8. System of Linear Equations

Exercise 8A

1. Question

Show that each one of the following systems of equations is inconsistent.

x + 2y = 9;

2x + 4y = 7.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + 2y = 9

2x + 4y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}9\\7\end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix}1 & 2\\0 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}9\\-11\end{bmatrix}$$

Again converting into equation form, we get

x + 2y = 9

0x + 0y = - 11

∴ 0 = - 11

which is not true

∴ x + 2y = 9

2x + 4y = 7 are inconsistent.

2. Question

Show that each one of the following systems of equations is inconsistent.

2x + 3y = 5;

6x + 9y = 10.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

2x + 3y = 5

6x + 9y = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}2 & 3\\6 & 9\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}5\\10\end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

Again converting into equation form, we get

2x + 3y = 5

0x + 0y = - 5

∴ 0 = - 5

which is not true

∴ 2x + 3y = 5

6x + 9y = 10 are inconsistent.

3. Question

Show that each one of the following systems of equations is inconsistent.

4x – 2y = 3;

6x – 3y = 5.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

4x – 2y = 3

6x – 3y = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$4R_2 - 6R_1$

$$\begin{bmatrix} 4 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Again converting into equation form, we get

4x – 2y = 3

0x + 0y = 2

∴ 0 = 2

which is not true

∴ 4x – 2y = 3

6x – 3y = 5 are inconsistent.

4. Question

Show that each one of the following systems of equations is inconsistent.

6x + 4y = 5;

9x + 6y = 8.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

6x + 4y = 5

9x + 6y = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$2R_2 - 3R_1$

$$\begin{bmatrix} 6 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Again converting into equation form, we get

6x + 4y = 5

0x + 0y = 3

∴ 0 = 3

which is not true

∴ 6x + 4y = 5

9x + 6y = 8 are inconsistent.

5. Question

Show that each one of the following systems of equations is inconsistent.

x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$R_2 - R_1$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 14 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 7 \end{bmatrix}$$

Converting back into equation form we get,

x + y – 2z = 5;

0x – 3y + 3z = - 7;

0x + 0y + 0z = 7

∴ 0 = 7

Which is not true.

∴x + y – 2z = 5;

x – 2y + z = - 2;

- 2x + y + z = 4

are inconsistent.

6. Question

Show that each one of the following systems of equations is inconsistent.

2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -1 & 3 \\ 3 & -2 & 5 \\ 5 & -4 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 14 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - 5R_1$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 23 \end{bmatrix}$$

$R_3 - 3R_2$

$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 56 \end{bmatrix}$$

Converting back into equation form we get,

2x – y + 3z = 1;

0x – 1y + 1z = - 11;

0x + 0y + 0z = 56

∴ 0 = 56

Which is not true.

∴2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

are inconsistent.

7. Question

Show that each one of the following systems of equations is inconsistent.

x + 2y + 4z = 12;

y + 2z = - 1;

3x + 2y + 4z = 4.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

x + 2y + 4z = 12;

y + 2z = - 1;

3x + 2y + 4z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\3 & 2 & 4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\4\end{bmatrix}$$

R_3 - $3R_1$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & -4 & -8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-32\end{bmatrix}$$

R_3 + $4R_2$

$$\begin{bmatrix}1 & 2 & 4\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\-1\\-36\end{bmatrix}$$

Converting back into equation form we get,

x + 2y + 4z = 12;

y + 2z = - 1;

0x + 0y + 0z = - 36

∴ 0 = - 36

Which is not true.

∴2x – y + 3z = 1;

3x – 2y + 5z = - 4;

5x – 4y + 9z = 14

are inconsistent.

8. Question

Show that each one of the following systems of equations is inconsistent.

3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3.

Answer

To prove: Set of given lines are inconsistent.

Given set of lines are : -

3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

R_3 - R_1

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & -4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

R_3 + 2R_2

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

Converting back into equation form we get,

3x – y – 2z = 2;

2y – z = - 1;

0x + 0y + 0z = - 1

∴ 0 = - 1

Which is not true.

∴3x – y – 2z = 2;

2y – z = - 1;

3x – 5y = 3

are inconsistent.

9. Question

Solve each of the following systems of equations using matrix method.

5x + 2y = 4;

7x + 3y = 5.

