RD SHARMA

Solutions

Class 9 Maths

Chapter 4

Ex *4.1*

1) Evaluate each of the following using identities:

(i) $(2x - \frac{1}{x})^2$

Solution:

Given,

$(2x - \frac{1}{x})^2 = (2x)^2 + (\frac{1}{x})^2 - 2 * 2x * \frac{1}{x}$

$(2x - \frac{1}{x})^2 = 4x^2 + \frac{1}{x^2} - 4 \qquad [\because (a - b)^2 = a^2 + b^2 - 2ab\ ]$

Where, $a = 2x,\ b = \frac{1}{x}$

$\therefore (2x - \frac{1}{x})^2 = 4x^2 + \frac{1}{x^2} - 4$

(ii) (2x+y) (2x-y)

Solution:

Given, (2x+y) (2x-y)

= $(2x)^2 - (y)^2 \qquad [\because (a + b)(a - b) = a^2 - b^2]$

= $4x^2 - y^2$

$\therefore (2x + y)(2x - y) = 4x^2 - y^2$

(iii) $(a^2b - ab^2)^2$

Solution:

Given, $(a^2b - ab^2)^2$

$= (a^2b)^2 + (ab^2)^2 - 2 * a^2b * ab^2 \qquad [\because (a - b)^2 = a^2 + b^2 - 2ab]$

Where, $a = a^2b,\ b = ab^2$

$= a^4b^2 + b^4a^2 - 2a^3b^3$

$\therefore (a^2b - ab^2)^2 = a^4b^2 + b^4a^2 - 2a^3b^3$

(iv) (a-0.1) (a+0.1)

Solution:

Given, (a-0.1) (a+0.1)

$= a^2 - (0.1)^2 \qquad [\because (a + b)(a - b) = a^2 - b^2]$

Where, a = a and b = 0.1

$= a^2 - 0.01$

$\therefore (a - 0.1)(a + 0.1) = a^2 - 0.01$

(v) $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

Solution:

Given, $(1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2)$

$= (1.5x^2)^2 - (0.3y^2)^2 \qquad [\because (a+b)(a-b) = a^2 - b^2]$

Where, $a = 1.5x^2,\ b = 0.3y^2$

$= 2.25x^4 - 0.09y^4$

$\therefore (1.5x^2 - 0.3y^2)(1.5x^2 + 0.3y^2) = 2.25x^4 - 0.09y^4$

2) Evaluate each of the following using identities:

(i) $(399)^2$

Solution:

We have,

$399^2 = (400-1)^2$

$= (400)^2 + (1)^2 - 2x400x1 \qquad [(a-b)^2 = a^2 + b^2 - 2ab]$

Where, a = 400 and b = 1

= 160000 + 1 − 8000

= 159201

Therefore, $(399)^2 = 159201$.

(ii) (0.98)²

Solution:

We have,

$(0.98)^2 = (1-0.02)^2$

$= (1)^2 + (0.02)^2 - 2x1x0.02$

= 1 + 0.0004 − 0.04 [Where, a=1 and b=0.02]

= 1.0004 − 0.04

= 0.9604

Therefore, $(0.98)^2 = 0.9604$

(iii) 991×1009

Solution:

We have,

991×1009

= (1000-9) (1000+9)

$= (1000)^2 - (9)^2 \qquad [(a+b)(a-b) = a^2 - b^2]$

= 1000000 − 81 [Where a=1000 and b=9]

= 999919

Therefore, $991 \times 1009 = 999919$

(iv) 117×83

Solution:

We have,

117×83

$= (100+17)(100-17)$

$= (100)^2 - (17)^2$ [$(a+b)(a-b) = a^2 - b^2$]

$= 10000 - 289$ [Where a=100 and b=17]

$= 9711$

Therefore, $117 \times 83 = 9711$

3) Simplify each of the following:

(i) 175 x 175 +2 x 175 x 25 + 25 x 25

Solution:

We have,

$175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = (175)^2 + 2(175)(25) + (25)^2$

$= (175+25)^2$ [$a^2 + b^2 + 2ab = (a+b)^2$]

$= (200)^2$ [Where a=175 and b=25]

$= 40000$

Therefore, $175 \times 175 + 2 \times 175 \times 25 + 25 \times 25 = 40000$.

