RD SHARMA

Solutions

Class 9 Maths

Chapter 4

Ex *4.2*

Q1. Find the cube of each of the following binomial expression

(a) $(\frac{1}{x} + \frac{y}{3})$

(b) $(\frac{3}{x} - \frac{2}{x^2})$

(c) $(2x + \frac{3}{x})$

(d) $(4 - \frac{1}{3x})$

Sol :

(a) $(\frac{1}{x} + \frac{y}{3}))^3$

Given, $(\frac{1}{x} + \frac{y}{3}))^3$

The above equation is in the form of $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

We know that, $a = \frac{1}{x}$, $b = \frac{y}{3}$

By using $(a + b)^3$ formula

$(\frac{1}{x} + \frac{y}{3}))^3 = (\frac{1}{x})^3 + (\frac{y}{3})^3 + 3(\frac{1}{x})(\frac{y}{3})(\frac{1}{x} + \frac{y}{3})$

$= \frac{1}{x^3} + \frac{y^3}{27} + 3 * \frac{1}{x} * \frac{y}{3}(\frac{1}{x} + \frac{y}{3})$

$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x}(\frac{1}{x} + \frac{y}{3})$

$= \frac{1}{x^3} + \frac{y^3}{27} + (\frac{y}{x} * \frac{1}{x}) +$

$(\frac{y}{x} * \frac{y}{3})$

$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x})$

Hence,

$(\frac{1}{x} + \frac{y}{3}))^3 = \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x})$

(b) $((\frac{3}{x} - \frac{2}{x^2}))^3$

Given, $((\frac{3}{x} - \frac{2}{x^2}))^3$

The above equation is in the form of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We know that, $a = \frac{3}{x}$, $b = \frac{2}{x^2}$

By using $(a - b)^3$ formula

$((\frac{3}{x} - \frac{2}{x^2}))^3 = (\frac{3}{x})^3 - (\frac{2}{x^2})^3 - 3(\frac{3}{x})(\frac{2}{x^2})(\frac{3}{x} - \frac{2}{x^2})$

$= \frac{27}{x^3} - \frac{8}{x^6} - 3 * \frac{3}{x} * \frac{2}{x^2}(\frac{3}{x} - \frac{2}{x^2})$

= $\frac{27}{x^3} - \frac{8}{x^6} - \frac{18}{x^3}(\frac{3}{x} - \frac{2}{x^2})$

= $\frac{27}{x^3} - \frac{8}{x^6} - (\frac{18}{x^3} * \frac{3}{x}) + (\frac{18}{x^3} * \frac{2}{x^2})$

= $\frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$

Hence, $((\frac{3}{x} - \frac{2}{x^2}))^3 = \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$

(c) $(2x + \frac{3}{x})^3$

Given, $(2x + \frac{3}{x})^3$

The above equation is in the form of $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

We know that, a = 2x , b = $\frac{3}{x}$

By using $(a + b)^3$ formula

= $8x^3 + \frac{27}{x^3} + \frac{18x}{x}(2x + \frac{3}{x})$

= $8x^3 + \frac{27}{x^3} + \frac{18x}{x}(2x + \frac{3}{x})$

= $8x^3 + \frac{27}{x^3} +$

$(18 * 2x) + (18 * \frac{3}{x})$

= $8x^3 + \frac{27}{x^3} + 36x \frac{54}{x})$

Hence,

The cube of $(2x + \frac{3}{x})^3 = 8x^3 + \frac{27}{x^3} + 36x \frac{54}{x})$

(d) $(4 - \frac{1}{3x})^3$

Given, $(4 - \frac{1}{3x})^3$

The above equation is in the form of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We know that, a = 4 , b = $\frac{1}{3x}$

By using $(a - b)^3$ formula

$(4 - \frac{1}{3x})^3 = 4^3 - (\frac{1}{3x})^3 - 3(4)(\frac{1}{3x})(4 - \frac{1}{3x})$

