1. Relation

Exercise 1A

1. Question

Find the domain and range of the relation

R = {(-1, 1), (1, 1), (-2, 4), (2, 4)}.

Answer

dom (R) = {-1, 1, -2, 2} and range (R) = {1, 4}

2. Question

Let R = {(a, a^3) : a is a prime number less than 5}.

Find the range of R.

Answer

range (R) = {8 27}

3. Question

Let R = {(a, a^3) : a is a prime number less than 10}.

Find (i) R (ii) dom (R) (iii) range (R).

Answer

(i) R = {(2, 8),(3, 27),(5, 125), (7, 343)}

(ii) dom (R) = {2, 3, 5, 7}

(iii) range (R) = {8, 27, 125, 343}

4. Question

Let R = (x, y) : x + 2y = be are relation on N.

Write the range of R.

Answer

{3, 2, 1}

5. Question

Let R ={(a, b): a, b ∈ N and a + 3b = 12}.

Find the domain and range of R.

Answer

dom (R) = {3, 6, 9} and range (R) = {3, 2, 1}

6. Question

Let R = {(a, b) : b = |a – 1|, a ∈ Z and |a| < 3}.

Find the domain and range of R.

Answer

dom (R) = {-2, -1, 0, 1, 2} and range (R) = {3, 2, 1, 0}

7. Question

Let $R = \left\{ \left(a, \frac{1}{a} \right) : a \in N \text{ and } 1 < a < 5 \right\}$.

Find the domain and range of R.

Answer

dom (R) = {2, 3, 4} and range $(R) = \left\{ \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}$

8. Question

Let R = {(a, b) : a, b ∈ N and b = a + 5, a < 4}.

Find the domain and range of R.

Answer

dom (R) = {1, 2, 3} and range (R) = {6, 7, 8}

9. Question

Let S be the set of all sets and let R = {(A, B) : A ⊂ B)}, i.e., A is a proper subset of B. Show that R is (i) transitive (ii) not reflexive (iii) not symmetric.

Answer

Let R = {(A, B) : A ⊂ B)}, i.e., A is a proper subset of B, be a relation defined on S.

Now,

Any set is a subset of itself, but not a proper subset.

⇒ (A,A) ∉ R ∀ A ∈ S

⇒ R is not reflexive.

Let (A,B) ∈ R ∀ A, B ∈ S

⇒ A is a proper subset of B

⇒ all elements of A are in B, but B contains at least one element that is not in A.

⇒ B cannot be a proper subset of A

⇒ (B,A) ∉ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B . we observe that B is not a proper subset of A.

⇒ R is not symmetric

Let (A,B) ∈ R and (B,C) ∈ R ∀ A, B,C ∈ S

⇒ A is a proper subset of B and B is a proper subset of C

⇒ A is a proper subset of C

⇒ (A,C) ∈ R

For e.g. , if B = {1,2,5} then A = {1,5} is a proper subset of B .

And if C = {1,2,5,7} then B = {1,2,5} is a proper subset of C.

We observe that A = {1,5} is a proper subset of C also.

⇒ R is transitive.

Thus, R is transitive but not reflexive and not symmetric.

10. Question

Let A be the set of all points in a plane and let O be the origin. Show that the relation R = {(P, Q) : P, Q∈ A and OP = OQ) is an equivalence relation.

Answer

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, A be the set of all points in a plane and O be the origin. Then, R = {(P, Q) : P, Q ∈ A and OP = OQ)}

Now,

R is Reflexive if (P,P) ∈ R ∀ P ∈ A

∀ P ∈ A , we have

OP=OP

⇒ (P,P) ∈ R

Thus, R is reflexive.

R is Symmetric if (P,Q) ∈ R ⇒ (Q,P) ∈ R ∀ P, Q ∈ A

Let P, Q ∈ A such that,

(P,Q) ∈ R

⇒ OP = OQ

⇒ OQ = OP

⇒ (Q,P) ∈ R

Thus, R is symmetric.

R is Transitive if (P,Q) ∈ R and (Q,S) ∈ R ⇒ (P,S) ∈ R ∀ P, Q, S ∈ A

Let (P,Q) ∈ R and (Q,S) ∈ R ∀ P, Q, S ∈ A

⇒ OP = OQ and OQ = OS

⇒ OP = OS

⇒ (P,S) ∈ R

Thus, R is transitive.

Since R is reflexive, symmetric and transitive it is an equivalence relation on A.

11. Question

On the set S of all real numbers, define a relation R = {(a, b) : a ≤ b}.

Show that R is (i) reflexive (ii) transitive (iii) not symmetric.

Answer

Let R = {(a, b) : a ≤ b} be a relation defined on S.

Now,

We observe that any element x ∈ S is less than or equal to itself.

⇒ (x,x) ∈ R ∀ x ∈ S

⇒ R is reflexive.

Let (x,y) ∈ R ∀ x, y ∈ S

⇒ x is less than or equal to y

But y cannot be less than or equal to x if x is less than or equal to y.

⇒ (y,x) ∉ R

For e.g. , we observe that (2,5) ∈ R i.e. 2 < 5 but 5 is not less than or equal to 2⇒ (5,2) ∉ R

⇒ R is not symmetric

Let (x,y) ∈ R and (y,z) ∈ R ∀ x, y ,z ∈ S

⇒ x ≤ y and y ≤ z

⇒ x ≤ z

⇒ (x,z) ∈ R

For e.g. , we observe that

(4,5) ∈ R ⇒ 4 ≤ 5 and (5,6) ∈ R ⇒ 5 ≤ 6

And we know that 4 ≤ 6 ∴ (4,6) ∈ R

⇒ R is transitive.

