

RD SHARMA

Solutions

Class 9 Maths

Chapter 6

Ex 6.5

Using factor theorem, factorize each of the following polynomials :

Q1. $x^3 + 6x^2 + 11x + 6$

Sol:

Given polynomial, $f(x) = x^3 + 6x^2 + 11x + 6$

The constant term in $f(x)$ is 6

The factors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$

Let, $x + 1 = 0$

$\Rightarrow x = -1$

Substitute the value of x in $f(x)$

$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$

$= -1 + 6 - 11 + 6$

$= 12 - 12$

$= 0$

So, $(x + 1)$ is the factor of $f(x)$

Similarly, $(x + 2)$ and $(x + 3)$ are also the factors of $f(x)$

Since, $f(x)$ is a polynomial having a degree 3, it cannot have more than three linear factors.

$\therefore f(x) = k(x + 1)(x + 2)(x + 3)$

$\Rightarrow x^3 + 6x^2 + 11x + 6 = k(x + 1)(x + 2)(x + 3)$

Substitute $x = 0$ on both the sides

$\Rightarrow 0 + 0 + 0 + 6 = k(0 + 1)(0 + 2)(0 + 3)$

$\Rightarrow 6 = k(1 \cdot 2 \cdot 3)$

$\Rightarrow 6 = 6k$

$\Rightarrow k = 1$

Substitute k value in $f(x) = k(x + 1)(x + 2)(x + 3)$

$\Rightarrow f(x) = (1)(x + 1)(x + 2)(x + 3)$

$\Rightarrow f(x) = (x + 1)(x + 2)(x + 3)$

$\therefore x^3 + 6x^2 + 11x + 6 = (x + 1)(x + 2)(x + 3)$

Q2. $x^3 + 2x^2 - x - 2$

Sol:

Given, $f(x) = x^3 + 2x^2 - x - 2$

The constant term in $f(x)$ is -2

The factors of (-2) are $\pm 1, \pm 2$

Let, $x - 1 = 0$

$$\Rightarrow x = 1$$

Substitute the value of x in $f(x)$

$$f(1) = (1)^3 + 2(1)^2 - 1 - 2$$

$$= 1 + 2 - 1 - 2$$

$$= 0$$

Similarly, the other factors $(x + 1)$ and $(x + 2)$ of $f(x)$

Since, $f(x)$ is a polynomial having a degree 3, it cannot have more than three linear factors.

$$\therefore f(x) = k(x - 1)(x + 2)(x + 1)$$

$$x^3 + 2x^2 - x - 2 = k(x - 1)(x + 2)(x + 1)$$

Substitute $x = 0$ on both the sides

$$0 + 0 - 0 - 2 = k(-1)(1)(2)$$

$$\Rightarrow -2 = -2k$$

$$\Rightarrow k = 1$$

Substitute k value in $f(x) = k(x - 1)(x + 2)(x + 1)$

$$f(x) = (1)(x - 1)(x + 2)(x + 1)$$

$$\Rightarrow f(x) = (x - 1)(x + 2)(x + 1)$$

$$\text{So, } x^3 + 2x^2 - x - 2 = (x - 1)(x + 2)(x + 1)$$

$$\mathbf{Q3. } x^3 - 6x^2 + 3x + 10$$

Sol:

$$\text{Let, } f(x) = x^3 - 6x^2 + 3x + 10$$

The constant term in $f(x)$ is 10

The factors of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute the value of x in $f(x)$

$$f(-1) = (-1)^3 - 6(-1)^2 + 3(-1) + 10$$

$$= -1 - 6 - 3 + 10$$

$$= 0$$

Similarly, the other factors $(x - 2)$ and $(x - 5)$ of $f(x)$

Since, $f(x)$ is a polynomial having a degree 3, it cannot have more than three linear factors.