(ii) 322 x 322 − 2 x 322 x 22 + 22 x 22

Solution:

We have,

$322 \times 322 - 2 \times 322 \times 22 + 22 \times 22$

$= (322-22)^2$ [$a^2 + b^2 - 2ab = (a-b)^2$]

$= (300)^2$ [Where a=322 and b=22]

$= 90000$

Therefore, $322 \times 322 - 2 \times 322 \times 22 + 22 \times 22 = 90000$.

(iii) 0.76 x 0.76 + 2 x 0.76 x 0.24 + 0.24 x 0.24

Solution:

We have,

$0.76 \times 0.76 + 2 \times 0.76 \times 0.24 + 0.24 \times 0.24$

$= (0.76+0.24)^2$ [$a^2 + b^2 + 2ab = (a+b)^2$]

$= (1.00)^2$ [Where a=0.76 and b=0.24]

$= 1$

Therefore, 0.76 x 0.76 + 2 x 0.76 x 0.24 + 0.24 x 0.24 = 1.

(iv) $\frac{7.83*7.83-1.17*1.17}{6.66}$

Solution:

We have,

$\frac{7.83*7.83-1.17*1.17}{6.66}$

$= \frac{(7.83+1.17)(7.83-1.17)}{6.66}$ $\quad [\because (a-b)^2 = (a+b)(a-b)]$

$= \frac{(9.00)(6.66)}{(6.66)}$

= 9

$\therefore \frac{7.83*7.83-1.17*1.17}{6.66} = 9$

4) If $x + \frac{1}{x} = 11$, find the value of $x^2 + \frac{1}{x^2}$

Solution:

We have, $x + \frac{1}{x} = 11$

Now, $(x + \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 + 2 * x * \frac{1}{x}$

$\Rightarrow (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow (11)^2 = x^2 + \frac{1}{x^2} + 2 \quad [\because x + \frac{1}{x} = 11]$

$\Rightarrow 121 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 119$

5) If $x - \frac{1}{x} = -1$, find the value of $x^2 + \frac{1}{x^2}$

Solution:

We have, $x - \frac{1}{x} = -1$

Now, $(x - \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 - 2 * x * \frac{1}{x}$

$\Rightarrow (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow (-1)^2 = x^2 + \frac{1}{x^2} - 2 \quad [\because x - \frac{1}{x} = -1]$

$\Rightarrow 2 + 1 = x^2 + \frac{1}{x^2}$

$\Rightarrow x^2 + \frac{1}{x^2} = 3$

6) If $x + \frac{1}{x} = \sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$

Solution:

We have,

$(x + \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 + 2 * x * \frac{1}{x}$

$\Rightarrow (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow (\sqrt{5})^2 = x^2 + \frac{1}{x^2} + 2 \quad [\because x + \frac{1}{x} = \sqrt{5}]$

$\Rightarrow 5 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow x^2 + \frac{1}{x^2} = 3$ .....(1)

Now, $(x^2 + \frac{1}{x^2})^2 = x^4 + \frac{1}{x^4} + 2 * x^2 * \frac{1}{x^2}$

$\Rightarrow (x^2 + \frac{1}{x^2})^2 = x^4 + \frac{1}{x^4} + 2$

$\Rightarrow 9 = x^4 + \frac{1}{x^4} + 2 \qquad [\because x^2 + \frac{1}{x^2} = 3]$

$\Rightarrow x^4 + \frac{1}{x^4} = 7$

Hence, $x^2 + \frac{1}{x^2} = 3;\ x^4 + \frac{1}{x^4} = 7.$

7) If $x^2 + \frac{1}{x^2} = 66,$ find the value of $x - \frac{1}{x}$

Solution:

We have,

$(x - \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 - 2 * x * \frac{1}{x}$

$\Rightarrow (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$

$\Rightarrow (x - \frac{1}{x})^2 = 66 - 2 \quad [\because x^2 + \frac{1}{x^2} = 66]$