= $64 - \frac{1}{27x^3} - \frac{12}{3x}(4 - \frac{1}{3x})$

= $64 - \frac{1}{27x^3} - \frac{4}{x}(4 - \frac{1}{3x})$

= $64 - \frac{1}{27x^3} - (\frac{4}{3x} * 4) + (\frac{4}{3x} * \frac{1}{3x})$

= $64 - \frac{1}{27x^3} - \frac{16}{x} + (\frac{4}{3x^2}$

Hence ,

The cube of $(4-\frac{1}{3x})^3$ = 64 − $\frac{1}{27x^3}$ − $\frac{16}{x}$ + $(\frac{4}{3x^2})$

Q2. Simplify each of the following

(a) $(x + 3)^3 + (x - 3)^3$

(b) $(\frac{x}{2}+\frac{y}{3})^3 - (\frac{x}{2}-\frac{y}{3})^3$

(c) $(x+\frac{2}{x})^3 + (x-\frac{2}{x})^3$

(d) $(2x - 5y)^3 - (2x + 5y)^3$

Sol :

(a) $(x + 3)^3 + (x - 3)^3$

The above equation is in the form of $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

We know that, a = (x + 3) , b = (x − 3)

By using $(a^3 + b^3)$ formula

$= (x + 3 + x - 3)[(x + 3)^3 + (x - 3)^3 - (x + 3)(x - 3)]$

$= 2x[(x^2 + 3^2 + 2*x*3) + (x^2 + 3^2 - 2*x*3) - (x^2 - 3^2)]$

$= 2x[(x^2 + 9 + 6x) + (x^2 + 9 - 6x) - x^2 + 9]$

$= 2x[(x^2 + 9 + 6x + x^2 - 9 - 6x - x^2 + 9)]$

$= 2x (x^2 + 27)$

$= 2x^3 + 54x$

Hence, the result of $(x + 3)^3 + (x - 3)^3$ is $2x^3 + 54x$

(b) $(\frac{x}{2}+\frac{y}{3})^3 - (\frac{x}{2}-\frac{y}{3})^3$

The above equation is in the form of $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

We know that, a = $(\frac{x}{2}+\frac{y}{3})^3$, b = $(\frac{x}{2}-\frac{y}{3})^3$

By using $(a^3 - b^3)$ formula

$= [((\frac{x}{2}+\frac{y}{3})^3 - ((\frac{x}{2}-\frac{y}{3})^3)][((\frac{x}{2}+\frac{y}{3})^3)^2 ((\frac{x}{2}-\frac{y}{3})^3)^2 - ((\frac{x}{2}+\frac{y}{3})^3)((\frac{x}{2}-\frac{y}{3})^3)$

$= (\frac{x}{3}+\frac{y}{3}-\frac{x}{2}+\frac{y}{3})[((\frac{x}{2})^2+(\frac{y}{3})^2+(\frac{2xy}{6}))+((\frac{x}{2})^2+(\frac{y}{3})^2-(\frac{2xy}{6}))+((\frac{x}{2})^2-(\frac{y}{3})^2)]$

$= \frac{2y}{3}[(\frac{x^2}{4}+\frac{y^2}{9}+\frac{2xy}{6})+(\frac{x^2}{4}+\frac{y^2}{9}-\frac{2xy}{6})+\frac{x^2}{4}-\frac{y^2}{9}]$

$= \frac{2y}{3}[\frac{x^2}{4}+\frac{y^2}{9}+\frac{2xy}{6}+\frac{x^2}{4}+\frac{y^2}{9}-\frac{2xy}{6}+\frac{x^2}{4}-\frac{y^2}{9}]$

$= \frac{2y}{3}[\frac{x^2}{4}+\frac{y^2}{9}+\frac{x^2}{4}+\frac{x^2}{4}]$

$= \frac{2y}{3}[\frac{3x^2}{4}+\frac{y^2}{9}]$

$= \frac{x^2y}{2}+\frac{2y^3}{27}$

Hence, the result of $(\frac{x}{2}+\frac{y}{3})^3 - (\frac{x}{2}-\frac{y}{3})^3 = \frac{x^2y}{2}+\frac{2y^3}{27}$

