

RD SHARMA

Solutions

Class 9 Maths

Chapter 3

Ex 3.2

1. Rationalize the denominator of each of the following:

(i) $\frac{3}{\sqrt{5}}$

(ii) $\frac{3}{2\sqrt{5}}$

(iii) $\frac{1}{\sqrt{12}}$

(iv) $\frac{\sqrt{2}}{\sqrt{3}}$

(v) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vi) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(vii) $\frac{3\sqrt{2}}{\sqrt{5}}$

Solution:

(i) $\frac{3}{\sqrt{5}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$$\begin{aligned} &= \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{3 \times \sqrt{5}}{5} \end{aligned}$$

(ii) $\frac{3}{2\sqrt{5}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$$\begin{aligned} &= \frac{3 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} \\ &= \frac{3\sqrt{5}}{2 \times \sqrt{5} \times 5} \\ &= \frac{3\sqrt{5}}{2 \times 5} \\ &= \frac{3\sqrt{5}}{10} \end{aligned}$$

(iii) $\frac{1}{\sqrt{12}}$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{12}$

$$\begin{aligned}
 &= \frac{1 \times \sqrt{12}}{\sqrt{12} \times \sqrt{12}} \\
 &= \frac{\sqrt{12}}{\sqrt{12} \times 12} \\
 &= \frac{\sqrt{12}}{12}
 \end{aligned}$$

$$(iv) \frac{\sqrt{2}}{\sqrt{3}}$$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{3}$

$$\begin{aligned}
 &= \frac{\sqrt{2} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{\sqrt{2 \times 3}}{\sqrt{3 \times 3}} \\
 &= \frac{\sqrt{6}}{3}
 \end{aligned}$$

$$(v) \frac{\sqrt{3}+1}{\sqrt{2}}$$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{2}$

$$\begin{aligned}
 &= \frac{(\sqrt{3}+1) \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\
 &= \frac{(\sqrt{3} \times \sqrt{2}) + \sqrt{2}}{\sqrt{2} \times 2} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{2}
 \end{aligned}$$

$$(vi) \frac{\sqrt{2} + \sqrt{5}}{\sqrt{3}}$$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{3}$

$$\begin{aligned}
 &= \frac{(\sqrt{2} + \sqrt{5}) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{(\sqrt{2} \times \sqrt{3}) + (\sqrt{5} \times \sqrt{3})}{\sqrt{3} \times \sqrt{3}} \\
 &= \frac{\sqrt{6} + \sqrt{15}}{3}
 \end{aligned}$$

$$(vii) \frac{3\sqrt{2}}{\sqrt{5}}$$

For rationalizing the denominator, multiply both numerator and denominator with $\sqrt{5}$

$$= \frac{3\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{3\sqrt{2 \times 5}}{\sqrt{5 \times 5}}$$

$$= \frac{3\sqrt{10}}{5}$$

2. Find the value to three places of decimals of each of the following.

It is given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{10} = 3.162$.

(i) $\frac{2}{\sqrt{3}}$

(ii) $\frac{3}{\sqrt{10}}$

(iii) $\frac{\sqrt{5}+1}{\sqrt{2}}$

(iv) $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$

(v) $\frac{2+\sqrt{3}}{3}$

(vi) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution:

Given, $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{10} = 3.162$.

(i) $\frac{2}{\sqrt{3}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{3}$

$$= \frac{2\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$= \frac{2\sqrt{3}}{\sqrt{3 \times 3}}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= \frac{2 \times 1.732}{3}$$

$$= \frac{3.464}{3}$$

$$= 1.154666666$$

(ii) $\frac{3}{\sqrt{10}}$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{10}$

$$= \frac{3\sqrt{10}}{\sqrt{10} \times \sqrt{10}}$$

$$= \frac{3\sqrt{10}}{\sqrt{10} \times 10}$$

$$= \frac{3\sqrt{10}}{10}$$

$$= \frac{9.486}{10}$$

$$= 0.9486$$

$$(iii) \frac{\sqrt{5}+1}{\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{2}$

$$= \frac{(\sqrt{5} \times \sqrt{2}) + \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{\sqrt{10} + \sqrt{2}}{2}$$

$$= \frac{4.576}{2}$$

$$= 2.288$$

$$(iv) \frac{\sqrt{10} + \sqrt{15}}{\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{2}$

