac{2}{3}} y^{-\frac{1}{2}})^2$$

$$= (x^{-\frac{2}{3}} y^{-\frac{1}{2}})^2 = \left(\frac{1}{x^{\frac{2}{3}} y^{\frac{1}{2}}}\right)^2$$

$$= \left(\frac{1}{x^{\frac{2 \times 2}{3}} y^{\frac{1 \times 2}{2}}}\right)$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

(iv)

$$\begin{aligned} & (\sqrt{x})^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy}^{\frac{1}{2}} \\ &= (x^{\frac{1}{2}})^{-\frac{2}{3}} (y^2) \div \sqrt{xy}^{\frac{1}{2}} \\ &= \frac{x^{\frac{1}{2} \times \frac{2}{3}} y^2}{(xy^{\frac{1}{2}})^{\frac{1}{2}}} \\ &= \frac{x^{-\frac{1}{3}} y^2}{x^{\frac{1}{2}} y^{-\frac{1}{2} \times \frac{1}{2}}} \\ &= (x^{-\frac{1}{3}} \times x^{-\frac{1}{2}}) \times (y^2 \times y^{\frac{1}{4}}) \\ &= (x^{-\frac{1}{3} - \frac{1}{2}}) (y^{2 + \frac{1}{4}}) \\ &= (x^{-\frac{-2-3}{6}}) (y^{\frac{8+1}{4}}) \\ &= (x^{-\frac{5}{6}}) (y^{-\frac{9}{4}}) \\ &= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}} \end{aligned}$$

(v)

$$\begin{aligned} & \sqrt[5]{243x^{10}y^5z^{10}} \\ &= (243x^{10}y^5z^{10})^{\frac{1}{5}} \\ &= (243)^{\frac{1}{5}} x^{\frac{10}{5}} y^{\frac{5}{5}} z^{\frac{10}{5}} \\ &= (3^5)^{\frac{1}{5}} x^2 y z^2 \\ &= 3x^2 y z^2 \end{aligned}$$

(vi)

$$\begin{aligned} & \left( \frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}} \\ &= \left( \frac{y^{10}}{x^4} \right)^{\frac{5}{4}} \\ &= \left( \frac{y^{10 \times \frac{5}{4}}}{x^{4 \times \frac{5}{4}}} \right) \\ &= \left( \frac{y^{\frac{25}{2}}}{x^5} \right) \end{aligned}$$

(vii)

$$\begin{aligned} & \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^5 \left(\frac{6}{7}\right)^2 \\ &= \left(\sqrt{\frac{2}{3}}\right)^5 \left(\frac{6}{7}\right)^{\frac{4}{2}} \\ &= \left(\frac{2}{3}\right)^{\frac{5}{2}} \left(\frac{6}{7}\right)^{\frac{4}{2}} \\ &= \left(\frac{2^5}{3^5}\right)^{\frac{1}{2}} \left(\frac{6^4}{7^4}\right)^{\frac{1}{2}} \\ &= \left(\frac{2^5}{3^5} \times \frac{6^4}{7^4}\right)^{\frac{1}{2}} \\ &= \left(\frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} \times \frac{6 \times 6 \times 6 \times 6}{7 \times 7 \times 7 \times 7}\right) \\ &= \left(\frac{512}{7203}\right) \end{aligned}$$

2. Simplify

(i)

$$\begin{aligned} & (16^{-\frac{1}{5}})^{\frac{5}{2}} \\ &= (16)^{-\frac{1}{5} \times \frac{5}{2}} \\ &= (16)^{-\frac{1}{2}} \\ &= (4^2)^{-\frac{1}{2}} \\ &= (4^{2 \times -\frac{1}{2}}) \\ &= (4^{-1}) \\ &= \frac{1}{4} \end{aligned}$$

(ii)  $\sqrt[5]{(32)^{-3}}$

$$\begin{aligned} & \sqrt[5]{(32)^{-3}} \\ &= [(2^5)^{-3}]^{\frac{1}{5}} \\ &= (2^{-15})^{\frac{1}{5}} \\ &= 2^{-3} \\ &= \frac{1}{2^3} \\ &= \frac{1}{8} \end{aligned}$$

(iii)

$$\begin{aligned} & \sqrt[3]{(343)^{-2}} \\ &= [(343)^{-2}]^{\frac{1}{3}} \\ &= (343)^{-2 \times \frac{1}{3}} \\ &= (7^3)^{-\frac{2}{3}} \\ &= (7^{-2}) \\ &= \left(\frac{1}{7^2}\right) \\ &= \left(\frac{1}{49}\right) \end{aligned}$$

(iv)

$$\begin{aligned} & (0.001)^{\frac{1}{3}} \\ &= \left(\frac{1}{1000}\right)^{\frac{1}{3}} \\ &= \left(\frac{1}{10^3}\right)^{\frac{1}{3}} \\ &= \left(\frac{1^{\frac{1}{3}}}{(10^3)^{\frac{1}{3}}}\right) \\ &= \frac{1}{10^{3 \times \frac{1}{3}}} \\ &= \frac{1}{10} = 0.1 \end{aligned}$$

(v)

$$\begin{aligned} & \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}} \\ &= \frac{((5^2))^{\frac{3}{2}} \times ((3^5))^{\frac{3}{5}}}{((4^2))^{\frac{5}{4}} \times ((4^2))^{\frac{4}{3}}} \\ &= \frac{5^{2 \times \frac{3}{2}} \times 3^{5 \times \frac{3}{5}}}{2^{4 \times \frac{5}{4}} \times 2^{3 \times \frac{4}{3}}} \\ &= \frac{5^3 \times 3^3}{2^5 \times 2^4} \\ &= \frac{125 \times 27}{32 \times 16} \\ &= \frac{3375}{512} \end{aligned}$$

$$(vi) \left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$$

$$= \frac{\left(\frac{\sqrt{2}}{5}\right)^8}{\left(\frac{\sqrt{2}}{5}\right)^{13}}$$

