

RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.3

In each of the following, using the remainder theorem, find the remainder when $f(x)$ is divided by $g(x)$ and verify the by actual division : (1 - 8)

$$Q1. f(x) = x^3 + 4x^2 - 3x + 10, g(x) = x + 4$$

Sol :

$$\text{Here, } f(x) = x^3 + 4x^2 - 3x + 10$$

$$g(x) = x + 4$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = x - (-4)$ the remainder will be equal to $f(-4)$

$$\text{Let, } g(x) = 0$$

$$\Rightarrow x + 4 = 0$$

$$\Rightarrow x = -4$$

Substitute the value of x in $f(x)$

$$f(-4) = (-4)^3 + 4(-4)^2 - 3(-4) + 10$$

$$= -64 + (4 * 16) + 12 + 10$$

$$= -64 + 64 + 12 + 10$$

$$= 12 + 10$$

$$= 22$$

Therefore, the remainder is 22

$$Q2. f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7, g(x) = x - 1$$

Sol :

$$\text{Here, } f(x) = 4x^4 - 3x^3 - 2x^2 + x - 7$$

$$g(x) = x - 1$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = x - (-1)$ the remainder will be equal to $f(1)$

$$\text{Let, } g(x) = 0$$

$$\Rightarrow x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute the value of x in $f(x)$

$$f(1) = 4(1)^4 - 3(1)^3 - 2(1)^2 + 1 - 7$$

$$= 4 - 3 - 2 + 1 - 7$$

$$= 5 - 12$$

$$= -7$$

Therefore, the remainder is 7

$$Q3. f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2, g(x) = x + 2$$

Sol :

$$\text{Here, } f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$$

$$g(x) = x + 2$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = x - (-2)$ the remainder will be equal to $f(-2)$

$$\text{Let, } g(x) = 0$$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

Substitute the value of x in $f(x)$

$$f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$$

$$= (2 * 16) - (6 * (-8)) + (2 * 4) + 2 + 2$$

$$= 32 + 48 + 8 + 2 + 2$$

$$= 92$$

Therefore, the remainder is 92

$$\text{Q4. } f(x) = 4x^3 - 12x^2 + 14x - 3, g(x) = 2x - 1$$

Sol:

$$\text{Here, } f(x) = 4x^3 - 12x^2 + 14x - 3$$

$$g(x) = 2x - 1$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = 2(x - \frac{1}{2})$, the remainder is equal to $f(\frac{1}{2})$

$$\text{Let, } g(x) = 0$$

$$\Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

Substitute the value of x in $f(x)$

$$f(\frac{1}{2}) = 4(\frac{1}{2})^3 - 12(\frac{1}{2})^2 + 14(\frac{1}{2}) - 3$$

$$= 4(\frac{1}{8}) - 12(\frac{1}{4}) + 4(\frac{1}{2}) - 3$$

$$= (\frac{1}{2}) - 3 + 2 - 3$$

$$= (\frac{1}{2}) + 1$$

Taking L.C.M

$$= (\frac{2+1}{2})$$

$$= (\frac{3}{2})$$

Therefore, the remainder is $(\frac{3}{2})$

$$\text{Q5. } f(x) = x^3 - 6x^2 + 2x - 4, g(x) = 1 - 2x$$

Sol :

$$\text{Here, } f(x) = x^3 - 6x^2 + 2x - 4$$

$$g(x) = 1 - 2x$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = -2(x - \frac{1}{2})$, the remainder is equal to $f(\frac{1}{2})$

$$\text{Let, } g(x) = 0$$

$$\Rightarrow 1 - 2x = 0$$

$$\Rightarrow -2x = -1$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

Substitute the value of x in $f(x)$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - 8\left(\frac{1}{4}\right) + 2\left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - \left(\frac{1}{2}\right) + 1 - 4$$

$$= \frac{1}{8} - \left(\frac{1}{2}\right) - 3$$

Taking L.C.M

$$= \frac{1 - 4 + 8 - 32}{8}$$

$$= \frac{1 - 36}{8}$$

$$= \frac{1 - 36}{8}$$

$$= \frac{-35}{8}$$

Therefore, the remainder is $\frac{-35}{8}$

$$\text{Q6. } f(x) = x^4 - 3x^2 + 4, g(x) = x - 2$$

Sol :

$$\text{Here, } f(x) = x^4 - 3x^2 + 4$$

$$g(x) = x - 2$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = x - 2$ the remainder will be equal to $f(2)$