(c) $(x+\frac{2}{x})^3 + (x-\frac{2}{x})^3$

The above equation is in the form of $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

We know that, $a = (x+\frac{2}{x})^3$, $b = (x-\frac{2}{x})^3$

By using $(a^3 + b^3)$ formula

$= (x+\frac{2}{x}+x-\frac{2}{x})[(x+\frac{2}{x})^2+(x-\frac{2}{x})^2-((x+\frac{2}{x})(x-\frac{2}{x}))]$

$= (2x)[(x^2+\frac{4}{x^2}+\frac{4x}{x})+(x^2+\frac{4}{x^2}-\frac{4x}{x})-(x^2-\frac{4}{x^2})$

$= (2x)[(x^2+\frac{4}{x^2}+\frac{4x}{x}+x^2+\frac{4}{x^2}-\frac{4x}{x}-x^2+\frac{4}{x^2})$

$= (2x)[(x^2+\frac{4}{x^2}+\frac{4}{x^2}+\frac{4}{x^2})$

$= (2x)[(x^2+\frac{12}{x^2})$

$= 2x^3+\frac{24}{x}$

Hence, the result of $(x+\frac{2}{x})^3 + (x-\frac{2}{x})^3 = (2x)[(x^2+\frac{12}{x^2})$

(d) $(2x - 5y)^3 - (2x + 5y)^3$

Given, $(2x - 5y)^3 - (2x + 5y)^3$

The above equation is in the form of $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

We know that, $a = (2x - 5y)$, $b = (2x + 5y)$

By using $(a^3 - b^3)$ formula

$= (2x - 5y - 2x - 5y)[(2x - 5y)^2 + (2x + 5y)^2 + ((2x - 5y) * (2x + 5y))]$

$= (-10y)[(4x^2 + 25y^2 - 20xy) + (4x^2 + 25y^2 + 20xy) + 4x^2 - 25y^2]$

$= (-10y)[4x^2 + 25y^2 - 20xy + 4x^2 + 25y^2 + 20xy + 4x^2 - 25y^2]$

$= (-10y)[4x^2 + 4x^2 + 4x^2 + 25y^2]$

$= (-10y)[12x^2 + 25y^2\}$

$= -120x^2y - 250y^3$

Hence, the result of $(2x - 5y)^3 - (2x + 5y)^3 = -120x^2y - 250y^3$

Q3. If a + b = 10 and ab = 21, Find the value of $a^3 + b^3$

Sol :

Given,

$a + b = 10$, $ab = 21$

we know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ ——— 1

substitute $a + b = 10$, $ab = 21$ in eq 1

=> $(10)^3 = a^3 + b^3 + 3(21)(10)$

=> $1000 = a^3 + b^3 + 630$

=> $1000 - 630 = a^3 + b^3$

=> $370 = a^3 + b^3$

Hence, the value of $a^3 + b^3 = 370$

Q4. If a − b = 4 and ab = 21, Find the value of $a^3 - b^3$

Sol :

Given,

$a - b = 4$, $ab = 21$

we know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ ——— 1

substitute $a - b = 4$, $ab = 21$ in eq 1

=> $(4)^3 = a^3 - b^3 - 3(21)(4)$

=> $64 = a^3 - b^3 - 252$

=> $64 + 252 = a^3 - b^3$

=> $316 = a^3 - b^3$

Hence, the value of $a^3 - b^3 = 316$

Q5. If $(x + \frac{1}{x}) = 5$, Find the value of $x^3 + \frac{1}{x^3}$

Sol :

Given, $(x + \frac{1}{x}) = 5$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ —– 1

Substitute $(x + \frac{1}{x}) = 5$ in eq 1

$(x + \frac{1}{x})^3 = x^3 + \frac{1}{x^3} + 3(x * \frac{1}{x})(x + \frac{1}{x})$