$$= \left(\frac{\sqrt{2}}{5}\right)^{8-13}$$

$$= \left(\frac{\sqrt{2}}{5}\right)^{-5}$$

$$= \frac{(2^{\frac{1}{2}})^{-5}}{(5)^{-5}}$$

$$= \frac{(2^{\frac{1}{2} \times -5})}{(5)^{-5}}$$

$$= \frac{1}{2^{\frac{5}{2}}} \times \frac{5^5}{1}$$

$$= \frac{5^5}{2^{\frac{5}{2}}}$$

$$= \frac{3125}{4\sqrt{2}}$$

(vii)

$$\left[\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right]^{\frac{7}{2}} \times \left[\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right]^{\frac{-5}{2}}$$

$$= \frac{(5^{-1} \times 7^2)^{\frac{7}{2}}}{(5^2 \times 7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2} \times 7^3)^{\frac{-5}{2}}}{(5^3 \times 7^{-5})^{\frac{-5}{2}}}$$

$$= \frac{(5^{-1})^{\frac{7}{2}} \times (7^2)^{\frac{7}{2}}}{(5^2)^{\frac{7}{2}} \times (7^{-4})^{\frac{7}{2}}} \times \frac{(5^{-2})^{\frac{-5}{2}} \times (7^3)^{\frac{-5}{2}}}{(5^3)^{\frac{-5}{2}} \times (7^{-5})^{\frac{-5}{2}}}$$

$$= \frac{5^{-\frac{7}{2}} \times 7^7}{5^7 \times 7^{-14}} \times \frac{5^5 \times 7^{-\frac{15}{2}}}{5^{-\frac{15}{2}} \times 7^{-\frac{25}{2}}}$$

$$= \frac{7^{7+\frac{7}{2}}}{5^{7+\frac{7}{2}}} \times \frac{5^{5+\frac{15}{2}}}{7^{\frac{15}{2}+\frac{25}{2}}}$$

$$\begin{aligned}
&= \frac{7^{21}}{5^{\frac{21}{2}}} \times \frac{5^{\frac{25}{2}}}{7^{\frac{40}{2}}} \\
&= \frac{7^{21}}{7^{20}} \times \frac{5^{\frac{25}{2}}}{5^{\frac{21}{2}}} \\
&= 7^{21-20} \times 5^{\frac{25}{2}-\frac{21}{2}} \\
&= 7^1 \times 5^{\frac{4}{2}} \\
&= 7^1 \times 5^2 \\
&= 7 \times 25 \\
&= 175
\end{aligned}$$

3. Prove that

$$\begin{aligned}
\text{(i)} \quad &(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}\sqrt{5}}) \times \sqrt[6]{3 \times 5^6} = \frac{3}{5} \\
&(\sqrt{3 \times 5^{-3}} \div \sqrt[3]{3^{-1}\sqrt{5}}) \times \sqrt[6]{3 \times 5^6} \\
&= ((3 \times 5^{-3})^{\frac{1}{2}} \div (3^{-1})^{\frac{1}{3}}(5)^{\frac{1}{2}}) \times (3 \times 5^6)^{\frac{1}{6}} \\
&= ((3)^{\frac{1}{2}}(5^{-3})^{\frac{1}{2}} \div (3^{-1})^{\frac{1}{3}}(5)^{\frac{1}{2}}) \times (3 \times 5^6)^{\frac{1}{6}} \\
&= ((3)^{\frac{1}{2}}(5)^{\frac{-3}{2}} \div (3)^{\frac{-1}{3}}(5)^{\frac{1}{2}}) \times ((3)^{\frac{1}{6}} \times (5)^{\frac{6}{6}}) \\
&= ((3)^{\frac{1}{2}-(-\frac{1}{3})} \times (5)^{-\frac{3}{2}-\frac{1}{2}}) \times ((3)^{\frac{1}{6}} \times (5)) \\
&= ((3)^{\frac{3+2}{6}} \times (5)^{-\frac{4}{2}}) \times ((3)^{\frac{1}{6}} \times (5)) \\
&= ((3)^{\frac{5}{6}} \times (5)^{-2}) \times ((3)^{\frac{1}{6}} \times (5)) \\
&= ((3)^{\frac{5}{6}+\frac{1}{6}} \times (5)^{-2+1}) \\
&= ((3)^{\frac{6}{6}} \times (5)^{-1}) \\
&= ((3)^1 \times (5)^{-1}) \\
&= ((3) \times (5)^{-1}) \\
&= ((3) \times (\frac{1}{5})) \\
&= (\frac{3}{5})
\end{aligned}$$

(ii)

$$\begin{aligned} & 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} \\ &= (3^2)^{\frac{3}{2}} - 3 - \left(\frac{1}{9^2}\right)^{-\frac{1}{2}} \\ &= 3^{2 \times \frac{3}{2}} - 3 - (9^{-2})^{-\frac{1}{2}} \\ &= 3^3 - 3 - (9)^{-2 \times -\frac{1}{2}} \\ &= 27 - 3 - 9 \\ &= 15 \end{aligned}$$

(iii)

$$\begin{aligned} & \frac{1}{4}^2 - 3 \times 8^{\frac{2}{3}} \times 4^0 + \left(\frac{9}{16}\right)^{-\frac{1}{2}} \\ &= \left(\frac{1}{2^2}\right)^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^2}{4^2}\right)^{-\frac{1}{2}} \\ &= (2^{-2})^{-2} - 3 \times 8^{\frac{2}{3}} \times 1 + \left(\frac{3^{2 \times -\frac{1}{2}}}{4^{2 \times -\frac{1}{2}}}\right) \\ &= 2^4 - 3 \times 2^{3 \times \frac{2}{3}} + \frac{4}{3} \\ &= 16 - 3 \times 2^2 + \frac{4}{3} \\ &= 16 - 3 \times 4 + \frac{4}{3} \\ &= 16 - 12 + \frac{4}{3} \\ &= \frac{12+4}{3} \\ &= \frac{16}{3} \end{aligned}$$

