RD SHARMA

Solutions

Class 9 Maths

Chapter 5

Ex 5.4

$Q\,1.\ a^3 + 8b^3 + 64c^3 - 24abc$

SOLUTION :

$= (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$

$= (a + 2b + 4c)(a^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a)$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$

$\therefore\ a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ac)$

$Q\,2.\ x^3 - 8y^3 + 27z^3 + 18xyz$

SOLUTION :

$= x^3 - (2y)^3 + (3z)^3 - 3 \times x \times (-2y)(3z)$

$= (x + (-2y) + 3z)(x^2 + (-2y)^2 + (3z)^2 - x(-2y) - (-2y)(3z) - 3z(x))$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

$= (x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

$\therefore\ x^3 - 8y^3 + 27z^3 + 18xyz = (x + (-2y) + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

$Q\,3.\ \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

SOLUTION :

$= \left(\frac{x}{3}\right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3}(-y)(5z)$

$= \left(\frac{x}{3} + (-y) + 5z\right)\left(\left(\frac{x}{3}\right)^2 + (-y)^2 + (5z)^2 - \frac{x}{3}(-y) - (-y)5z - 5z\left(\frac{x}{3}\right)\right)$

$= \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3}\right)$

$\therefore\ \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{x}{3} - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5zx}{3}\right)$

$Q\,4.\ 8x^3 + 27y^3 - 216z^3 + 108xyz$

SOLUTION :

$= (2x)^3 + (3y)^3 + (-6y)^3 - 3(2x)(3y)(-6z)$

$= (2x + 3y + (-6z))\left((2x)^2 + (3y)^2 + (-6z)^2 - 2x \times 3y - 3y(-6z) - (-6z)\,2x\right)$

$= (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

$\therefore\ 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x + 3y + (-6z))(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

$Q\,5.\ 125 + 8x^3 - 27y^3 + 90xy$

SOLUTION :

$= (5)^3 + (2x)^3 + (-3y)^3 - 3 \times 5 \times 2x \times (-3y)$

$= (5 + 2x + (-3y))\left(5^2 + (2x)^2 + (-3y)^2 - 5(2x) - 2x(-3y) - (-3y)\,5\right)$

$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

$\therefore\ 125 + 8x^3 - 27y^3 + 90xy = (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

$Q\,6.\ (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3$

SOLUTION :

Let $(3x - 2y) = a$, $(2y - 4z) = b$, $(4z - 3x) = c$

$\therefore a + b + c = 3x - 2y + 2y - 4z + 4z - 3x = 0$

$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$

$= 3(3x - 2y)(2y - 4z)(4z - 3x)$

$\therefore\ (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$

$Q\,7.\ (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3$

SOLUTION :

Let $2x - 3y = a$, $4z - 2x = b$, $3y - 4z = c$

$\therefore a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$

$\because a + b + c = 0 \therefore a^3 + b^3 + c^3 = 3abc$

$= 3(2x - 3y)(4z - 2x)(3y - 4z)$

$\therefore\ (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z)$

$Q\,8.\ \left[\frac{x}{2} + y + \frac{z}{3}\right]^3 + \left[\frac{x}{3} - \frac{2y}{3} + z\right]^3 + \left[-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right]^3$

SOLUTION :

Let $[\frac{x}{2} + y + \frac{z}{3}] = a, [\frac{x}{3} - \frac{2y}{3} + z] = b, [-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}] = c$

$a + b + c = \frac{x}{2} + y + \frac{z}{3} + \frac{x}{3} - \frac{2y}{3} + z - \frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}$

$a + b + c = (\frac{x}{2} + \frac{x}{3} - \frac{5x}{6}) + (y - \frac{2y}{3} - \frac{y}{3}) + (\frac{z}{3} + z - \frac{4z}{3})$

$a + b + c = \frac{3x}{6} + \frac{2x}{6} - \frac{5x}{6} + \frac{3y}{3} - \frac{2y}{3} - \frac{y}{3} + \frac{z}{3} + \frac{3z}{3} - \frac{4z}{3}$

$a + b + c = \frac{5x - 5x}{6} + \frac{3y - 3y}{3} + \frac{4z - 4z}{3}$

$a + b + c = 0$

$\because a + b + c = 0 \qquad \therefore a^3 + b^3 + c^3 = 3abc$

$= 3 (\frac{x}{2} + y + \frac{z}{3})(\frac{x}{3} - \frac{2y}{3} + z)(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3})$

$\therefore [\frac{x}{2} + y + \frac{z}{3}]^3 + [\frac{x}{3} - \frac{2y}{3} + z]^3 + [-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}]^3 = 3 (\frac{x}{2} + y + \frac{z}{3})(\frac{x}{3} - \frac{2y}{3} + z)(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3})$

$Q\,9.\ (a - 3b)^3 + (3b - c)^3 + (c - a)^3$

SOLUTION :