$5^3 = x^3 + \frac{1}{x^3} + 3(x * \frac{1}{x})(x + \frac{1}{x})$

$125 = x^3 + \frac{1}{x^3} + 3(x + \frac{1}{x})$

$125 = x^3 + \frac{1}{x^3} + 3(5)$

$125 = x^3 + \frac{1}{x^3} + 15$

$125 - 15 = x^3 + \frac{1}{x^3}$

$x^3 + \frac{1}{x^3} = 110$

hence, the result is $x^3 + \frac{1}{x^3} = 110$

Q6. If $(x - \frac{1}{x}) = 7$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, If $(x - \frac{1}{x}) = 7$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ --- 1

Substitute $(x - \frac{1}{x}) = 7$ in eq 1

$(x - \frac{1}{x})^3 = x^3 - \frac{1}{x^3} - 3(x * \frac{1}{x})(x - \frac{1}{x})$

$7^3 = x^3 - \frac{1}{x^3} - 3(x - \frac{1}{x})$

$343 = x^3 - \frac{1}{x^3} - (3 * 7)$

$343 = x^3 - \frac{1}{x^3} - 21$

$343 + 21 = x^3 - \frac{1}{x^3}$

$x^3 - \frac{1}{x^3} = 364$

hence, the result is $x^3 - \frac{1}{x^3} = 364$

Q7. If $(x - \frac{1}{x}) = 5$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, If $(x - \frac{1}{x}) = 5$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ --- 1

Substitute $(x - \frac{1}{x}) = 5$ in eq 1

$(x - \frac{1}{x})^3 = x^3 - \frac{1}{x^3} - 3(x * \frac{1}{x})(x - \frac{1}{x})$

$5^3 = x^3 - \frac{1}{x^3} - 3(x - \frac{1}{x})$

$125 = x^3 - \frac{1}{x^3} - (3 * 5)$

$125 = x^3 - \frac{1}{x^3} - 15$

$125 + 15 = x^3 - \frac{1}{x^3}$

$x^3 - \frac{1}{x^3} = 140$

hence, the result is $x^3 - \frac{1}{x^3} = 140$

Q8. If $(x^2 + \frac{1}{x^2}) = 51$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, $(x^2 + \frac{1}{x^2}) = 51$

We know that , $(x - y)^2 = x^2 + y^2 - 2xy$ ——— 1

Substitute $(x^2 + \frac{1}{x^2}) = 51$ in eq 1

$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2 * x * \frac{1}{x}$

$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$

$(x - \frac{1}{x})^2 = 51 - 2$

$(x - \frac{1}{x})^2 = 49$

$(x - \frac{1}{x}) = \sqrt{49}$

$(x - \frac{1}{x}) = \pm 7$

We need to find $x^3 - \frac{1}{x^3}$

So, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$x^3 - \frac{1}{x^3} = (x - \frac{1}{x})(x^{2} + \frac{1}{x^2} + (x * \frac{1}{x})$

We know that ,

$(x - \frac{1}{x}) = 7$ and $(x^2 + \frac{1}{x^2}) = 51$

$x^3 - \frac{1}{x^3} = 7(51 + 1)$

$x^3 - \frac{1}{x^3} = 7(52)$

$x^3 - \frac{1}{x^3} = 364$

Hence, the value of $x^3 - \frac{1}{x^3} = 364$

Q9. If $(x^2 + \frac{1}{x^2}) = 98$, Find the value of $x^3 + \frac{1}{x^3}$

Sol :