Answer

To find: - x , y

Given set of lines are : -

5x + 2y = 4;

7x + 3y = 5.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$5R_2 - 7R_1$

$$\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Again converting into equation form, we get

5x + 2y = 4;

y = - 3

5x + 2× - 3 = 4

5x = 10

X = 2

∴ x = 2 , y = - 3

10. Question

Solve each of the following systems of equations using matrix method.

3x + 4y – 5 = 0;

x - y + 3 = 0.

Answer

To find: - x , y

Given set of lines are : -

3x + 4y – 5 = 0;

x - y + 3 = 0

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$3R_2 - R_1$

$$\begin{bmatrix} 3 & 4 \\ 0 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -14 \end{bmatrix}$$

Again converting into equation form, we get

3x + 4y = 5

- 7 y = - 14

Y = 2

3x + 4y = 5

3x + 4×2 = 5

3x = - 3

X = - 1

∴ x = - 1 , y = 2

11. Question

Solve each of the following systems of equations using matrix method.

x + 2y = 1;

3x + y = 4.

Answer

To find: - x , y

Given set of lines are : -

x + 2y = 1

3x + y = 4

Converting the following equations in matrix form,

AX = B

$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$R_2 - 3R_1$

$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Again converting into equation form, we get

x + 2y = 1

- 5y = 1

Y = $-\frac{1}{5}$

x + 2× $-\frac{1}{5}$ = 1

x + $-\frac{2}{5}$ = 1

x = 1 + $\frac{2}{5}$

X = $\frac{7}{5}$

∴ x = $\frac{7}{5}$, y = $-\frac{1}{5}$

12. Question

Solve each of the following systems of equations using matrix method.

5x + 7y + 2 = 0;

4x + 6y + 3 = 0.

Answer

To find: - x , y

Given set of lines are : -

5x + 7y + 2 = 0;

4x + 6y + 3 = 0.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$5R_2 - 4R_1$

$$\begin{bmatrix} 5 & 7 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -7 \end{bmatrix}$$

Again converting into equation form, we get

5x + 7y = - 2

2y = - 7

Y = $-\frac{7}{2}$

5x + 7$\times -\frac{7}{2}$ = - 2

5x = - 2 + $\frac{49}{2}$

5x = $\frac{45}{2}$

X = $\frac{9}{2}$

$\therefore x = \frac{9}{2}, y = -\frac{7}{2}$

13. Question

Solve each of the following systems of equations using matrix method.

2x - 3y + 1 = 0;

x + 4y + 3 = 0.

Answer

To find: - x , y

Given set of lines are : -

2x - 3y + 1 = 0;

x + 4y + 3 = 0

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \end{bmatrix}$$

$2R_2 - R_1$

$$\begin{bmatrix} 2 & -3 \\ 0 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

Again converting into equation form we get

2x - 3y = - 1

11 y = - 5

$Y = -\frac{5}{11}$

$2x - 3 \times -\frac{5}{11} = -1$

$2x = -1 - \frac{15}{11}$

$X = -\frac{13}{11}$

$\therefore x = -\frac{13}{11}, y = -\frac{5}{11}$

14. Question

Solve each of the following systems of equations using matrix method.

4x - 3y = 3;

3x - 5y = 7.

Answer

To find: - x , y

Given set of lines are : -

4x - 3y = 3;

3x - 5y = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$4R_2 - 3R_1$

$$\begin{bmatrix} 4 & -3 \\ 0 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 19 \end{bmatrix}$$

Again converting into equation form, we get

4x – 3y = 3

- 11y = 19

$Y = -\frac{19}{11}$

$4x - 3 \times -\frac{19}{11} = 3$

$4x = 3 - \frac{57}{11}$

$4x = -\frac{24}{11}$

$X = -\frac{6}{11}$

$\therefore x = -\frac{6}{11}, y = -\frac{19}{11}$

15. Question

Solve each of the following systems of equations using matrix method.

2x + 8y + 5z = 5;

x + y + z = - 2;

x + 2y - z = 2.

Answer

To find: - x , y , z

Given set of lines are : -

2x + 8y + 5z = 5;

x + y + z = - 2;

x + 2y - z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 8 & 5 \\ 1 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$2R_2 – R_1$

$2R_3 – R_1$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & -4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ -1 \end{bmatrix}$$

$3R_3 – 2R_2$

$$\begin{bmatrix} 2 & 8 & 5 \\ 0 & -6 & -3 \\ 0 & 0 & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \\ 15 \end{bmatrix}$$

Again converting into equations, we get

2x + 8y + 5z = 5

- 6y - 3z = - 9

- 15z = 15

Z = - 1

- 6y - 3 × - 1 = - 9

- 6y = - 9 - 3

Y = 2

2x + 8×2 + 5× - 1 = 5

2x = 5 - 16 + 5

X = - 3

$\therefore x = -3, y = 2, z = -1$

16. Question

Solve each of the following systems of equations using matrix method.

x – y + z = 1;

2x + y – z = 2;

X – 2y – z = 4.