$$\therefore f(x) = k(x + 1)(x - 2)(x - 5)$$

Substitute $x = 0$ on both sides

$$\Rightarrow x^3 - 6x^2 + 3x + 10 = k(x + 1)(x - 2)(x - 5)$$

$$\Rightarrow 0 - 0 + 0 + 10 = k(1)(-2)(-5)$$

$$\Rightarrow 10 = k(10)$$

$$\Rightarrow k = 1$$

Substitute $k = 1$ in $f(x) = k(x + 1)(x - 2)(x - 5)$

$$f(x) = (1)(x + 1)(x - 2)(x - 5)$$

$$\text{so, } x^3 - 6x^2 + 3x + 10 = (x + 1)(x - 2)(x - 5)$$

$$\mathbf{Q4. } x^4 - 7x^3 + 9x^2 + 7x - 10$$

Sol:

$$\text{Given, } f(x) = x^4 - 7x^3 + 9x^2 + 7x - 10$$

The constant term in $f(x)$ is 10

The factors of 10 are $\pm 1, \pm 2, \pm 5, \pm 10$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

Substitute the value of x in $f(x)$

$$f(x) = 1^4 - 7(1)^3 + 9(1)^2 + 7(1) - 10$$

$$= 1 - 7 + 9 + 7 - 10$$

$$= 10 - 10$$

$$= 0$$

$(x - 1)$ is the factor of $f(x)$

Similarly, the other factors are $(x + 1), (x - 2), (x - 5)$

Since, $f(x)$ is a polynomial of degree 4. So, it cannot have more than four linear factor.

$$\text{So, } f(x) = k(x - 1)(x + 1)(x - 2)(x - 5)$$

$$\Rightarrow x^4 - 7x^3 + 9x^2 + 7x - 10 = k(x - 1)(x + 1)(x - 2)(x - 5)$$

Put $x = 0$ on both sides

$$0 - 0 + 0 - 10 = k(-1)(1)(-2)(-5)$$

$$- 10 = k(-10)$$

$$\Rightarrow k = 1$$

Substitute $k = 1$ in $f(x) = k(x - 1)(x + 1)(x - 2)(x - 5)$

$$f(x) = (1)(x - 1)(x + 1)(x - 2)(x - 5)$$

$$= (x - 1)(x + 1)(x - 2)(x - 5)$$

$$\text{So, } x^4 - 7x^3 + 9x^2 + 7x - 10 = (x - 1)(x + 1)(x - 2)(x - 5)$$

$$\mathbf{Q5. } x^4 - 2x^3 - 7x^2 + 8x + 12$$

Sol:

$$\text{Given, } f(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$$

The constant term $f(x)$ is equal to 12

The factors of 12 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute the value of x in $f(x)$

$$f(-1) = (-1)^4 - 2(-1)^3 - 7(-1)^2 + 8(-1) + 12$$

$$= 1 + 2 - 7 - 8 + 12$$

$$= 0$$

So, $x + 1$ is a factor of $f(x)$

Similarly, $(x + 2), (x - 2), (x - 3)$ are also the factors of $f(x)$

Since, $f(x)$ is a polynomial of degree 4, it cannot have more than four linear factors.

$$\Rightarrow f(x) = k(x + 1)(x + 2)(x - 3)(x - 2)$$

$$\Rightarrow x^4 - 2x^3 - 7x^2 + 8x + 12 = k(x + 1)(x + 2)(x - 3)(x - 2)$$

Substitute $x = 0$ on both sides,

$$\Rightarrow 0 - 0 - 0 + 12 = k(1)(2)(-2)(-3)$$

$$\Rightarrow 12 = k12$$

$$\Rightarrow k = 1$$

Substitute $k = 1$ in $f(x) = k(x - 2)(x + 1)(x + 2)(x - 3)$

$$f(x) = (x - 2)(x + 1)(x + 2)(x - 3)$$

$$\text{so, } x^4 - 2x^3 - 7x^2 + 8x + 12 = (x - 2)(x + 1)(x + 2)(x - 3)$$

$$\mathbf{Q6. } x^4 + 10x^3 + 35x^2 + 50x + 24$$

Sol:

$$\text{Given, } f(x) = x^4 + 10x^3 + 35x^2 + 50x + 24$$

The constant term in $f(x)$ is equal to 24

The factors of 24 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

Substitute the value of x in $f(x)$

$$f(-1) = (-1)^4 + 10(-1)^3 + 35(-1)^2 + 50(-1) + 24$$

$$= 1 - 10 + 35 - 50 + 24$$

$$= 0$$

$\Rightarrow (x + 1)$ is the factor of $f(x)$

Similarly, $(x + 2), (x + 3), (x + 4)$ are also the factors of $f(x)$

Since, $f(x)$ is a polynomial of degree 4, it cannot have more than four linear factors.