Given, $(x^2 + \frac{1}{x^2}) = 98$

We know that , $(x + y)^2 = x^2 + y^2 + 2xy$ ——— 1

Substitute $(x^2 + \frac{1}{x^2}) = 98$ in eq 1

$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2 * x * \frac{1}{x}$

$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2$

$(x + \frac{1}{x})^2 = 98 + 2$

$(x + \frac{1}{x})^2 = 100$

$(x + \frac{1}{x}) = \sqrt{100}$

$(x + \frac{1}{x}) = \pm 10$

We need to find $x^3 + \frac{1}{x^3}$

So, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$x^3 + \frac{1}{x^3} = (x + \frac{1}{x})(x^2 + \frac{1}{x^2} - (x * \frac{1}{x})$

We know that,

$(x + \frac{1}{x}) = 10$ and $(x^2 + \frac{1}{x^2}) = 98$

$x^3 + \frac{1}{x^3} = 10(98 - 1)$

$x^3 + \frac{1}{x^3} = 10(97)$

$x^3 + \frac{1}{x^3} = 970$

Hence, the value of $x^3 + \frac{1}{x^3} = 970$

Q10. If $2x + 3y = 13$ and $xy = 6$, Find the value of $8x^3 + 27y^3$

Sol :

Given , $2x + 3y = 13$, $xy = 6$

We know that,

$(2x + 3y)^3 = 13^2$

=> $8x^3 + 27y^3 + 3(2x)(3y)(2x + 3y) = 2197$

=> $8x^3 + 27y^3 + 18xy(2x + 3y) = 2197$

Substitute $2x + 3y = 13$, $xy = 6$

=> $8x^3 + 27y^3 + 18(6)(13) = 2197$

=> $8x^3 + 27y^3 + 1404 = 2197$

=> $8x^3 + 27y^3 = 2197 - 1404$

=> $8x^3 + 27y^3 = 793$

Hence, the value of $8x^3 + 27y^3 = 793$

Q11. If $3x - 2y = 11$ and $xy = 12$, Find the value of $27x^3 - 8y^3$

Sol :

Given, $3x - 2y = 11$, $xy = 12$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a + b)$

$(3x - 2y)^3 = 11^3$

=> $27x^3 - 8y^3 - (18 * 12 * 11) = 1331$

=> $27x^3 - 8y^3 - 2376 = 1331$

=> $27x^3 - 8y^3 = 1331 + 2376$

=> $27x^3 - 8y^3 = 3707$

Hence, the value of $27x^3 - 8y^3 = 3707$

Q12. If $x^4 + (\frac{1}{x^4})$ = 119 , Find the value of $x^3 - (\frac{1}{x^3})$

Sol:

Given, $x^4 + (\frac{1}{x^4}) = 119$ ---- 1

We know that $(x + y)^2 = x^2 + y^2 + 2xy$

Substitute $x^4 + (\frac{1}{x^4}) = 119$ in eq 1

$(x^2 + (\frac{1}{x^2}))^2 = x^4 + (\frac{1}{x^4}) + (2 * x^2 * \frac{1}{x^2})$

$= x^4 + (\frac{1}{x^4}) + 2$

$= 119 + 2$

$= 121$

$(x^2 + (\frac{1}{x^2}))^2 = 121$

$x^2 + (\frac{1}{x^2}) = \sqrt{121}$

$x^2 + (\frac{1}{x^2}) = \pm 11$

Now, find $(x - \frac{1}{x})$

We know that $(x - y)^2 = x^2 + y^2 - 2xy$

$(x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - (2 * x * \frac{1}{x}$

$= x^2 + \frac{1}{x^2} - 2$

$= 11 - 2$

$= 9$

$(x - \frac{1}{x}) = \sqrt{9}$

$= \pm 3$

We need to find $x^3 - (\frac{1}{x^3})$

We know that, $a^3 - b^3 = (a - b)(a^2 + b^2 - ab)$

$x^3 - (\frac{1}{x^3}) = (x - \frac{1}{x})(x^2 + (\frac{1}{x^2}) + x * \frac{1}{x}$

Here, $x^2 + (\frac{1}{x^2}) = 11$ and $(x - \frac{1}{x}) = 3$

$x^3 - (\frac{1}{x^3}) = 3(11 + 1)$

= 3(12)