$\Rightarrow (x - \frac{1}{x})^2 = 64$

$\Rightarrow (x - \frac{1}{x})^2 = (\pm 8)^2$

$\Rightarrow x - \frac{1}{x} = \pm 8$

8) If $x^2 + \frac{1}{x^2} = 79,$ find the value of $x + \frac{1}{x}$

Solution:

We have,

$(x + \frac{1}{x})^2 = x^2 + (\frac{1}{x})^2 + 2 * x * \frac{1}{x}$

$\Rightarrow (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow (x + \frac{1}{x})^2 = 79 + 2 \quad [\because x^2 + \frac{1}{x^2} = 79]$

$\Rightarrow (x + \frac{1}{x})^2 = 81$

$\Rightarrow (x + \frac{1}{x})^2 = (\pm 9)^2$

$\Rightarrow x + \frac{1}{x} = \pm 9$

9) If $9x^2 + 25y^2 = 181$ and $xy = -6$, find the value of $3x + 5y$.

Solution:

We have,

$(3x + 5y)^2 = (3x)^2 + (5y)^2 + 2*3x*5y$

=>$(3x + 5y)^2 = 9x^2 + 25y^2 + 30xy$

= 181 + 30(-6) [Since, $9x^2 + 25y^2 = 181$ and xy = -6]

=>$(3x+5y)^2 = 1$

=>$(3x + 5y)^2 = (\pm 1)^2$

=>$3x + 5y = \pm 1$

10) If $2x + 3y = 8$ and $xy = 2$, find the value of $4x^2 + 9y^2$.

Solution:

We have,

$(2x + 3y)^2 = (2x)^2 + (3y)^2 + 2*2x*3y$

=> $(2x + 3y)^2 = 4x^2 + 9y^2 + 12xy$ [Since, 2x + 3y = 8 and xy = 24]

=> $(8)^2 = 4x^2 + 9y^2 + 24$

=> $64 - 24 = 4x^2 + 9y^2$

=> $4x^2 + 9y^2 = 40$

11) If $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$.

Solution:

We have,

$(2 - 7y)^2 = (3x)^2 + (-7y)^2 - 2*3x*7y$

=> $(3x - 7y)^2 = 9x^2 + 49y^2 - 42xy$ [Since, 3x − 7y = 10 and xy = -1]

=> $(10)^2 = 9x^2 + 49y^2 + 42$

=> $100 - 42 = 9x^2 + 49y^2$

=> $9x^2 + 49y^2 = 58$

12) Simplify each of the following products:

(i) $(\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2)$

Solution:

$(\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2)$

$\Rightarrow [(\frac{1}{2}a)^2 - (3b)^2)]\ [(\frac{1}{4}a^2 + 9b^2)]$ $\quad [\because (a+b)(a-b) = a^2 - b^2)]$

$\Rightarrow [\frac{1}{4}a^2 - 9b^2)]\ [\frac{1}{4}a^2 + 9b^2]$ $\quad [\because (ab)^2 = a^2b^2)]$

$= [(\frac{1}{4}a^2)^2 - (9b^2)^2]$ $\quad [\because (a+b)(a-b) = a^2 - b^2)]$

$= \frac{1}{16}a^4 - 81b^4$

$\therefore (\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2) = \frac{1}{16}a^4 - 81b^4$

(ii) $(m + \frac{n}{7})^3\ (m - \frac{n}{7})$

Solution:

We have,

$(m + \frac{n}{7})^3\ (m - \frac{n}{7})$

$= (m + \frac{n}{7})\ (m + \frac{n}{7})\ (m + \frac{n}{7})\ (m - \frac{n}{7})$

$= (m + \frac{n}{7})^2\ (m^2 - (\frac{n}{7})^2)$ $\quad [\because (a+b)(a+b) = (a+b)^2$ and $(a+b)(a-b) = a^2 - b^2]$

$= (m + \frac{n}{7})^2\ (m^2 - \frac{n^2}{49})$

$\therefore (m + \frac{n}{7})^3\ (m - \frac{n}{7}) = (m + \frac{n}{7})^2\ (m^2 - \frac{n^2}{49})$