$$= \frac{(\sqrt{10} \times \sqrt{2}) + (\sqrt{15} \times \sqrt{2})}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{\sqrt{20} + \sqrt{30}}{2}$$

$$= \frac{(\sqrt{10} \times \sqrt{2}) + (\sqrt{10} \times \sqrt{3})}{2}$$

$$= \frac{(3.162 \times 1.414) + (3.162 \times 1.732)}{2}$$

$$= \frac{(4.471068) + (5.476584)}{2}$$

$$= \frac{9.947652}{2}$$

$$= 4.973826$$

$$(v) \frac{2 + \sqrt{3}}{3}$$

$$= \frac{2+1.732}{3}$$

$$= \frac{3.732}{3}$$

$$= 1.244$$

$$(vi) \frac{\sqrt{2}-1}{\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with $\sqrt{5}$

$$= \frac{(\sqrt{2} \times \sqrt{5}) - \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{\sqrt{10} - \sqrt{5}}{5}$$

$$= \frac{3.162 - 2.236}{5}$$

$$= \frac{0.926}{5}$$

$$= 0.1852$$

3. Express each one of the following with rational denominator:

$$(i) \frac{1}{3+\sqrt{2}}$$

$$(ii) \frac{1}{\sqrt{6}-\sqrt{5}}$$

$$(iii) \frac{16}{\sqrt{41}-5}$$

$$(iv) \frac{30}{5\sqrt{3}-3\sqrt{5}}$$

$$(v) \frac{1}{2\sqrt{5}-\sqrt{3}}$$

$$(vi) \frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$$

$$(vii) \frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$(viii) \frac{3\sqrt{2}+1}{2\sqrt{5}-3}$$

$$(ix) \frac{b^2}{\sqrt{(a^2+b^2)+a}}$$

Solution:

$$(i) \frac{1}{3+\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 - \sqrt{2}$

$$= \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{3-\sqrt{2}}{9-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

$$(ii) \frac{1}{\sqrt{6}-\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$\sqrt{6} + \sqrt{5}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{\sqrt{6}+\sqrt{2}}{6-2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$(iii) \frac{16}{\sqrt{41}-5}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$\sqrt{41} + 5$$

$$= \frac{16 \times (\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{16\sqrt{41}+80}{41-5}$$

$$= \frac{16\sqrt{41}+80}{16}$$

$$= \frac{16(\sqrt{41}+5)}{16}$$

$$= \sqrt{41} + 5$$

$$(iv) \frac{30}{5\sqrt{3}-3\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$5\sqrt{3} + 3\sqrt{5}$$

$$= \frac{30 \times (5\sqrt{3} + 3\sqrt{5})}{(5\sqrt{3} - 3\sqrt{5})(5\sqrt{3} + 3\sqrt{5})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{30 \times (5\sqrt{3} + 3\sqrt{5})}{75 - 45}$$

$$= \frac{30 \times (5\sqrt{3} + 3\sqrt{5})}{30}$$

$$= 5\sqrt{3} + 3\sqrt{5}$$

$$(v) \frac{1}{2\sqrt{5} - \sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{5} + \sqrt{3}$

$$= \frac{2\sqrt{5} + \sqrt{3}}{(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{2\sqrt{5} + \sqrt{3}}{20 - 3}$$

$$= \frac{2\sqrt{5} + \sqrt{3}}{17}$$

$$(vi) \frac{\sqrt{3} + 1}{2\sqrt{2} - \sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{2} + \sqrt{3}$

$$= \frac{(\sqrt{3} + 1)(2\sqrt{2} + \sqrt{3})}{(2\sqrt{2} + \sqrt{3})(2\sqrt{2} - \sqrt{3})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(2\sqrt{6} + 3 + 2\sqrt{2} + \sqrt{3})}{8 - 3}$$

$$= \frac{(2\sqrt{6} + 3 + 2\sqrt{2} + \sqrt{3})}{5}$$

$$(vii) \frac{6 - 4\sqrt{2}}{6 + 4\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $6 - 4\sqrt{2}$

$$= \frac{(6 - 4\sqrt{2})(6 - 4\sqrt{2})}{(6 + 4\sqrt{2})(6 - 4\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(6-4\sqrt{2})^2}{36-32}$$