= 36

Hence, the value of $x^3 - (\frac{1}{x^3}) = 36$

Q13. Evaluate each of the following

(a) $(103)^3$

(b) $(98)^3$

(c) $(9.9)^3$

(d) $(10.4)^3$

(e) $(598)^3$

(f) $(99)^3$

Sol :

Given,

(a) $(103)^3$

we know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

=> $(103)^3$ can be written as $(100 + 3)^3$

Here , a = 100 and b = 3

$(103)^3 = (100 + 3)^3$

$= (100)^3 + (3)^3 + 3(100)(3)(100 + 3)$

= 1000000 + 27 + (900*103)

= 1000000 + 27 + 92700

= 1092727

The value of $(103)^3$ = 1092727

(b) $(98)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

=> $(98)^3$ can be written as $(100 - 2)^3$

Here , a = 100 and b = 2

$(98)^3 = (100 - 2)^3$

$= (100)^3 - (2)^3 - 3(100)(2)(100 - 2)$

= 1000000 − 8 − (600*102)

= 1000000 − 8 − 58800

= 941192

The value of $(98)^3$ = 941192

(c) $(9.9)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

=> $(9.9)^3$ can be written as $(10 - 0.1)^3$

Here , a = 10 and b = 0.1

$(9.9)^3 = (10 - 0.1)^3$

$= (10)^3 - (0.1)^3 - 3(10)(0.1)(10 - 0.1)$

$= 1000 - 0.001 - (3*9.9)$

$= 1000 - 0.001 - 29.7$

$= 1000 - 29.701$

$= 970.299$

The value of $(9.9)^3 = 970.299$

(d) $(10.4)^3$

we know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

=> $(10.4)^3$ can be written as $(10 + 0.4)^3$

Here , a = 10 and b = 0.4

$(10.4)^3 = (10 + 0.4)^3$

$= (10)^3 + (0.4)^3 + 3(10)(0.4)(10 + 0.4)$

$= 1000 + 0.064 + (12*10.4)$

$= 1000 + 0.064 + 124.8$

$= 1000 + 124.864$

$= 1124.864$

The value of $(10.4)^3 = 1124.864$

(e) $(598)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

=> $(598)^3$ can be written as $(600 - 2)^3$

Here , a = 600 and b = 2

$(598)^3 = (600 - 2)^3$

$= (600)^3 - (2)^3 - 3(600)(2)(600 - 2)$

$= 216000000 - 8 - (3600*598)$

$= 216000000 - 8 - 2152800$

$= 216000000 - 2152808$

$= 213847192$

The value of $(598)^3 = 213847192$

(f) $(99)^3$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

=> $(99)^3$ can be written as $(100 - 1)^3$

Here , a = 100 and b = 1

$(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - (300*99)$

$= 1000000 - 1 - 29700$

$= 1000000 - 29701$

$= 970299$

The value of $(99)^3 = 970299$

Q14. Evaluate each of the following

(a) $111^3 - 89^3$

(b) $46^3 + 34^3$

(c) $104^3 + 96^3$

(d) $93^3 - 107^3$

Sol:

Given,

(a) $111^3 - 89^3$

the above equation can be written as $(100 + 11)^3 - (100 - 11)^3$

we know that, $(a + b)^3 - (a - b)^3 = 2[b^3 + 3ab^2]$

here, a= 100 b = 11

$(100 + 11)^3 - (100 - 11)^3 = 2[11^3 + 3(100)^2(11)]$

$= 2[1331 + 330000]$

$= 2[331331]$

$= 662662$

The value of $111^3 - 89^3 = 662662$

(b) $46^3 + 34^3$

the above equation can be written as $(40 + 6)^3 + (40 - 6)^3$

we know that, $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$

here, a= 40 , b = 4

$(40 + 6)^3 + (40 - 6)^3 = 2[40^3 + 3(6)^2(40)]$

$= 2[64000 + 4320]$

$= 2[68320]$

$= 1366340$

The value of $46^3 + 34^3 = 1366340$

(c) $104^3 + 96^3$

the above equation can be written as $(100 + 4)^3 + (100 - 4)^3$

we know that, $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$

here, a= 100 b = 4

$(100 + 4)^3 - (100 - 4)^3 \quad = 2[100^3 + 3(4)^2(100)]$

= 2[1000000 + 4800]