$$\Rightarrow f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)$$

$$\Rightarrow x^4 + 10x^3 + 35x^2 + 50x + 24 = k(x + 1)(x + 2)(x + 3)(x + 4)$$

Substitute $x = 0$ on both sides

$$\Rightarrow 0 + 0 + 0 + 0 + 24 = k(1)(2)(3)(4)$$

$$\Rightarrow 24 = k(24)$$

$$\Rightarrow k = 1$$

Substitute $k = 1$ in $f(x) = k(x + 1)(x + 2)(x + 3)(x + 4)$

$$f(x) = (1)(x + 1)(x + 2)(x + 3)(x + 4)$$

$$f(x) = (x + 1)(x + 2)(x + 3)(x + 4)$$

$$\text{hence, } x^4 + 10x^3 + 35x^2 + 50x + 24 = (x + 1)(x + 2)(x + 3)(x + 4)$$

$$\mathbf{Q7. } 2x^4 - 7x^3 - 13x^2 + 63x - 45$$

Sol :

$$\text{Given, } f(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$$

The factors of constant term -45 are $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$

The factors of the coefficient of x^4 is 2. Hence possible rational roots of $f(x)$ are

$$\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{9}{2}, \pm \frac{15}{2}, \pm \frac{45}{2}$$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

$$f(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$$

$$= 2 - 7 - 13 + 63 - 45$$

$$= 0$$

$$\text{Let, } x - 3 = 0$$

$$\Rightarrow x = 3$$

$$f(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$$

$$= 162 - 189 - 117 + 189 - 45$$

$$= 0$$

So, $(x - 1)$ and $(x - 3)$ are the roots of $f(x)$

$$\Rightarrow x^2 - 4x + 3 \text{ is the factor of } f(x)$$

Divide $f(x)$ with $x^2 - 4x + 3$ to get other three factors

By long division,

$$2x^2 + x - 15$$

$$x^2 - 4x + 3 \quad 2x^4 - 7x^3 - 13x^2 + 63x - 45$$

$$2x^4 - 8x^3 + 6x^2$$

$$(-) \quad (+) \quad (-)$$

$$x^3 - 19x^2 + 63x$$

$$x^3 - 4x^2 + 3x$$

$$(-) \quad (+) \quad (-)$$

$$-15x^2 + 60x - 45$$

$$-15x^2 + 60x - 45$$

$$(+) \quad (-) \quad (+)$$

$$0$$

$$\Rightarrow 2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x^2 - 4x + 3)(2x^2 + x - 15)$$

$$\Rightarrow 2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x - 1)(x - 3)(2x^2 + x - 15)$$

Now,

$$2x^2 + x - 15 = 2x^2 + 6x - 5x - 15$$

$$= 2x(x + 3) - 5(x + 3)$$

$$= (2x - 5)(x + 3)$$

$$\text{So, } 2x^4 - 7x^3 - 13x^2 + 63x - 45 = (x - 1)(x - 3)(x + 3)(2x - 5)$$

$$\text{Q8. } 3x^3 - x^2 - 3x + 1$$

Sol :

$$\text{Given, } f(x) = 3x^3 - x^2 - 3x + 1$$

The factors of constant term 1 is ± 1

The factors of the coefficient of $x^2 = 3$

The possible rational roots are $\pm 1, \frac{1}{3}$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