Thus, R is reflexive and transitive but not symmetric.

12. Question

Let A = {1, 2, 3, 4, 5, 6) and let R = {(a, b) : a, b∈ A and b = a + 1}.

Show that R is (i) not reflexive, (ii) not symmetric and (iii) not transitive.

Answer

Given that,

A = {1, 2, 3, 4, 5, 6) and R = {(a, b) : a, b∈ A and b = a + 1}.

∴ R = {(1,2),(2,3),(3,4),(4,5),(5,6)}

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ A

Since, (1,1),(2,2),(3,3),(4,4),(5,5),(6,6) ∉ R

Thus, R is not reflexive .

R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ A

We observe that (1,2) ∈ R but (2,1) ∉ R .

Thus, R is not symmetric .

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ A

We observe that (1,2) ∈ R and (2,3) ∈ R but (1,3) ∉ R

Thus, R is not transitive.

Exercise 1B

1. Question

Define a relation on a set. What do you mean by the domain and range of a relation? Give an example.

Answer

Relation: Let A and B be two sets. Then a relation R from set A to set B is a subset of A x B. Thus, R is a relation to A to B ⇔ R ⊆ A x B.

If R is a relation from a non-void set B and if (a,b) ∈ R, then we write a R b which is read as ‘a is related to b by the relation R’. if (a,b) ∉ R, then we write a R b, and we say that a is not related to b by the relation R.

Domain: Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R.

Thus, domain of R ={a : (a,b) ∈ R}. The domain of R ⊆ A.

Range: let R be a relation from a set A to a set B. then the set of all second component or coordinates of the ordered pairs belonging to R is called the range of R.

Example 1: R = {(-1, 1), (1, 1), (-2, 4), (2, 4)}.

dom (R) = {-1, 1, -2, 2} and range (R) = {1, 4}

Example 2: R ={(a, b): a, b ∈ N and a + 3b = 12}

dom (R) = {3, 6, 9} and range (R) = {3, 2, 1}

2. Question

Let A be the set of all triangles in a plane. Show that the relation

R = {(Δ_1, Δ_2) : $\Delta_1 \sim \Delta_2$} is an equivalence relation on A.

Answer

Let R = {(Δ_1, Δ_2) : $\Delta_1 \sim \Delta_2$} be a relation defined on A.

Now,

R is Reflexive if (Δ, Δ) ∈ R ∀ Δ ∈ A

We observe that for each Δ ∈ A we have,

Δ ~ Δ since, every triangle is similar to itself.

⇒ (Δ, Δ) ∈ R ∀ Δ ∈ A

⇒ R is reflexive.

R is Symmetric if (Δ_1, Δ_2) ∈ R ⇒ (Δ_2, Δ_1) ∈ R ∀ Δ_1, Δ_2 ∈ A

Let (Δ_1, Δ_2) ∈ R ∀ Δ_1, Δ_2 ∈ A

⇒ $\Delta_1 \sim \Delta_2$

⇒ $\Delta_2 \sim \Delta_1$

⇒ (Δ_2, Δ_1) ∈ R

⇒ R is symmetric

R is Transitive if (Δ_1, Δ_2) ∈ R and (Δ_2, Δ_3)∈ R ⇒ (Δ_1, Δ_3) ∈ R ∀ Δ_1, Δ_2, Δ_3 ∈ A

Let (Δ_1, Δ_2) ∈ R and ((Δ_2, Δ_3) ∈ R ∀ Δ_1, Δ_2, Δ_3 ∈ A

⇒ $\Delta_1 \sim \Delta_2$ and $\Delta_2 \sim \Delta_3$

⇒ $\Delta_1 \sim \Delta_3$

⇒ (Δ_1, Δ_3) ∈ R

⇒ R is transitive.

Since R is reflexive, symmetric and transitive, it is an equivalence relation on A.

3. Question

Let R = {(a, b) : a, b ∈ Z and (a + b) is even}.

Show that R is an equivalence relation on Z.

Answer

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, ∀ a, b ∈ Z, R = {(a, b) : (a + b) is even }.

Now,

R is Reflexive if (a,a) $\in$ R $\forall$ a $\in$ Z

For any a $\in$ A, we have

a+a = 2a, which is even.

$\Rightarrow$ (a,a) $\in$ R

Thus, R is reflexive.

R is Symmetric if (a,b) $\in$ R $\Rightarrow$ (b,a) $\in$ R $\forall$ a,b $\in$ Z

(a,b) $\in$ R

$\Rightarrow$ a+b is even.

$\Rightarrow$ b+a is even.

$\Rightarrow$ (b,a) $\in$ R

Thus, R is symmetric .

R is Transitive if (a,b) $\in$ R and (b,c) $\in$ R $\Rightarrow$ (a,c) $\in$ R $\forall$ a,b,c $\in$ Z

Let (a,b) $\in$ R and (b,c) $\in$ R $\forall$ a, b,c $\in$ Z

$\Rightarrow$ a+b = 2P and b+c = 2Q

Adding both, we get

a+c+2b = 2(P+Q)

$\Rightarrow$ a+c = 2(P+Q)-2b

$\Rightarrow$ a+c is an even number

$\Rightarrow$ (a, c) $\in$ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

4. Question

Let R = {(a, b) : a, b $\in$ Z and (a - b) is divisible by 5}.

Show that R is an equivalence relation on Z.

Answer

In order to show R is an equivalence relation, we need to show R is Reflexive, Symmetric and Transitive.