$$\text{let, } g(x) = 0$$

$$\Rightarrow x - 2 = 0$$

$$\Rightarrow x = 2$$

Substitute the value of x in $f(x)$

$$f(2) = 2^4 - 3(2)^2 + 4$$

$$= 16 - (3 \cdot 4) + 4$$

$$= 16 - 12 + 4$$

$$= 20 - 12$$

$$= 8$$

Therefore, the remainder is 8

$$Q7. f(x) = 9x^3 - 3x^2 + x - 5, g(x) = x - \frac{2}{3}$$

Sol :

$$\text{Here, } f(x) = 9x^3 - 3x^2 + x - 5$$

$$g(x) = x - \frac{2}{3}$$

from, the remainder theorem when $f(x)$ is divided by $g(x) = x - \frac{2}{3}$ the remainder will be equal to $f(\frac{2}{3})$

substitute the value of x in $f(x)$

$$f(\frac{2}{3}) = 9(\frac{2}{3})^3 - 3(\frac{2}{3})^2 + (\frac{2}{3}) - 5$$

$$= 9(\frac{8}{27}) - 3(\frac{4}{9}) + \frac{2}{3} - 5$$

$$= (\frac{8}{3}) - (\frac{4}{3}) + \frac{2}{3} - 5$$

$$= \frac{8-4+2-15}{3}$$

$$= \frac{10-19}{3}$$

$$= \frac{-9}{3}$$

$$= -3$$

Therefore, the remainder is -3

$$Q8. f(x) = 3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}, g(x) = x + \frac{2}{3}$$

Sol :

$$\text{Here, } f(x) = 3x^4 + 2x^3 - \frac{x^3}{3} - \frac{x}{9} + \frac{2}{27}$$

$$g(x) = x + \frac{2}{3}$$

from remainder theorem when $f(x)$ is divided by $g(x) = x + \frac{2}{3}$, the remainder is equal to $f(-\frac{2}{3})$

substitute the value of x in $f(x)$

$$f(-\frac{2}{3}) = 3(-\frac{2}{3})^4 + 2(-\frac{2}{3})^3 - \frac{(-\frac{2}{3})^3}{3} - \frac{(-\frac{2}{3})}{9} + \frac{2}{27}$$

$$= 3(\frac{16}{81}) + 2(\frac{-8}{27}) - \frac{4}{(9 \cdot 3)} - (\frac{-2}{(9 \cdot 3)}) + \frac{2}{27}$$

$$= (\frac{16}{27}) - (\frac{16}{27}) - \frac{4}{27} + (\frac{2}{27}) + \frac{2}{27}$$

$$= \left(\frac{4}{27}\right) - \left(\frac{4}{27}\right)$$

$$= 0$$

Therefore, the remainder is 0

Q9. If the polynomial $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x + a$ leave the same remainder when divided by $x - 2$, Find the value of a

Sol :

Given , the polynomials are

$$f(x) = 2x^3 + ax^2 + 3x - 5$$

$$p(x) = x^3 + x^2 - 4x + a$$

The remainders are $f(2)$ and $p(2)$ when $f(x)$ and $p(x)$ are divided by $x - 2$

We know that,

$$f(2) = p(2) \quad (\text{given in problem})$$

we need to calculate $f(2)$ and $p(2)$

for, $f(2)$

substitute $(x = 2)$ in $f(x)$

$$f(2) = 2(2)^3 + a(2)^2 + 3(2) - 5$$

$$= (2 * 8) + a4 + 6 - 5$$

$$= 16 + 4a + 1$$

$$= 4a + 17 \text{ --- 1}$$

for, $p(2)$

substitute $(x = 2)$ in $p(x)$

$$p(2) = 2^3 + 2^2 - 4(2) + a$$

$$= 8 + 4 - 8 + a$$

$$= 4 + a \text{ --- 2}$$

Since, $f(2) = p(2)$

Equate eqn 1 and 2

$$\Rightarrow 4a + 17 = 4 + a$$

$$\Rightarrow 4a - a = 4 - 17$$

$$\Rightarrow 3a = -13$$

$$\Rightarrow a = \frac{-13}{3}$$

The value of $a = \frac{-13}{3}$

Q10. If polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ when divided by $(x - 4)$ leave the remainders as R_1 and R_2 respectively. Find the values of a in each of the following cases, if

1. $R_1 = R_2$
2. $R_1 + R_2 = 0$
3. $2R_1 - R_2 = 0$

Sol :

