# 6. Determinants

## Exercise 6A

**1. Question**

If A is a 2 × 2 matrix such that |A| ≠ 0 and |A| = 5, write the value of |4A|.

**Answer**

Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given, p=4,k=2 and |A|=5.

$|4A|=4^2 \times 5$

$=16 \times 5$

$=80$

**2. Question**

If A is a 3 × 3 matrix such that |A| ≠ 0 and |3A| = k|A| then write the value of k.

**Answer**

Theorem: If Let A be k × k matrix then $|pA|=p^k|A|$.

Given: k=3 and p=3.

$|3A|=3^3 \times |A|$

$=27|A|$.

Comparing above with k|A| gives k=27.

**3. Question**

Let A be a square matrix of order 3, write the value of |2A|, where |A| = 4.

**Answer**

Theorem: If A be k × k matrix then $|pA|=p^k|A|$.

Given: p=2, k=3 and|A|=4

$|2A|=2^3 \times |A|$

$=8 \times 4$

$= 32$

**4. Question**

If $A_{ij}$ is the cofactor of the element $a_{ij}$ of $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then write the value of $(a_{32}A_{32})$.

**Answer**

Theorem: $A_{ij}$ is found by deleting $i^{th}$ rowand $j^{th}$ column, the determinant of left matrix is called cofactor with multiplied by $(-1)^{(i+j)}$.

Given: i=3 and j=2.

$A_{32}=(-1)^{(3+2)}(2 \times 4-6 \times 5)$

$=-1 \times (-22)$

=22

$a_{32}$=5

$a_{32}A_{32}$= 5 × 22

=110

**5. Question**

Evaluate $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$.

**Answer**

Theorem: This evaluation can be done in two different ways either by taking out the common things and then calculating the determinants or simply take determinant.

I will prefer first method because with that chances of silly mistakes reduces.

Take out x+1 from second row.

(x+1) × $\begin{vmatrix} x^2-x+1 & x-1 \\ 1 & 1 \end{vmatrix}$

⇒(x+1) × ($x^2$-x+1-(x-1))

⇒ (x+1) × ($x^2$-2x+2)

⇒ $x^3$-2$x^2$+2x+$x^2$-2x+2

⇒ $x^3$-$x^2$+2 .

**6. Question**

Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

**Answer**

This we can very simply go through directly.

((a+ib)(a-ib))-((-c+id)(c+id)).

⇒ ($a^2$+$b^2$)-(-$c^2$ -$d^2$).

⇒ $a^2$+$b^2$ + $c^2$+$d^2$

∵ i × i=-1

**7. Question**

If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, write the value of x.

**Answer**

Here the determinant is compared so we need to take determinant both sides then find x.

12x+14=32-42

⇒ 12x=-10-14

⇒ 12x=-24

⇒ x=-2

**8. Question**

If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, write the value of x.

**Answer**

this question is having the same logic as above.

$2x^2$-40=18+14

⇒ $2x^2$=72

⇒ $x^2$=36

⇒ x=$\pm 6$.

**9. Question**

If $\begin{vmatrix} 2x & x+3 \\ 2(x+1) & x+1 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 3 & 3 \end{vmatrix}$, write the value of x.

**Answer**

Simply by equating both sides we can get the value of x.

$2x^2+2x-2(x^2+4x+3)=-12$

⇒ -6x-6=-12

⇒ -6x=-6

⇒ $x = 1$

**10. Question**

If $A = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, find the value of 3|A|.

**Answer**

Find the determinant of A and then multiply it by 3

|A|=2

3|A|=3 × 2

=6

**11. Question**

Evaluate $2\begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.

**Answer**

It is determinant multiplied by a scalar number 2, just find determinant of matrix and multiply it by 2.

2 × (35-20)

2 × 15= 30

**12. Question**

Evaluate $\begin{vmatrix} \sqrt{6} & \sqrt{5} \\ \sqrt{20} & \sqrt{24} \end{vmatrix}$.

**Answer**

Find determinant

$\sqrt{6} \times \sqrt{24} - \sqrt{20} \times \sqrt{5}$

$\sqrt{144} - \sqrt{100}$.

=12-10

=2.

**13. Question**

Evaluate $\begin{vmatrix} 2\cos\theta & -2\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$.

**Answer**

After finding determinant we will get a trigonometric identity.

$2\cos^2\theta + 2\sin^2\theta$

=2

$\because \sin^2\theta + \cos^2\theta = 1$

**14. Question**

Evaluate $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$.

**Answer**

After finding determinant we will get a trigonometric identity.

$\cos^2\alpha + \sin^2\alpha$

=1

$\because \sin^2\theta + \cos^2\theta = 1$

**15. Question**

Evaluate $\begin{vmatrix} \sin 60^\circ & \cos 60^\circ \\ -\sin 30^\circ & \cos 30^\circ \end{vmatrix}$.

**Answer**

After finding determinant we will get,

Sin60° $= \frac{\sqrt{3}}{2} =$ cos30°

Cos60° $= \frac{1}{2} =$ sin30°

sin 60° × cos30° + sin30° × cos60°

$\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

= 1.

**16. Question**

Evaluate $\begin{vmatrix} \cos 65^\circ & \sin 65^\circ \\ \sin 25^\circ & \cos 25^\circ \end{vmatrix}$.

**Answer**

By directly opening this determinant

cos65° × cos25° -sin25° × sin65°

= cos(65°+25°) ∵ cosAcosB-sinAsinB=cos(A+B)

= cos90°

= 0

∵ cosAcosB-sinAsinB=cos(A+B)

**17. Question**

Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$.

**Answer**

cos15°cos75° - sin75°sin15°

= cos(15°+75°) ∵ cosAcosB-sinAsinB=cos(A+B)

= cos90°

= 0

∵ cosAcosB-sinAsinB=cos(A+B)

**18. Question**

Evaluate $\begin{vmatrix} 0 & 2 & 0 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{vmatrix}$.

**Answer**

We know that expansion of determinant with respect to first row is $a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$.

0(3 × 6-5 × 4)-2(2 × 6-4 × 4)+0(2 × 5-4 × 3)

= 8.

**19. Question**

Without expanding the determinant, prove that $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix} = 0$.

**SINGULAR MATRIX** A square matrix A is said to be singular if |A| = 0.

Also, A is called non singular if |A| ≠ 0.

**Answer**

We know that $C_1 \Rightarrow C_1 - C_2$, would not change anything for the determinant.

Applying the same in above determinant, we get

$\begin{bmatrix} 40 & 1 & 5 \\ 72 & 7 & 9 \\ 24 & 5 & 3 \end{bmatrix}$ Now it can clearly be seen that $C_1 = 8 \times C_3$

Applying above equation we get,

$$\begin{bmatrix} 0 & 1 & 5 \\ 0 & 7 & 9 \\ 0 & 3 & 3 \end{bmatrix}$$

We know that if a row or column of a determinant is 0. Then it is singular determinant.

**20. Question**

For what value of x, the given matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is a singular matrix?

**Answer**

For A to be singular matrix its determinant should be equal to 0.

0= (3-2x) × 4-(x+1) × 2

0= 12-8x-2x-2

0=10-10x

X=1.

**21. Question**

Evaluate $\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$.

**Answer**

$\begin{vmatrix} 14 & 9 \\ -8 & -7 \end{vmatrix}$.=14 × (-7)-9 × (-8)

= -26

**22. Question**

Evaluate $\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$.

**Answer**

$\begin{vmatrix} \sqrt{3} & \sqrt{5} \\ -\sqrt{5} & 3\sqrt{3} \end{vmatrix}$. = 3√3 × √3 – (-√5 × √5)

= 14.

## Exercise 6B

**1. Question**

Evaluate :

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 67 & 19 & 21 \\ 78 & 26 & 28 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_2' = (1/2)R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 67 & 19 & 21 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81 & 24 & 26 \end{vmatrix}$$ [$R_1' = R_1 - R_3$]

$$= \begin{vmatrix} -14 & -5 & -5 \\ -3 & 2 & 2 \\ 81/2 & 12 & 13 \end{vmatrix}$$ [$R_3' = 2R_3$]

$= (-14)\{(2 \times 13) - (2 \times 12)\} - 5\{(2 \times {}^{81}/_{2}) - (-3) \times 13\} - 5\{(-3) \times 12 - 2 \times {}^{81}/_{2}\}$

[expanding by the first row]

$= -14 \times (26 - 24) - 5(81 + 39) - 5(-36 - 81)$

$= -14 \times 2 - 5 \times 120 - 5 \times (-117) = -28 - 600 + 585 =$ **$-43$**

**2. Question**

Evaluate :

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 29 & 26 & 22 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & -5 & -5 \\ 25 & 31 & 27 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 4 & -5 & -5 \\ 50 & 62 & 54 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_2' = 2R_2$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63 & 54 & 46 \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 4 & -5 & -5 \\ -13 & 8 & 8 \\ 63/2 & 27 & 23 \end{vmatrix}$$ [$R_3' = 2R_3$]

$= 4(8 \times 23 - 8 \times 27) - 5\{8 \times {}^{63}/_{2} - (-13) \times 23\} - 5\{(-13) \times 27 - 8 \times {}^{63}/_{2}\}$

[expansion by first row]

**= 132**

**3. Question**

Evaluate :

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} = 6 \times \begin{vmatrix} 17 & 18 & 6 \\ 1 & 6 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$ [$R_1' = R_1/6$]

Now, for any determinant, if at least two rows are identical, then the value of the determinant becomes zero.

Here, the first and third rows are identical.

So, the value of the above determinant evaluated = **0**

**4. Question**

Evaluate :

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 2^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$$

**Answer**

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Expanding by first row, we get,

1(9 × 25 - 16 × 16) + 4(16 × 9 - 4 × 25) + 9(4 × 16 - 9 × 9) = - 31 + 176 - 153 =**- 8**

**5. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a).$$

**Answer**

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$$ [$C_1' = C_1/(a - b)$ & $C_2' = C_2/(b - c)$]

= (a - b)(b - c)[0 + 0 + 1{ - a - ( - c)}] [expansion by first row]

= **(a - b)(b - c)(c - a)**

**6. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a).$$

**Answer**

$$\begin{vmatrix} 1 & b + c & b^2 + c^2 \\ 1 & c + a & c^2 + a^2 \\ 1 & a + b & a^2 + b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & b-a & b^2 - a^2 \\ 0 & c-b & c^2 - b^2 \\ 1 & a + b & a^2 + b^2 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 0 & b-a & (b-a)(b + a) \\ 0 & c-b & (c-b)(c + b) \\ 1 & a + b & a^2 + b^2 \end{vmatrix}$$

$$= (b-a)(c-b) \begin{vmatrix} 0 & 1 & b + a \\ 0 & 1 & c + b \\ 1 & a + b & a^2 + b^2 \end{vmatrix}$$ [$R_1' = R_1/(b - a)$ & $R_2' = R_2/(c - b)$]

= (b - a)(c - b)[0 + 0 + 1{(c + b) - (b + a)}][expansion by first column]