(iv)

$$\begin{aligned} & \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 4^{\frac{1}{4}}}{10^{-\frac{1}{5}} \times 5^{\frac{3}{5}}} \div \frac{4^{\frac{4}{3}} \times 5^{-\frac{7}{5}}}{4^{-\frac{3}{5}} \times 6} \\ &= \frac{2^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times (2^2)^{\frac{1}{4}} (2^2)^{-\frac{3}{5}} \times (2 \times 3)}{(2 \times 5)^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}} \\ &= \frac{2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times (2^2)^{-\frac{6}{5}} \times 2^1 \times 3^{\frac{1}{3}} \times 3}{2^{-\frac{1}{5}} \times 5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 3^{\frac{4}{3}} \times 5^{-\frac{7}{5}}} \\ &= \frac{2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 2^{-\frac{6}{5}} \times 2 \times 3^{\frac{1}{3}} \times 3 \times 3^{-\frac{4}{3}}}{5^{-\frac{1}{5}} \times 5^{\frac{3}{5}} \times 5^{-\frac{7}{5}}} \\ &= \frac{(2)^{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{6}{5} + 1 + 1} \times (3)^{\frac{1}{3} + 1 - \frac{4}{3}}}{5^{-\frac{1}{5} + \frac{3}{5} + 5^{-\frac{7}{5}}}} \end{aligned}$$



$$= \frac{(2)^{\frac{1}{5}+1-\frac{6}{5}+1} \times (3)^{1-\frac{3}{3}}}{5^{-\frac{5}{5}}}$$

$$= \frac{(2)^{\frac{1}{5}+2-\frac{6}{5}} \times (3)^{1-1}}{5^{-1}}$$

$$= \frac{(2)^{2-1} \times (3)^{1-1}}{5^{-1}}$$

$$= \frac{(2)^1 \times (3)^0}{5^{-1}}$$

$$= 2 \times 1 \times 5$$

$$= 10$$

(v)

$$\sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.01)^{\frac{1}{2}}} - (3^3)^{\frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.1)^{2 \times \frac{1}{2}}} - (3)^{3 \times \frac{2}{3}}$$

$$= \frac{1}{2} + \frac{1}{(0.1)^1} - (3)^2$$

$$= \frac{1}{2} + \frac{1}{(0.1)} - 9$$

$$= \frac{1}{2} + 10 - 9$$

$$= \frac{1}{2} + 1$$

$$= \frac{3}{2}$$

(vi)

$$\frac{2^{n+2^{n-1}}}{2^{n+1}-2^n}$$

$$= \frac{2^n + 2^n \times 2^{-1}}{2^n \times 2^1 - 2^n}$$

$$= \frac{2^n [1 + 2^{-1}]}{2^n [2 - 1]}$$

$$= \frac{1 + \frac{1}{2}}{1}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

(vii)

$$\begin{aligned} & \left(\frac{64}{125}\right)^{-\frac{2}{3}} + \frac{1}{\left(\frac{256}{625}\right)^{-\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right) \\ &= \left(\frac{125}{64}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + \left(\frac{5}{(64)^{\frac{1}{3}}}\right) \\ &= \left(\frac{5^3}{4^3}\right)^{\frac{2}{3}} + \frac{1}{\left(\frac{4}{5}\right)} + \left(\frac{5}{(4^3)^{\frac{1}{3}}}\right) \\ &= \left(\frac{5}{4}\right)^2 + \frac{5}{4} + \frac{5}{4} \\ &= \frac{25}{16} + \frac{10}{4} \\ &= \frac{25}{16} + \frac{40}{16} \\ &= \frac{26+40}{16} \\ &= \frac{65}{16} \end{aligned}$$

(viii)

$$\begin{aligned} & \frac{3^{-3} \times 6^2 \times \sqrt{98}}{5^2 \times \sqrt[3]{\frac{1}{25}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} \\ &= \frac{3^{-3} \times 36 \times \sqrt{7 \times 7 \times 2}}{5^2 \times \left(\frac{1}{25}\right)^{\frac{1}{3}} \times (15)^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} \\ &= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times \left(\frac{1}{5^{2 \times \frac{1}{3}}}\right) \times \frac{1}{(15)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}} \\ &= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times \frac{1}{(5 \times 3)^{\frac{4}{3}}} \times 3^{\frac{1}{3}}} \\ &= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{5^2 \times 5^{-\frac{2}{3}} \times 5^{\frac{4}{3}} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}} \end{aligned}$$

$$\begin{aligned}
&= \frac{3^{-3} \times 36 \times 7\sqrt{2}}{(5^2 \times 5^{-\frac{2}{3}} \times 5^{-\frac{4}{3}}) \times 3^{-\frac{4}{3}} \times 3^{\frac{1}{3}}} \\
&= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}}{(5)^{2-\frac{2}{3}-\frac{4}{3}}} \\
&= \frac{3^{-3} \times 36 \times 7\sqrt{2} \times 3^{\frac{4}{3}} \times 3^{\frac{1}{3}}}{(5)^{\frac{6-2-4}{3}}} \\
&= \frac{3^{-3+\frac{4}{3}+\frac{1}{3}} \times 36 \times 7\sqrt{2}}{(5)^0} \\
&= 3^{-3+\frac{5}{3}} \times 36 \times 7\sqrt{2} \\
&= 3^{-3+1} \times 36 \times 7\sqrt{2} \\
&= 3^{-2} \times 36 \times 7\sqrt{2} \\
&= \frac{1}{3^2} \times 36 \times 7\sqrt{2} \\
&= \frac{1}{9} \times 36 \times 7\sqrt{2} \\
&= 4 \times 7\sqrt{2} \\
&= 28\sqrt{2}
\end{aligned}$$

(ix)

$$\begin{aligned}
&\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1} - \left(\frac{3}{2}\right)^3 + (-3)^1} \\
&= \frac{1 - \frac{1}{0.1}}{\frac{8}{3} \times \left(\frac{3}{2}\right)^3 - 3} \\
&= \frac{1 - 10}{\frac{8}{3} \times \frac{3^3}{2^3} - 3} \\
&= \frac{-9}{3^2 - 3} \\
&= \frac{-9}{9 - 3} \\
&= \frac{-9}{6} \\
&= -\frac{3}{2}
\end{aligned}$$

4. Show that

$$(i) \frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1 + \frac{x^a}{x^b}} + \frac{1}{1 + \frac{x^b}{x^a}}$$

$$\frac{x^b}{x^b + x^a} + \frac{x^a}{x^a + x^b}$$

$$\frac{x^b + x^a}{x^a + x^b}$$

$$1$$

Therefore, LHS = RHS

Hence proved

$$(ii) \frac{1}{1 + x^{b-a} + x^{c-a}} + \frac{1}{1 + x^{a-b} + x^{c-b}} + \frac{1}{1 + x^{b-c} + x^{a-c}}$$