= 2[1004800]

= 2009600

The value of $104^3 + 96^3 = 2009600$

(a) $93^3 - 107^3$

the above equation can be written as $(100 - 7)^3 - (100 + 7)^3$

we know that, $(a - b)^3 - (a + b)^3 = -2[b^3 + 3ba^2]$

here, a= 93, b = 107

$(100 - 7)^3 - (100 + 7)^3 \quad = -2[7^3 + 3(100)^2(7)]$

= -2[343 + 210000]

= -2[210343]

= -420686

The value of $93^3 - 107^3 = -420686$

Q15. If $x + \frac{1}{x} = 3$, calculate $x^2 + \frac{1}{x^2}$, $x^3 + \frac{1}{x^3}$, $x^4 + \frac{1}{x^4}$

Sol :

Given, $x + \frac{1}{x} = 3$

We know that $(x + y)^2 = x^2 + y^2 + 2xy$

$(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + (2 * x * \frac{1}{x})$

$3^2 \qquad = x^2 + \frac{1}{x^2} + 2$

$9 - 2 \quad = x^2 + \frac{1}{x^2}$

$x^2 + \frac{1}{x^2} = 7$

squaring on both sides

$(x^2 + \frac{1}{x^2})^2 = 7^2$

$x^4 + \frac{1}{x^4} + 2 * x^2 * \frac{1}{x^2} = 49$

$x^4 + \frac{1}{x^4} + 2 = 49$

$x^4 + \frac{1}{x^4} = 49 - 2$

$x^4 + \frac{1}{x^4} = 47$

again , cubing on both sides

$(x + \frac{1}{x})^3 = 3^3$

$x^3 + \frac{1}{x^3} + 3x*\frac{1}{x}(x + \frac{1}{x}) = 27$

$x^3 + \frac{1}{x^3} + (3*3) = 27$

$x^3 + \frac{1}{x^3} + 9 = 27$

$x^3 + \frac{1}{x^3} = 27 - 9$

$x^3 + \frac{1}{x^3} = 18$

hence, the values are $x^2 + \frac{1}{x^2} = 7$, $x^4 + \frac{1}{x^4} = 47$, $x^3 + \frac{1}{x^3} = 18$

Q16. If $x^4 + \frac{1}{x^4} = 194$, calculate $x^2 + \frac{1}{x^2}, x^3 + \frac{1}{x^3}, x + \frac{1}{x}$

Sol :

Given,

$x^4 + \frac{1}{x^4} = 194$ ——— 1

add and subtract $(2*x^2 * \frac{1}{x^2})$ on left side in above given equation

$x^4 + \frac{1}{x^4} + (2*x^2 * \frac{1}{x^2}) - 2(2*x^2 * \frac{1}{x^2}) = 194$

$x^4 + \frac{1}{x^4} + (2*x^2 * \frac{1}{x^2}) - 2 = 194$

$(x^2 + \frac{1}{x^2})^2 - 2 = 194$

$(x^2 + \frac{1}{x^2})^2 = 194 + 2$

$(x^2 + \frac{1}{x^2})^2 = 196$

$(x^2 + \frac{1}{x^2}) = \sqrt{196}$

$(x^2 + \frac{1}{x^2}) = 14$ ———— 2

Add and subtract $(2*x* \frac{1}{x})$ on left side in eq 2

$(x^2 + \frac{1}{x^2}) + (2*x* \frac{1}{x}) - (2*x* \frac{1}{x}) = 14$

$(x + \frac{1}{x})^2 - 2 = 14$

$(x + \frac{1}{x})^2 = 14 + 2$

$(x + \frac{1}{x})^2 = 16$

$(x + \frac{1}{x}) = \sqrt{16}$

$(x + \frac{1}{x}) = 4$ ———- 3

Now, cubing eq 3 on both sides

$(x + \frac{1}{x})^3 = 4^3$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$x^3 + \frac{1}{x^3} + 3*x*\frac{1}{x}(x + \frac{1}{x}) = 64$