Answer

To find: - x , y , z

Given set of lines are : -

x – y + z = 1;

2x + y – z = 2;

X – 2y – z = 4

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & 1\\2 & 1 & -1\\1 & -2 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\2\\4\end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & -1 & 1\\0 & 3 & -3\\0 & -1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\3\end{bmatrix}$$

$3R_3 + R_2$

$$\begin{bmatrix}1 & -1 & 1\\0 & 3 & -3\\0 & 0 & -9\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\9\end{bmatrix}$$

Again converting into equations we get

X – y + z = 1

3y - 3z = 0

- 9z = 9

Z = - 1

Y = z

Y = - 1

X + 1 - 1 = 1

X = 1

$\therefore x = 1, y = -1, z = -1$

17. Question

Solve each of the following systems of equations using matrix method.

3X + 4y + 7z = 4;

2x – y + 3z = - 3;

x + 2y – 3z = 8.

Answer

To find: - x , y , z

Given set of lines are : -

3X + 4y + 7z = 4;

2x – y + 3z = - 3;

x + 2y – 3z = 8

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 8 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 2 & -16 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 20 \end{bmatrix}$$

$11R_3 + 2R_2$

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & -11 & -5 \\ 0 & 0 & -186 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -17 \\ 186 \end{bmatrix}$$

Again converting into equations we get

3x + 4y + 7z = 4

- 11y - 5z = - 17

- 186z = 186

Z = - 1

- 11y + 5 = - 17

- 11y = - 22

Y = 2

3x + 4×2 + 7× - 1 = 4

3x = 4 - 8 + 7

X = 1

$\therefore x = 1, y = 2, z = -1$

18. Question

Solve each of the following systems of equations using matrix method.

x + 2y + z = 7;

x + 3z = 11;

2x – 3y = 1.

Answer

To find: - x , y , z

Given set of lines are : -

x + 2y + z = 7;

x + 3z = 11;

2x – 3y = 1

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -7 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -13 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -9 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \\ -9 \end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 7

- 2y + 2z = 4

- 9y = - 9

Y = 1

- 2×1 + 2z = 4

2z = 6

Z = 3

X + 2×1 + 3 = 7

X = 7 - 2 - 3

X = 2

$\therefore x = 2, y = 1, z = 3$

19. Question

Solve each of the following systems of equations using matrix method.

2x - 3y + 5z = 16;

3x + 2y – 4z = - 4

x + y – 2z = - 3.

Answer

To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 16;

3x + 2y – 4z = - 4

x + y – 2z = - 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -22 \end{bmatrix}$$

$13R_3$ - $5R_2$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ -56 \\ -6 \end{bmatrix}$$

Again converting into equations, we get

2x - 3y + 5z = 16

13y - 23z = - 56

- 2z = - 6

Z = 3

13y – 23×3 = - 56

13y = - 56 + 69

Y = 1

2x - 3×1 + 5×3 = 16

2x = 16 + 3 - 15

2x = 4

X = 2

$\therefore x = 2, y = 1, z = 3$

20. Question

Solve each of the following systems of equations using matrix method.

x + y + z = 4;

2x – y + z = - 1;

2x + y – 3z = - 9.

Answer

To find: - x , y , z

Given set of lines are : -

x + y + z = 4;

2x – y + z = - 1;

2x + y – 3z = - 9.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -9 \end{bmatrix}$$

R_2 – $2R_1$

R_3 –$2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ -17 \end{bmatrix}$$

$3R_3$ - R_2

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ -42 \end{bmatrix}$$

Again converting into equations, we get

X + y + z = 4

- 3y - z = - 9

- 14z = - 42

Z = 3

- 3y - 3 = - 9

- 3y = - 6

Y = 2

X + 2 + 3 = 4

X = 4 - 5

X = - 1

$\therefore x = -1, y = 2, z = 3$

21. Question

Solve each of the following systems of equations using matrix method.