Given that, $\forall$ a, b $\in$Z, aRb if and only if a – b is divisible by 5.

Now,

R is Reflexive if (a,a) $\in$ R $\forall$ a $\in$ Z

aRa $\Rightarrow$ (a-a) is divisible by 5.

a-a = 0 = 0 × 5 [since 0 is multiple of 5 it is divisible by 5]

$\Rightarrow$ a-a is divisible by 5

$\Rightarrow$ (a,a) $\in$ R

Thus, R is reflexive on Z.

R is Symmetric if (a,b) $\in$ R $\Rightarrow$ (b,a) $\in$ R $\forall$ a,b $\in$ Z

(a,b) $\in$ R $\Rightarrow$ (a-b) is divisible by 5

⇒ (a-b) = 5z for some z ∈ Z

⇒ -(b-a) = 5z

⇒ b-a = 5(-z) [∵ z ∈ Z ⇒ -z ∈ Z]

⇒ (b-a) is divisible by 5

⇒ (b,a) ∈ R

Thus, R is symmetric on Z.

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ Z

(a,b) ∈ R ⇒ (a-b) is divisible by 5

⇒ a-b = $5z_1$ for some z_1∈ Z

(b,c) ∈ R ⇒ (b-c) is divisible by 5

⇒ b-c = $5z_2$ for some z_2∈ Z

Now,

a-b = $5z_1$ and b-c = $5z_2$

⇒ (a-b) + (b-c) = $5z_1 + 5z_2$

⇒ a-c = $5(z_1 + z_2) = 5z_3$ where $z_1 + z_2 = z_3$

⇒ a-c = $5z_3$ [∵ z_1,z_2 ∈ Z ⇒ z_3∈ Z]

⇒ (a-c) is divisible by 5.

⇒ (a, c) ∈ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

5. Question

Show that the relation R defined on the set A = (1, 2, 3, 4, 5), given by

R = {(a, b) : |a – b| is even} is an equivalence relation.

Answer

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, ∀ a, b ∈A, R = {(a, b) : |a – b| is even}.

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ A

For any a ∈ A, we have

|a-a| = 0, which is even.

⇒ (a,a) ∈ R

Thus, R is reflexive.

R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ A

(a,b) ∈ R

⇒ |a-b| is even.

⇒ |b-a| is even.

⇒ (b,a) ∈ R

Thus, R is symmetric .

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ A

Let (a,b) ∈ R and (b,c) ∈ R ∀ a, b,c ∈ A

⇒ |a – b| is even and |b – c| is even

⇒ (a and b both are even or both odd) and (b and c both are even or both odd)

Now two cases arise:

Case 1 : when b is even

Let (a,b) ∈ R and (b,c) ∈ R

⇒ |a – b| is even and |b – c| is even

⇒ a is even and c is even [∵ b is even]

⇒ |a – c| is even [∵ difference of any two even natural numbers is even]

⇒ (a, c) ∈ R

Case 2 : when b is odd

Let (a,b) ∈ R and (b,c) ∈ R

⇒ |a – b| is even and |b – c| is even

⇒ a is odd and c is odd [∵ b is odd]

⇒ |a – c| is even [∵ difference of any two odd

natural numbers is even]

⇒ (a, c) ∈ R

Thus, R is transitive on Z.

Since R is reflexive, symmetric and transitive it is an equivalence relation on Z.

6. Question

Show that the relation R on N × N, defined by

(a, b) R (c, d) ⇔ a + d = b + c

is an equivalent relation.

Answer

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, R be the relation in N ×N defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in N ×N.

R is Reflexive if (a, b) R (a, b) for (a, b) in N ×N

Let (a,b) R (a,b)

⇒ a+b = b+a

which is true since addition is commutative on N.

⇒ R is reflexive.

R is Symmetric if (a,b) R (c,d) ⇒ (c,d) R (a,b) for (a, b), (c, d) in N ×N

Let (a,b) R (c,d)

⇒ a+d = b+c

⇒ b+c = a+d

⇒ c+b = d+a [since addition is commutative on N]

⇒ (c,d) R (a,b)

⇒ R is symmetric.

R is Transitive if (a,b) R (c,d) and (c,d) R (e,f) ⇒ (a,b) R (e,f) for (a, b), (c, d),(e,f) in N ×N

Let (a,b) R (c,d) and (c,d) R (e,f)

⇒ a+d = b+c and c+f = d+e

⇒ (a+d) – (d+e) = (b+c) – (c+f)

⇒ a-e= b-f

⇒ a+f = b+e

⇒ (a,b) R (e,f)

⇒ R is transitive.

Hence, R is an equivalence relation.

7. Question

Let S be the set of all real numbers and let

R = {(a, b) : a, b ∈ S and a = ± b}.

Show that R is an equivalence relation on S.

Answer

In order to show R is an equivalence relation we need to show R is Reflexive, Symmetric and Transitive.

Given that, ∀ a, b ∈ S, R = {(a, b) : a = ± b }

Now,

R is Reflexive if (a,a) ∈ R ∀ a ∈ S

For any a ∈ S, we have

a = ±a

⇒ (a,a) ∈ R

Thus, R is reflexive.

R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ S

(a,b) ∈ R

⇒ a = ± b

⇒ b = ± a

⇒ (b,a) ∈ R

Thus, R is symmetric .

R is Transitive if (a,b) ∈ R and (b,c) ∈ R ⇒ (a,c) ∈ R ∀ a,b,c ∈ S

Let (a,b) ∈ R and (b,c) ∈ R ∀ a, b,c ∈ S

⇒ a = ± b and b = ± c

⇒ a = ± c

⇒ (a, c) ∈ R

Thus, R is transitive.