Left hand side (LHS) = Right hand side (RHS)

Considering LHS,

$$\frac{1}{1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}} + \frac{1}{1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}} + \frac{1}{1 + \frac{x^b}{x^c} + \frac{x^a}{x^c}}$$

$$\frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^b + x^a + x^c} + \frac{x^c}{x^c + x^b + x^a}$$

$$\frac{x^a + x^b + x^c}{x^a + x^b + x^c}$$

$$1$$

Therefore, LHS = RHS

Hence proved

(ii)

$$\left[ \left( \frac{x^{a(a-b)}}{x^{a(a+b)}} \right) \div \left( \frac{x^{b(b-a)}}{x^{b(b+a)}} \right) \right]^{a+b}$$

$$\left[ \left( \frac{x^{a(a-b)}}{x^{a(a+b)}} \right) \div \left( \frac{x^{b(b-a)}}{x^{b(b+a)}} \right) \right]^{a+b}$$

$$= \left[ \left( \frac{x^{a^2-ab}}{x^{a^2+ab}} \right) \div \left( \frac{x^{b^2-ab}}{x^{b^2+ab}} \right) \right]^{a+b}$$

$$= \left[ x^{(a^2-ab)-(a^2+ab)} \div x^{(b^2-ab)-(b^2+ab)} \right]^{a+b}$$

$$= \left[ x^{-2ab} \div x^{-2ab} \right]^{a+b}$$

$$= \left[ x^{-2ab-(-2ab)} \right]^{a+b}$$

$$= \left[ x^{-2ab+2ab} \right]^{a+b}$$

$$= \left[ x^0 \right]^{a+b}$$

$$= \left[ 1 \right]^{a+b}$$

$$= 1$$

(iii)

$$\begin{aligned} & \left(X^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \left(X^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \left(X^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}} \\ &= \left(X^{\frac{1}{(a-b)(a-c)}}\right) \left(X^{\frac{1}{(b-c)(b-a)}}\right) \left(X^{\frac{1}{(c-a)(c-b)}}\right) \\ &= X^{\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}} \\ &= X^{\frac{1}{(a-b)(a-c)} + \frac{-1}{(b-c)(a-b)} + \frac{1}{(a-c)(b-c)}} \\ &= X^{\frac{(b-c)}{(a-b)(a-c)(b-c)} + \frac{-(a-c)}{(b-c)(a-b)(a-c)} + \frac{(a-b)}{(a-c)(b-c)(a-b)}} \\ &= X^{\frac{b-c-a+c+a-b}{(a-b)(a-c)(b-c)}} \\ &= X^{\frac{0}{(a-b)(a-c)(b-c)}} \\ &= X^0 = 1 \end{aligned}$$

(iv)  $\left(\frac{X^{a^2+b^2}}{X^{ab}}\right)^{a+b} \left(\frac{X^{b^2+c^2}}{X^{bc}}\right)^{b+c} \left(\frac{X^{c^2+a^2}}{X^{ac}}\right)^{a+c} = 2(a^3 + b^3 + c^3)$

$$\begin{aligned} & \left(\frac{X^{a^2+b^2}}{X^{ab}}\right)^{a+b} \left(\frac{X^{b^2+c^2}}{X^{bc}}\right)^{b+c} \left(\frac{X^{c^2+a^2}}{X^{ac}}\right)^{a+c} \\ &= \left(X^{a^2+b^2-ab}\right)^{a+b} \left(X^{b^2+c^2-bc}\right)^{b+c} \left(X^{c^2+a^2-ac}\right)^{a+c} \\ &= \left(X^{a+b(a^2+b^2-ab)}\right) \left(X^{b+c(b^2+c^2-bc)}\right) \left(X^{a+c(c^2+a^2-ac)}\right) \\ &= \left(X^{a^3+ab^2-a^2b+ab^2+b^3-ab^2}\right) \left(X^{b^3+bc^2-b^2c+cb^2+c^3-bc^2}\right) \left(X^{ac^2+a^3-a^2c+c^3+a^2c-ac^2}\right) \\ &= \left(X^{a^3+b^3}\right) \left(X^{b^3+c^3}\right) \left(X^{a^3+c^3}\right) \\ &= \left(X^{a^3+b^3+b^3+c^3+a^3+c^3}\right) \\ &= \left(X^{2a^3+2b^3+2c^3}\right) \\ &= \left(X^{2(a^3+b^3+c^3)}\right) \end{aligned}$$

(v)  $(X^{a-b})^{a+b} (X^{b-c})^{b+c} (X^{c-a})^{c+a} = 1$

$$\begin{aligned} & (X^{a-b})^{a+b} (X^{b-c})^{b+c} (X^{c-a})^{c+a} \\ &= X^{a^2-b^2} X^{b^2-c^2} X^{c^2-a^2} \\ &= X^{a^2-b^2+b^2-c^2+c^2-a^2} \\ &= X^0 \\ &= 1 \end{aligned}$$

$$(vi) [(X^{a-a^{-1}})^{\frac{1}{a-1}}]^{\frac{a}{a+1}} = X$$

$$[(X^{a-a^{-1}})^{\frac{1}{a-1}}]^{\frac{a}{a+1}}$$

$$= [(X^{\frac{a-a^{-1}}{a-1}})]^{\frac{a}{a+1}}$$

$$= [(X^{\frac{a-a^{-1}}{a-1}})]^{\frac{a}{a+1}}$$

$$= (X^{\frac{a(a-a^{-1})}{a^2-1}})$$

$$= (X^{\frac{a^2-a^{-1+1}}{a^2-1}})$$

$$= (X^{\frac{a^2-a^0}{a^2-1}})$$

$$= (X^{\frac{a^2-1}{a^2-1}})$$

$$= X^1 = X$$

$$(vii) [\frac{a^{x+1}}{a^{y+1}}]^{x+y} [\frac{a^{y+2}}{a^{z+2}}]^{y+z} [\frac{a^{z+3}}{a^{x+3}}]^{z+x} = 1$$