2x - 3y + 5z = 11;

3x + 2y - 4z = - 5;

x + y – 2z = - 3.

Answer

To find: - x , y , z

Given set of lines are : -

2x - 3y + 5z = 11;

3x + 2y - 4z = - 5;

x + y – 2z = - 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$2R_2 - 3R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 5 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -43 \\ -17 \end{bmatrix}$$

$13R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & 13 & -23 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -43 \\ -6 \end{bmatrix}$$

Again converting into equations we get

2x – 3y + 5z = 11

13y - 23 z = - 43

- 2z = - 6

Z = 3

13y - 23×3 = - 43

13y = - 43 + 69

13y = 26

Y = 2

2x – 3×2 + 5×3 = 11

2x = 11 + 6 - 15

X = 1

$\therefore x = 1, y = 2, z = 3$

22. Question

Solve each of the following systems of equations using matrix method.

x + y + z = 1;

x – 2y + 3z = 2;

5x – 3y + z = 3.

Answer

To find: - x , y , z

Given set of lines are : -

x + y + z = 1;

x – 2y + 3z = 2;

5x – 3y + z = 3.

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$R_2 – R_1$

$R_3 – 5R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & -8 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & -14 & -0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Again converting into equations we get

X + y + z = 1

- 3y + 2z = 1

- 14 y = 0

Y = 0

- 3 × 0 + 2z = 1

Z = $\frac{1}{2}$

X + 0 + $\frac{1}{2}$ = 1

X = $\frac{1}{2}$

$\therefore x = \frac{1}{2}, y = 0, z = \frac{1}{2}$

23. Question

Solve each of the following systems of equations using matrix method.

x + y + z = 6;

x + 2z = 7;

3x + y + z = 12.

Answer

To find: - x , y , z

Given set of lines are : -

x + y + z = 6;

x + 2z = 7;

3x + y + z = 12

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -6 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$$

Again converting into equations we get

X + y + z = 6

- y + z = 1

- 4 y = - 4

Y = 1

- 1 + z = 1

Z = 2

X + 1 + 2 = 6

X = 6 - 3

X = 3

$\therefore x = 3, y = 1, z = 2$

24. Question

Solve each of the following systems of equations using matrix method.

2x + 3y + 3z = 5;

x – 2y + z = - 4;

3x – y – 2z = 3.

Answer

To find: - x , y , z

Given set of lines are : -

2x + 3y + 3z = 5;

x – 2y + z = - 4;

3x – y – 2z = 3

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & -11 & -13 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ -9 \end{bmatrix}$$

$R_3 - 13R_2$

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & -7 & -1 \\ 0 & 80 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -13 \\ 160 \end{bmatrix}$$

Again converting into equations we get

2X + 3y + 3z = 5

- 7y - z = - 13

80 y = 160

Y = 2

- 7×2 - z = - 13

Z = - 1

2x + 3×2 + 3× - 1 = 5

2x = 5 - 6 + 3

X = 1

$\therefore x = 1, y = 2, z = -1$

25. Question

Solve each of the following systems of equations using matrix method.

4x - 5y – 11z = 12;

X – 3y + z = 1;

2x + 3y – 7z = 2.

Answer

To find: - x , y , z

Given set of lines are : -

4x - 5y - 11z = 12

X – 3y + z = 1;

2x + 3y – 7z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - R_1$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 11 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -8 \end{bmatrix}$$

$5R_3 + R_2$

$$\begin{bmatrix} 4 & -5 & -11 \\ 0 & -7 & 15 \\ 0 & 48 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \\ -48 \end{bmatrix}$$

Again converting into equations we get

4x – 5y - 11z = 12

- 7y + 15z = - 8

48 y = - 48

Y = - 1

7 + 15z = - 8

15z = - 15

Z = - 1

4x + 5 + 11 = 12

4x = 12 - 5 - 11

4x = - 4

X = - 1

$\therefore x = -1, y = -1, z = -1$

26. Question

Solve each of the following systems of equations using matrix method.

x – y + 2z = 7;

3x + 4y – 5z = - 5:

2x – y + 3z = 12.