= **(a - b)(b - c)(c - a)**

**7. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1.$$

**Answer**

$$\begin{vmatrix} 1 & 1 + p & 1 + p + q \\ 2 & 3 + 2p & 1 + 3p + 2q \\ 3 & 6 + 3p & 1 + 6p + 3q \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2-p & -2p-q \\ -1 & -3-p & -3p-q \\ 3 & 6 + 3p & 1 + 6p + 3q \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 0 & 1 & p \\ -1 & -3-p & -3p-q \\ 3 & 6 + 3p & 1 + 6p + 3q \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ -2 & -6-2p & -6p-2q \\ 3 & 6 + 3p & 1 + 6p + 3q \end{vmatrix}$$ [$R_2' = R_2*2$]

$$= \left(\frac{1}{2}\right) \begin{vmatrix} 0 & 1 & p \\ 1 & p & 1 + q \\ 3 & 6 + 3p & 1 + 6p + 3q \end{vmatrix}$$ [$R_2' = R_2 + R_3$]

= (1/2)[0 + 3(1 + q) - (1 + 6p + 3q) + p(6 + 3p - 3p)] [expansion by first row]

= (1/2)(3 + 3q - 1 - 6p - 3q + 6p) = **1**

**8. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z).$$

**Answer**

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

$= \begin{vmatrix} a & -a & 0 \\ 0 & a & -a \\ x & y & a+z \end{vmatrix}$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$= a^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ x & y & a+z \end{vmatrix}$ [$R_1' = R_1/a$ & $R_2' = R_2/a$]

= $a^2$[a + z - ( - y) - ( - x)] [expansion by first row]

= $\mathbf{a^2(a + x + y + z)}$

**9. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x+2a)(x-a)^2.$$

**Answer**

$$\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$$

$= \begin{vmatrix} x+2a & x+2a & x+2a \\ a & x & a \\ a & a & x \end{vmatrix}$ [$R_1' = R_1 + R_2 + R_3$]

$= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ a & x & a \\ a & a & x \end{vmatrix}$ [$R_1' = R_1/(x + 2a)$]

$= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & a-x \\ a & a & x \end{vmatrix}$ [$R_2' = R_2 - R_3$]

$= (x+2a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & x-a & -(x-a) \\ a & a & x \end{vmatrix}$

$= (x+2a)(x-a) \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ a & a & x \end{vmatrix}$ [$R_2' = R_2/(x - a)$]

= (x + 2a)(x - a)[x - ( - a) + ( - a - 0) + ( - a)] [expansion by first row]

= (x + 2a)(x - a)(x + a - a - a) = $\mathbf{(x + 2a)(x - a)^2}$

**10. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(x-4)^2.$$

**Answer**

$$\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= \begin{vmatrix} 5x+4 & 5x+4 & 5x+4 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} [R_1' = R_1 + R_2 + R_3]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} [R_1' = R_1/(5x+4)]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+4 & x-4 \\ 2x & 2x & x+4 \end{vmatrix} [R_2' = R_2 - R_3]$$

$$= (5x+4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-4) & x-4 \\ 2x & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)(x-4)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+4 \end{vmatrix} [R_2' = R_2/(x-4)]$$

$= (5x + 4)(x - 4)[ - (x + 4) - 2x + 2x - 0 + 0 - ( - 2x)]$ [expansion by first row]

$= (5x + 4)(x - 4)( - x - 4 + 2x) =$ $\mathbf{(5x + 4)(x - 4)^2}$

**11. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} = (5x+\lambda)(\lambda-x)^2.$$

**Answer**

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} 5x+\lambda & 5x+\lambda & 5x+\lambda \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [R_1' = R_1 + R_2 + R_3]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [R_1' = R_1/(5x+\lambda)]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -x+\lambda & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix} [R_2' = R_2 - R_3]$$

$$= (5x+\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(x-\lambda) & x-\lambda \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= (5x+\lambda)(x-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 2x & 2x & x+\lambda \end{vmatrix} [R_2' = R_2/(x-\lambda)]$$

$= (5x + \lambda)(x - \lambda)[ - (x + \lambda) - 2x + 2x - 0 + 0 - ( - 2x)]$ [expansion by first row]

$= (5x + \lambda)(x - \lambda)( - x - \lambda + 2x) =$ $\mathbf{(5x + \lambda)(x - \lambda)^2}$

**12. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3.$$

**Answer**

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2a - 2 & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2(a - 1) & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= (a-1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ [$R_1' = R_1/(a - 1)$ & $R_2' = R_2/(a - 1)$]

$= (a - 1)^2[a + 1 - 0 - 2]$ [expansion by first row]

$= \mathbf{(a - 1)^3}$

**13. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y).$$

**Answer**

$$\begin{vmatrix} x & x + y & x + 2y \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix}$$

$$= \begin{vmatrix} 3(x + y) & 3(x + y) & 3(x + y) \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= 3(x + y)\begin{vmatrix} 1 & 1 & 1 \\ x + 2y & x & x + y \\ x + y & x + 2y & x \end{vmatrix}$$ [$R_1' = R_1/3(x + y)$]

$$= 3(x + y)\begin{vmatrix} 1 & 1 & 1 \\ y & -2y & y \\ x + y & x + 2y & x \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= 3y(x + y)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ x + y & x + 2y & x \end{vmatrix}$$ [$R_2' = R_2/y$]

$$= 3y(x+y)\begin{vmatrix} 0 & 3 & 0 \\ 1 & -2 & 1 \\ x+y & x+2y & x \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

= 3y(x + y)[0 + 3(x + y) - x + 0] [expansion by first row]

= 3y(x + y)(3y) = $\mathbf{9y^2(x+y)}$

**14. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx).$$

**Answer**

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$$

$$= \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$ [$C_1' = C_1 + C_2 + C_3$]

$$= (x+y+z)\begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$ [$C_1' = C_1/(x + y + z)$]

$$= (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ -x+y & 3y & y-z \\ -x+z & z-y & 3z \end{vmatrix}$$ [transforming row and column]

$$= (x+y+z)\begin{vmatrix} 0 & 0 & 1 \\ -x-2y & 2y+z & y-z \\ -x+y & -y-2z & x \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

= (x + y + z)[0 + 0 + ( - x - 2y)( - y - 2z) - ( - x + y)(2y + z)] [expansion by first row]

= (x + y + z)(xy + 2y$^2$ + 2xz + 4yz + 2xy - 2y$^2$ + xz - yz)

= (x + y + z)(3xy + 3yz + 3xz)

= **3(x + y + z)(xy + yz + zx)**

**15. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

**Answer**

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

$$= xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$ [$C_1' = C_1/x$, $C_2' = C_2/y$ & $C_3' = C_3/z$]

$$= xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= xyz\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ (x+y)(x-y) & (y+z)(y-z) & z^2 \end{vmatrix}$$

$$= xyz(x-y)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2 \end{vmatrix}$$ [$C_1' = C_1/(x - y)$& $C_2' = C_2/(y - z)$]

= xyz(x - y)(y - z)(0 + 0 + y + z - x - y) [expansion by first row]

**= xyz(x - y)(y - z)( z - x)**

**16. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3.$$

**Answer**

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= (a+b+c)\begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$ [$R_1' = R_1/(a + b + c)$]

= (a + b + c)[2(b - c)c - b(c - a) + (c + a)(c - a) - (a + b)(b - c)][expansion by first row]

= (a + b + c)(2bc - 2$c^2$ - bc + ab + $c^2$ - $a^2$ - ab - $b^2$ + ac + bc

= (a + b + c)(ab + bc + ac - $a^2$ - $b^2$ - $c^2$)

**= 3abc - $a^3$ - $b^3$ - $c^3$**

**17. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

**Answer**

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

$$= \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$ [$R_1' = R_1 + R_2 + R_3$]

$$= 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} [R_1' = R_1/2]$$

$$= 2\begin{vmatrix} c & 0 & a \\ b-c & a & -a \\ c & c & a+b \end{vmatrix} [R_1' = R_1 - R_2 \,\&\, R_2' = R_2 - R_3]$$

= 2[c{a(a + b) - ( - ac)} + 0 + a{c(b - c) - ac}][expansion by first row]

= $2(a^2c + abc + ac^2 + abc - ac^2 - a^2c)$

= **4abc**

**18. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = -a^3.$$

**Answer**

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$$

$$= \left(\frac{1}{3}\right)\begin{vmatrix} 3a & 3a+6b & 3a+6b+9c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = 3R_1]$$

$$= \left(\frac{1}{3}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 6a & 8a+12b & 10a+14b+18c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = 2R_2]$$

$$= \left(\frac{1}{6}\right)\begin{vmatrix} 0 & -a & -2a-b \\ 0 & -a & -a-b \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} [R_2' = R_2 - R_3]$$

= (1/6)[0 + 0 + 6a{a(a + b) - a(2a + b)}][expansion by first column]

= **- $a^3$**

**19. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

**Answer**

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

$$= \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} [R_1' = R_1 + R_2]$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} [R_1' = R_1/(a+b)]$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c-b & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix} [R_2' = R_2 + R_3]$$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}$$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix} [R_2' = R_1/(b+c)]$$

$$= (a+b)(b+c)\begin{vmatrix} 0 & 2 & 0 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix} [R_1' = R_1 + R_2]$$

= (a + b)(b + c){0 + 2( - b + a + b + c) + 0}[expansion by first row]

**= 2(a + b)(b + c)(c + a)**

**20. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = \left(b^2-ac\right)\left(ax^2+3bxy+cy^2\right).$$

**Answer**

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix}$$

$$= \left(\frac{1}{xy}\right)\begin{vmatrix} ax & bx & ax^2+bxy \\ by & cy & bxy+cy^2 \\ ax+by & bx+cy & 0 \end{vmatrix} [R_1' = xR_1 \ \& \ R_2' = yR_2]$$

$$= \left(\frac{1}{xy}\right)\begin{vmatrix} 0 & 0 & ax^2+2bxy+cy^2 \\ by & cy & bxy+cy^2 \\ ax+by & bx+cy & 0 \end{vmatrix} [R_1' = R_1 + R_2 - R_3]$$

= $(1/xy)[0 + 0 + (ax^2 + 2bxy + cy^2)\{by(bx + cy) - cy(ax + by)\}]$[expansion by first row] .