As we know, $(a - b)^2 = (a^2 - 2 \times a \times b + b^2)$

$$= \frac{36-48\sqrt{2}+32}{4}$$

$$= \frac{68-48\sqrt{2}}{4}$$

$$= \frac{4(17-12\sqrt{2})}{4}$$

$$= 17 - 12\sqrt{2}$$

(viii) $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{5} - 3$

$$= \frac{(3\sqrt{2}+1) \times (2\sqrt{5}-3)}{(2\sqrt{5}-3)(2\sqrt{5}-3)}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{6\sqrt{10}-9\sqrt{2}+2\sqrt{5}-3}{(20-9)}$$

$$= \frac{6\sqrt{10}-9\sqrt{2}+2\sqrt{5}-3}{11}$$

(ix) $\frac{b^2}{\sqrt{(a^2+b^2)}+a}$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{(a^2 + b^2)} - a$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(\sqrt{(a^2+b^2)}+a)(\sqrt{(a^2+b^2)}-a)}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{(a^2+b^2)-a^2}$$

$$= \frac{b^2(\sqrt{(a^2+b^2)}-a)}{b^2}$$

4. Rationalize the denominator and simplify:

$$(i) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$(ii) \frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

$$(iii) \frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

$$(iv) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

$$(v) \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$(vi) \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Solution:

$$(i) \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{3} - \sqrt{2}$

$$= \frac{(\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2}$$

As we know, $(a - b)^2 = (a^2 - 2 \times a \times b + b^2)$

$$= \frac{3-2\sqrt{3}\sqrt{2}+2}{1}$$

$$= 5 - 2\sqrt{6}$$

$$(ii) \frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $7 - 4\sqrt{3}$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$= 35 - 20\sqrt{3} + 14\sqrt{3} - 24$$

$$= 11 - 6\sqrt{3}$$

$$(iii) \frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 + 2\sqrt{2}$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$$

$$= 3 + 2\sqrt{2} + 3\sqrt{2} + 4$$

$$= 7 + 5\sqrt{2}$$

$$(iv) \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3\sqrt{5} + 2\sqrt{6}$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{45-24}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21}$$

$$= \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{21}$$

$$= \frac{6\sqrt{30}+24-15-2\sqrt{30}}{21}$$

$$= \frac{4\sqrt{30}+9}{21}$$

$$(v) \frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{48} - \sqrt{18}$

$$= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$\begin{aligned}
&= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18} \\
&= \frac{48-12\sqrt{6}+20\sqrt{6}-30}{30} \\
&= \frac{18+8\sqrt{6}}{30} \\
&= \frac{9+4\sqrt{6}}{15}
\end{aligned}$$

$$(vi) \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2\sqrt{2} - 3\sqrt{3}$

$$\begin{aligned}
&= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})} \\
&= \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{8-27} \\
&= \frac{(4\sqrt{6}-2\sqrt{10})-18+3\sqrt{15}}{-19} \\
&= \frac{(18-4\sqrt{6}+2\sqrt{10}-3\sqrt{15})}{19}
\end{aligned}$$

5. Simplify:

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

$$(ii) \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$(iii) \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$(iv) \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$$

$$(v) \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$

Solution:

$$(i) \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} + \frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3\sqrt{2} - 2\sqrt{3}$ for $\frac{1}{3\sqrt{2}+2\sqrt{3}}$ and the rationalizing factor $\sqrt{3} + \sqrt{2}$ for $\frac{1}{\sqrt{3}-\sqrt{2}}$

$$= \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}-2\sqrt{3})} + \frac{\sqrt{12}(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$\text{Now, } (a + b)(a - b) = (a^2 - b^2)$$

$$= \frac{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}-2\sqrt{3})}{18-12} + \frac{\sqrt{12}(\sqrt{3}+\sqrt{2})}{3-2}$$

$$\text{As we know, } (a - b)^2 = (a^2 - 2 \times a \times b + b^2)$$

$$= \frac{(3\sqrt{2})^2 - (2 \times 3\sqrt{2} \times 2\sqrt{3}) + (2\sqrt{3})^2}{6} + 2\sqrt{3}(\sqrt{3} + \sqrt{2})$$

$$= \frac{(18-12\sqrt{6}+12)}{6} + (6 + 2\sqrt{6})$$

$$= 3 - 2\sqrt{6} + 2 + (6 + 2\sqrt{6})$$

$$= 5 - 2\sqrt{6} + (6 + 2\sqrt{6})$$

$$= 11$$

$$\text{(ii) } \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5} + \sqrt{3}$ for $\frac{1}{\sqrt{5}-\sqrt{3}}$ and the rationalizing factor $\sqrt{5} - \sqrt{3}$ for $\frac{1}{\sqrt{5}+\sqrt{3}}$

$$= \frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{(\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