Answer

To find: - x , y , z

Given set of lines are : -

x - y + 2z = 7

3x + 4y – 5z = - 5

2x – y + 3z = 12

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - 2R_1$

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 7 & -11\\ 0 & 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}7\\ -26\\ -2\end{bmatrix}$$

$7R_3 - R_2$

$$\begin{bmatrix}1 & -1 & 2\\ 0 & 7 & -11\\ 0 & 0 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}7\\ -26\\ 12\end{bmatrix}$$

Again converting into equations we get

x – y + 2z = 7

7y - 11 z = - 26

4z = 12

Z = 3

7y – 11 ×3 = - 26

7y = - 26 + 33

7y = 7

Y = 1

X – 1 + 2×3 = 7

X = 7 + 1 - 6

X = 2

$\therefore x = 2, y = 1, z = 3$

27. Question

Solve each of the following systems of equations using matrix method.

6X - 9y – 20z = - 4;

4x – 15y + 10z = - 1;

2x - 3y - 5z = - 1.

Answer

To find: - x , y , z

Given set of lines are : -

6x - 9y - 20z = - 4

4x – 15y + 10z = - 1

2x - 3y - 5z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 6 & -9 & -20 \\ 4 & -15 & 10 \\ 2 & -3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ -1 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 6 & -9 & -20 \\ 0 & -27 & 70 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 5 \\ 1 \end{bmatrix}$$

Again converting into equations, we get

6x - 9y - 20 z = - 4

- 27y + 70z = 5

5z = 1

Z = $\frac{1}{5}$

- 27y + 70 $\times \frac{1}{5}$ = 5

- 27y = 5 – 14

- 27y = - 9

Y = $\frac{1}{3}$

6x – 9 $\times \frac{1}{3}$ - 20 $\times \frac{1}{5}$ = - 4

6x = - 4 + 3 + 4

X = $\frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

28. Question

Solve each of the following systems of equations using matrix method.

3x - 4y + 2z = - 1;

2x + 3y + 5z = 7;

X + z = 2.

Answer

To find: - x , y , z

Given set of lines are : -

3x - 4y + 2z = - 1

2x + 3y + 5z = 7;

x + z = 2

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$3R_2 - 2R_1$

$3R_3 - R_1$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 7 \end{bmatrix}$$

$11R_3 - R_2$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 17 & 11 \\ 0 & 27 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 23 \\ 54 \end{bmatrix}$$

Again converting into equations, we get

3x - 4y + 2z = - 1

17y + 11z = 23

27 y = 54

Y = 2

17×2 + 11z = 23

11z = 23 – 34

Z = - 1

3x – 4 × 2 + 2× - 1 = - 1

3x = - 1 + 8 + 2

3x = 9

X = 3

$\therefore x = 3, y = 2, z = -1$

29. Question

Solve each of the following systems of equations using matrix method.

X + y - z = 1;

3x + y – 2z = 3;

X – y – z = - 1.

Answer

To find: - x , y , z

Given set of lines are : -

x + y - z = 1

3x + y - 2z = 3

x - y - z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & -1\\3 & 1 & -2\\1 & -1 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\3\\-1\end{bmatrix}$$

$R_2 - 3R_1$

$R_3 - R_1$

$$\begin{bmatrix}1 & 1 & -1\\0 & -2 & 1\\0 & -2 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\-2\end{bmatrix}$$

Again converting into equations we get

X + y – z = 1

- 2y + z = 0

- 2y = - 2

Y = 1

- 2 + z = 0

Z = 2

X + 1 - 2 = 1

X = 2

$\therefore x = 2, y = 1, z = 2$

30. Question

Solve each of the following systems of equations using matrix method.

2x + y - z = 1;

x – y + z = 2;

3x + y – 2z = - 1.

Answer

To find: - x , y , z

Given set of lines are : -

2x + y - z = 1

x - y + z = 2

3x + y - 2z = - 1

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}2 & 1 & -1\\1 & -1 & 1\\3 & 1 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$$

$3R_3$ - R_2

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & -3 & 3 \\ 0 & 0 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -18 \end{bmatrix}$$

Again converting into equations we get

2x + y – z = 1

- 3y + 3z = 3

- 6z = - 18

Z = 3

- 3y + 3×3 = 3

- 3y = 3 - 9

- 3y = - 6

Y = 2

2x + 2 - 3 = 1

2x = 1 + 1

X = 1

∴ x = 1 , y = 2 , z = 3

31. Question

Solve each of the following systems of equations using matrix method.

X + 2y + z = 4;

- x + y + z = 0;

x - 3y + z = 4.