(iii) $(\frac{x}{2} - \frac{2}{5})\ (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$

Solution:

We have,

$(\frac{x}{2} - \frac{2}{5})\ (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$

$\Rightarrow -(\frac{2}{5} - \frac{x}{2})\ (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x$

$\Rightarrow -(\frac{2}{5} - \frac{x}{2})^2 - x^2 + 2x$ $\quad [\because (a-b)(a-b) = (a-b)^2]$

$\Rightarrow -[(\frac{2}{5})^2 + (\frac{x}{2})^2 - 2(\frac{2}{5})(\frac{x}{2})] - x^2 + 2x$

$\Rightarrow -(\frac{4}{25} + \frac{x^2}{4} - \frac{2x}{5}) - x^2 + 2x$

$\Rightarrow -\frac{x^2}{4} - x^2 + \frac{2x}{5} + 2x - \frac{4}{25}$

$\Rightarrow -\frac{5x^2}{4} + \frac{12x}{5} - \frac{4}{25}$

$\therefore (\frac{x}{2} - \frac{2}{5})\ (\frac{2}{5} - \frac{x}{2}) - x^2 + 2x = -\frac{5x^2}{4} + \frac{12x}{5} - \frac{4}{25}$

(iv) $(x^2 + x - 2)\ (x^2 - x + 2)$

Solution:

$(x^2 + x - 2)\ (x^2 - x + 2)$

$[(x)^2 + (x - 2)]\ [(x^2 - (x + 2)]$

=>$(x^2)^2 - (x - 2)^2$ $[(a - b)(a + b) = a^2 - b^2]$

=>$x^4 - (x^2 + 4 - 4x)$ $[\because (a - b)^2 = a^2 + b^2 - 2ab]$

=>$x^4 - x^2 + 4x - 4$

$\therefore (x^2 + x - 2)(x^2 - x + 2) = x^4 - x^2 + 4x - 4$

(v) $(x^3 - 3x - x)(x^2 - 3x + 1)$

Solution:

We have,

$(x^3 - 3x - x)(x^2 - 3x + 1)$

=>$x(x^2 - 3x - 1)(x^2 - 3x + 1)$

=>$x[(x^2 - 3x)^2 - (1)^2]$ $[\because (a + b)(a - b) = a^2 - b^2]$

=>$x[(x^2)^2 + (-3x)^2 - 2(3x)(x^2) - 1]$

=>$x[x^4 + 9x^2 - 6x^3 - 1]$

=>$x^5 - 6x^4 + 9x^3 - x$

$\therefore (x^3 - 3x - x)(x^2 - 3x + 1) = x^5 - 6x^4 + 9x^3 - x$

(vi) $(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$

Solution:

We have,

$(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$

=>$[(2x^4 - 4x^2)^2 - (1)^2]$ $[\because (a + b)(a - b) = a^2 - b^2]$

=>$[(2x^4)^2 + (4x^2)^2 - 2(2x^4)(4x^2) - 1]$

=>$4x^8 - 16x^6 + 16x^4 - 1$ $[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$\therefore (2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1) = 4x^8 - 16x^6 + 16x^4 - 1$

13) Prove that $a^2 + b^2 + c^2 - ab - bc - ca$ ***is always non-negative for all values of a, b and c.***

Solution:

We have,

$a^2 + b^2 + c^2 - ab - bc - ca$

Multiply and divide by '2'

= $\frac{2}{2}\ [a^2 + b^2 + c^2 - ab - bc - ca]$

= $\frac{1}{2}\ [2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$

$= \frac{1}{2}\ [a^2 + a^2 + b^2 + b^2 + c^2 + c^2 - 2ab - 2bc - 2ca]$

$= \frac{1}{2}\ [(a^2 + b^2 - 2ab) + (a^2 + c^2 - 2ca) + (b^2 + c^2 - 2bc)]$

$= \frac{1}{2}\ [(a - b)^2 + (b - c)^2 + (c - a)^2] \qquad [\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= \frac{(a-b)^2+(b-c)^2+(c-a)^2}{2} \geq 0$

$\therefore a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

Hence, $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$ is always non-negative for all values of a, b and c.