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Solutions

Class 9 Maths

Chapter 4

Ex *4.2*

Question 1: Write the following in the expand form:

(i): $(a + 2b + c)^2$

(ii): $(2a - 3b - c)^2$

(iii): $(-3x + y + z)^2$

(iv): $(m + 2n - 5p)^2$

(v): $(2 + x - 2y)^2$

(vi): $(a^2 + b^2 + c^2)^2$

(vii): $(ab + bc + ca)^2$

(viii): $(\frac{x}{y} + \frac{y}{z} + \frac{z}{x})^2$

(ix): $(\frac{a}{bc} + \frac{b}{ac} + \frac{c}{ab})^2$

(x): $(x + 2y + 4z)^2$

(xi): $(2x - y + z)^2$

(xii): $(-2x + 3y + 2z)^2$

Solution 1(i):

We have,

$(a + 2b + c)^2 = a^2 + (2b)^2 + c^2 + 2a(2b) + 2ac + 2(2b)c$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$\therefore (a + 2b + c)^2 = a^2 + 4b^2 + c^2 + 4ab + 2ac + 4bc$

Solution 1(ii):

We have,

$(2a - 3b - c)^2 = [(2a) + (-3b) + (-c)]^2$

$(2a)^2 + (-3b)^2 + (-c)^2 + 2(2a)(-3b) + 2(-3b)(-c) + 2(2a)(-c)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca$

∴ (2a-3b-c)^{2}=4x^{2}+9y^{2}+c^{2}-12ab+6bc-4ca

Solution 1(iii):

We have,

$(-3x + y + z)^2 = [(-3x)^2 + y^2 + z^2 + 2(-3x)y + 2yz + 2(-3x)z$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$

$(-3x + y + z)^2 = 9x^2 + y^2 + z^2 - 6xy + 2xy - 6xy$

Solution 1(iv):

We have,

$(m + 2n - 5p)^2 = m^2 + (2n)^2 + (-5p)^2 + 2m \times 2n + (2 \times 2n \times -5p) + 2m \times -5p$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(m + 2n - 5p)^2 = m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$

Solution 1(v):

We have,

$(2 + x - 2y)^2 = 2^2 + x^2 + (-2y)^2 + 2(2)(x) + 2(x)(-2y) + 2(2)(-2y)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$= 4 + x^2 + 4y^2 + 4x - 4xy - 8y$

$(2 + x - 2y)^2 = 4 + x^2 + 4y^2 + 4x - 4xy - 8y$

Solution 1(vi):

We have,

$(a^2 + b^2 + c^2)^2 = (a^2)^2 + (b^2)^2 + (c^2)^2 + 2a^2b^2 + 2b^2c^2 + 2a^2c^2$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(a^2 + b^2 + c^2)^2 = a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$

Solution 1(vii):

We have,

$(ab + bc + ca)^2 = (ab)^2 + (bc)^2 + (ca)^2 + 2(ab)(bc) + 2(bc)(ca) + 2(ab)(ca)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$= a^2b^2 + b^2c^2 + c^2a^2 + 2(ac)b^2 + 2(ab)(c)^2 + 2(bc)(a)^2$

$(ab + bc + ca)^2 = a^2b^2 + b^2c^2 + c^2a^2 + 2acb^2 + 2abc^2 + 2bca^2$

Solution 1(viii):

We have,

$(\frac{x}{y} + \frac{y}{z} + \frac{z}{x})^2 = (\frac{x}{y})^2 + (\frac{y}{z})^2 + (\frac{z}{x})^2 + 2\frac{x}{y}\frac{y}{z} + 2\frac{y}{z}\frac{z}{x} + 2\frac{z}{x}\frac{x}{y}$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$\therefore (\frac{x}{y} + \frac{y}{z} + \frac{z}{x})^2 = (\frac{x^2}{y^2}) + (\frac{y^2}{z^2}) + (\frac{z^2}{x^2}) + 2\frac{x}{z} + 2\frac{y}{x} + 2\frac{x}{y}$

Solution 1(ix):

We have,

$(\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab})^2 = (\frac{a}{bc})^2 + (\frac{b}{ca})^2 + (\frac{c}{ab})^2 + 2(\frac{a}{bc})(\frac{b}{ca}) + 2(\frac{b}{ca})(\frac{c}{ab}) + 2(\frac{a}{bc})(\frac{c}{ab})$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab})^2 = (\frac{a^2}{b^2c^2}) + (\frac{b^2}{c^2a^2}) + (\frac{c^2}{a^2b^2}) + \frac{2}{a^2} + \frac{2}{b^2} + \frac{2}{c^2}$

Solution 1(x):

We have,

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2x \times 2y + 2 \times 2y \times 4z + 2x \times 4z$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(x + 2y + 4z)^2 = x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

Solution 1(xi):

We have,

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(2x - y + z)^2 = 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

Solution 1 (xii):

We have,

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$(-4x + 6y + 4z)^2 = 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

Question 2: Use algebraic identities to expand the following algebraic equations.