Answer

To find: - x , y , z

Given set of lines are : -

x + 2y + z = 4

- x + y + z = 0

x - 3y + z = 4

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

R_2 + R_1

R_3 – R_1

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

Again converting into equations we get

X + 2y + z = 4

3y + 2z = 4

- 5y = 0

Y = 0

0 + 2z = 4

Z = 2

X + 0 + 2 = 4

X = 2

∴ x = 2 , y = 0 , z =2

32. Question

Solve each of the following systems of equations using matrix method.

x - y – 2z = 3;

x + y = 1;

x + z = - 6.

Answer

To find: - x , y , z

Given set of lines are : -

x - y - 2z = 3

x + y = 1

x + z = - 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ -6 \end{bmatrix}$$

R_2 – R_1

R_3 – R_1

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -9 \end{bmatrix}$$

$2R_3$ - R_2

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -16 \end{bmatrix}$$

Again converting into equations we get

X + y - 2z = 3

2y + 2z = - 2

4z = - 16

Z = - 4

2y – 8 = - 2

2y = - 2 + 8

2y = 6

Y = 3

X - 3 + 8 = 3

X = - 2

∴ x = - 2 , y = 3 , z = - 4

33. Question

Solve each of the following systems of equations using matrix method.

5x - y = - 7;

2x + 3z = 1;

3y – z = 5.

Answer

To find: - x , y , z

Given set of lines are : -

5x - y = - 7

2x + 3z = 1

3y - z = 5

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 5 & -1 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ 5 \end{bmatrix}$$

$5R_2 - 2R_1$

$$\begin{bmatrix} 5 & -1 & 0 \\ 0 & 2 & 15 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 19 \\ 5 \end{bmatrix}$$

$2R_3 - 3R_2$

$$\begin{bmatrix} 5 & -1 & 0 \\ 0 & 2 & 15 \\ 0 & 0 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 19 \\ -47 \end{bmatrix}$$

Again converting into equations we get

5x – y = - 7

2y + 15z = 19

- 47z = - 47

Z = 1

2y + 15 = 19

2y = 19 - 15

Y = 2

5x – 2 = - 7

5x = - 5

X = - 1

∴ x = - 1 , y = 2 , z = 1

34. Question

Solve each of the following systems of equations using matrix method.

x - 2y + z = 0;

y – z = 2;

2x – 3z = 10.

Answer

To find: - x , y , z

Given set of lines are : -

x - 2y + z = 0

y - z = 2

2x - 3z = 10

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

R_3 – $2R_1$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 10 \end{bmatrix}$$

R_3 - $4R_2$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

Again converting into equations we get

X - 2y + z = 0

Y – z = 2

- z = 2

Z = - 2

Y + 2 = 2

Y = 0

X + 0 - 2 = 0

X = 2

∴ x = 2 , y = 0 , z = - 2

35. Question

Solve each of the following systems of equations using matrix method.

x - y = 3;

2x + 3y + 4z = 17;

y + 2z = 7.

Answer

To find: - x , y , z

Given set of lines are : -

x - y = 3

2x + 3y + 4z = 17

y + 2z = 7

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 7 \end{bmatrix}$$

$2R_3 - R_2$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & 4 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 11 \\ 3 \end{bmatrix}$$

Again converting into equations we get

X – y = 3

5y + 4z = 11

- 3y = 3

Y = - 1

5× - 1 + 4z = 11

4z = 16

Z = 4

X + 1 = 3

X = 2

∴ x = 2 , y = - 1 , z = 4

36. Question

Solve each of the following systems of equations using matrix method.

4x + 3y + 2z = 60;

x + 2y + 3z = 45;

6x + 2y + 3z = 70.

Answer

To find: - x , y , z

Given set of lines are : -

4x + 3y + 2z = 60

x + 2y + 3z = 45

6x + 2y + 3z = 70

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Again converting into equations, we get

4x + 3y + 2z = 60

5y + 10z = 120

- 5y = - 40

Y = 8

5×8 + 10z = 120

10z = 120 - 40

10z = 80

Z = 8

4x + 3×8 + 2×8 = 60

4x = 60 – 24 - 16

4x = 20

X = 5

∴ x = 5 , y = 8 , z = 8

37. Question

If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equations:

2x – 3y + 5z = 11;

3x + 2y – 4z = - 5;

x + y – 2z = - 3.