= $(1/xy)( ax^2 + 2bxy + cy^2)(b^2xy + bcy^2 - acxy - bcy^2)$

**= $(b^2 - ac)(ax^2 + 2bxy + cy^2)$**

**21. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4(a-b)(b-c)(c-a)$$

**Answer**

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^2+2a+1 & b^2+2b+1 & c^2+2c+1 \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}$$ [$R_2' = R_2 - R_3$]

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ a^2-2a+1 & b^2-2b+1 & c^2-2c+1 \end{vmatrix}$$ [$R_2' = R_2/4$]

$$= 4\begin{vmatrix} a^2 & a & a^2-2a+1 \\ b^2 & b & b^2-2b+1 \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$ [transforming row and column]

$$= 4\begin{vmatrix} a^2-b^2 & a-b & (a^2-b^2)-2(a-b) \\ b^2-c^2 & b-c & (b^2-c^2)-2(b-c) \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= 4\begin{vmatrix} (a-b)(a+b) & a-b & (a-b)(a+b-2) \\ (b-c)(b+c) & b-c & (b-c)(b+c-2) \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$

$$= 4(a-b)(b-c)\begin{vmatrix} a+b & 1 & a+b-2 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$ [$R_1' = R_1/(a - b)$ & $R_2' = R_2/(b - c)$]

$$= 4(a-b)(b-c)\begin{vmatrix} a-c & 0 & a-c \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$ [$R_1' = R_1 - R_2$]

$$= 4(a-b)(b-c)(a-c)\begin{vmatrix} 1 & 0 & 1 \\ b+c & 1 & b+c-2 \\ c^2 & c & c^2-2c+1 \end{vmatrix}$$ [$R_1' = R_1/(a - c)$]

$= 4(a - b)(b - c)(a - c)(c^2 - 2c + 1 - bc - c^2 + 2c + 0 + bc + c^2 - c^2)$ [expansion by first row]

**$= 4(a - b)(b - c)(c - a)$**

**22. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix} = -8$$

**Answer**

$$\begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ x^2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$$

$$= \begin{vmatrix} x^2-4x+4 & x^2-2x+1 & x^2 \\ x^2-2x+1 & x^2 & x^2+2x+1 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$

$$= \begin{vmatrix} -2x+3 & -2x+1 & -2x-1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix}$$ [$R_1' = R_1 - R_2$ & $R_2' = R_2 - R_3$]

$$= \begin{vmatrix} 2 & 2 & 2 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2]$$

$$= 2\begin{vmatrix} 1 & 1 & 1 \\ -2x+1 & -2x-1 & -2x-3 \\ x^2 & x^2+2x+1 & x^2+4x+4 \end{vmatrix} [R_1' = R_1/2]$$

$$= 2\begin{vmatrix} 1 & -2x+1 & x^2 \\ 1 & -2x-1 & x^2+2x+1 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} \text{[transforming row and column]}$$

$$= 2\begin{vmatrix} 0 & 2 & -2x-1 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2 \ \& \ R_2' = R_2 - R_3]$$

$$= 2\begin{vmatrix} 0 & 0 & 2 \\ 0 & 2 & -2x-3 \\ 1 & -2x-3 & x^2+4x+4 \end{vmatrix} [R_1' = R_1 - R_2]$$

= 2{0 + 0 + 2(0 - 2)} [expansion by first row]

. = **- 8**

**23. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix} = (l^2+m^2+n^2)(l-m)$$

$$(m-n)(n-l).$$

**Answer**

$$\begin{vmatrix} (m+n)^2 & l^2 & mn \\ (n+l)^2 & m^2 & ln \\ (l+m)^2 & n^2 & lm \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} m^2+2mn+n^2 & l^2 & 2mn \\ n^2+2nl+l^2 & m^2 & 2ln \\ l^2+2lm+m^2 & n^2 & 2lm \end{vmatrix} [C_3' = 2C_3]$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} m^2+n^2 & l^2 & 2mn \\ n^2+l^2 & m^2 & 2ln \\ l^2+m^2 & n^2 & 2lm \end{vmatrix} [C_1' = C_1 - C_3]$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} l^2+m^2+n^2 & l^2 & 2mn \\ l^2+m^2+n^2 & m^2 & 2ln \\ l^2+m^2+n^2 & n^2 & 2lm \end{vmatrix} [C_1' = C_1 + C_2]$$

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & l^2 & 2mn \\ 1 & m^2 & 2ln \\ 1 & n^2 & 2lm \end{vmatrix} [C_1' = C_1/(l^2+m^2+n^2)]$$

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 1 & 1 & 1 \\ l^2 & m^2 & n^2 \\ 2mn & 2ln & 2lm \end{vmatrix} \text{[transforming row and column]}$$

$$= \left(\frac{1}{2}\right)(l^2+m^2+n^2)\begin{vmatrix} 0 & 0 & 1 \\ l^2-m^2 & m^2-n^2 & n^2 \\ -2n(l-m) & -2l(m-n) & 2lm \end{vmatrix} [C_1' = C_1 - C_2 \ \& \ C_2' = C_2 - C_3]$$

$$= (l^2 + m^2 + n^2)(l-m)(m-n)\begin{vmatrix} 0 & 0 & 1 \\ l+m & m+n & n^2 \\ -n & -l & lm \end{vmatrix}$$ [$C_1' = C_1/(l - m)$ & $R_2' = C_2/(l - m)$]

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{0 + 0 - l(l + m) + n(m + n)\}$ [expansion by first row]

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{0 + 0 - l(l + m) + n(m + n)\}$

$= (l^2 + m^2 + n^2)(l - m)(m - n)( - l^2 - ml + mn + n^2)$

$= (l^2 + m^2 + n^2)(l - m)(m - n)\{(n^2 - l^2) + m(n - l)\}$

$= \mathbf{(l^2 + m^2 + n^2)(l - m)(m - n)(n - l)(l + m + n)}$

**24. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a-b)\ (b-c)(c-a)(a+b+c).$$

**Answer**

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$$

$$= \left(\frac{1}{2}\right)\begin{vmatrix} b^2+2bc+c^2 & a^2 & 2bc \\ c^2+2ac+a^2 & b^2 & 2ca \\ a^2+2ab+b^2 & c^2 & 2ab \end{vmatrix}$$ [$C_3' = 2C_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} b^2+c^2 & a^2 & 2bc \\ c^2+a^2 & b^2 & 2ca \\ a^2+b^2 & c^2 & 2ab \end{vmatrix}$$ [$C_1' = C_1 - C_3$]

$$= \left(\frac{1}{2}\right)\begin{vmatrix} a^2+b^2+c^2 & a^2 & 2bc \\ a^2+b^2+c^2 & b^2 & 2ca \\ a^2+b^2+c^2 & c^2 & 2ab \end{vmatrix}$$ [$C_1' = C_1 + C_2$]

$$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 2bc \\ 1 & b^2 & 2ca \\ 1 & c^2 & 2ab \end{vmatrix}$$ [$C_1' = C_1/(a^2 + b^2 + c^2)$]

$$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ 2bc & 2ca & 2ab \end{vmatrix}$$ [transforming row and column]

$$= \left(\frac{1}{2}\right)(a^2+b^2+c^2)\begin{vmatrix} 0 & 0 & 1 \\ a^2-b^2 & b^2-c^2 & c^2 \\ -2c(a-b) & -2a(b-c) & 2ab \end{vmatrix}$$ [$C_1' = C_1 - C_2$ & $C_2' = C_2 - C_3$]

$$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ a+b & b+c & c^2 \\ -c & -a & ab \end{vmatrix}$$ [$C_1' = C_1/(a - b)$ & $C_2' = C_2/(b - c)$]

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{0 + 0 - a(a + b) + c(b + c)\}$ [expansion by first row]

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{0 + 0 - a(a + b) + c(b + c)\}$

$= (a^2 + b^2 + c^2)(a - b)(b - c)( - a^2 - ba + bc + c^2)$

$= (a^2 + b^2 + c^2)(a - b)(b - c)\{(c^2 - a^2) + b(c - a)\}$

$= \mathbf{(a^2 + b^2 + c^2)(a - b)(b - c)(c - a)(a + b + c)}$

**25. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$$

**Answer**

$$\begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 2(b^2 + c^2) & 2(c^2 + a^2) & 2(a^2 + b^2) \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1 + R_2 + R_3]$$

$$= 2\begin{vmatrix} (b^2 + c^2) & (c^2 + a^2) & (a^2 + b^2) \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1/2]$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix} [R_1' = R_1 - R_2]$$

$= 2[c^2\{(c^2 + a^2)(a^2 + b^2) - b^2c^2\} + 0 + a^2\{b^2c^2 - c^2(c^2 + a^2)\}]$ [expansion by first row]

$= 2[c^2(c^2a^2 + a^4 + b^2c^2 + a^2b^2 - b^2c^2) + a^2(b^2c^2 - c^4 - a^2c^2)]$

$= 2[a^2c^4 + a^4c^2 + a^2b^2c^2 + a^2b^2c^2 - a^2c^4 - a^4c^2]$

$= \mathbf{4a^2b^2c^2}$

**26. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3.$$

**Answer**

Operating $R_1 \to R_1 + bR$ �$_3$, $R_2 \to R_2 - aR_3$

$$\begin{vmatrix} 1 + a^2 - b^2 + 2b^2 & 2ab - 2ab & -2b + b - a^2b - b^3 \\ 2ab - 2ab & 1 - a^2 + b^2 + 2a^2 & 2a - a + a^3 + ab^2 \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 + a^2 + b^2 & 0 & -b - a^2b - b^3 \\ 0 & 1 + a^2 + b^2 & a + a^3 + ab^2 \\ 2b & -2a & 1 - a^2 + b^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 + a^2 + b^2 & 0 & -b(1 + a^2 + b^2) \\ 0 & 1 + a^2 + b^2 & a(1 + a^2 + b^2) \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$$

Taking $(1+ a^2+b^2)$ from $R_1$ and $R_2$

$$= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix}$$

Operating $R_3 \to R_3 - 2bR_1 + 2aR_2$

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1+a^2+b^2 \end{vmatrix}$$

Taking $(1+a^2+b^2)$ from $R_3$

$$(1+a^2+b^2)^3 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with respect to $C_1$

$= (1+a^2+b^2)^3\ 1\times[\ 1-0]$

$= (1+a^2+b^2)^3$

Hence proved

**27. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)\left(a^2+b^2+c^2\right).$$

**Answer**

Operating $C_1 \to aC_1$

$$\frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & a+b & c \end{vmatrix}$$

Operating $C_1 \to C_1 + bC_2 + cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+\ b^2-bc+c^2+bc & b-c & c+b \\ a^2+ac+b^2+c^2-ac & b & c-a \\ a^2-ab+ab+b^2+c^2 & a+b & c \end{vmatrix}$$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & a+b & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from $C_1$

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 1 & b-c & c+b \\ 1 & b & c-a \\ 1 & a+b & c \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -c-a & b \\ 0 & -a & -a \\ 1 & a+b & c \end{vmatrix}$$

Operating $C_2 \to C_2 - C_3$

$$= \frac{1}{a}(a^2+b^2+c^2)\begin{vmatrix} 0 & -(a+b+c) & b \\ 0 & 0 & -a \\ 1 & (a+b+c) & c \end{vmatrix}$$

Taking $(a+b+c)$ common from $C_2$

$$=\frac{1}{a}(a^2+b^2+c^2)(a+b+c)\begin{vmatrix}0 & -1 & b\\ 0 & 0 & -a\\ 1 & 1 & c\end{vmatrix}$$

Expanding with respect to $C_1$

$$=\frac{1}{a}(a^2+b^2+c^2)(a+b+c)\times 1\times\left(0—(-a)\right)$$

$$=\frac{1}{a}(a^2+b^2+c^2)(a+b+c)\ (a)$$

$= (a^2+b^2+c^2)(a+b+c)$

**28. Question**

Using properties of determinants prove that:

$$\begin{vmatrix}b^2c^2 & bc & b+c\\ c^2a^2 & ca & c+a\\ a^2b^2 & ab & a+b\end{vmatrix}=0.$$

**Answer**

Expanding with R1

$=b^2c^2(a^2c+abc-abc-a^2b)-bc(a^3c^2+a^2bc^2-a^2b^2c-a^3b^2)+(b+c)(a^3bc^2-a^3b^2c)$

$=a^2b^3c^2-a^2b^3c^2-a^3bc^2-a^2b^3c^2+a^2b^3c^2+a^3b^3c+a^3b^2c^2-a^3b^3c+a^3bc^3-a^3b^2c^2$

$=0$

**29. Question**

Using properties of determinants prove that:

$$\begin{vmatrix}(b+c)^2 & ab & ca\\ ab & (a+c)^2 & bc\\ ac & bc & (a+b)^2\end{vmatrix}=2abc(a+b+c)^3.$$