Q 2.1: $(a + b + c)^2 + (a - b + c)^2$

Ans : We have,

$(a + b + c)^2 + (a - b + c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) + (a^2 + (-b)^2 + c^2 - 2ab - 2bc + 2ca)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$= 2a^2 + 2b^2 + 2c^2 + 4ca$

$(a + b + c)^2 + (a - b + c)^2 = 2a^2 + 2b^2 + 2c^2 + 4ca$

Q 2.2: $(a + b + c)^2 - (a - b + c)^2$

Ans: We have,

$(a + b + c)^2 - (a - b + c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$
$- (a^2 + (-b)^2 + c^2 - 2ab - 2bc + 2ca)$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - b^2 - c^2 + 2ab + 2bc - 2ca)$

$= 4ab + 4bc$

$(a + b + c)^2 - (a - b + c)^2 = 4ab + 4bc$

Q 2.3: $(a + b + c)^2 + (a + b - c)^2 + (a + b - c)^2$

Ans: We have,

$$(a + b + c)^2 + (a + b - c)^2 + (a + b - c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$+ (a^2 + b^2 + (z)^2 - 2bc - 2ab + 2ca) + (a^2 + b^2 + c^2 - 2ca - 2bc + 2ab)$$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$= 3a^2 + 3b^2 + 3c^2 + 2ab + 2bc + 2ca - 2bc - 2ab - 2ca - 2bc + 2ab$

$= 3x^2 + 3y^2 + 3z^2 + 2ab - 2bc + 2ca$

$(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 = 3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca$

$(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 = 3(a^2 + b^2 + c^{2)} + 2(ab - bc + ca)$

Q 2.4: $(2x + p - c)^2 - (2x - p + c)^2$

Ans: We have,

$$(2x + p - c)^2 - (2x - p + c)^2 = [2x^2 + p^2 + (-c)^2 + 2(2x)p + 2p(-c) + 2(2x)(-c)]$$
$$- [4x^2 + (-p)^2 + c^2 + 2(2x)(-p) + 2c(-p) + 2(2x)c]$$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

$$(2x + p - c)^2 - (2x - p + c)^2 = [4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc]$$
$$- [4x^2 + p^2 + c^2 - 4xp - 2pc + 4xc]$$

Opening the bracket,

$$(2x + p - c)^2 - (2x - p + c)^2 = 4x^2 + p^2 + c^2 + 4xp - 2pc - 4cx - 4x^2 - p^2 - c^2 + 4xp$$
$$+ 2pc - 4cx]$$

$(2x + p - c)^2 - (2x - p + c)^2 = 8xp - 8xc$

$=8x(p - c)$

Hence, $(2x + p - c)^2 - (2x - p + c)^2 = 8x(p - c)$

Q 2.5: $(x^2 + y^2 + (-z)^2) - (x^2 - y^2 + z^2)^2$

Ans: We have,

$(x^2 + y^2 + (-z)^2)^2 - (x^2(-y)^2 + z^2)^2$

=

$$[x^4 + y^4 + (-z)^4 + 2x^2y^2 + 2y^2(-z)^2 + 2x^2(-z)^2]$$
$$- [x^4 + (-y)^4 + z^4 - 2x^2y^2 - 2y^2z^2 + 2x^2z^2]$$

$[\because (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz]$

Taking the negative sign inside,

=

$$[x^4 + y^4 + (-z)^4 + 2x^2y^2 + 2y^2(-z)^2 + 2x^2(-z)^2]$$
$$- [x^4 + (-y)^4 + z^4 - 2x^2y^2 - 2y^2z^2 + 2x^2z^2]$$