Let $a - 3b = x$, $3b - c = y$, $c - a = z$

$x + y + z = a - 3b + 3b - c + c - a = 0$

$(\because x + y + z = 0) \qquad \therefore x^3 + y^3 + z^3 = 3xyz$

$= 3 (a - 3b)(3b - c)(c - a)$

$\therefore\ (a - 3b)^3 + (3b - c)^3 + (c - a)^3 = 3 (a - 3b)(3b - c)(c - a)$

$Q\,10.\ 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$

SOLUTION :

$= (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + c^3 - 3 \times \sqrt{2a} \times \sqrt{3b} \times c$

$= (\sqrt{2a} + \sqrt{3b} + c)((\sqrt{2a})^2 + (\sqrt{3b})^2 + c^2 - (\sqrt{2a})(\sqrt{3b}) - (\sqrt{3b})c - (\sqrt{2ac}))$

$= (\sqrt{2a} + \sqrt{3b} + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$

$\therefore\ 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = (\sqrt{2a} + \sqrt{3b} + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ac)$

$Q\,11.\ 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$

SOLUTION :

$= (\sqrt{3}a)^3 + (-b)^3 - (\sqrt{5}c)^3 - 3(\sqrt{3}a)(-b)(-\sqrt{5}c)$

$=$

$(\sqrt{3}a + (-b) + (-\sqrt{5}c))((\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - \sqrt{3}a(-b) - (-b)(-\sqrt{5}c)$

$- (-\sqrt{5}c)\sqrt{3}a)$

$= (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

$\therefore \ 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = (\sqrt{3}a - b - \sqrt{5}c)(3a^2 + b^2 + 5c^2 + \sqrt{3}ab - \sqrt{5}bc + \sqrt{15}ac)$

Q 12 . $8x^3 - 125y^3 + 216 + 180xy$

SOLUTION :

$= (2x)^3 + (-5y)^3 + 6^3 - 3 \times (2x)(-5y)(6)$

$= (2x + (-5y) + 6)((2x)^2 + (-5y)^2 + 6^2 - 2x \times (-5y) - (-5y)6 - 6(2x))$

$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$

$\therefore \ 8x^3 - 125y^3 + 216 + 180xy = (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$

Q 13 . $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

SOLUTION :

$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3 \times \sqrt{2}a \times 2\sqrt{2}b \times c$

$= (\sqrt{2}a + 2\sqrt{2}b + c)((\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b) - (2\sqrt{2}b)c - (\sqrt{2}a)c)$

$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

$\therefore \ 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ac)$

Q 14 . Find the value of $x^3 + y^3 - 12xy + 64$ *when x + y = -4.*

SOLUTION :

$= x^3 + y^3 + 64 - 12xy$

$= x^3 + y^3 + 4^3 - 3(x)(y)(4)$

$= (x + y + 4)(x^2 + y^2 + 4^2 - xy - y \times 4 - 4 \times x)$

$= (-4 + 4)(x^2 + y^2 + 16 - xy - 4y - 4x)$ $[\because x + y = -4]$

$= 0$

$\therefore \quad x^3 + y^3 - 12xy + 64 = 0$

Q 15 . MULTIPLY :

(i) . $x^2 + y^2 + z^2 - xy + xz + yz$ *by* $x + y - z$

SOLUTION :

$= (x^2 + y^2 + z^2 - xy + xz + yz)(x + y - z)$

$= x^3 + y^3 + z^3 - 3xyz$

(ii) . $x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$ *by* $x - 2y - z$

SOLUTION :

$x^2 + (-2y)^2 + (-z)^2 - (-2y)(-z) - (-z)(x) = x^3 + (-2y)^3 + (-z)^3 - 3x(-2y)(-z)$

$\Rightarrow x^2 + 4y^2 + z^2 + 2xy - 2yz + zx = x^3 - 8y^3 - z^3 - 6xyz$

(iii) . $x^2 + 4y^2 + 2xy - 3x + 6y + 9$ *by* $(x - 2y + 3)$

SOLUTION :

$(x)^2 + (-2y)^2 + (3)^2 - (x)(-2y) - (-2y)(3) - 3(x) = (x)^3 + (-2y)^3 + 3^3 - 3(x)(-2y)(3)$

$\Rightarrow x^2 + 4y^2 + 9 + 2xy + 6y - 3x = x^3 - 8y^3 + 27 + 18xy$

(iv) . $9x^2 + 25y^2 + 15xy + 12x - 20y + 16$ *by* $3x - 5y + 4$

SOLUTION :

$(3x)^2 + (5y)^2 + 4^2 - (-3x)(5y) - (5y)(4) - (4)(-3x) = (-3x)^3 + (5y)^3 + 4^3$
$- 3(-3x)(5y)(4)$

$\Rightarrow 9x^2 + 25y^2 + 16 + 15xy - 20y + 12x = -27x^3 + 125y^3 + 64 + 180xy$