**Answer**

$$=\begin{vmatrix}b^2+c^2+2bc & ab & ac\\ ab & a^2+c^2+2ac & bc\\ ac & bc & a^2+b^2+2ab\end{vmatrix}$$

Operating $R_1\to aR_1$, $R_2\to bR_2$, $R_3\to cR_3$

$$=\frac{1}{abc}\begin{vmatrix}a(b^2+c^2+2bc) & a^2b & a^2c\\ ab^2 & b(a^2+c^2+2ac) & b^2c\\ ac^2 & bc^2 & c(a^2+b^2+2ab)\end{vmatrix}$$

Taking a, b, c common from $C_1$, $C_2$, $C_3$ respectively

$$=\frac{abc}{abc}\begin{vmatrix}(b+c)^2 & a^2 & a^2\\ b^2 & (a+c)^2 & b^2\\ c^2 & c^2 & (a+b)^2\end{vmatrix}$$

Operating $R_1\to R_1-R_3$, $R_2\to R_2-R_3$

$$= \begin{vmatrix} (b+c)^2-a^2 & 0 & a^2 \\ 0 & (a+c)^2-b^2 & b^2 \\ c^2-(a+b)^2 & c^2-(a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (a+c+b)(a+c-b) & b^2 \\ (c-a-b)(c+a+b) & (c-a-b)(c+a+b) & (a+b)^2 \end{vmatrix}$$

Taking (a+b+c) common from $R_1$, $R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ c-a-b & c-a-b & (a+b)^2 \end{vmatrix}$$

Operating $R_3 \rightarrow R_3 - R_1 - R_2$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & a^2+b^2+2ab-a^2-b^2 \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & a+c-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$$

Operating $C_1 \rightarrow aC_1$, $C_2 \rightarrow bC_2$

$$\frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(a+c-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$$

Operating $C_1 \rightarrow C_1 + C_3$, $C_2 \rightarrow C_2 + C_3$

$$= \frac{(a+b+c)^2}{ab} \begin{vmatrix} a(b+c) & a^2 & a^2 \\ b^2 & b(a+c) & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$$

Taking a, b, 2ab from $R_1$, $R_2$, $R_3$

$$= \frac{(a+b+c)^2 a.b.2ab}{ab} \begin{vmatrix} b+c & a & a \\ b & a+c & b \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding with $R_3$

$$= 2ab(a+b+c)^2 \times 1 \times (ab+ac+bc+c^2-ab)$$

$$= 2ab(a+b+c)^2\big(c(a+b+c)\big)$$

$$= 2abc(a+b+c)^3$$

**30. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} b^2-ab & b-c & bc-ac \\ ab-a^2 & a-b & b^2-ab \\ bc-ac & c-a & ab-a^2 \end{vmatrix} = 0.$$

**Answer**

$$= \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$$

Taking (b-a) common from $C_1$, $C_3$

$$= (b-a)^2 \begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$$

Operating $R_2 \rightarrow R_2 - R_1 + R_3$

$$= \begin{vmatrix} b & b-c-b+c & c \\ a & a-b-a+b & b \\ c & c-a-c+a & a \end{vmatrix}$$

$$= (b-a)^2 \begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix}$$

[Properties of determinants say that if 1 row or column has only 0 as its elements, the value of the determinant is 0]

= 0

Hence Proved

**31. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & ab^3 & -c\left(a^2+b^2+c^2\right) \end{vmatrix} = (abc)\left(a^2+b^2+c^2\right)^3.$$

**Answer**

Taking a, b, c from $C_1$, $C_2$, $C_3$

$$= abc \begin{vmatrix} -b^2-c^2+a^2 & 2b^2 & 2c^2 \\ 2a^2 & b^2-c^2-a^2 & 2c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2) \end{vmatrix}$$

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$= abc \begin{vmatrix} -b^2-c^2-a^2 & 0 & a^2+b^2+c^2 \\ 0 & -(a^2+b^2+c^2) & a^2+b^2+c^2 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from $R_1$, $R_2$

$$= abc(a^2+b^2+c^2)^2 \begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -a^2-b^2+c^2 \end{vmatrix}$$

Operating $R_3 \rightarrow R_3 + R_1 + R_2$

$$= abc(a^2+b^2+c^2)^2 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & a^2+b^2+c^2 \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$ common from $C_3$ �

$$= abc(a^2+b^2+c^2)^3 \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 2a^2 & 2b^2 & 1 \end{vmatrix}$$

Expanding with $C_3$

$= abc(a^2+b^2+c^2)^3 \times 1 \times (1-0)$

$= abc\ (a^2+b^2+c^2)^3$

Hence proved

**32. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0,$$ where α, β, γ are in AP.

**Answer**

Given that α, β, γ are in an AP, which means $2\beta=\alpha+\gamma$

Operating $R_3 \to R_3-2R_2+R_1$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1-2x+4+x-3 & x-2-2x+6+x-4 & x-\gamma-2x+2\beta+x-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+2\beta-\alpha \end{vmatrix}$$ [we know that $2\beta=\alpha+\gamma$]

Operating $R_1 \to R_1-R_3$, $R_2 \to R$ �$_2$-$R_3$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & -\gamma+\alpha+\gamma-\alpha \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ 0 & 0 & 0 \end{vmatrix}$$

[By the properties of determinants, we know that if all the elements of a row or column is 0, then the value of the determinant is also 0]

=0

Hence proved

**33. Question**

Using properties of determinants prove that:

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

**Answer**

Operating $R_1 \to R_1-R_2$, $R_2 \to R_2-R_3$

$$= \begin{vmatrix} (a+1)(a+2)-(a+2)(a+3) & a+2-a-3 & 0 \\ (a+2)(a+3)-(a+3)(a+4) & a+3-a-4 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+2)(a+1-a-3) & -1 & 0 \\ (a+3)(a+2-a-4) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} -2(a+2) & -1 & 0 \\ -2(a+3) & -1 & 0 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Expanding with $C_3$

= (2(a+2) – 2(a+3))

= (2a+4-2a-6)

= -2

**34. Question**

If x ≠ y ≠ z and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0,$ prove that xyz (xy + yz + zx) = (x + y + z).

**Answer**

By properties of determinants, we can split the given determinant into 2 parts

$$\to 0 = \begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Taking x, y, z common from $R_1$, $R_2$, $R_3$ respectively

$$\to 0 = xyz\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$\to 0 = xyz\begin{vmatrix} 0 & x^2-z^2 & x^3-z^3 \\ 0 & y^2-z^2 & y^3-z^3 \\ 1 & z^2 & z^3 \end{vmatrix} - \begin{vmatrix} x-z & x^3-z^3 & 0 \\ y-z & y^3-z^3 & 0 \\ z & z^3 & 1 \end{vmatrix}$$

$$\to \begin{vmatrix} x-z & (x-z)(x^2+xz+z^2) & 0 \\ y-z & (y-z)(y^2+yx+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = xyz\begin{vmatrix} 0 & (x-z)(x+z) & (x-z)(x^2+xz+z^2) \\ 0 & (y-z)(y+z) & (y-z)(y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Taking (x-z) and (y-z) common from $R_1$, $R_2$

$$\to (x-z)(y-z)\begin{vmatrix} 1 & (x^2+xz+z^2) & 0 \\ 1 & (y^2+yz+z^2) & 0 \\ z & z^3 & 1 \end{vmatrix} = (x-z)(y-z)\begin{vmatrix} 0 & x+z & (x^2+xz+z^2) \\ 0 & y+z & (y^2+yz+z^2) \\ 1 & z^2 & z^3 \end{vmatrix}$$

Expanding with $R_3$

$\to y^2+yz+z^2-x^2-xz-z^2 = xyz(xy^2+xyz+xz^2+zy^2+yz^2+z^3-x^2y-xyz-yz^2-x^2z-xz^2-z^3)$

$\to$ (y-x)(y+x) +z(y-x) =xyz($xy^2+zy^2$ -$x^2y$ -$x^2z$)

$\to$(y-x)(x+y+z)=xyz(xy(y-x)+z($y^2-x^2$))

$\to$(y-x)(x+y+z)= xyz(xy(y-x)+z(x+y)(y-x))

$\to$(y-x)(x+y+z) = xyz(xy(y-x)+(xz+yz)(y-x))

$\to$(y-x)(x+y+z)= xyz(y-x)(xy+xz+yz)

$\to$x+y+z = xyz(xy+xz+yz)

Hence Proved

**35. Question**

Prove that $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$ = - (a – b) (b – c) (c – a) ($a^2$ + $b^2$ + $c^2$).

**Answer**

Operating $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 0 & a^2+bc-b^2-ac & a^3-b^3 \\ 0 & b^2+ca-c^2-ab & b^3-c^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & (a-b)(a+b)-c(a-b) & (a-b)(a^2+ab+b^2) \\ 0 & (b-c)(b+c)-a(b-c) & (b-c)(b^2+bc+c^2) \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-b), (b-c) common from $R_1$, $R_2$ respectively

$$= (a-b)(b-c)\begin{vmatrix} 0 & a+b-c & a^2+ab+b^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_2$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2a-2c & a^2+ab-bc-c^2 \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 2(a-c) & (a+c)(a-c)+b(a-c) \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Taking (a-c) common from $R_1$

$$= (a-c)(a-b)(b-c)\begin{vmatrix} 0 & 2 & a+b+c \\ 0 & b+c-a & b^2+bc+c^2 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Expanding with $C_1$

$= (a-c)(a-b)(b-c)\times(2b^2+2bc+2c^2-ab-b^2-bc-ac-bc-c^2+a^2+ab+ac)$

$=-(c-a)(b-c)(a-b)(a^2+b^2+c^2)$

Hence Proved

**36. Question**

Without expanding the determinant, prove that:

$$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

**Answer**

Operating $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\to \begin{vmatrix} 0 & a-b & bc-ac \\ 0 & b-c & ac-ab \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\rightarrow\begin{vmatrix}0 & a-b & -c(a-b)\\0 & b-c & -a(b-c)\\1 & c & ab\end{vmatrix}=\begin{vmatrix}0 & a-b & (a-b)(a+b)\\0 & b-c & (b-c)(b+c)\\1 & c & c^2\end{vmatrix}$$

Taking (a-b) and (b-c) from $R_1$, $R_2$

$$\rightarrow(a-b)(b-c)\begin{vmatrix}0 & 1 & -c\\0 & 1 & -a\\1 & c & ab\end{vmatrix}=(a-b)(b-c)\begin{vmatrix}0 & 1 & (a+b)\\0 & 1 & (b+c)\\1 & c & c^2\end{vmatrix}$$

Method 1:

For the two determinants to be equal, their difference must be 0.

$$=\begin{vmatrix}0 & 1 & -c\\0 & 1 & -a\\1 & c & ab\end{vmatrix}-\begin{vmatrix}0 & 1 & a+b\\0 & 1 & b+c\\1 & c & c^2\end{vmatrix}$$

$$=\begin{vmatrix}0-0 & 1-1 & -(a+b+c)\\0-0 & 1-1 & -(a+b+c)\\1-1 & c-c & ab-c^2\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & -(a+b+c)\\0 & 0 & -(a+b+c)\\0 & 0 & ab-c^2\end{vmatrix}$$

Since 2 columns have only 0 as their elements, by properties of determinants

=0

Method 2:

Expanding both with $C_1$

LHS

=(a-b)(b-c)(-a+c)

RHS

=(a-b)(b-c)(b+c-a-b)

=(a-b)(b-c)(-a+c)

∴ LHS = RHS

**37. Question**

Without expanding the determinant, prove that:

$$\begin{vmatrix}1 & a & a^2\\1 & b & b^2\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}1 & bc & b+c\\1 & ca & c+a\\1 & ab & a+b\end{vmatrix}$$

**Answer**

Operating $R_1 \rightarrow R_1 - R_3$, $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix}0 & a-c & a^2-c^2\\0 & b-c & b^2-c^2\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}0 & bc-ab & b+c-a-b\\0 & ac-ab & c+a-a-b\\1 & ab & a+b\end{vmatrix}$$

$$\rightarrow\begin{vmatrix}0 & a-c & (a-c)(a+c)\\0 & b-c & (b-c)(b+c)\\1 & c & c^2\end{vmatrix}=\begin{vmatrix}0 & -b(a-c) & -(a-c)\\0 & -a(b-c) & -(b-c)\\1 & ab & a+b\end{vmatrix}$$

Taking (a-c) and (b-c) common from $R_1$, $R_2$

$$\rightarrow(a-c)(b-c)\begin{vmatrix}0 & 1 & a+c\\0 & 1 & b+c\\1 & c & c^2\end{vmatrix}=(a-c)(b-c)\begin{vmatrix}0 & -b & -1\\0 & -a & -1\\1 & ab & a+b\end{vmatrix}$$

Method 1:

If the determinants are equal, their difference must also be equal.