$$f(1) = 3(1)^3 - (1)^2 - 3(1) + 1$$

$$= 3 - 1 - 3 + 1$$

$$= 0$$

So, $x - 1$ is the factor of $f(x)$

Now, divide $f(x)$ with $(x - 1)$ to get other factors

By long division method,

$$3x^2 + 2x - 1$$

$$x - 1 \quad 3x^3 - x^2 - 3x + 1$$

$$3x^3 - x^2$$

$$(-) \quad (+)$$

$$2x^2 - 3x$$

$$2x^2 - 2x$$

$$(-) \quad (+)$$

$$-x + 1$$

$$-x + 1$$

$$(+) \quad (-)$$

$$0$$

$$\Rightarrow 3x^3 - x^2 - 3x + 1 = (x - 1)(3x^2 + 2x - 1)$$

Now,

$$3x^2 + 2x - 1 = 3x^2 + 3x - x - 1$$

$$= 3x(x + 1) - 1(x + 1)$$

$$= (3x - 1)(x + 1)$$

$$\text{Hence, } 3x^3 - x^2 - 3x + 1 = (x - 1)(3x - 1)(x + 1)$$

$$\mathbf{Q9.} \quad x^3 - 23x^2 + 142x - 120$$

Sol :

$$\text{Let, } f(x) = x^3 - 23x^2 + 142x - 120$$

The constant term in $f(x)$ is -120

The factors of -120 are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 8, \pm 10, \pm 12, \pm 15, \pm 20, \pm 24, \pm 30, \pm 40, \pm 60, \pm 120$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

$$f(1) = (1)^3 - 23(1)^2 + 142(1) - 120$$

$$= 1 - 23 + 142 - 120$$

$$= 0$$

So, $(x - 1)$ is the factor of $f(x)$

Now, divide $f(x)$ with $(x - 1)$ to get other factors

By long division,

$$x^2 - 22x + 120$$

$$x - 1 \quad x^3 - 23x^2 + 142x - 120$$

$$x^3 - x^2$$

$$(-) \quad (+)$$

$$-22x^2 + 142x$$

$$-22x^2 + 22x$$

$$(+) \quad (-)$$

$$120x - 120$$

$$120x - 120$$

$$(-) \quad (+)$$

$$0$$

$$\Rightarrow x^3 - 23x^2 + 142x - 120 = (x - 1)(x^2 - 22x + 120)$$

Now,

$$x^2 - 22x + 120 = x^2 - 10x - 12x + 120$$

$$= x(x - 10) - 12(x - 10)$$

$$= (x - 10)(x - 12)$$

$$\text{Hence, } x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 10)(x - 12)$$

$$\mathbf{Q10. } y^3 - 7y + 6$$

Sol :

$$\text{Given, } f(y) = y^3 - 7y + 6$$

The constant term in $f(y)$ is 6

The factors are $\pm 1, \pm 2, \pm 3, \pm 6$

$$\text{Let, } y - 1 = 0$$

$$\Rightarrow y = 1$$

$$f(1) = (1)^3 - 7(1) + 6$$

$$= 1 - 7 + 6$$

$$= 0$$

So, $(y - 1)$ is the factor of $f(y)$

Similarly, $(y - 2)$ and $(y + 3)$ are also the factors

Since, $f(y)$ is a polynomial which has degree 3, it cannot have more than 3 linear factors

$$\Rightarrow f(y) = k(y - 1)(y - 2)(y + 3)$$

$$\Rightarrow y^3 - 7y + 6 = k(y - 1)(y - 2)(y + 3) \text{ ----- 1}$$

Substitute $k = 0$ in eq 1

$$\Rightarrow 0 - 0 + 6 = k(-1)(-2)(3)$$

$$\Rightarrow 6 = 6k$$

$$\Rightarrow k = 1$$

$$y^3 - 7y + 6 = (1)(y - 1)(y - 2)(y + 3)$$

$$y^3 - 7y + 6 = (y - 1)(y - 2)(y + 3)$$

$$\text{Hence, } y^3 - 7y + 6 = (y - 1)(y - 2)(y + 3)$$

$$\mathbf{Q11. } x^3 - 10x^2 - 53x - 42$$

Sol :