Hence, R is an equivalence relation.

8. Question

Let S be the set of all points in a plane and let R be a relation in S defined by R = {(A, B) : d(A, B) < 2 units}, where d(A, B) is the distance between the points A and B.

Show that R is reflexive and symmetric but not transitive.

Answer

Given that, ∀ A, B ∈ S, R = {(A, B) : d(A, B) < 2 units}.

Now,

<u>R is Reflexive if (A,A) ∈ R ∀ A ∈ S</u>

For any A ∈ S, we have

d(A,A) = 0, which is less than 2 units

⇒ (A,A) ∈ R

Thus, R is reflexive.

<u>R is Symmetric if (A, B) ∈ R ⇒ (B,A) ∈ R ∀ A,B ∈ S</u>

(A, B) ∈ R

⇒ d(A, B) < 2 units

⇒ d(B, A) < 2 units

⇒ (B,A) ∈ R

Thus, R is symmetric .

<u>R is Transitive if (A, B) ∈ R and (B,C) ∈ R ⇒ (A,C) ∈ R ∀ A,B,C ∈ S</u>

Consider points A(0,0),B(1.5,0) and C(3.2,0).

d(A,B)=1.5 units < 2 units and d(B,C)=1.7 units < 2 units

d(A,C)= 3.2 ≮ 2

⇒ (A, B) ∈ R and (B,C) ∈ R ⇒ (A,C) ∉ R

Thus, R is not transitive.

Thus, R is reflexive, symmetric but not transitive.

9. Question

Let S be the set of all real numbers. Show that the relation R = {(a, b) : $a^2 + b^2 = 1$} is symmetric but neither reflexive nor transitive.

Answer

Given that, ∀ a, b ∈ S, R = {(a, b) : $a^2 + b^2 = 1$ }

Now,

<u>R is Reflexive if (a,a) ∈ R ∀ a ∈ S</u>

For any a ∈ S, we have

$a^2+a^2 = 2\ a^2 \neq 1$

⇒ (a,a) ∉ R

Thus, R is not reflexive.

<u>R is Symmetric if (a,b) ∈ R ⇒ (b,a) ∈ R ∀ a,b ∈ S</u>

$(a,b) \in R$

$\Rightarrow a^2 + b^2 = 1$

$\Rightarrow b^2 + a^2 = 1$

$\Rightarrow (b,a) \in R$

Thus, R is symmetric .

<u>R is Transitive if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R\ \forall\ a,b,c \in S$</u>

Let $(a,b) \in R$ and $(b,c) \in R\ \forall$ a, b,c $\in S$

$\Rightarrow a^2 + b^2 = 1$ and $b^2 + c^2 = 1$

Adding both, we get

$a^2 + c^2 + 2b^2 = 2$

$\Rightarrow a^2 + c^2 = 2 - 2b^2 \neq 1$

$\Rightarrow (a, c) \notin R$

Thus, R is not transitive.

Thus, R is symmetric but neither reflexive nor transitive.

10. Question

Let R = $\{(a, b) : a = b^2\}$ for all a, b $\in$ N.

Show that R satisfies none of reflexivity, symmetry and transitivity.

Answer

We have, R = $\{(a, b) : a = b^2\}$ relation defined on N.

Now,

We observe that, any element a $\in$ N cannot be equal to its square except 1.

$\Rightarrow (a,a) \notin R\ \forall\ a \in N$

For e.g. $(2,2) \notin R \because 2 \neq 2^2$

$\Rightarrow$ R is not reflexive.

Let $(a,b) \in R\ \forall$ a, b $\in$ N

$\Rightarrow a = b^2$

But b cannot be equal to square of a if a is equal to square of b.

$\Rightarrow (b,a) \notin R$

For e.g., we observe that $(4,2) \in R$ i.e $4 = 2^2$ but $2 \neq 4^2 \Rightarrow (2,4) \notin R$

$\Rightarrow$ R is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R\ \forall$ a, b,c $\in$ N

$\Rightarrow a = b^2$ and $b = c^2$

$\Rightarrow a \neq c^2$

$\Rightarrow (a,c) \notin R$

For e.g., we observe that

$(16,4) \in R \Rightarrow 16 = 4^2$ and $(4,2) \in R \Rightarrow 4 = 2^2$

But $16 \neq 2^2$

⇒ $(16,2) \notin R$

⇒ R is not transitive.

Thus, R is neither reflexive nor symmetric nor transitive.

11. Question

Show that the relation R = {(a, b) : a > b} on N is transitive but neither reflexive nor symmetric.

Answer

We have, R = {(a, b) : a > b} relation defined on N.

Now,

We observe that, any element $a \in N$ cannot be greater than itself.

⇒ $(a,a) \notin R \ \forall\ a \in N$

⇒ R is not reflexive.

Let $(a,b) \in R \ \forall\ a, b \in N$

⇒ a is greater than b

But b cannot be greater than a if a is greater than b.

⇒ $(b,a) \notin R$

For e.g., we observe that $(5,2) \in R$ i.e $5 > 2$ but $2 \ngtr 5 \Rightarrow (2,5) \notin R$

⇒ R is not symmetric

Let $(a,b) \in R$ and $(b,c) \in R \ \forall\ a, b,c \in N$

⇒ $a > b$ and $b > c$

⇒ $a > c$

⇒ $(a,c) \in R$

For e.g., we observe that

$(5,4) \in R \Rightarrow 5 > 4$ and $(4,3) \in R \Rightarrow 4 > 3$

And we know that $5 > 3 \therefore (5,3) \in R$

⇒ R is transitive.