$$[\frac{a^{x+1}}{a^{y+1}}]^{x+y} [\frac{a^{y+2}}{a^{z+2}}]^{y+z} [\frac{a^{z+3}}{a^{x+3}}]^{z+x}$$

$$= [a^{(x+1)-(y+1)}]^{x+y} [a^{(y+2)-(z+2)}]^{y+z} [a^{(z+3)-(x+3)}]^{z+x}$$

$$= [a^{x-y}]^{x+y} [a^{y-z}]^{y+z} [a^{z-x}]^{z+x}$$

$$= [a^{x^2-y^2}] [a^{y^2-z^2}] [a^{z^2-x^2}]$$

$$= a^{x^2-y^2+y^2-z^2+z^2-x^2} = a^0$$

$$= 1$$

$$(viii) (\frac{3^a}{3^b})^{a+b} (\frac{3^b}{3^c})^{b+c} (\frac{3^c}{3^a})^{c+a} = 1$$

$$\begin{aligned}
& \left(\frac{3^a}{3^b}\right)^{a+b} \left(\frac{3^b}{3^c}\right)^{b+c} \left(\frac{3^c}{3^a}\right)^{c+a} \\
&= (3^{a-b})^{a+b} (3^{b-c})^{b+c} (3^{c-a})^{c+a} \\
&= 3^{a^2-b^2} \times 3^{b^2-c^2} \times 3^{c^2-a^2} \\
&= 3^{a^2-b^2+b^2-c^2+c^2-a^2} \\
&= 3^0 = 1
\end{aligned}$$

## Level 2

5. If  $2^x = 3^y = 12^z$ , show that  $\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$

$$2^x = 3^y = (2 \times 3 \times 2)^z$$

$$2^x = 3^y = (2^2 \times 3)^z$$

$$2^x = 3^y = (2^{2z} \times 3^z)$$

$$2^x = 3^y = 12^z = k$$

$$2 = k^{\frac{1}{x}}$$

$$3 = k^{\frac{1}{y}}$$

$$12 = k^{\frac{1}{z}}$$

$$12 = 2 \times 3 \times 2$$

$$12 = k^{\frac{1}{z}} = k^{\frac{1}{y}} \times k^{\frac{1}{x}} \times k^{\frac{1}{x}}$$

$$k^{\frac{1}{z}} = k^{\frac{2}{x} + \frac{1}{y}}$$

$$\frac{1}{z} = \frac{1}{y} + \frac{2}{x}$$

6. If  $2^x = 3^y = 6^{-z}$ , show that  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$$2^x = 3^y = 6^{-z}$$

$$2^x = k$$

$$2 = k^{\frac{1}{x}}$$

$$3^y = k$$

$$3 = k^{\frac{1}{y}}$$

$$6^{-z} = k$$

$$k = \frac{1}{6^z}$$

$$6 = k^{-\frac{1}{z}}$$

$$2 \times 3 = 6$$

$$k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{-\frac{1}{z}}$$

$$\frac{1}{x} + \frac{1}{y} = -\frac{1}{z} \quad \text{[by equating exponents]}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

7. If  $a^x = b^y = c^z$  and  $b^2 = ac$ , then show that  $y = \frac{2zx}{z+x}$

$$\text{Let } a^x = b^y = c^z = k$$

$$a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$$

Now,

$$b^2 = ac$$

$$(k^{\frac{1}{y}})^2 = k^{\frac{1}{x}} \times k^{\frac{1}{z}}$$

$$k^{\frac{2}{y}} = k^{\frac{1}{x} + \frac{1}{z}}$$

$$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\frac{2}{y} = \frac{x+z}{xz}$$

$$y = \frac{2xz}{x+z}$$

8. If  $3^x = 5^y = (75)^z$ , show that  $z = \frac{xy}{2x+y}$



$$3^x = k$$

$$3 = k^{\frac{1}{x}}$$

$$5^y = k$$

$$5 = k^{\frac{1}{y}}$$

$$75^z = k$$

$$75 = k^{\frac{1}{z}}$$

$$3^1 \times 5^2 = 75^1$$

$$k^{\frac{1}{x}} \times k^{\frac{2}{y}} = k^{\frac{1}{z}}$$

$$\frac{1}{x} + \frac{2}{y} = \frac{1}{z}$$

$$\frac{y+2x}{xy} = \frac{1}{z}$$

$$z = \frac{xy}{2x+y}$$

9. If  $(27)^x = \frac{9}{3^x}$ , find  $x$

We have,

$$(27)^x = \frac{9}{3^x}$$

$$(3^3)^x = \frac{9}{3^x}$$

$$3^{3x} = \frac{9}{3^x}$$

$$3^{3x} = \frac{3^2}{3^x}$$

$$3^{3x} = 3^{2-x}$$

$$3x = 2 - x \quad \text{[On equating exponents]}$$

$$3x + x = 2$$

$$4x = 2$$

$$x = \frac{2}{4}$$

$$x = \frac{1}{2}$$

Here the value of  $x$  is  $\frac{1}{2}$

10. Find the values of  $x$  in each of the following

(i).  $2^{5x} \div 2^x = \sqrt[5]{2^{20}}$

We have

$$\begin{aligned}
2^{5x} \div 2^x &= \sqrt[5]{2^{20}} \\
&= \frac{2^{5x}}{2^x} = (2^{20})^{\frac{1}{5}} \\
&= 2^{5x-x} = 2^{20 \times \frac{1}{5}} \\
&= 2^{4x} = 2^4 \\
&= 4x = 4 \quad \text{[On equating exponent]} \\
x &= 1
\end{aligned}$$

Hence the value of x is 1

$$(ii). (2^3)^4 = (2^2)^x$$

We have

$$\begin{aligned}
(2^3)^4 &= (2^2)^x \\
&= 2^{3 \times 4} = 2^{2 \times x} \\
12 &= 2x \\
2x &= 12 \quad \text{[On equating exponents]} \\
x &= 6
\end{aligned}$$

Hence the value of x is 6

$$(iii). \left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} = \frac{125}{27}$$