= $4x^2y^2 – 4z^2x^2$

Hence, $(x^2 + y^2 + (-z)^2)^2 - (x^2(-y)^2 + z^2)^2 = 4x^2y^2 – 4z^2x^2$

Q3: If a+b+c=0 and $a^2 + b^2 + c^2 = 16$***, find the value of ab+bc+ca:***

Ans: We know that,

$[\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$(0)^2$=16+2(ab+bc+ca)
2(ab+bc+ca)=-16
ab+bc+ca=-8

Hence, value of required express ab+bc+ca =-8

Q4: If $a^2 + b^2 + c^2 = 16$ ***and ab+bc+ca=10, find the value of a+b+c?***

Ans: We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
$$(x + y + z)^2 = 16 + 2(10)$$
$$(x + y + z)^2 = 36$$
$$(x + y + z) = \sqrt{36}$$
$$(x + y + z) = \pm 6$$

Hence, value of required expression I; $(a + b + c) = \pm 8$

Q5: If a+b+c=9 and ab+bc+ca=23, find value of $a^2 + b^2 + c^2$

Ans: We know that,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
$$9^2 = a^2 + b^2 + c^2 + 2(23)$$
$$81 = a^2 + b^2 + c^2 + 46$$
$$a^2 + b^2 + c^2 = 81 - 46$$
$$a^2 + b^2 + c^2 = 35$$

Hence, value of required expression $a^2 + b^2 + c^2 = 35$

Q6: Find the value of the equation : $4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20zx$ ***when x=4,y=3,z=2***

Ans: $4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20zx$

$$(2x)^2 + y^2 + (-5z)^2 + 2(2x)(y) + 2(y)(-5z) + 2(-5z)(2x)$$
$$(2x + y - 5z)^2$$
$$(2(4) + 3 - 5(2))^2$$
$$(8 + 3 - 10)^2$$
$$(1)^2$$

1

Hence value of the equation is equals to 1

Q7: Simplify each of the following expressions:

Q 7.1: $(x+y+z)^2+(x+\frac{y}{2}+\frac{2}{3})^2-(\frac{x}{2}+\frac{y}{3}+\frac{z}{4})^2$

Ans: Expanding, we get

$$=[x^2+y^2+z^2+2xy+2yz+2zx]+[x^2+\frac{y^2}{4}+\frac{z^2}{9}+2x\frac{y}{2}+2\frac{zx}{3}+\frac{yz}{3}]$$

$$-[\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{10}+\frac{xy}{3}+\frac{yz}{6}+\frac{xz}{4}]$$

$[\because (x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2xz]$

$$=x^2+y^2+z^2+2xy+2yz+2zx+x^2+\frac{y^2}{4}+\frac{z^2}{9}+2x\frac{y}{2}+\frac{xy}{3}+\frac{2zx}{3}-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{10}-\frac{xy}{3}$$
$$-\frac{yz}{6}-\frac{xz}{4}$$

Rearranging coefficients ,

$$=\frac{8x^2-x^2}{4}+\frac{36y^2+9y^2-4y^2}{36}+\frac{144z^2+16z^2-9z^2}{144}+\frac{6xy+3xy-xy}{3}+\frac{13yz}{6}+\frac{29xz}{12}$$

$$=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

$$(x+y+z)^2+(x+\frac{y}{2}+\frac{z}{3})^2-(\frac{x}{2}+\frac{y}{3}+\frac{z}{4})^2=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

Q 7.2: $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

Ans: $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

$$=[x^2+y^2+4z^2+2xy+2y(-2z)+2a(-2c)]-x^2-y^2-3z^2+4xy$$

$$=z^2+6xy-4yz-4zx$$

$$(x+y-2z)^2-x^2-y^2-3z^2+4xy=z^2+6xy-4yz-4zx$$

Q 7.3: $[x^2-x+1]^2-[x^2+x+1]^2$

Ans: $[x^2-x+1]^2-[x^2+x+1]^2$

$$=(x^2)^2+(-x)^2+1^2+2(x^2)(-x)+2(-x)(1)+2x^2)$$
$$-[(x^2)^2+x^2+1+2x^2x+2x(1)+2x^2(1)]$$

$[\because (x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2xz]$

$$=x^4+y^2+1-2x^3-2x+2x^2-x^2-x^4-1-2x^3-2x-2x^2$$

$$=-4x^3-4x$$

$$=-4x(x^2+1)$$

Hence simplified equation $=[x^2-x+1]^2-[x^2+x+1]^2=-4x(x^2+1)$