Thus, R is transitive but not reflexive not symmetric.

12. Question

Let A = {1, 2, 3} and R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}.

Show that R is reflexive but neither symmetric nor transitive.

Answer

Given that, A = {1, 2, 3} and R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3)}.

Now,

R is reflexive $\because (1,1),(2,2),(3,3) \in R$

R is not symmetric $\because (1,2),(2,3) \in R$ but $(2,1),(3,2) \notin R$

R is not transitive $\because (1,2) \in R$ and $(2,3) \in R \Rightarrow (1,3) \notin R$

Thus, R is reflexive but neither symmetric nor transitive.

13. Question

Let A = (1, 2, 3, 4) and R = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)}. Show that R is reflexive and transitive but not symmetric.

Answer

Given that, A = {1, 2, 3} and R = {1, 1), (2, 2), (3, 3),(4, 4), (1, 2), (1, 3), (3, 2)}.

Now,

R is reflexive ∵ (1,1),(2,2),(3,3),(4,4) ∈ R

R is not symmetric ∵ (1,2),(1,3),(3,2) ∈ R but (2,1),(3,1),(2,3) ∉ R

R is transitive ∵ (1,3) ∈ R and (3,2) ∈ R ⇒ (1,2) ∈ R

Thus, R is reflexive and transitive but not symmetric.

Objective Questions

1. Question

Mark the tick against the correct answer in the following:

Let A = {1, 2, 3} and let R = {(1, 1),(2, 2), (3, 3), (1, 3), (3, 2), (1, 2)}. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

Given set A = {1, 2, 3}

And R = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 2), (1, 2)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (1,1) ∈ R , (2,2) ∈ R , (3,3) ∈ R

Therefore , R is reflexive .... (1)

Check for symmetric

Since (1,3) ∈ R but (3,1) ∉ R

Therefore , R is not symmetric .... (2)

Check for transitive

Here , (1,3) ∈ R and (3,2) ∈ R and (1,2) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

2. Question

Mark the tick against the correct answer in the following:

Let A = {a, b, c} and let R = {(a, a), (a, b), (b, a)}. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

Given set A = {a, b, c}

And R = {(a, a), (a, b), (b, a)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (b,b) ∉ R and (c,c) ∉ R

Therefore , R is not reflexive ……. (1)

Check for symmetric

Since , (a,b) ∈ R and (b,a) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (a,b) ∈ R and (b,a) ∈ R and (a,a) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

3. Question

Mark the tick against the correct answer in the following:

Let A = {1, 2, 3} and let R = {(1, 1),(2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}. Then, R is

A. reflexive and symmetric but not transitive

B. symmetric and transitive but not reflexive

C. reflexive and transitive but not symmetric

D. an equivalence relation

Answer

Given set A = {1, 2, 3}

And R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (1,1) ∈ R , (2,2) ∈ R , (3,3) ∈ R

Therefore , R is reflexive .... (1)

Check for symmetric

Since , (1,2) ∈ R and (2,1) ∈ R

(2,3) ∈ R and (3,2) ∈ R

Therefore , R is symmetric .... (2)

Check for transitive

Here , (1,2) ∈ R and (2,3) ∈ R but (1,3) ∉ R

Therefore , R is not transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

4. Question

Mark the tick against the correct answer in the following:

Let S be the set of all straight lines in a plane. Let R be a relation on S defined by a R b ⇔ a ⊥ b. Then, R is

A. reflexive but neither symmetric nor transitive

B. symmetric but neither reflexive nor transitive

C. transitive but neither reflexive nor symmetric

D. an equivalence relation

Answer

According to the question ,

Given set S = {x, y, z}

And R = {(x, y), (y, z), (x, z) , (y, x), (z, y), (z, x)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (x,x) ∉ R , (y,y) ∉ R , (z,z) ∉ R

Therefore , R is not reflexive ……. (1)

Check for symmetric

Since , (x,y) ∈ R and (y,x) ∈ R

(z,y) ∈ R and (y,z) ∈ R

(x,z) ∈ R and (z,x) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (x,y) ∈ R and (y,x) ∈ R but (x,x) ∉ R

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

5. Question

Mark the tick against the correct answer in the following:

Let S be the set of all straight lines in a plane. Let R be a relation on S defined by a R b ⇔ a || b. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

Given set S = {x, y, z}

And R = {(x, x), (y, y), (z, z)}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Since , (x,x) ∈ R , (y,y) ∈ R , (z,z) ∈ R

Therefore , R is reflexive ……. (1)

Check for symmetric

Since , (x,x) ∈ R and (x,x) ∈ R

(y,y) ∈ R and (y,y) ∈ R

(z,z) ∈ R and (z,z) ∈ R

Therefore , R is symmetric ……. (2)

Check for transitive

Here , (x,x) ∈ R and (y,y) ∈ R and (z,z) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

6. Question

Mark the tick against the correct answer in the following:

Let Z be the set of all integers and let R be a relation on Z defined by a R b ⇔ (a - b) is divisible by 3. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

Given set Z = {1, 2, 3 ,4 …..}

And R = {(a, b) : a,b ∈ Z and (a-b) is divisible by 3}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) $\in$ R for every a $\in$ A

Symmetric

The relation is Symmetric if (a , b) $\in$ R , then (b , a) $\in$ R

Transitive

Relation is Transitive if (a , b) $\in$ R & (b , c) $\in$ R , then (a , c) $\in$ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

(a - a) = 0 which is divisible by 3

(a,a) $\in$ R where a $\in$ Z

Therefore , R is reflexive .... (1)