We have

$$\begin{aligned}
\left(\frac{3}{5}\right)^x \left(\frac{5}{3}\right)^{2x} &= \frac{125}{27} \\
\Rightarrow \frac{(3)^x (5)^{2x}}{(5)^x (3)^{2x}} &= \frac{5^3}{3^3} \\
\Rightarrow \frac{5^{2x-x}}{3^{2x-x}} &= \frac{5^3}{3^3} \\
\Rightarrow \frac{5^x}{3^x} &= \frac{5^3}{3^3} \\
\Rightarrow \left(\frac{5}{3}\right)^x &= \left(\frac{5}{3}\right)^3 \\
x &= 3 \quad \text{[on equating exponents]}
\end{aligned}$$

Hence the value of x is 3

$$(iv) \quad 5^{x-2} \times 3^{2x-3} = 135$$

We have,

$$\begin{aligned}
5^{x-2} \times 3^{2x-3} &= 135 \\
\Rightarrow 5^{x-2} \times 3^{2x-3} &= 5 \times 27 \\
\Rightarrow 5^{x-2} \times 3^{2x-3} &= 5^1 \times 3^3 \\
\Rightarrow x-2 &= 1, 2x-3 = 3 \text{ [On equating exponents]} \\
\Rightarrow x &= 2+1, 2x = 3+3 \\
\Rightarrow x &= 3, 2x = 6 \Rightarrow x = 3
\end{aligned}$$

Hence the value of x is 3

(v).  $2^{x-7} \times 5^{x-4} = 1250$

We have

$$\begin{aligned}
2^{x-7} \times 5^{x-4} &= 1250 \\
\Rightarrow 2^{x-7} \times 5^{x-4} &= 2 \times 625 \\
\Rightarrow 2^{x-7} \times 5^{x-4} &= 2 \times 5^4 \\
\Rightarrow x-7 &= 1 \Rightarrow x = 8, x-4 = 4 \Rightarrow x = 8
\end{aligned}$$

Hence the value of x is 8

(vi).

$$\begin{aligned}
(\sqrt[3]{4})^{2x+\frac{1}{2}} &= \frac{1}{32} \\
(4^{\frac{1}{3}})^{2x+\frac{1}{2}} &= \frac{1}{32} \\
(4)^{\frac{1}{3}(2x+\frac{1}{2})} &= \frac{1}{32} \\
(4)^{\frac{1}{3}(2x+\frac{1}{2})} &= \frac{1}{2^5} \\
(4)^{\frac{2}{3}x+\frac{1}{6}} &= \frac{1}{2^5} \\
(2^2)^{\frac{2}{3}x+\frac{1}{6}} &= \frac{1}{2^5} \\
(2)^{2(\frac{2}{3}x+\frac{1}{6})} &= \frac{1}{2^5} \\
(2)^{\frac{4}{3}x+\frac{2}{6}} &= \frac{1}{2^5} \\
(2)^{\frac{4}{3}x+\frac{1}{3}} &= 2^{-5} \\
\frac{4}{3}x + \frac{1}{3} &= -5
\end{aligned}$$

$$4x + 1 = -15$$

$$4x = -15 - 1$$

$$4x = -16$$

$$x = \frac{-16}{4}$$

$$x = -4$$

Hence the value of x is 4

(vii).

$$5^{2x+3} = 1$$

$$5^{2x+3} = 1 \times 5^0$$

$$2x + 3 = 0 \quad \text{[By equating exponents]}$$

$$2x = -3$$

$$x = \frac{-3}{2}$$

Hence the value of x is  $\frac{-3}{2}$

(viii).

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$(13)^{\sqrt{x}} = 256 - 81 - 6$$

$$(13)^{\sqrt{x}} = 256 - 87$$

$$(13)^{\sqrt{x}} = 169$$

$$(13)^{\sqrt{x}} = 13^2$$

$$\sqrt{x} = 2 \quad \text{[By equating exponents]}$$

$$(\sqrt{x})^2 = (2)^2$$

$$x = 4$$

Hence the value of x is 4

(ix).

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{5^3}{3^3}$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \left(\frac{5}{3}\right)^3$$

$$\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \left(\frac{3}{5}\right)^{-3}$$

$$\left(\frac{3}{5}\right)^{\frac{1}{2}(x+1)} = \left(\frac{3}{5}\right)^{-3}$$

$$\frac{1}{2}(x+1) = -3$$

$$x+1 = -6$$

$$x = -6 - 1$$

$$x = -7$$

Hence the value of x is 7

11. If  $x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$ , show that  $x^3 - 6x = 6$

$$x^3 - 6x = 6$$

$$x = 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$$

Putting cube on both the sides, we get

$$x^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

As we know,  $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3\left(2^{\frac{1}{3}}\right)\left(2^{\frac{2}{3}}\right)\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$$

$$x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3\left(2^{\frac{1}{3}+\frac{2}{3}}\right)(x)$$

$$x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(2^{\frac{2}{3}}\right)^3 + 3(2)(x)$$

$$x^3 = 6 + 6x$$

$$x^3 - 6x = 6$$

Hence proved

12. Determine  $(8x)^x$ , if  $9^{x+2} = 240 + 9^x$ .

$$9^{x+2} = 240 + 9^x$$

$$9^x \cdot 9^2 = 240 + 9^x$$

Let  $9^x$  be y

$$81y = 240 + y$$

$$81y - y = 240$$

$$80y = 240$$

$$y = 3$$

Since,  $y = 3$

Then,

$$9^x = 3$$

$$3^{2x} = 3$$

Therefore,  $x = \frac{1}{2}$

$$(8x)^x = \left(8 \times \frac{1}{2}\right)^{\frac{1}{2}}$$

$$= (4)^{\frac{1}{2}}$$

$$= 2$$

Therefore  $(8x)^x = 2$

13. If  $3^{x+1} = 9^{x-2}$ , find the value of  $2^{1+x}$

$$3^{x+1} = 9^{x-2}$$

$$3^{x+1} = 3^{2x-4}$$

$$x + 1 = 2x - 4$$

$$x = 5$$

Therefore the value of  $2^{1+x} = 2^{1+5} = 2^6 = 64$

14. If  $3^{4x} = (81)^{-1}$  and  $(10)^{\frac{1}{y}} = 0.0001$ , find the value of  $2^{-x+4y}$ .

$$3^{4x} = (81)^{-1} \text{ and } (10)^{\frac{1}{y}} = 0.0001$$

$$3^{4x} = (3)^{-4}$$

$$x = -1$$

And,  $(10)^{\frac{1}{y}} = 0.0001$

$$(10)^{\frac{1}{y}} = (10)^{-4}$$

$$\frac{1}{y} = -4$$

$$y = \frac{1}{-4}$$

To find the value of  $2^{-x+4y}$ , we need to substitute the value of  $x$  and  $y$