(a-c) and (b-c) get cancelled.

$$=\begin{vmatrix}0 & 1 & a+c\\0 & 1 & b+c\\1 & c & c^2\end{vmatrix}-\begin{vmatrix}0 & -b & -1\\0 & -a & -1\\1 & ab & a+b\end{vmatrix}$$

$$=\begin{vmatrix}0-0 & 1+b & a+c+1\\0-0 & 1+a & b+c+1\\1-1 & c-ab & c^2+a+b\end{vmatrix}$$

$$=\begin{vmatrix}0 & 1+b & a+c+1\\0 & 1+a & b+c+1\\0 & c-ab & c^2+a+b\end{vmatrix}$$

Since all elements of $C_1$ are 0, by properties of determinants,

=0

∴ The 2 determinants are equal.

Method 2:

Expanding with $C_1$

→(a-c)(b-c)(b+c-a-c) = (a-c)(b-c)(b-a)

→(a-c)(b-c)(b-a)=(a-c)(b-c)(b-a)

∴ RHS and LHS are equal

**38. Question**

Show that x = 2 is a root of the equation $\begin{vmatrix}x & -6 & -1\\2 & -3x & x-3\\-3 & 2x & 2+x\end{vmatrix}=0.$

**Answer**

Operating $R_1 \rightarrow R_1 - R_2$

$$0=\begin{vmatrix}x-2 & -6+3x & -1-x+3\\2 & -3x & x-3\\-3 & 2x & 2+x\end{vmatrix}$$

$$0=\begin{vmatrix}x-2 & 3(x-2) & -(x-2)\\2 & -3x & x-3\\-3 & 2x & 2+x\end{vmatrix}$$

Taking (x-2) common from $R_1$

$$0=(x-2)\begin{vmatrix}1 & 1 & -1\\2 & -3x & x-3\\-3 & 2x & 2+x\end{vmatrix}$$

Here, we can see that x-2 is a factor of the determinant.

We can say that when x-2 is put in the equation, we get 0.

x-2=0

$\rightarrow x=2$

**39. Question**

Solve the following equations:

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & x^3 \end{vmatrix} = 0$$

**Answer**

Operating $R_1 \rightarrow R_1 - R_2$, $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} 0 & x-b & x^3-b^3 \\ 0 & b-c & b^3-c^3 \\ 1 & c & c^3 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} 0 & x-c & (x-b)^3+3xb(x-b) \\ 0 & b-c & (b-c)^3+3bc(b-c) \\ 1 & c & c^3 \end{vmatrix}$$

$$0 = (x-c)(b-c)\begin{vmatrix} 0 & 1 & (x-b)^2+3xb \\ 0 & 1 & (b-c)^2+3bc \\ 1 & c & c^3 \end{vmatrix}$$

Expanding with $C_1$

$0=(x-c)(b-c)(b^2-2bc+c^2+3bc-x^2+2xb-b^2-3xb)$

$0= (x-c)(b-c)(bc+c^2-x^2-xb)$

$0=(x-c)(b-c)(-b(-c+x)-(c-x)(-c-x))$

$0=(x-c)^2(b-c)(-b-c-x)$

Either $x-c=0$ or $b-c=0$ or $(-b-c-x)=0$

$\therefore x=c$ or $b=c$ or $x=-(b+c)$

If $b=c$, $x=b$

$\therefore$ $x=c$ or $x=b$ or $x=-(b+c)$

**40. Question**

Solve the following equations:

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ b & b & x+c \end{vmatrix} = 0$$

**Answer**

Operating $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

Taking $(x+a+b+c)$ common from $C_1$

$$(x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$(x+a+b+c)\begin{vmatrix} 0 & 0 & -x \\ 0 & x & -x \\ 1 & b & x+c \end{vmatrix} = 0$$

Expanding with $C_1$

$0=(x+a+b+c)(0+x^2)$

$0=x^2(x+a+b+c)$

Either $x^2=0$ or $(x+a+b+c)=0$

$\therefore$ $x=0$ or $x=-(a+b+c)$

**41. Question**

Solve the following equations:

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

**Answer**

Operating $C1 \to C1 + C_2 + C_3$

$$0 = \begin{vmatrix} 3x-8+3+3 & 3 & 3 \\ 3+3x-8+3 & 3x-8 & 3 \\ 3+3+3x-8 & 3 & 3x-8 \end{vmatrix}$$

$$0 = \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix}$$

Taking (3x-2) common from $C_1$

$$0 = (3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$0 = (3x-2)\begin{vmatrix} 0 & 0 & -(3x-11) \\ 0 & 3x-11 & -3x+11 \\ 1 & 3 & 3x-8 \end{vmatrix}$$

Expanding with $C_1$

$0= (3x-2)(0+(3x-11)^2)$

$0=(3x-2)(3x-11)^2$

Either $3x-2=0$ or $3x-11=0$

$\therefore \mathbf{x = \frac{2}{3} \text{ or } x = \frac{11}{3}}$

**42. Question**

Solve the following equations:

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

**Answer**

Operating $C_1 \rightarrow C_1+C_2+C_3$

$$0=\begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix}$$

Taking (x+9) common from $C_1$

$$0=(x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix}$$

Operating $R_1 \rightarrow R_1-R_3$, $R_2 \rightarrow R_2-R_3$

$$0=(x+9)\begin{vmatrix} 0 & 0 & 1-x \\ 0 & x-1 & 1-x \\ 1 & 3 & x+4 \end{vmatrix}$$

$0= (x+9)(0-x+x^2+1-x)$

$0=(x+9)(x^2-2x+1)$

$0=(x+9)(x-1)^2$

∴ Either x+9=0 or x-1=0

x=-9, x=1

**43. Question**

Solve the following equations:

$$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}=0$$

**Answer**

Operating $R1 \rightarrow R1+R_2+R_3$

$$0=\begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Taking (x+9) common from $R_1$

$$0=(x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$$

Operating $C_1 \rightarrow C_1-C_3$, $C_2 \rightarrow C_2-C_3$

$$0=(x+9)\begin{vmatrix} 0 & 0 & 1 \\ 0 & x-2 & 2 \\ 7-x & 6-x & x \end{vmatrix}$$

Expanding with $R_1$

0=(x+9)(0-(x-2)(7-x))

0=(x+9)(7-x)(2-x)

Either x+9=0 or 7-x=0 or 2-x=0

∴ x=-9 or x=7 or x=2

**44. Question**

Solve the following equations:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

**Answer**

Expanding with R1

$0= x(-3x^2-6x-2x^2+6x)+6(2x+4+3x-9)-1(4x-9x)$

$0=x(-5x^2)+6(5x-5)-1(-5x)$

$0= -5x^3+30x-30+5x$

$0=-5x^3+35x-30$

$x^3-7x+6=0$

$x^3-x-6x+6=0$

$x(x^2-1)-6(x-1)=0$

$x(x-1)(x+1)-6(x-1)=0$

$(x-1)(x^2+x-6)=0$

$(x-1)(x^2+3x-2x-6)=0$

$(x-1)(x(x+3)-2(x+3)(=0$

$(x-1)(x+3)(x-2)=0$

Either $x-1=0$ or $x+3=0$ or $x-2=0$

$\therefore$ $x=1$ or $x=-3$ or $x=2$

**45. Question**

Prove that

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

**Answer**

Operating $C_1 \rightarrow aC_1$

$$= \frac{1}{a}\begin{vmatrix} a^2 & b-c & c+b \\ a^2+ac & b & c-a \\ a^2-ab & b+a & c \end{vmatrix}$$

Operating $C_1 \rightarrow C_1+bC_2+cC_3$

$$= \frac{1}{a}\begin{vmatrix} a^2+b^2+c^2 & b-c & c+b \\ a^2+b^2+c^2 & b & c-a \\ a^2+b^2+c^2 & b+a & c \end{vmatrix}$$

Taking $(a^2+b^2+c^2)$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix}1 & b-c & c+b\\1 & b & c-a\\1 & b+a & c\end{vmatrix}$$

Operating $C_2 \rightarrow C_2 - bC_1$, $C_3 \rightarrow C_3 - cC_3$

$$= \frac{a^2+b^2+c^2}{a}\begin{vmatrix}1 & -c & b\\1 & 0 & -a\\1 & a & 0\end{vmatrix}$$

Expanding with $R_3$

$$= \frac{a^2+b^2+c^2}{a}(ac-0+a^2+ab)$$

$$= \frac{a^2+b^2+c^2}{a}a(a+b+c)$$

$= (a^2+b^2+c^2)(a+b+c)$

Hence Proved

## Exercise 6C

### 1 A. Question

Find the area of the triangle whose vertices are:

A(3, 8), B(-4, 2) and C(5, -1)

**Answer**

Area of a triangle $= \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix}3 & 8 & 1\\-4 & 2 & 1\\5 & -1 & 1\end{vmatrix}$$

Expanding with $C_3$

$$= \frac{1}{2}[\,(4-10)-(-3-40)+(6+32)]$$

$$= \frac{1}{2}[-6+43+38]$$

$$= \frac{75}{2}$$

= 37.5 sq. units

### 1 B. Question

Find the area of the triangle whose vertices are:

A(-2, 4), B(2, -6) and C(5, 4)

**Answer**

**Area of a triangle** $= \frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\x_2 & y_2 & 1\\x_3 & y_3 & 1\end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix}-2 & 4 & 1\\2 & -6 & 1\\5 & 4 & 1\end{vmatrix}$$

Expanding with $C_3$

$= \frac{1}{2}[(8+30)-(-8-20)+(12-8)]$

$= \frac{1}{2}[38+28+4]$

$= \frac{68}{2}$

= 34 sq. units

**1 C. Question**

Find the area of the triangle whose vertices are:

A(-8, -2), B(-4, -6) and C(-1, 5)

**Answer**

**Area of a triangle** $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -8 & -2 & 1 \\ -4 & -6 & 1 \\ -1 & 5 & 1 \end{vmatrix}$

Expanding with $R_3$

$= \frac{1}{2}[(-20-6)-(-40-2)+(48-8)]$

$= \frac{1}{2}[-26+42+40]$

$= \frac{56}{2}$

=28 sq. units

**1 D. Question**

Find the area of the triangle whose vertices are:

P(0, 0), Q(6, 0) and R(4, 3)

**Answer**

**Area of a triangle** $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

Expanding with $R_1$

$= \frac{1}{2}[18]$

= 9 sq. units

**1 E. Question**

Find the area of the triangle whose vertices are:

P(1, 1), Q(2, 7) and R(10, 8)

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 7 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$$= \frac{1}{2}\begin{vmatrix} -9 & -7 & 0 \\ -8 & -1 & 0 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding with $C_3$

$$= \frac{1}{2}[\,9 - 56\,]$$

$$= \frac{1}{2}[-47]$$

$$= \frac{-47}{2}$$

= -23.5 sq. units = 23.5 sq units

**2 A. Question**

Use determinants to show that the following points are collinear.