Here, the polynomials are

$$f(x) = ax^3 + 3x^2 - 3$$

$$p(x) = 2x^3 - 5x + a$$

let,

R_1 is the remainder when $f(x)$ is divided by $x - 4$

$$\Rightarrow R_1 = f(4)$$

$$\Rightarrow R_1 = a(4)^3 + 3(4)^2 - 3$$

$$= 64a + 48 - 3$$

$$= 64a + 45 \quad \text{--- 1}$$

Now , let

R_2 is the remainder when $p(x)$ is divided by $x - 4$

$$\Rightarrow R_2 = p(4)$$

$$\Rightarrow R_2 = 2(4)^3 - 5(4) + a$$

$$= 128 - 20 + a$$

$$= 108 + a \quad \text{--- 2}$$

$$1. \text{ Given , } R_1 = R_2$$

$$\Rightarrow 64a + 45 = 108 + a$$

$$\Rightarrow 63a = 63$$

$$\Rightarrow a = 1$$

$$2. \text{ Given, } R_1 + R_2 = 0$$

$$\Rightarrow 64a + 45 + 108 + a = 0$$

$$\Rightarrow 65a + 153 = 0$$

$$\Rightarrow a = \frac{-153}{65}$$

$$3. \text{ Given, } 2R_1 - R_2 = 0$$

$$\Rightarrow 2(64a + 45) - 108 - a = 0$$

$$\Rightarrow 128a + 90 - 108 - a = 0$$

$$\Rightarrow 127a - 18 = 0$$

$$\Rightarrow a = \frac{18}{127}$$

Q11. If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ when divided by $(x - 2)$ leave the same remainder, Find the value of a

Sol :

Here , the polynomials are

$$f(x) = ax^3 + 3x^2 - 13$$

$$p(x) = 2x^3 - 5x + a$$

equate , $x - 2 = 0$

$$x = 2$$

substitute the value of x in $f(x)$ and $p(x)$

$$f(2) = (2)^3 + 3(2)^2 - 13$$

$$= 8a + 12 - 13$$

$$= 8a - 1 \quad \text{----- 1}$$

$$p(2) = 2(2)^3 - 5(2) + a$$

$$= 16 - 10 + a$$

$$= 6 + a \quad \text{----- 2}$$

$$f(2) = p(2)$$

$$\Rightarrow 8a - 1 = 6 + a$$

$$\Rightarrow 8a - a = 6 + 1$$

$$\Rightarrow 7a = 7$$

$$\Rightarrow a = 1$$

The value of $a = 1$

Q12. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by,

1. $x + 1$
2. $x - \frac{1}{2}$
3. x
4. $x + \pi$
5. $5 + 2x$

Sol :

$$\text{Here, } f(x) = x^3 + 3x^2 + 3x + 1$$

by remainder theorem

$$1. \Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

substitute the value of x in $f(x)$

$$f(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$$

$$= -1 + 3 - 3 + 1$$

$$= 0$$

$$2. x - \frac{1}{2}$$

Sol :

$$\text{Here, } f(x) = x^3 + 3x^2 + 3x + 1$$

By remainder theorem

$$\Rightarrow x - \frac{1}{2} = 0$$

$$\Rightarrow x = \frac{1}{2}$$

substitute the value of x in f(x)

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{8} + \frac{3}{4} + \frac{3}{2} + 1$$

$$= \frac{1+6+12+8}{8}$$

$$= \frac{27}{8}$$

$$3. x$$

Sol :

$$\text{Here, } f(x) = x^3 + 3x^2 + 3x + 1$$

by remainder theorem

$$\Rightarrow x = 0$$

substitute the value of x in f(x)

$$f(0) = 0^3 + 3(0)^2 + 3(0) + 1$$

$$= 0 + 0 + 0 + 1$$

$$= 1$$

$$4. x + \pi$$

Sol :

$$\text{Here, } f(x) = x^3 + 3x^2 + 3x + 1$$

by remainder theorem

$$\Rightarrow x + \pi = 0$$

$$\Rightarrow x = -\pi$$

Substitute the value of x in f(x)

$$f(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$$

$$= -(\pi)^3 + 3(\pi)^2 - 3(\pi) + 1$$

$$5. 5 + 2x$$

Sol :

$$\text{Here, } f(x) = x^3 + 3x^2 + 3x + 1$$

by remainder theorem

$$5 + 2x = 0$$

$$2x = -5$$

$$x = \frac{-5}{2}$$

substitute the value of x in f(x)

$$f\left(\frac{-5}{2}\right) = \left(\frac{-5}{2}\right)^3 + 3\left(\frac{-5}{2}\right)^2 + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8} + 3\left(\frac{25}{4}\right) + 3\left(\frac{-5}{2}\right) + 1$$

$$= \frac{-125}{8} + \frac{75}{4} - \frac{15}{2} + 1$$

Taking L.C.M

$$= \frac{-125+150-50+8}{8}$$

$$= \frac{-27}{8}$$