$$\text{Given, } f(x) = x^3 - 10x^2 - 53x - 42$$

The constant in $f(x)$ is -42

The factors of -42 are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

$$f(-1) = (-1)^3 - 10(-1)^2 - 53(-1) - 42$$

$$= -1 - 10 + 53 - 42$$

$$= 0$$

So., $(x + 1)$ is the factor of $f(x)$

Now, divide $f(x)$ with $(x + 1)$ to get other factors

By long division,

$$x^2 - 11x - 42$$

$$x + 1 \quad x^3 - 10x^2 - 53x - 42$$

$$x^3 + x^2$$

$$(-) \quad (-)$$

$$-11x^2 - 53x$$

$$-11x^2 - 11x$$

$$(+)$$

$$-42x - 42$$

$$-42x - 42$$

$$(+)$$

$$0$$

$$\Rightarrow x^3 - 10x^2 - 53x - 42 = (x + 1)(x^2 - 11x - 42)$$

Now,

$$x^2 - 11x - 42 = x^2 - 14x + 3x - 42$$

$$= x(x - 14) + 3(x - 14)$$

$$= (x + 3)(x - 14)$$

$$\text{Hence, } x^3 - 10x^2 - 53x - 42 = (x + 1)(x + 3)(x - 14)$$

$$\mathbf{Q12. } y^3 - 2y^2 - 29y - 42$$

Sol :

$$\text{Given, } f(x) = y^3 - 2y^2 - 29y - 42$$

The constant in $f(x)$ is -42

The factors of -42 are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

$$\text{Let, } y + 2 = 0$$

$$\Rightarrow y = -2$$

$$f(-2) = (-2)^3 - 2(-2)^2 - 29(-2) - 42$$

$$= -8 - 8 + 58 - 42$$

$$= 0$$

So, $(y + 2)$ is the factor of $f(y)$

Now, divide $f(y)$ with $(y + 2)$ to get other factors

By, long division

$$y^2 - 4y - 21$$

$$y + 2 \quad y^3 - 2y^2 - 29y - 42$$

$$y^3 + 2y^2$$

(-) (-)

$$-4y^2 - 29y$$

$$-4y^2 - 8y$$

(+) (+)

$$-21y - 42$$

$$-21y - 42$$

(+) (+)

$$0$$

$$\Rightarrow y^3 - 2y^2 - 29y - 42 = (y + 2)(y^2 - 4y - 21)$$

Now,

$$y^2 - 4y - 21 = y^2 - 7y + 3y - 21$$

$$= y(y - 7) + 3(y - 7)$$

$$= (y - 7)(y + 3)$$

$$\text{Hence, } y^3 - 2y^2 - 29y - 42 = (y + 2)(y - 7)(y + 3)$$

$$\mathbf{Q13.} \quad 2y^3 - 5y^2 - 19y + 42$$

Sol :

$$\text{Given, } f(x) = 2y^3 - 5y^2 - 19y + 42$$

The constant in $f(x)$ is $+42$

The factors of 42 are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 7, \pm 14, \pm 21, \pm 42$

$$\text{Let, } y - 2 = 0$$

$$\Rightarrow y = 2$$

$$f(2) = 2(2)^3 - 5(2)^2 - 19(2) + 42$$

$$= 16 - 20 - 38 + 42$$

$$= 0$$

So, $(y - 2)$ is the factor of $f(y)$

Now, divide $f(y)$ with $(y - 2)$ to get other factors

By, long division method

$$2y^2 - y - 21$$

$$y - 2 \quad 2y^3 - 5y^2 - 19y + 42$$

$$2y^3 - 4y^2$$

$$(-) \quad (+)$$

$$-y^2 - 19y$$

$$-y^2 + 2y$$

$$(+) \quad (-)$$

$$-21y + 42$$

$$-21y + 42$$

$$(+) \quad (-)$$

$$0$$

$$\Rightarrow 2y^3 - 5y^2 - 19y + 42 = (y - 2)(2y^2 - y - 21)$$

Now,

$$2y^2 - y - 21$$

The factors are $(y + 3)(2y - 7)$

$$\text{Hence, } 2y^3 - 5y^2 - 19y + 42 = (y - 2)(y + 3)(2y - 7)$$

$$\mathbf{Q14.} \quad x^3 + 13x^2 + 32x + 20$$

Sol:

$$\text{Given, } f(x) = x^3 + 13x^2 + 32x + 20$$

The constant in $f(x)$ is 20

The factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

$$f(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$$

$$= -1 + 13 - 32 + 20$$

$$= 0$$

So, $(x + 1)$ is the factor of $f(x)$

Divide $f(x)$ with $(x + 1)$ to get other factors

By, long division

$$x^2 + 12x + 20$$

$$x + 1 \quad x^3 + 13x^2 + 32x + 20$$

$$x^3 + x^2$$

$$(-) \quad (-)$$

$$12x^2 + 32x$$

$$12x^2 + 12x$$

$$(-) \quad (-)$$

$$20x - 20$$

$$20x - 20$$

$$(-) \quad (-)$$

$$0$$

$$\Rightarrow x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20)$$

Now,

$$x^2 + 12x + 20 = x^2 + 10x + 2x + 20$$

$$= x(x + 10) + 2(x + 10)$$

The factors are $(x + 10)$ and $(x + 2)$

$$\text{Hence, } x^3 + 13x^2 + 32x + 20 = (x + 1)(x + 10)(x + 2)$$

$$\mathbf{Q15. } x^3 - 3x^2 - 9x - 5$$

Sol :

$$\text{Given, } f(x) = x^3 - 3x^2 - 9x - 5$$

The constant in $f(x)$ is -5

The factors of -5 are $\pm 1, \pm 5$

$$\text{Let, } x + 1 = 0$$

$$\Rightarrow x = -1$$

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$$

$$= -1 - 3 + 9 - 5$$

$$= 0$$

So, $(x + 1)$ is the factor of $f(x)$

Divide $f(x)$ with $(x + 1)$ to get other factors

By, long division

$$x^2 - 4x - 5$$

$$x + 1 \quad x^3 - 3x^2 - 9x - 5$$

$$x^3 + x^2$$

$$(-) \quad (-)$$

$$-4x^2 - 9x$$

$$-4x^2 - 4x$$

$$(+)$$

$$-5x - 5$$

$$-5x - 5$$

$$(+)$$

$$0$$

$$\Rightarrow x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5)$$

Now,

$$x^2 - 4x - 5 = x^2 - 5x + x - 5$$

$$= x(x - 5) + 1(x - 5)$$

The factors are $(x - 5)$ and $(x + 1)$

$$\text{Hence, } x^3 - 3x^2 - 9x - 5 = (x + 1)(x - 5)(x + 1)$$

$$\mathbf{Q16.} \quad 2y^3 + y^2 - 2y - 1$$

Sol :

$$\text{Given, } f(y) = 2y^3 + y^2 - 2y - 1$$

The constant term is 2

$$\text{The factors of 2 are } \pm 1, \pm \frac{1}{2}$$

$$\text{Let, } y - 1 = 0$$

$$\Rightarrow y = 1$$

$$f(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 0$$

So, $(y - 1)$ is the factor of $f(y)$

Divide $f(y)$ with $(y - 1)$ to get other factors

By, long division

$$2y^2 + 3y + 1$$

$$y - 1 \quad 2y^3 + y^2 - 2y - 1$$

$$2y^3 - 2y^2$$

(-) (+)

$$3y^2 - 2y$$

$$3y^2 - 3y$$

(-) (+)

$$y - 1$$

$$y - 1$$

(-) (+)

$$0$$

$$\Rightarrow 2y^3 + y^2 - 2y - 1 = (y - 1)(2y^2 + 3y + 1)$$

Now,

$$2y^2 + 3y + 1 = 2y^2 + 2y + y + 1$$

$$= 2y(y + 1) + 1(y + 1)$$

$$= (2y + 1)(y + 1) \text{ are the factors}$$

$$\text{Hence, } 2y^3 + y^2 - 2y - 1 = (y - 1)(2y + 1)(y + 1)$$

$$\mathbf{Q17.} \quad x^3 - 2x^2 - x + 2$$

Sol :