$$2^{-x+4y} = 2^{1+4\left(\frac{1}{-4}\right)} = 2^{1-1} = 2^0 = 1$$

15. If  $5^{3x} = 125$  and  $10^y = 0.001$ . Find  $x$  and  $y$ .

$$5^{3x} = 125 \text{ and } 10^y = 0.001$$

$$5^{3x} = 5^3$$

$$x = 1$$

Now,

$$10^y = 0.001$$

$$10^y = 10^{-3}$$

$$y = -3$$

Therefore, the value of  $x = 1$  and the value of  $y = -3$

16. Solve the following equations

(i)

$$3^{x+1} = 27 \times 3^4$$

$$3^{x+1} = 3^3 \times 3^4$$

$$3^{x+1} = 3^{3+4}$$

$$x + 1 = 3 + 4 \quad [\text{By equating exponents}]$$

$$x + 1 = 7$$

$$x = 7 - 1$$

$$x = 6$$

(ii)

$$4^{2x} = (\sqrt[3]{16})^{-\frac{6}{y}} = (\sqrt{8})^2$$

$$(2^2)^{2x} = (16^{\frac{1}{3}})^{-\frac{6}{y}} = (\sqrt{8})^2$$

$$2^{4x} = [(2^4)^{\frac{1}{3}}]^{-\frac{6}{y}} = (2^{\frac{3}{2}})^2$$

$$2^{4x} = (2^{\frac{4}{3}})^{-\frac{6}{y}} = (2^{\frac{3}{2}})^2$$

$$2^{4x} = (2^{\frac{4}{3}})^{-\frac{6}{y}} = 2^3$$

$$2^{4x} = 2^3$$

$$4x = 3 \quad (\text{By equating exponents})$$

$$x = \frac{3}{4}$$

$$2^{-\frac{8}{y}} = 2^3$$

$$-\frac{8}{y} = 3 \quad (\text{By equating exponents})$$

$$y = \frac{-8}{3}$$

(iii).

$$3^{x-1} \times 5^{2y-3} = 225$$

$$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$

$$x - 1 = 2 \quad [\text{By equating exponents}]$$

$$x = 3$$

$$3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$

$$2y - 3 = 2 \quad [\text{By equating exponents}]$$

$$2y = 5$$

$$y = \frac{5}{2}$$

(iv).

$$8^{x+1} = 16^{y+2} \text{ and } \left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$$

$$(2^3)^{x+1} \text{ and } (2^{-1})^{3+x} = (2^{-2})^{3y}$$

$$3x + 3 = 4y + 8 \text{ and } -3 - x = -6y$$

$$3x + 3 = 4y + 8 \text{ and } 3 + x = 6y$$

$$3x + 3 = 4y + 8 \text{ and } y = \frac{3+x}{6}$$

$$3x+3=4y+8\text{---eq1}$$

$$y = \frac{3+x}{6}\text{---eq2}$$

Substitute eq2 in eq1

$$3x + 3 = 4\left(\frac{3+x}{6}\right) + 8$$

$$3x + 3 = 2\left(\frac{3+x}{3}\right) + 8$$

$$3x + 3 = \left(\frac{6+2x}{3}\right) + \frac{24}{3}$$

$$3(3x + 3) = 6 + 2x + 24$$

$$9x + 9 = 30 + 2x$$

$$7x = 21$$

$$x = \frac{21}{7}$$

$$x = 3$$

Putting value of x in eq2

$$\frac{3+3}{6} = yy = 1$$



(v).

$$4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$2^{2x-2} \times \left(\frac{5}{10}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$$

$$2^{2x-2} \times \left(\frac{1}{2}\right)^{3-2x} = 2^{-3x}$$

$$2^{2x-2} \times 2^{-3+2x} = 2^{-3x}$$

$$2x - 2 - 3 + 2x = -3x \quad \text{[By equating exponents]}$$

$$4x + 3x = 5$$

$$7x = 5$$

$$x = \frac{5}{7}$$

(vi).

$$\sqrt{\frac{a}{b}} = \left(\frac{b}{a}\right)^{1-2x}$$

$$\left(\frac{a}{b}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-(1-2x)} \frac{1}{2} = -1 + 2x \quad \text{[By equating exponents]}$$

$$\frac{1}{2} + 1 = 2x$$

$$2x = \frac{3}{2}$$

$$x = \frac{3}{4}$$

17. If  $a$  and  $b$  are distinct positive primes such that  $\sqrt[3]{a^6b^{-4}} = a^x b^{2y}$ , find  $x$  and  $y$

$$\sqrt[3]{a^6b^{-4}} = a^x b^{2y}$$

$$(a^6b^{-4})^{\frac{1}{3}} = a^x b^{2y}$$

$$a^{\frac{6}{3}} b^{\frac{-4}{3}} = a^x b^{2y}$$

$$a^2 b^{\frac{-4}{3}} = a^x b^{2y}$$

$$x = 2, 2y = \frac{-4}{3}$$

$$y = \frac{\frac{-4}{3}}{2}$$

$$y = -\frac{2}{3}$$

18. If  $a$  and  $b$  are different positive primes such that

(i).