$x^3 + \frac{1}{x^3} + (3*4) = 64$

$x^3 + \frac{1}{x^3} = 64 - 12$

$x^3 + \frac{1}{x^3} = 52$

hence, the values of $(x^2 + \frac{1}{x^2})^2 = 196$, $(x + \frac{1}{x}) = 4$, $x^3 + \frac{1}{x^3} = 52$

Q17. Find the values of $27x^3 + 8y^3$, if

(a) $3x + 2y = 14$ and $xy = 8$

(b) $3x + 2y = 20$ and $xy = \frac{14}{9}$

Sol:

(a) Given, $3x + 2y = 14$ and $xy = 8$

cubing on both sides

$(3x + 2y)^3 = 14^3$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 2744$

$27x^3 + 8y^3 + 18xy(3x + 2y) = 2744$

$27x^3 + 8y^3 + 18(8)(14) = 2744$

$27x^3 + 8y^3 + 2016 = 2744$

$27x^3 + 8y^3 = 2744 - 2016$

$27x^3 + 8y^3 = 728$

Hence, the value of $27x^3 + 8y^3 = 728$

(b) Given, $3x + 2y = 20$ and $xy = \frac{14}{9}$

cubing on both sides

$(3x + 2y)^3 = 20^3$

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 8000$

$27x^3 + 8y^3 + 18xy(3x + 2y) = 8000$

$27x^3 + 8y^3 + 18(\frac{14}{9})(20) = 8000$

$27x^3 + 8y^3 + 560 = 8000$

$27x^3 + 8y^3 \quad = 8000 - 560$

$27x^3 + 8y^3 \quad = 7440$

Hence, the value of $27x^3 + 8y^3 = 7440$

Q18. Find the value of $64x^3 - 125z^3$, if $4x - 5z = 16$ and $xz = 12$

Sol:

Given, $64x^3 - 125z^3$

Here, $4x - 5z = 16$ and $xz = 12$

Cubing $4x - 5z = 16$ on both sides

$(4x - 5z)^3 = 16^3$

We know that, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(4x)^3 - (5z)^3 - 3(4x)(5z)(4x - 5z) = 16^3$

$64x^3 - 125z^3 - 60(xz)(16) = 4096$

$64x^3 - 125z^3 - 60(12)(16) = 4096$

$64x^3 - 125z^3 - 11520 = 4096$

$64x^3 - 125z^3 = 4096 + 11520$

$64x^3 - 125z^3 = 15616$

The value of $64x^3 - 125z^3 = 15616$

Q19. If $x - \frac{1}{x} = 3 + 2\sqrt{2}$, Find the value of $x^3 - \frac{1}{x^3}$

Sol :

Given, $x - \frac{1}{x} = 3 + 2\sqrt{2}$

Cubing $x - \frac{1}{x} = 3 + 2\sqrt{2}$ on both sides

We know that, $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(x - \frac{1}{x})^3 = (3 + 2\sqrt{2})^3$

$x^3 - \frac{1}{x^3} - 3*x*\frac{1}{x}(x - \frac{1}{x}) = 3^2 + (2\sqrt{2})^3 + 3*3*2\sqrt{2}(3 + 2\sqrt{2})$

$x^3 - \frac{1}{x^3} - 3(3 + 2\sqrt{2}) = 27 + 16\sqrt{2} + 18\sqrt{2}(3 + 2\sqrt{2})$

$x^3 - \frac{1}{x^3} = 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$

$x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}$

hence, the value of $x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}$