A(2, 3), B(-1, -2) and C(5, 8)

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

Expanding with $C_3$

$$= \frac{1}{2}[\,(-8+10) - (16-15) + (-4+3)] = \frac{1}{2}\,[\,2 - 1 - 1]$$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

**2 B. Question**

Use determinants to show that the following points are collinear.

A(3, 8), B(-4, 2) and C(10, 14)

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$$

Expanding with $C_3$

$= \frac{1}{2}[(-56-20)-(42-80)+(6+32)]$

$= \frac{1}{2}[-76+38+38]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

**2 C. Question**

Use determinants to show that the following points are collinear.

P(-2, 5), Q(-6, -7) and R(-5, -4)

**Answer**

$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} -2 & 5 & 1 \\ -6 & -7 & 1 \\ -5 & -4 & 1 \end{vmatrix}$

Expanding with $C_3$

$= \frac{1}{2}[(24-35)-(8+25)+(14+30)] = \frac{1}{2}[-11-33+44]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

**3. Question**

Find the value of k for which thepoints A( 3, -2), B(k, 2) and C(8, 8) are collinear.

**Answer**

$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since they are collinear, the area will be 0

$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$

Expanding with $C_3$

$\rightarrow 0 = \frac{1}{2}[(8k-16)-(24+16)+(6+2k)]$

$\rightarrow 0 = \frac{1}{2}[10k-50]$

→ 10k – 50=0

→ 10k=50

∴ k=5

**4. Question**

Find the value of k for which thepoints P(5, 5), Q(k, 1) and R(11, 7) are collinear.

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$$

Expanding with $C_3$

→ 0 = (7k-11)-(35-55)+(5-5k)

→ 0= 2k-14

→ 2k=14

∴ k=7

**5. Question**

Find the value of k for which thepoints A(1, -1), B(2, k) and C(4, 5) are collinear.

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since they are collinear, the area will be 0

$$\rightarrow 0 = \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & k & 1 \\ 4 & 5 & 1 \end{vmatrix}$$

Expanding with $C_3$

→ 0 = (10-4k)-(5+4)+(k+2)

→ 0=-3k+3

→ 3k=3

∴ k= 1

**6. Question**

Find the value of k for which the area of aABC having vertices A(2, -6), B(5, 4) and C(k, 4) is 35 sq units.

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

Expanding with $C_3$

→ 70 = (20-4k)-(8+6k)+(8+30)

→ 70= -10k+50

→ 20=-2k

→ k=-2

**7. Question**

If A(-2, 0), B(0, 4) and C(0, k) be three points such that area of a ABC is 4 sq units, find the value of k.

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$4 = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$$

Expanding with $C_1$

→ 8=-2(4-k)

→ -4=4-k

→ k=8

**8. Question**

If the points A(a, 0), B(0, b) and C(1, 1) are collinear, prove that $\frac{1}{a}+\frac{1}{b}=1$.

**Answer**

$$\textbf{Area of a triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since the points are collinear, the area they enclose is 0

$$0 = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding with $C_1$

→ 0 = a(b-1)+(-b)

→ 0= ab-a-b

→ a+b=ab

→ $\frac{a+b}{ab} = 1$

→ $\frac{1}{a}+\frac{1}{b} = 1$

Hence proved

## Objective Questions

**1. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 70^o & \sin 20^o \\ \sin 70^o & \cos 20^o \end{vmatrix} = ?$$

A. 1

B. 0

C. cos $50^o$

D. sin $50^o$

**Answer**

To find: Value of $\begin{vmatrix} \cos 70° & \sin 20° \\ \sin 70° & \cos 20° \end{vmatrix}$

Formula used: (i) $\cos \theta = \sin(90 - \theta)$

We have, $\begin{vmatrix} \cos 70° & \sin 20° \\ \sin 70° & \cos 20° \end{vmatrix}$

On expanding the above,

⇒ {cos 70°} {cos 20°} – {sin 70°} {sin 20°}

On applying formula $\cos \theta = \sin(90 - \theta)$

⇒ {sin (90 – 70)} {sin (90 – 20)} - {sin 70°} {sin 20°}

⇒ {sin 20°} {sin 70°} - {sin 70°} {sin 20°}

= 0

**2. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \cos 15^{\circ} & \sin 15^{\circ} \\ \sin 15^{\circ} & \cos 15^{\circ} \end{vmatrix} = ?$$

A. 1

B. $\frac{1}{2}$

C. $\frac{\sqrt{3}}{2}$

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 15° & \cos 15° \end{vmatrix}$

Formula used: (i) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

We have, $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 15° & \cos 15° \end{vmatrix}$

On expanding the above,

⇒ {cos 15°} {cos 15°} – {sin 15°} {sin 15°}

On applying formula $\cos(A + B) = \cos A \cos B - \sin A \sin B$

= cos (15 + 15)

= cos (30°)

$= \frac{\sqrt{3}}{2}$

**3. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin 23^{\circ} & -\sin 7^{\circ} \\ \cos 23^{\circ} & \cos 7^{\circ} \end{vmatrix} = ?$$

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1}{2}$

C. sin $16^{o}$

D. cos $16^{o}$

**Answer**

To find: Value of $\begin{vmatrix} \sin 23° & -\sin 7° \\ \cos 23° & \cos 7° \end{vmatrix}$

Formula used: (i) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

We have, $\begin{vmatrix} \sin 23° & -\sin 7° \\ \cos 23° & \cos 7° \end{vmatrix}$

On expanding the above,

⇒ (sin 23°) (cos 7°) – (cos 23°) (-sin 7°)

⇒ (sin 23°) (cos 7°) + (cos 23°) (sin 7°)

On applying formula $\sin(A + B) = \sin A \cos B + \cos A \sin B$

= sin (23 + 7)

= sin (30°)

$= \frac{1}{2}$

**4. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-id \end{vmatrix} = ?$$

A. $(a^2 + b^2 - c^2 - d^2)$

B. $(a^2 - b^2 + c^2 - d^2)$

C. $(a^2 + b^2 + c^2 + d^2)$

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

Formula used: $i^2 = -1$

We have, $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

On expanding the above,

⇒ (a + ib) (a – ib) – (-c + id) (c + id)

$\Rightarrow (a^2 - iab + iba - i^2b^2) - (-c^2 - icd + icd + i^2d^2)$

$\Rightarrow \{a^2 - iab + iba - (-1)b^2\} - \{-c^2 - icd + icd + (-1)d^2\}$

$\Rightarrow \{a^2 - iab + iba + 1b^2\} - \{-c^2 - icd + icd - 1d^2\}$

$\Rightarrow a^2 + b^2 + c^2 + d^2$

**5. Question**

Mark the tick against the correct answer in the following:

If ω is a complex root of unity then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$

A. 1

B. -1

C. 0

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

Formula used: $\omega^3 = 1$

We have, $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$

On expanding the above along 1st column

$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$\Rightarrow \left[1\left\{(\omega^2)(\omega)-(1)(1)\right\}\right] - \left[\omega\left\{(\omega)(\omega)-(\omega^2)(1)\right\}\right] + \left[\omega^2\left\{(\omega)(1)-(\omega^2)(\omega^2)\right\}\right]$

$\Rightarrow \left[1\left\{\omega^3-1\right\}\right]-\left[\omega\left\{\omega^2-\omega^2\right\}\right]+\left[\omega^2\left\{\omega-\omega^4\right\}\right]$ ... (i)

As $\omega^3 = 1$,

$\Rightarrow \omega^3.\omega=1.\,\omega$

$\Rightarrow \omega^4 = \omega$

Using the above obtained value of $\omega^4$ in eqn. (i)

$\Rightarrow \left[1\left\{\omega^3-1\right\}\right] - \left[\omega\left\{\omega^2-\omega^2\right\}\right] + \left[\omega^2\{\omega-\omega\}\right]$

$\Rightarrow 1\left\{\omega^3-1\right\}$

$\Rightarrow \omega^3-1$

$\Rightarrow 1 - 1 = 0$

**6. Question**

Mark the tick against the correct answer in the following:

If ω is a complex cube root of unity then the value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$ is

A. 2

B. 4

C. 0

D. -3

**Answer**

To find: Value of $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

Formula used: (i) $\omega^3 = 1$

(ii) $1+\omega+\omega^2 = 0$

We have, $\begin{vmatrix} 1 & \omega & 1+\omega \\ 1+\omega & 1 & \omega \\ \omega & 1+\omega & 1 \end{vmatrix}$

On expanding the above along 1$^{st}$ column

$\Rightarrow 1 \begin{vmatrix} \omega^2 & 1 \\ 1 & \omega \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega^2 \\ 1 & \omega \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$\Rightarrow [1\{(\omega^2)(\omega)-(1)(1)\}]-[\omega\{(\omega)(\omega)-(\omega^2)(1)\}]+[\omega^2\{(\omega)1-(\omega^2)(\omega^2)\}]$

$\Rightarrow [1\{\omega^3-1\}]-[\omega\{\omega^2-\omega^2\}]+[\omega^2\{\omega-\omega^4\}]$ ... (i)

As $\omega^3 = 1$,

$\Rightarrow \omega^3.\omega=1.\,\omega$

$\Rightarrow \omega^4= \omega$

Using the above obtained value of $\omega^4$ in eqn. (i)

$\Rightarrow [1\{\omega^3-1\}]-[\omega\{\omega^2-\omega^2\}]+[\omega^2\{\omega-\omega\}]$

$\Rightarrow 1\{\omega^3-1\}$

$\Rightarrow \omega^3-1$

$\Rightarrow 1 - 1 = 0$

**7. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = ?$$

A. 8

B. -8

C. 16

D. 142

**Answer**

To find: Value of $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

We have, $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_2 \to R_1 - R_2$

$\Rightarrow \begin{vmatrix} 8 & 12 & 16 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Taking 4 common from $R_1$

$\Rightarrow 4 \begin{vmatrix} 2 & 3 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$

$\Rightarrow 4 \begin{vmatrix} -2 & 0 & 4 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Taking -2 common from $R_1$

$\Rightarrow (4)(-2) \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_1 \to 9R_1$

$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 9 & 16 & 25 \end{vmatrix}$

Applying $R_3 \to R_3 - R_1$

$\Rightarrow \frac{-8}{9} \begin{vmatrix} 9 & 0 & -18 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$