$$\text{Let, } f(x) = x^3 - 2x^2 - x + 2$$

The constant term is 2

The factors of 2 are $\pm 1, \pm \frac{1}{2}$

$$\text{Let, } x - 1 = 0$$

$$\Rightarrow x = 1$$

$$f(1) = (1)^3 - 2(1)^2 - (1) + 2$$

$$= 1 - 2 - 1 + 2$$

$$= 0$$

So, $(x - 1)$ is the factor of $f(x)$

Divide $f(x)$ with $(x - 1)$ to get other factors

By, long division

$$x^2 - x - 2$$

$$x - 1 \quad x^3 - 2x^2 - y + 2$$

$$x^3 - x^2$$

(-) (+)

$$-x^2 - x$$

$$-x^2 + x$$

(+) (-)

$$- 2x + 2$$

$$- 2x + 2$$

(+) (-)

$$0$$

$$\Rightarrow x^3 - 2x^2 - y + 2 = (x - 1)(x^2 - x - 2)$$

Now,

$$x^2 - x - 2 = x^2 - 2x + x - 2$$

$$= x(x - 2) + 1(x - 2)$$

$$= (x - 2)(x + 1) \text{ are the factors}$$

$$\text{Hence, } x^3 - 2x^2 - y + 2 = (x - 1)(x + 1)(x - 2)$$

Q18. Factorize each of the following polynomials :

1. $x^3 + 13x^2 + 31x - 45$ given that $x + 9$ is a factor

2. $4x^3 + 20x^2 + 33x + 18$ given that $2x + 3$ is a factor

Sol :

1. $x^3 + 13x^2 + 31x - 45$ given that $x + 9$ is a factor

let, $f(x) = x^3 + 13x^2 + 31x - 45$

given that $(x + 9)$ is the factor

divide $f(x)$ with $(x + 9)$ to get other factors

by , long division

$$x^2 + 4x - 5$$

$$x + 9 \quad x^3 + 13x^2 + 31x - 45$$

$$x^3 + 9x^2$$

(-) (-)

$$4x^2 + 31x$$

$$4x^2 + 36x$$

(-) (-)

$$-5x - 45$$

$$-5x - 45$$

(+) (+)

$$0$$

$$\Rightarrow x^3 + 13x^2 + 31x - 45 = (x + 9)(x^2 + 4x - 5)$$

Now,

$$x^2 + 4x - 5 = x^2 + 5x - x - 5$$

$$= x(x + 5) - 1(x + 5)$$

$$= (x + 5)(x - 1) \text{ are the factors}$$

$$\text{Hence, } x^3 + 13x^2 + 31x - 45 = (x + 9)(x + 5)(x - 1)$$

$$2. 4x^3 + 20x^2 + 33x + 18 \text{ given that } 2x + 3 \text{ is a factor}$$

$$\text{let, } f(x) = 4x^3 + 20x^2 + 33x + 18$$

given that $2x + 3$ is a factor

divide $f(x)$ with $(2x + 3)$ to get other factors

by, long division

$$2x^2 + 7x + 6$$

$$2x + 3 \quad 4x^3 + 20x^2 + 33x + 18$$

$$4x^3 + 6x^2$$

$$(-) \quad (-)$$

$$14x^2 - 33x$$

$$14x^2 - 21x$$

$$(-) \quad (+)$$

$$12x + 18$$

$$12x + 18$$

$$(-) \quad (-)$$

$$0$$

$$\Rightarrow 4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x^2 + 7x + 6)$$

Now,

$$2x^2 + 7x + 6 = 2x^2 + 4x + 3x + 6$$

$$= 2x(x + 2) + 3(x + 2)$$

$$= (2x + 3)(x + 2) \text{ are the factors}$$

$$\text{Hence, } 4x^3 + 20x^2 + 33x + 18 = (2x + 3)(2x + 3)(x + 2)$$