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^x b^y$$

find  $x$  and  $y$

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right) = a^x b^y$$

$$(a^{-1-2}b^{2+4})^7 \div (a^{3+2}b^{-5-3}) = a^x b^y$$

$$(a^{-3}b^6)^7 \div (a^5b^{-8}) = a^x b^y$$

$$(a^{-21}b^{42}) \div (a^5b^{-8}) = a^x b^y$$

$$(a^{-21-5}b^{42+8}) = a^x b^y$$

$$(a^{-26}b^{50}) = a^x b^y$$

$$x = -26, y = 50$$

(ii)  $(a + b)^{-1} (a^{-1} + b^{-1}) = a^x b^y$ , find x and y

$$(a + b)^{-1} (a^{-1} + b^{-1})$$

$$= \left(\frac{1}{a+b}\right) \left(\frac{1}{a} + \frac{1}{b}\right)$$

$$= \left(\frac{1}{a+b}\right) \left(\frac{b+a}{ab}\right)$$

$$= \frac{1}{ab}$$

$$= (ab)^{-1} = a^{-1} b^{-1}$$

By equating exponents

$$x = -1, y = -1$$

$$\text{Therefore } x + y + 2 = -1 - 1 + 2 = 0$$

19. If  $2^x \times 3^y \times 5^z = 2160$ , find x, y and z. Hence compute the value of  $3^x \times 2^{-y} \times 5^{-z}$

$$2^x \times 3^y \times 5^z = 2160$$

$$2^x \times 3^y \times 5^z = 2^4 \times 3^3 \times 5^1$$

$$x = 4, y = 3, z = 1$$

$$3^x \times 2^{-y} \times 5^{-z} = 3^4 \times 2^{-3} \times 5^{-1}$$

$$= \frac{3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5}$$

$$= \frac{81}{40}$$

20. If  $1176 = 2^a \times 3^b \times 7^c$ , find the values of a, b and c. Hence compute the value of  $2^a \times 3^b \times 7^{-c}$  as a fraction

$$1176 = 2^a \times 3^b \times 7^c$$

$$2^3 \times 3^1 \times 7^2 = 2^a \times 3^b \times 7^c$$

$$a = 3, b = 1, c = 2$$

We have to find the value of  $2^a \times 3^b \times 7^{-c}$

$$2^a \times 3^b \times 7^{-c} = 2^3 \times 3^1 \times 7^{-2}$$

$$= \frac{2 \times 2 \times 2 \times 3}{7 \times 7}$$

$$= \frac{24}{49}$$

21. Simplify

(i)

$$\left(\frac{x^{a+b}}{x^c}\right)^{a-b} \left(\frac{x^{b+c}}{x^a}\right)^{b-c} \left(\frac{x^{c+a}}{x^b}\right)^{c-a}$$

$$(x^{a+b-c})^{a-b} (x^{b+c-a})^{b-c} (x^{c+a-b})^{c-a}$$

$$(x^{a^2-b^2-ca+cb})(x^{b^2-c^2-ab+ac})(x^{c^2-a^2-bc+ab})$$

$$x^{a^2-b^2-ca+cb+b^2-c^2-ab+ac+c^2-a^2-bc+ab}$$

$$x^0 = 1$$

(ii)

$$\sqrt[l]{\frac{x^l}{x^m}} \times \sqrt[m]{\frac{x^m}{x^n}} \times \sqrt[n]{\frac{x^n}{x^l}}$$

$$\sqrt[l]{x^{l-m}} \times \sqrt[m]{x^{m-n}} \times \sqrt[n]{x^{n-l}}$$

$$(x^{l-m})^{\frac{1}{lm}} \times (x^{m-n})^{\frac{1}{mn}} \times (x^{n-l})^{\frac{1}{nl}}$$

$$(x)^{\frac{l-m}{lm}} \times (x)^{\frac{m-n}{mn}} \times (x)^{\frac{n-l}{nl}}$$

$$(x)^{\frac{l-m}{lm} + \frac{m-n}{mn} + \frac{n-l}{nl}}$$

$$(x)^{n\left(\frac{l-m}{lm}\right) + l\left(\frac{m-n}{mn}\right) + m\left(\frac{n-l}{nl}\right)}$$

$$(x)^{\frac{nl-mn+lm-nl+mn-ml}{mnl}}$$

$$(x)^{\frac{0}{mnl}}$$

$$x^0 = 1$$

22. Show that

$$\begin{aligned}
& \frac{\left(\frac{a+1}{b}\right)^m \times \left(\frac{a-1}{b}\right)^n}{\left(\frac{b+1}{a}\right)^m \times \left(\frac{b-1}{a}\right)^n} = \left(\frac{a}{b}\right)^{m+n} \\
& = \frac{\left(\frac{ab+1}{b}\right)^m \times \left(\frac{ab-1}{b}\right)^n}{\left(\frac{ab+1}{a}\right)^m \times \left(\frac{ab-1}{a}\right)^n} \\
& = \left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n \\
& = \left(\frac{a}{b}\right)^{m+n}
\end{aligned}$$

Hence LHS = RHS

23.

(i). If  $a = x^{m+n}y^l$ ,  $b = x^{n+1}y^m$  and  $c = x^{l+m}y^n$ , prove that  $a^{m-n}b^{n-1}c^{l-m} = 1$

$$\begin{aligned}
& (x^{m+n}y^l)^{m-n} (x^{n+1}y^m)^{n-1} (x^{l+m}y^n)^{l-m} \\
& = (x^{(m+n)(m-n)}y^{l(m-n)}) (x^{(n+1)(n-1)}y^{m(n-1)}) (x^{(l+m)(l-m)}y^{n(l-m)}) \\
& = (x^{m^2-n^2}y^{lm-ln}) (x^{n^2-l^2}y^{mn-ml}) (x^{l^2-m^2}y^{nl-nm}) \\
& = x^{m^2-n^2+n^2-l^2+l^2-m^2}y^{lm-ln+mn-ml+nl-nm} \\
& = x^0y^0 \\
& = 1
\end{aligned}$$

(ii). If  $x = a^{m+n}$ ,  $y = a^{n+1}$  and  $z = a^{l+m}$ , prove that  $x^m y^n z^l = x^n y^l z^m$

$$\begin{aligned}
& \text{LHS} = x^m y^n z^l \\
& (a^{m+n})^m (a^{n+1})^n (a^{l+m})^l \\
& = a^{m^2+nm} \times a^{n^2+ln} \times a^{l^2+ml} \\
& = a^{n^2+nm} \times a^{l^2+ln} \times a^{m^2+ml} \\
& = a^{(m+n)n} a^{(n+1)l} a^{(l+m)m} \\
& = x^n y^l z^m
\end{aligned}$$