Check for symmetric

Consider , (a,b) $\in$ R

$\therefore$ (a - b) which is divisible by 3

- (a - b) which is divisible by 3

(since if 6 is divisible by 3 then -6 will also be divisible by 3)

$\therefore$ (b - a) which is divisible by 3 $\Rightarrow$ (b,a) $\in$ R

For any (a,b) $\in$ R ; (b,a) $\in$ R

Therefore , R is symmetric .... (2)

Check for transitive

Consider , (a,b) $\in$ R and (b,c) $\in$ R

$\therefore$ (a - b) which is divisible by 3

and (b - c) which is divisible by 3

[(a-b)+(b-c)] is divisible by 3] (if 6 is divisible by 3 and 9 is divisible by 3 then 6+9 will also be divisible by 3)

$\therefore$ (a - c) which is divisible by 3 $\Rightarrow$ (a,c) $\in$ R

Therefore (a,b) $\in$ R and (b,c) $\in$ R then (a,c) $\in$ R

Therefore , R is transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

7. Question

Mark the tick against the correct answer in the following:

Let R be a relation on the set N of all natural numbers, defined by a R b $\Leftrightarrow$ a is a factor of b. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

Given set N = {1, 2, 3 ,4 …..}

And R = {(a, b) : a,b ∈ N and a is a factor of b}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

a is a factor of a

(2,2) , (3,3)… (a,a) where a ∈ N

Therefore , R is reflexive ……. (1)

Check for symmetric

a R b ⇒ a is factor of b

b R a ⇒ b is factor of a as well

Ex _ (2,6) ∈ R

But (6,2) ∉ R

Therefore , R is not symmetric ……. (2)

Check for transitive

a R b ⇒ a is factor of b

b R c ⇒ b is a factor of c

a R c ⇒ b is a factor of c also

Ex _(2,6) , (6,18)

∴ (2,18) ∈ R

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (B)

8. Question

Mark the tick against the correct answer in the following:

Let Z be the set of all integers and let R be a relation on Z defined by a R b ⇔ a≥ b. Then, R is

A. symmetric and transitive but not reflexive

B. reflexive and symmetric but not transitive

C. reflexive and transitive but not symmetric

D. an equivalence relation

Answer

According to the question ,

Given set Z = {1, 2, 3 ,4 .....}

And R = {(a, b) : a,b ∈ Z and a ≥ b}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a) (b,b)

∴ a ≥ a and b ≥ b which is always true.

Therefore , R is reflexive .... (1)

Check for symmetric

a R b ⇒ a ≥ b

b R a ⇒ b ≥ a

Both cannot be true.

Ex _ If a=2 and b=1

∴ 2 ≥ 1 is true but 1 ≥ 2 which is false.

Therefore , R is not symmetric .... (2)

Check for transitive

a R b ⇒ a ≥ b

b R c ⇒ b ≥ c

∴ a ≥ c

Ex _a=5 , b=4 and c=2

∴ 5≥4 , 4≥2 and hence 5≥2

Therefore , R is transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

9. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S defined by a R b ⇔ |a| ≤ b. Then, R is

A. reflexive but neither symmetric nor transitive

B. symmetric but neither reflexive nor transitive

C. transitive but neither reflexive nor symmetric

D. none of these

Answer

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and |a| ≤ b }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

∴ |a| ≤ a and which is not always true.

Ex_if a=-2

∴ |-2| ≤ -2 ⇒ 2 ≤ -2 which is false.

Therefore , R is not reflexive ……. (1)

Check for symmetric

a R b ⇒ |a| ≤ b

b R a ⇒ |b| ≤ a

Both cannot be true.

Ex _ If a=-2 and b=-1

∴ 2 ≤ -1 is false and 1 ≤ -2 which is also false.

Therefore , R is not symmetric ……. (2)

Check for transitive

a R b ⇒ $|a| \leq b$

b R c ⇒ $|b| \leq c$

∴ $|a| \leq c$

Ex _a=-5 , b= 7 and c=9

∴ $5 \leq 7$, $7 \leq 9$ and hence $5 \leq 9$

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (C)

10. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S, defined by a R b ⇔ $|a - b| \leq 1$. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and $|a - b| \leq 1$ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

∴ $|a - a| \leq 1$ and which is always true.

Ex_if a=2

∴ $|2-2| \leq 1 \Rightarrow 0 \leq 1$ which is true.

Therefore , R is reflexive ……. (1)

Check for symmetric

a R b ⇒ $|a - b| \leq 1$

$b\ R\ a \Rightarrow |b - a| \le 1$

Both can be true.

Ex _ If a=2 and b=1

$\therefore |2 - 1| \le 1$ is true and $|1-2| \le 1$ which is also true.

Therefore , R is symmetric ……. (2)

Check for transitive

$a\ R\ b \Rightarrow |a - b| \le 1$

$b\ R\ c \Rightarrow |b - c| \le 1$

$\therefore |a - c| \le 1$ will not always be true

Ex _a=-5 , b= -6 and c= -7

$\therefore |6-5| \le 1$, $|7 - 6| \le 1$ are true But $|7 - 5| \le 1$ is false.

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

11. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S, defined by $a\ R\ b \Leftrightarrow (1 + ab) > 0$. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. none of these

Answer

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and (1 + ab) > 0 }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore$ (1 + a×a) > 0 which is always true because a×a will always be positive.

Ex_if a=2

$\therefore$ (1 + 4) > 0 ⇒ (5) > 0 which is true.