Taking 9 common from $R_1$

$\Rightarrow -8 \begin{vmatrix} 1 & 0 & -2 \\ 4 & 3 & 0 \\ 0 & 16 & 43 \end{vmatrix}$

Expanding along $R_1$

$\Rightarrow -8\ [1[(3)(43)-(16)(0)] - 0\ [(4)(43)-(0)(0)] - 2\ [(4)(16)-(3)(0)]]$

⇒ -8 [[(129)-(0)] - 2 [(64)-(0)]]

⇒ -8 [129 – 128]

⇒ -8

**8. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = ?$$

A. 2

B. 6

C. 24

D. 120

**Answer**

To find: Value of $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$

We have, $\left|\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}\right|$

$\Rightarrow \begin{vmatrix} 1 & 2 & 6 \\ 2 & 6 & 24 \\ 6 & 24 & 120 \end{vmatrix}$

Taking 2 common from $R_2$

$\Rightarrow 2 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 6 & 24 & 120 \end{vmatrix}$

Taking 6 common from $R_3$

$\Rightarrow 2 \times 6 \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 1 & 4 & 20 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$

$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 1 & 4 & 20 \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - R_1$

$\Rightarrow 12 \begin{vmatrix} 1 & 2 & 6 \\ 0 & 1 & 6 \\ 0 & 2 & 14 \end{vmatrix}$

Expanding column 1

⇒ 12 [1{(1)(14)-(6)(2)}]

⇒ 12 [1{(14)-(12)}]

⇒ 12[2]

$\Rightarrow 24$

**9. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = ?$$

A. (a + b + c)

B. 3(a + b + c)

C. 3abc

D. 0

**Answer**

To find: Value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

We have, $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

Applying $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{vmatrix} a-b+b-c & b-c+c-a & c-a+a-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$

$$\Rightarrow \begin{vmatrix} a-c+c-a & b-a+a-b & c-b+b-c \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

If every element of a row is 0 then the value of the determinant will be 0

**10. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = ?$$

A. 0

B. 1

C. -1

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - 2R_1$

$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 3 & 6+3p & 1+6p+3q \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - 3R_1$

$\Rightarrow \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & p-2 \\ 0 & 3 & 3p-2 \end{vmatrix}$

Expanding along $C_1$

$\Rightarrow [1\{(1)(3p-2)-(3)(p-2)\}]$

$\Rightarrow 1$

**11. Question**

Mark the tick against the correct answer in the following:

$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = ?$

A. (a – b) (b – c) (c – a)

B. -(a – b) (b – c) (c – a)

C. (a – b) (b – c) (c – a) (a + b + c)

D. abc (a – b)(b – c) (c – a)

**Answer**

To find: Value of $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$

Applying $C_2 \rightarrow C_2 - C_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 1 \\ a & b-a & c \\ a^3 & b^3-a^3 & c^3 \end{vmatrix}$

Applying $C_3 \rightarrow C_3 - C_1$

$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$

We know, $x^3 - y^3 = (x - y)(x^2+xy+y^2)$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (b-a) common from $C_2$

$$\Rightarrow (b-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & c-a \\ a^3 & (b^2+ab+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

Taking (c-a) common from $C_2$

$$\Rightarrow (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & (b^2+ab+a^2) & (c^2+ca+a^2) \end{vmatrix}$$

Expanding along $C_1$

$\Rightarrow (b - a)(c - a)[1\{(1)(c^2 + ca + a^2) - (b^2 + ab + a^2)(1)\}]$

$\Rightarrow (b - a)(c - a)[c^2 + ca + a^2 - b^2 - ab - a^2]$

$\Rightarrow (b - a)(c - a)[c^2 - b^2 + ca - ab]$

$\Rightarrow (b - a)(c - a)[(c - b)(c + b) + a(c - b)]$

$\Rightarrow (b - a)(c - a)[(a + b + c)(c - b)]$

$\Rightarrow (a - b)(b - c)(c - a)(a + b + c)$

**12. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = ?$$

A. 0

B. 1

C. sin (α + δ) + sin (β + δ)+ sin (γ + δ)

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

Formula Used: sin(A+B) = sinAcosB+cosAsinB

We have, $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$

Applying $C_1 \rightarrow \cos(\delta)C_1$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta\cos\delta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma\cos\delta & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}$$

Applying $C_2 \to \sin(\delta)C_2$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin(\alpha+\delta) \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin(\beta+\delta) \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin(\gamma+\delta) \end{vmatrix}$$

We know, sin(A+B) = sinAcosB+cosAsinB

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin\alpha\cos\delta + \cos\alpha\sin\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin\beta\cos\delta + \cos\beta\sin\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin\gamma\cos\delta + \cos\gamma\sin\delta \end{vmatrix}$$

Applying $C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin\alpha\cos\delta + \cos\alpha\sin\delta - \sin\alpha\cos\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin\beta\cos\delta + \cos\beta\sin\delta - \sin\beta\cos\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin\gamma\cos\delta + \cos\gamma\sin\delta - \sin\gamma\cos\delta \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \cos\alpha\sin\delta \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \cos\beta\sin\delta \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \cos\gamma\sin\delta \end{vmatrix}$$

= 0

When two columns are identical then the value of determinant is 0

**13. Question**

Mark the tick against the correct answer in the following:

If a, b, c be distinct positive real numbers then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

A. positive

B. negative

C. a perfect square

D. 0

**Answer**

To find: Nature of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

We have, $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$$

Taking (a+b+c) common from $R_1$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

Expanding along $R_1$

$\Rightarrow (a+b+c)[1\{(b)(c)-(a)(a)\} - 1\{(b)(b)-(c)(a)\} + 1\{(a)(b)-(c)(c)\}]$

$\Rightarrow (a+b+c)[1\{bc-a^2\} - 1\{b^2-ca\} + 1\{ba - c^2\}]$

$\Rightarrow (a+b+c)[bc - a^2 - b^2 + ca + ab - c^2]$

$\Rightarrow -(a+b+c)[c^2 + a^2 + b^2 - ca - bc - ba]$

$\Rightarrow -\frac{1}{2}(a+b+c)\, 2[c^2 + a^2 + b^2 - ca - bc - ba]$

$\Rightarrow -\frac{1}{2}(a+b+c)\, [2c^2 + 2a^2 + 2b^2 - 2ca - 2bc - 2ba]$

$\Rightarrow -\frac{1}{2}(a+b+c)\, [c^2 + a^2 - 2ca + c^2 + b^2 - 2bc + a^2 + b^2 - 2ba]$

$\Rightarrow -\frac{1}{2}(a+b+c)\, [(c-a)^2 + (c-b)^2 + (a-b)^2]$

Clearly, we can see that the answer is negative

**14. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = ?$$

A. 0

B. $x^3$

C. $y^3$

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

We have, $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying $R_2 \to 2R_2$

$\Rightarrow \frac{1}{2}\begin{vmatrix} x+y & x & x \\ 10x+8y & 8x & 4x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$\Rightarrow \frac{1}{2}\begin{vmatrix} x+y & x & x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying $R_1 \to 8R_1$

$\Rightarrow \frac{1}{2\times 8}\begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 10x+8y & 8x & 3x \end{vmatrix}$

Applying $R_3 \to R_3 - R_1$

$\Rightarrow \frac{1}{16}\begin{vmatrix} 8x+8y & 8x & 8x \\ 0 & 0 & x \\ 2x & 0 & -5x \end{vmatrix}$

Expanding along $R_2$

$\Rightarrow \frac{1}{16}$ [x{(2x)(8x) - (8x+8y)(0)}]

$\Rightarrow \frac{1}{16}$ [x{16x$^2$}]

$\Rightarrow x^3$

**15. Question**

Mark the tick against the correct answer in the following:

$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = ?$

A. (a – 1)

B. $(a – 1)^2$

C. $(a – 1)^3$

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

We have, $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

Expanding along $C_3$

$\Rightarrow [1\{(a^2-1)(a-1) – (a-1)(2a – 2)\}]$

$\Rightarrow [1\{(a-1)(a+1)(a-1) – (a-1)2(a – 1)\}]$

$\Rightarrow [\{(a+1)(a-1)^2 – 2(a-1)^2\}]$

$\Rightarrow [\{(a-1)^2 (a+1-2)\}]$

$\Rightarrow [\{(a-1)^2 (a-1)\}]$

$\Rightarrow (a-1)^3$

**16. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix} = ?$$

A. $a^3$

B. $-a^3$

C. 0

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

We have, $\begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 6a & 9a+12b & 11a+15b+18c \end{vmatrix}$

Applying $R_3 \rightarrow R_3 - 2R_2$

$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 3a & 4a+6b & 5a+7b+9c \\ 0 & a & a+b \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - 3R_1$

$\Rightarrow \begin{vmatrix} a & a+2b & a+2b+3c \\ 0 & a & 2a+b \\ 0 & a & a+b \end{vmatrix}$

Expanding along $C_1$

$\Rightarrow [a\{(a)(a+b) - (a)(2a+b)\}]$

$\Rightarrow [a\{(a^2 + ab) - (2a^2+ab)\}]$

$\Rightarrow [a\{a^2 + ab - 2a^2 - ab\}]$

$\Rightarrow [a\{-a^2\}]$

$\Rightarrow -a^3$

**17. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = ?$$

A. $(a + b + c)(a - c)$

B. $(a + b + c)(b - c)$

C. $(a + b + c)(a - c)^2$

D. $(a + b + c)(b - c)^2$

**Answer**

To find: Value of $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

We have, $\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} b+c+c+a+a+b & a+c+b & b+a+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

$\Rightarrow (a+b+c)\begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

Expanding along $R_1$

$\Rightarrow (a+b+c)[2\{(c)(c) – (b)(a)\} -1\{(c+a)(c)-(a+b)(a)\} + 1\{(c+a)(b)-(a+b)(c)\}]$

$\Rightarrow (a+b+c)[2\{c^2 – ab\} -1\{c^2+ac-a^2-ab\} + 1\{bc+ba-ac-bc\}]$

$\Rightarrow (a+b+c)[2c^2 – 2ab - c^2 – ac + a^2 + ab + ba - ac]$

$\Rightarrow (a+b+c)[c^2 + a^2 - 2ac]$

$\Rightarrow (a+b+c)(c – a)^2$

**18. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} = ?$$

A. (x + y)

B. (x – y)

C. xy

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

We have, $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$\Rightarrow \begin{vmatrix} 0 & -x & 0 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Expanding along $R_1$

$\Rightarrow [x\{(1)(1+y)-(1)(1)\}]$

$\Rightarrow [x\{1+y-1\}]$

$\Rightarrow xy$

**19. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = ?$$

A. (a – b) (b – c) (c – a)

B. -(a – b) (b – c) (c – a)

C. (a + b) (b + c) (c + a)

D. None of these

**Answer**

To find: Value of $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

We have, $\begin{vmatrix} bc & b+c & 1 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} bc-ca & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} c(b-a) & b-a & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Taking (b – a) common

$$\Rightarrow (b-a)\begin{vmatrix} c & 1 & 0 \\ ca & a+c & 1 \\ ab & a+b & 1 \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\Rightarrow (b-a)\begin{vmatrix} c & 1 & 0 \\ ca-ab & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

$$\Rightarrow (b-a)\begin{vmatrix} c & 1 & 0 \\ a(c-b) & c-b & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Taking (c - b) common

$$\Rightarrow (b-a)(c-b)\begin{vmatrix} c & 1 & 0 \\ a & 1 & 0 \\ ab & a+b & 1 \end{vmatrix}$$