Now as we know, $(a + b)(a - b) = (a^2 - b^2)$, $(a - b)^2 = (a^2 - 2 \times a \times b + b^2)$ and $(a + b)^2 = (a^2 + 2 \times a \times b + b^2)$

$$= \frac{5+2 \times \sqrt{5} \times \sqrt{3}+3}{5-3} + \frac{5-2 \times \sqrt{3} \times \sqrt{5}+3}{5-3}$$

$$= \frac{8+2\sqrt{15}+8-2\sqrt{15}}{2}$$

$$= \frac{16}{2}$$

$$= 8$$

$$\text{(iii) } \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 - \sqrt{5}$ for $\frac{1}{3+\sqrt{5}}$ and the rationalizing factor $3 + \sqrt{5}$ for $\frac{1}{3-\sqrt{5}}$

$$\frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} - \frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3-\sqrt{5})}$$

Now as we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(7+3\sqrt{5})(3-\sqrt{5})}{9-5} - \frac{(7-3\sqrt{5})(3+\sqrt{5})}{9-5}$$

$$\begin{aligned}
&= \frac{(21-7\sqrt{5}+9\sqrt{5}-15)}{4} - \frac{(21+7\sqrt{5}-9\sqrt{5}-15)}{4} \\
&= \frac{(6+2\sqrt{5})}{4} - \frac{(6-2\sqrt{5})}{4} \\
&= \frac{4\sqrt{5}}{4} \\
&= \sqrt{5}
\end{aligned}$$

$$(iv) \frac{1}{2+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} + \frac{1}{2-\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2 - \sqrt{3}$ for $\frac{1}{2+\sqrt{3}}$, the rationalizing factor $\sqrt{5} + \sqrt{3}$ for $\frac{2}{\sqrt{5}-\sqrt{3}}$, and the rationalizing factor $2 + \sqrt{5}$ for $\frac{1}{2-\sqrt{5}}$

$$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})} + \frac{2\times(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} + \frac{2+\sqrt{5}}{(2-\sqrt{5})(2+\sqrt{5})}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$\begin{aligned}
&= \frac{2-\sqrt{3}}{4-3} + \frac{2\times(\sqrt{5}+\sqrt{3})}{5-3} + \frac{2+\sqrt{5}}{4-5} \\
&= \frac{2-\sqrt{3}}{1} + \frac{2\sqrt{5}+2\sqrt{3}}{2} + \frac{2+\sqrt{5}}{-1} \\
&= \frac{4-2\sqrt{3}+2\sqrt{5}+2\sqrt{3}-4-2\sqrt{5}}{2} \\
&= \frac{0}{2} \\
&= 0
\end{aligned}$$

$$(v) \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5} - \sqrt{3}$ for $\frac{2}{\sqrt{5}+\sqrt{3}}$, the rationalizing factor $\sqrt{3} - \sqrt{2}$ for $\frac{1}{\sqrt{3}+\sqrt{2}}$, and the rationalizing factor $\sqrt{5} - \sqrt{2}$ for

$$\begin{aligned}
&\frac{1}{\sqrt{5}+\sqrt{2}} \\
&= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} + \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} - \frac{3\times(\sqrt{5}-\sqrt{2})}{\sqrt{5}+\sqrt{2}(\sqrt{5}-\sqrt{2})}
\end{aligned}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$\begin{aligned}
&= \frac{2(\sqrt{5}-\sqrt{3})}{5-3} + \frac{\sqrt{3}-\sqrt{2}}{3-2} - \frac{3\times(\sqrt{5}-\sqrt{2})}{5-2} \\
&= \frac{2\sqrt{5}-2\sqrt{3}}{2} + \frac{\sqrt{3}-\sqrt{2}}{1} - \frac{3\times\sqrt{5}-3\sqrt{2}}{3} \\
&= \frac{6\sqrt{5}-6\sqrt{3}+6\sqrt{3}-6\sqrt{2}-6\sqrt{5}+6\sqrt{2}}{3}
\end{aligned}$$

$$= \frac{0}{3}$$

$$= 0$$

6. In each of the following determine rational numbers a and b:

$$(i) \frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

$$(ii) \frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$$

$$(iii) \frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

$$(iv) \frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

$$(v) \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$$

$$(vi) \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$$

Solution:

(i) Given,

$$\frac{\sqrt{3}-1}{\sqrt{3}+1} = a - b\sqrt{3}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{3} - 1$

$$= \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{3-2\sqrt{3}+1}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

$$2 - \sqrt{3} = a - b\sqrt{3}$$

On comparing the rational and irrational parts of the above equation, we get,

$$a = 2 \text{ and } b = 1$$

(ii) Given;

$$\frac{4+\sqrt{2}}{2+\sqrt{2}} = a - \sqrt{b}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2 - \sqrt{2}$

$$= \frac{(4+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(8-4\sqrt{2}+2\sqrt{2}-2)}{4-2}$$

$$= \frac{(6-2\sqrt{2})}{2}$$

$$= 3 - \sqrt{2}$$

$$3 - \sqrt{2} = a - \sqrt{b}$$

On comparing the rational and irrational parts of the above equation, we get,

$$a = 3 \text{ and } b = 2$$

(iii) Given,

$$\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 + \sqrt{2}$

$$= \frac{(3+\sqrt{2})(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(9+6\sqrt{2}+2)}{9-2}$$

$$= \frac{(11+6\sqrt{2})}{7}$$

$$= \frac{11}{7} + \frac{6\sqrt{2}}{7}$$

$$\frac{11}{7} + \frac{6\sqrt{2}}{7} = a + b\sqrt{2}$$

On comparing the rational and irrational parts of the above equation, we get,

$$a = \frac{11}{7} + \frac{6\sqrt{2}}{7} \text{ and}$$

$$b = \frac{6}{7}$$

(iv) Given,

$$\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a + b\sqrt{3}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$7 - 4\sqrt{3}$$

$$= \frac{(5+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(35-20\sqrt{3}+21\sqrt{3}-36)}{49-48}$$

$$= -1 + \sqrt{3}$$

$$-1 + \sqrt{3} = a + b\sqrt{3}$$

On comparing the rational and irrational parts of the above equation, we get,

$$a = -1 \text{ and}$$

$$b = 1$$

$$(v) \frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{11}-\sqrt{7}$

$$= \frac{(\sqrt{11}-\sqrt{7})(\sqrt{11}-\sqrt{7})}{(\sqrt{11}+\sqrt{7})(\sqrt{11}-\sqrt{7})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(11-\sqrt{77}-\sqrt{77}+7)}{11-7}$$

$$= \frac{(18-2\sqrt{77})}{4}$$

$$= \frac{9}{2} - \frac{\sqrt{77}}{2}$$

$$\frac{9}{2} - \frac{\sqrt{77}}{2} = a - b\sqrt{77}$$

On comparing the rational and irrational parts of the above equation, we get,

$$a = \frac{9}{2} \text{ and}$$

$$b = \frac{1}{2}$$

(vi) Given,

$$= \frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a + b\sqrt{5}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $4 + 3\sqrt{5}$

$$= \frac{(4+3\sqrt{5})(4+3\sqrt{5})}{(4-3\sqrt{5})(4+3\sqrt{5})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$\begin{aligned}
&= \frac{(16+24\sqrt{5}+45)}{-29} \\
&= \frac{(61+24\sqrt{5})}{-29} \\
&= \frac{-61}{29} - \frac{(24\sqrt{5})}{29} \\
&\frac{-61}{29} - \frac{(24\sqrt{5})}{29} = a + b\sqrt{5}
\end{aligned}$$

On comparing the rational and irrational parts of the above equation, we get,

$$\begin{aligned}
a &= \frac{-61}{29}, \text{ and} \\
b &= \frac{-24}{29}
\end{aligned}$$

7. If $x = 2 + \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3}$

Solution:

Given,

$$x = 2 + \sqrt{3},$$

To find the value of $x^3 + \frac{1}{x^3}$

We have, $x = 2 + \sqrt{3}$,

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $2 - \sqrt{3}$ for $\frac{1}{2+\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{4-3}$$

$$x + \frac{1}{x} = 2 + \sqrt{3} + 2 - \sqrt{3}$$

$$x + \frac{1}{x} = 4$$

We know that, $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

$$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2})$$

$$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} - 1)$$

$$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} + 2 - 2 - 1)$$

$$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} + 2(x \cdot \frac{1}{x}) - 2 - 1)$$

$$(x^3 + \frac{1}{x^3}) = (x + \frac{1}{x})((x + \frac{1}{x})^2 - 3)$$

Putting the value of $x + \frac{1}{x}$ in the above equation, we get,

$$(x^3 + \frac{1}{x^3}) = (4)(4^2 - 3)$$

$$(x^3 + \frac{1}{x^3}) = 52$$

8. If $x = 3 + \sqrt{8}$, find the value of $(x^2 + \frac{1}{x^2})$

Solution:

Given,

$$x = 3 + \sqrt{8},$$

To find the value of $(x^2 + \frac{1}{x^2})$

We have, $x = 3 + \sqrt{8}$,

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $3 - \sqrt{8}$ for $\frac{1}{3 + \sqrt{8}}$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{(3 + \sqrt{8})(3 - \sqrt{8})}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$\frac{1}{x} = \frac{3 - \sqrt{8}}{9 - 8}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

$$(x^2 + \frac{1}{x^2}) = ((3 + \sqrt{8})^2(3 - \sqrt{8})^2)$$

$$(x^2 + \frac{1}{x^2}) = ((9 + 8 + 6\sqrt{8}) + (9 + 8 - 6\sqrt{8}))$$

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9. Find the value of $\frac{6}{\sqrt{5} - \sqrt{3}}$, it being given that $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$.

Given,

$$\frac{6}{\sqrt{5} - \sqrt{3}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor $\sqrt{5} + \sqrt{3}$ for $\frac{1}{\sqrt{5} - \sqrt{3}}$

$$= \frac{6(\sqrt{5} + \sqrt{3})}{(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$\begin{aligned}
&= \frac{6\sqrt{5}+6\sqrt{3}}{5-3} \\
&= \frac{6\sqrt{5}+6\sqrt{3}}{2} \\
&= 3(\sqrt{5} + \sqrt{3}) \\
&= 3(2.236+1.732) \\
&= 3(3.968) \\
&= 11.904
\end{aligned}$$

10. Find the values of each of the following correct to three places of decimals, it being given that

$$\sqrt{2} = 1.414, \sqrt{3} = 1.732, \sqrt{5} = 2.236, \sqrt{6} = 2.4495, \sqrt{10} = 3.162$$

$$(i) \frac{3-\sqrt{5}}{3+2\sqrt{5}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$3 - 2\sqrt{5}$$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3+2\sqrt{5})(3+2\sqrt{5})}$$

Since, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{9-20}$$

$$= \frac{(9-6\sqrt{5}-3\sqrt{5}+10)}{-11}$$

$$= \frac{(19-9\sqrt{5})}{-11}$$

$$= \frac{(9\sqrt{5}-19)}{11}$$

$$= \frac{(9(2.236)-19)}{11}$$

$$= \frac{(20.124-19)}{11}$$

$$= \frac{1.124}{11}$$

$$= 0.102$$

$$(ii) \frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

Rationalizing the denominator by multiplying both numerator and denominator with the rationalizing factor

$$3 + 2\sqrt{2}$$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$

As we know, $(a + b)(a - b) = (a^2 - b^2)$

$$= \frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$$

$$= 3 + 2\sqrt{2} + 3\sqrt{2} + 4$$

$$= 7 + 5\sqrt{2}$$

$$= 7 + 7.07$$

$$= 14.07$$

11. If $x = \frac{\sqrt{3}+1}{2}$, find the value of $4x^3 + 2x^2 - 8x + 7$.

Solution:

Given,

$$x = \frac{\sqrt{3}+1}{2} \text{ and given to find the value of } 4x^3 + 2x^2 - 8x + 7$$

$$2x = \sqrt{3} + 1$$

$$2x - 1 = \sqrt{3}$$

Now, squaring on both the sides, we get,

$$(2x - 1)^2 = 3$$

$$4x^2 - 4x + 1 = 3$$

$$4x^2 - 4x + 1 - 3 = 0$$

$$4x^2 - 4x - 2 = 0$$

$$2x^2 - 2x - 1 = 0$$

Now taking $4x^3 + 2x^2 - 8x + 7$

$$2x(2x^2 - 2x - 1) + 4x^2 + 2x + 2x^2 - 8x + 7$$

$$2x(2x^2 - 2x - 1) + 6x^2 - 6x + 7$$

As, $2x^2 - 2x - 1 = 0$

$$2x(0) + 3(2x^2 - 2x - 1) + 7 + 3$$

$$0 + 3(0) + 10$$

$$10$$