Therefore , R is reflexive ……. (1)

Check for symmetric

a R b ⇒ (1 + ab) > 0

b R a ⇒ (1 + ba) > 0

Both the equation are the same and therefore will always be true.

Ex _ If a=2 and b=1

$\therefore$ (1 + 2×1) > 0 is true and (1+1×2) > which is also true.

Therefore , R is symmetric ……. (2)

Check for transitive

a R b ⇒ (1 + ab) > 0

b R c ⇒ (1 + bc) > 0

$\therefore$(1 + ac) > 0 will not always be true

Ex _a=-1 , b= 0 and c= 2

$\therefore$ (1 + -1×0) > 0 , (1 + 0×2) > 0 are true

But (1 + -1×2) > 0 is false.

Therefore , R is not transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

12. Question

Mark the tick against the correct answer in the following:

Let S be the set of all triangles in a plane and let R be a relation on S defined by Δ_1 S Δ_2⇔ $\Delta_1 \equiv A_2$. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

Given set S = {…All triangles in plane….}

And R = {(Δ_1 , Δ_2) : Δ_1 , Δ_2∈ S and $\Delta_1 \equiv \Delta_2$}

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (Δ_1, Δ_1)

∴ We know every triangle is congruent to itself.

$(\Delta_1, \Delta_1) \in R$ all $\Delta_1 \in S$

Therefore , R is reflexive ……. (1)

Check for symmetric

$(\Delta_1 , \Delta_2) \in R$ then Δ_1 is congruent to Δ_2

$(\Delta_2 , \Delta_1) \in R$ then Δ_2 is congruent to Δ_1

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

Check for transitive

Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

Then $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1$ is congruent to $\Delta_{2,}$ and Δ_2 is congruent to Δ_3

$\Rightarrow \Delta_1$ is congruent to Δ_3

$\therefore (\Delta_1, \Delta_3) \in R$

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

13. Question

Mark the tick against the correct answer in the following:

Let S be the set of all real numbers and let R be a relation on S defined by a R b ⇔ $a^2 + b^2 = 1$. Then, R is

A. symmetric but neither reflexive nor transitive

B. reflexive but neither symmetric nor transitive

C. transitive but neither reflexive nor symmetric

D. none of these

Answer

According to the question ,

Given set S = {…….,-2,-1,0,1,2 …..}

And R = {(a, b) : a,b ∈ S and $a^2 + b^2 = 1$ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a,a)

$\therefore a^2 + a^2 = 1$ which is not always true

Ex_if a=2

$\therefore 2^2 + 2^2 = 1 \Rightarrow 4 + 4 = 1$ which is false.

Therefore , R is not reflexive .... (1)

Check for symmetric

$a R b \Rightarrow a^2 + b^2 = 1$

$b R a \Rightarrow b^2 + a^2 = 1$

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric .... (2)

Check for transitive

$a R b \Rightarrow a^2 + b^2 = 1$

$b R c \Rightarrow b^2 + c^2 = 1$

$\therefore a^2 + c^2 = 1$ will not always be true

Ex _a=-1 , b= 0 and c= 1

$\therefore (-1)^2 + 0^2 = 1$, $0^2 + 1^2 = 1$ are true

But $(-1)^2 + 1^2 = 1$ is false.

Therefore , R is not transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (A)

14. Question

Mark the tick against the correct answer in the following:

Let R be a relation on N × N, defined by(a, b) R (c, d) ⇔ a + d = b + c. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

Answer

According to the question ,

R = {(a, b) , (c, d) : a + d = b + c }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (a, b) R (a, b)

(a, b) R (a, b) ⇔ a + b = a + b

which is always true .

Therefore , R is reflexive .... (1)

Check for symmetric

(a, b) R (c, d) ⇔ a + d = b + c

(c, d) R (a, b) ⇔ c + b = d + a

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric .... (2)

Check for transitive

(a, b) R (c, d) ⇔ a + d = b + c

(c, d) R (e, f) ⇔ c + f = d + e

On adding these both equations we get , a + f = b + e

Also,

(a, b) R (e, f) ⇔ a + f = b + e

∴ It will always be true

Therefore , R is transitive .... (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

15. Question

Mark the tick against the correct answer in the following:

Let A be the set of all points in a plane and let O be the origin. Let R = {(P, Q) : OP = QQ}. Then, R is

A. reflexive and symmetric but not transitive

B. reflexive and transitive but not symmetric

C. symmetric and transitive but not reflexive

D. an equivalence relation

There is printing mistake in the question…

R should be R = {(P, Q) : OP = OQ}

Instead of R = {(P, Q) : OP = QQ}

Answer

According to the question ,

O is the origin

R = {(P, Q) : OP = OQ }

Formula

For a relation R in set A

Reflexive

The relation is reflexive if (a , a) ∈ R for every a ∈ A

Symmetric

The relation is Symmetric if (a , b) ∈ R , then (b , a) ∈ R

Transitive

Relation is Transitive if (a , b) ∈ R & (b , c) ∈ R , then (a , c) ∈ R

Equivalence

If the relation is reflexive , symmetric and transitive , it is an equivalence relation.

Check for reflexive

Consider , (P , P) ∈ R ⇔ OP = OP

which is always true .

Therefore , R is reflexive ……. (1)

Check for symmetric

(P , Q) ∈ R ⇔ OP = OQ

(Q , P) ∈ R ⇔ OQ = OP

Both the equation are the same and therefore will always be true.