Expanding along $C_3$

$\Rightarrow$ (b – a) (c – b) [1{(c) (1) – (a) (1)}]

$\Rightarrow$ (b – a) (c – b) (c – a)

$\Rightarrow$ (a – b) (b – c) (c – a)

**20. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = ?$$

A. 4abc

B. 2(a + b + c)

C. (ab + bc + ca)

D. none of these

**Answer**

To find: Value of $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

We have, $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} b+c+b+c & a+c+a+c & a+b+a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common

$$\Rightarrow 2\begin{vmatrix} b+c & a+c & a+b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\Rightarrow 2\begin{vmatrix} c & 0 & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Expanding along $R_1$

$\Rightarrow$ 2 [c{(c + a) (a + b) – (b) (c)} + a{(b)(c) – (c) (c + a)}]

$\Rightarrow$ 2 [c{(ac + cb + $a^2$ + ab – bc} + a{(bc – $c^2$ - ac)}]

$\Rightarrow$ 2 [c{(ac + $a^2$ + ab)} + a{(bc – $c^2$ - ac)}]

$\Rightarrow$ 2 [$ac^2$ + $ca^2$ + abc + abc – $ac^2$ – $a^2c$]

$\Rightarrow$ 2 [2abc]

$\Rightarrow$ 4abc

**21. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = ?$$

A. a + b+ c

B. 2(a + b + c)

C. 4abc

D. $a^2b^2c^2$

**Answer**

To find: Value of $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

We have, $\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{vmatrix} a & 1 & b+c \\ b-a & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Taking (a - b) common

$$\Rightarrow (a-b)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c & 1 & a+b \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow (a-b)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ c-a & 0 & a-c \end{vmatrix}$$

Taking (c-a) common

$$\Rightarrow (b-a)(c-a)\begin{vmatrix} a & 1 & b+c \\ -1 & 0 & a-b \\ 1 & 0 & -1 \end{vmatrix}$$

Expanding along $R_1$

= (b – a)(c – a)[0 – 1(1 – (a – b)) + (b + c)(0)]

= (b – a)(c – a)(-1 + a – b)

= (b – a)(c – a)(a – b – 1)

= (b – a)(ac – bc – c – $a^2$ + ab + a)

= (abc – $b^2$c – bc – $a^2$b + a$b^2$ + ab – $a^2$c + abc + ac + $a^3$ + $a^2$b + $a^2$)

= 4abc

**22. Question**

Mark the tick against the correct answer in the following:

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix} = ?$$

A. -2

B. 2

C. $x^2 - 2$

D. $x^2 + 2$

**Answer**

To find: Value of $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

We have, $\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Applying $R_1 \rightarrow R_2 - R_1$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Applying $R_2 \rightarrow R_3 - R_2$

$\Rightarrow \begin{vmatrix} 2 & 3 & 4 \\ 4 & 5 & 6 \\ x+7 & x+10 & x+14 \end{vmatrix}$

Expanding along $R_1$

$\Rightarrow [2\{(5)(x+14) - (6)(x+10)\} - 3\{(4)(x+14) - (6)(x+7)\} + 4\{(4)(x+10) - (5)(x+7)\}]$

$\Rightarrow [2\{5x + 70 - 6x - 60\} - 3\{4x + 56 - 6x - 42\} + 4\{4x + 40 - 5x - 35\}]$

$\Rightarrow [2\{10 - x\} - 3\{14 - 2x\} + 4\{5 - x\}]$

$\Rightarrow [20 - 2x - 42 + 6x + 20 - 4x]$

$\Rightarrow -2$

**23. Question**

Mark the tick against the correct answer in the following:

If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$ then x = ?

A. 0

B. 6

C. -6

D. 9

**Answer**

To find: Value of x

We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Applying $R_1 \to 2R_1$

$\Rightarrow \begin{vmatrix} 10 & 6 & -2 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$

$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$

Expanding along $R_1$

⇒ [1{(x)(-2) – (6)(2)}] = 0

⇒ [1{-2x – 12}] = 0

⇒ -2x-12 = 0

⇒ -2x = 12

⇒ x = -6

**24. Question**

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$ is

A. {2, -3. 7}

B. {2, 7. -9}

C. [-2, 3, -7}

D. none of these

**Answer**

To find: Value of x

We have, $\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Applying $R_1 \to 2R_1$

$\Rightarrow \begin{vmatrix} 2x & 6 & 14 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$

$\Rightarrow \begin{vmatrix} 2x-7 & 0 & 14-x \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$

Expanding along $R_1$

⇒ [(2x-7){(x)(x) – (6)(2)} + (14-x){(2)(6) – (x)(7)] = 0

$\Rightarrow [(2x-7)\{x^2 - 12\} + (14-x)\{12 - 7x\}] = 0$

$\Rightarrow [2x^3 - 24x - 7x^2 + 84 + 168 - 98x - 12x + 7x^2] = 0$

$\Rightarrow [2x^3 - 134x + 252] = 0$

$\Rightarrow [x^3 - 67x + 126] = 0$

By Hit and trial x = -2, 3, -7

**25. Question**

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 2x-64 \end{vmatrix} = 0$ is

A. {4}

B. {2, 4}

C. {2, 8}

D. {4, 8}

**Answer**

To find: Value of x

We have, $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

Applying $C_2 \rightarrow C_2 - 2C_1$

$\Rightarrow \begin{vmatrix} x-2 & 1 & 3x-4 \\ x-4 & -1 & 3x-16 \\ x-8 & -11 & 3x-64 \end{vmatrix} = 0$

Applying $C_3 \rightarrow C_3 - 3C_1$

$\Rightarrow \begin{vmatrix} x-2 & 1 & 2 \\ x-4 & -1 & -4 \\ x-8 & -11 & -40 \end{vmatrix} = 0$

Expanding along $R_1$

$\Rightarrow [x-2\{(-1)(-40) - (-4)(-11)\} -1 \{(x-4)(-40) - (-4)(x-8)\} + 2 \{(x-4)(-11) - (-1)(x-8)] = 0$

$\Rightarrow [(x-2)\{40-44\} -1 \{(-40x + 160 + 4x - 32\} + 2 \{-11x + 44 + x - 8\}] = 0$

$\Rightarrow [(x-2)\{-4\} -1 \{(-36x + 128\} + 2 \{-10x+36\}] = 0$

$\Rightarrow [-4x + 8 + 36x - 128 - 20x + 72] = 0$

$\Rightarrow 12x - 48 = 0$

$\Rightarrow x = 4$

**26. Question**

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ is

A. {a, 0}

B. {3a, 0}

C. {a, 3a}

D. None of these

**Answer**

To find: Value of x

We have, $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} 2x & -2x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Taking 2 common from $R_1$

$\Rightarrow 2 \begin{vmatrix} x & -x & 0 \\ 0 & 2x & -2x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Taking 2 common from $R_2$

$\Rightarrow 2 \times 2 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a-x & a-x & a+x \end{vmatrix} = 0$

Applying $R_3 \rightarrow R_1 + R_3$

$\Rightarrow 4 \begin{vmatrix} x & -x & 0 \\ 0 & x & -x \\ a & a-2x & a+x \end{vmatrix} = 0$

Expanding along $R_1$

$\Rightarrow 4[x\{(x)(a+x) – (-x)(a-2x)\}] – (-x)\{(0)(a+x) – (-x)(a)\}] = 0$

$\Rightarrow 4[x\{ax + x^2 + ax - 2x^2\}] – (-x)\{ax\}] = 0$

$\Rightarrow 4[x\{2ax - x^2\}] + ax^2] = 0$

$\Rightarrow 4[2ax^2 – x^3 + ax^2] = 0$

$\Rightarrow – x^2 + 3ax = 0$

$\Rightarrow -x(x – 3a) = 0$

$\Rightarrow x = 0$ , or $x = 3a$

**27. Question**

Mark the tick against the correct answer in the following:

The solution set of the equation $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$ is

A. $\left\{\frac{2}{3}, \frac{8}{3}\right\}$

B. $\left\{\frac{2}{3}, \frac{11}{3}\right\}$

C. $\left\{\frac{3}{2}, \frac{8}{3}\right\}$

D. None of these

**Answer**

To find: Value of x

We have, $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying $R_1 \rightarrow R_1 - R_2$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_3$

$\Rightarrow \begin{vmatrix} 3x-11 & 11-3x & 0 \\ 0 & 3x-11 & 11-3x \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Expanding along $R_1$

$\Rightarrow (3x-11)\{(3x-11)(3x-8) - (3)(11-3x)\} - (11-3x)\{(0)((3x-8) - (11-3x)(3)\} = 0$

$\Rightarrow (3x-11)\{(3x-11)(3x-8+3)\} - (11-3x)\{-(11-3x)(3)\} = 0$

$\Rightarrow (3x-11)^2(3x-5)\} + (3x-11)\{(3x-11)(3)\} = 0$

$\Rightarrow (3x-11)^2(3x-5)\} + (3x-11)^2(3)\} = 0$

$\Rightarrow (3x-11)^2(3x-5+3) = 0$

$\Rightarrow (3x-11)^2(3x-2) = 0$

$\Rightarrow x = \frac{11}{3}$, Or, $x = \frac{2}{3}$

**28. Question**

Mark the tick against the correct answer in the following:

The vertices of a a ABC are A(-2, 4), B(2, -6) and C(5, 4). The area of a ABC is

A. 17.5 sq units

B. 35 sq units

C. 32 sq units

D. 28 sq units

**Answer**

To find: Area of ABC

Given: A(-2,4), B(2,-6) and C(5,4)

Formula used: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

We have, A(-2,4), B(2,6) and C(5,4)

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

Expanding along $R_1$

$\Rightarrow \frac{1}{2}$ [-2{(-6)(1)-(4)(1)-4 {(2)(1)-(5)(1)}+1 {(2)(4)-(5)(-6)}]

$\Rightarrow \frac{1}{2}$ [-2{-6-4} -4 {2-5}+1 {8+30}]

$\Rightarrow \frac{1}{2}$ [-2{-10} - 4{-3}+1 {38}]

$\Rightarrow \frac{1}{2}$ [20 + 12 + 38]

$\Rightarrow \frac{1}{2}$ [70]

$\Rightarrow$ 35 sq. units

**29. Question**

Mark the tick against the correct answer in the following:

If the points A(3, -2), B(k, 2) and C(8, 8) are collinear then the value of k is

A. 2

B. -3

C. 5

D. -4

**Answer**

To find: Area of ABC

Given: A(3,-2), B(k,2) and C(8,8)

The formula used: $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

We have, A(3,-2), B(k,2) and C(8,8)

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ k & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

Expanding along $R_1$

$\Rightarrow \frac{1}{2}$ [3{(2)(1)-(8)(1)} - (-2){(k)(1)-(8)(1)}+ 1{(k)(8)-(2)(8)}] = 0

$\Rightarrow \frac{1}{2}$ [3{2-8} + 2{k-8}+ 1{8k-16}]=0

$\Rightarrow$ -18 +2k - 16 + 8k -16 = 0

$\Rightarrow 10k - 50 = 0$

$\Rightarrow k = 5$