Therefore , R is symmetric ……. (2)

Check for transitive

(P , Q) ∈ R ⇔ OP = OQ

(Q , R) ∈ R ⇔ OQ = OR

On adding these both equations, we get , OP = OR

Also,

(P , R) ∈ R ⇔ OP = OR

∴ It will always be true

Therefore , R is transitive ……. (3)

Now , according to the equations (1) , (2) , (3)

Correct option will be (D)

16. Question

Mark the tick against the correct answer in the following:

Let Q be the set of all rational numbers, and * be the binary operation, defined by a * b = a + 2b, then

A. * is commutative but not associative

B. * is associative but not commutative

C. * is neither commutative nor associative

D. * is both commutative and associative

Answer

According to the question ,

Q is set of all rarional numbers

R = {(a, b) : a * b = a + 2b }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + 2b

And , b * a = b + 2a

Both equations will not always be true .

Therefore , * is not commutative .... (1)

Check for associative

Consider , (a * b) * c = (a + 2b) * c = a+2b + 2c

And , a * (b * c) = a * (b+2c) = a+2(b+2c) = a+2b+4c

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

17. Question

Mark the tick against the correct answer in the following:

Let a * b = a + ab for all a, b $\in$ Q. Then,

A. * is not a binary composition

B. * is not commutative

C. * is commutative but not associative

D. * is both commutative and associative

Answer

According to the question ,

Q = { a,b }

R = {(a, b) : a * b = a + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + ab

And , b * a = b + ba

Both equations will not always be true .

Therefore , * is not commutative .... (1)

Check for associative

Consider , (a * b) * c = (a + ab) * c = a+ab + (a+ab)c=a+ab+ac+abc

And , a * (b * c) = a * (b+bc) = a+a(b+bc) = a+ab+abc

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (B)

18. Question

Mark the tick against the correct answer in the following:

Let Q^+ be the set of all positive rationals. Then, the operation * on Q^+ defined by $a * b = \frac{ab}{2}$ for all a, b ∈ Q^+ is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

According to the question ,

Q = { Positive rationals }

R = {(a, b) : a * b = ab/2 }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = ab/2

And , b * a = ba/2

Both equations are the same and will always true .

Therefore , * is commutative .... (1)

Check for associative

Consider , (a * b) * c = (ab/2) * c = $\frac{\frac{ab}{2}\times c}{2}$ = abc/4

And , a * (b * c) = a * (bc/2) = $\frac{a\times\frac{bc}{2}}{2}$ = abc/4

Both the equation are the same and therefore will always be true.

Therefore , * is associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

19. Question

Mark the tick against the correct answer in the following:

let Z be the set of all integers and let a * b = a – b + ab. Then, * is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a – b + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a – b + ab

And , b * a = b – a + ba

Both equations are not the same and will not always be true .

Therefore , * is not commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a – b + ab) * c

= a – b + ab – c +(a – b + ab)c

=a – b +ab – c +ac – bc + abc

And , a * (b * c) = a * (b – c + bc)

= a - (b – c + bc) + a(b – c + bc)

=a – b + c – bc + ab – ac + abc

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative ……. (2)

Now , according to the equations (1) , (2) Correct option will be (C)

20. Question

Mark the tick against the correct answer in the following:

Let Z be the set of all integers. Then, the operation * on Z defined by

a * b = a + b - ab is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a + b - ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + b - ab

And , b * a = b + a - ba

Both equations are the same and will always be true .

Therefore , * is commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a + b - ab) * c

= a + b - ab + c -(a + b - ab)c

=a + b – ab + c – ac – bc + abc

And , a * (b * c) = a * (b + c - bc)

= a + (b + c - bc) - a(b + c - bc)

=a + b + c – bc - ab – ac + abc

Both the equation are the same and therefore will always be true.

Therefore , * is associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (D)

21. Question

Mark the tick against the correct answer in the following:

Let Z^+ be the set of all positive integers. Then, the operation * on Z^+ defined bya * b = a^b is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

According to the question ,

Q = { All integers }

R = {(a, b) : a * b = a^b }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a^b

And , b * a = b^a

Both equations are not the same and will not always be true .

Therefore , * is not commutative ……. (1)

Check for associative

Consider , (a * b) * c = (a^b) * c = $(a^b)^c$

And , a * (b * c) = a * (b^c)= $a^{(b^c)}$

Ex a=2 b=3 c=4

(a * b) * c = (2^3) * c = $(8)^4$

a * (b * c) = 2 * (3^4)= $2^{(81)}$

Both the equation are not the same and therefore will not always be true.

Therefore , * is not associative ……. (2)

Now , according to the equations (1) , (2)

Correct option will be (C)

22. Question

Mark the tick against the correct answer in the following:

Define * on Q - {-1} by a * b= a + b + ab. Then, * on Q – {-1} is

A. commutative but not associative

B. associative but not commutative

C. neither commutative nor associative

D. both commutative and associative

Answer

According to the question ,

R = {(a, b) : a * b = a + b + ab }

Formula

* is commutative if a * b = b * a

* is associative if (a * b) * c = a * (b * c)

Check for commutative

Consider , a * b = a + b + ab

And , b * a = b + a + ba

Both equations are same and will always be true .

Therefore , * is commutative .... (1)

Check for associative

Consider , (a * b) * c = (a + b + ab) * c

= a + b + ab + c + (a + b + ab)c

=a + b + c + ab + ac + bc + abc

And , a * (b * c) = a * (b + c + bc)

= a + b + c + bc + a(b + c + bc)

=a +b + c + ab + bc + ac + abc

Both the equation are same and therefore will always be true.

Therefore , * is associative .... (2)

Now , according to the equations (1) , (2)

Correct option will be (D)