Answer

Given,

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

The determinant of matrix A is

$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

= 2(2 × - 2 – (- 4)×1) + 3(3× - 2 – (- 4)×1) + 5(3×1 – 2×1)

= 2(- 4 + 4) + 3(- 6 + 4) + 5(3 – 2)

= 2(0) + 3(- 2) + 5(1)

= - 6 + 5

= - 1

|A| ≠ 0

∴ A^{-1} is possible.

$A^T = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$

$Adj(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Given set of lines are : -

2x – 3y + 5z = 11

3x + 2y – 4z = - 5

x + y – 2z = - 3

Converting following equations in matrix form,

AX = B

Where $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \times 11 - 5 \times 1 - 3 \times -2 \\ -2 \times 11 - 5 \times 9 - 3 \times -23 \\ -1 \times 11 - 5 \times 5 - 3 \times -13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

∴ x = 1 , y = 2 , z = 3

38. Question

If $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{pmatrix}$, find A^{-1}.

Using A^{-1}, solve the following

system of linear equations:

2x + y + z = 1;

X – 2y – z = $\frac{3}{2}$;

3y – 5z = 9.

HINT: Here $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{pmatrix}$,

$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 3/2 \\ 9 \end{pmatrix}$.

Answer

Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adj(A)$

The determinant of matrix A is

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

= 2(- 2× - 5 - (- 1)×3) – (1× - 5 - (- 1)×0) + (1×3 – (- 2)×0)

= 2(10 + 3) – (- 5) + (3)

= 26 + 5 + 3

= 34

|A| ≠ 0

∴ A^{-1} is possible.

$A^T = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 3 \\ 1 & -1 & -5 \end{bmatrix}$

$\text{Adj}(A) = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\, adj(A)$

$A^{-1} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

Given set of lines are : -

2x + y + z = 1

X – 2y – z = $\frac{3}{2}$

3y – 5z = 9

Converting the following equations in matrix form,

AX = B

Where $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

Pre - multiplying by A^{-1}

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 1 \times 13 + \frac{3}{2} \times 8 + 9 \times 1 \\ 1 \times 5 + \frac{3}{2} \times -10 + 9 \times 3 \\ 1 \times 3 + \frac{3}{2} \times -6 + 9 \times -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 + 12 + 9 \\ 5 - 15 + 27 \\ 3 - 9 - 45 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

∴ x = 1 , y = $\frac{1}{2}$, z = - $\frac{3}{2}$

39 Using. Question

If $A = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{pmatrix}$ and

B = $\begin{pmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{pmatrix}$, find *AB*.

Hence, solve the system of equations:

x – 2y = 10,

2x + y + 3z = 8 and

- 2y + z = 7.

HINT: $AB = (11)I = A\left(\frac{1}{11}B\right) = I$

$A^{-1} = \left(\frac{1}{11}\right)B.$

Answer

Given,

$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 7\times1-2\times-2-4\times0 & 2\times1+1\times-2+2\times0 & -6\times1-3\times-2+5\times0 \\ 7\times2-2\times1-4\times3 & 2\times2+1\times1+2\times3 & -6\times2-3\times1+5\times3 \\ 7\times0-2\times-2-4\times1 & 2\times0+1\times-2+2\times1 & -6\times0-3\times-2+5\times1 \end{bmatrix}$

$AB = \begin{bmatrix} 7+4+0 & 2-2+0 & -6+6+0 \\ 14-2-12 & 4+1+6 & -12-3+15 \\ 0+4-4 & 0-2+2 & 0+6+5 \end{bmatrix}$

$AB = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

$AB = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

AB = 11I

Pre - multiplying by A^{-1}

$A^{-1}AB = 11\,A^{-1}I$

$IB = 11\,A^{-1}$

$B = 11\,A^{-1}$

$A^{-1} = \frac{1}{11}B$

Given set of lines are : -

x – 2y = 10

x + y + 3z = 8

- 2y + z = 7

Converting following equations in matrix form,

AX = C

Where A = $\begin{bmatrix}1 & -2 & 0\\2 & 1 & 3\\0 & -2 & 1\end{bmatrix}$,X = $\begin{bmatrix}x\\y\\z\end{bmatrix}$, C = $\begin{bmatrix}10\\8\\7\end{bmatrix}$

Pre - multiplying by A $^{-1}$

A $^{-1}$AX = A $^{-1}$C

IX = A $^{-1}$C

X = A $^{-1}$C

X = $\frac{1}{11}$ BC

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{11}\begin{bmatrix}7 & 2 & -6\\-2 & 1 & -3\\-4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\8\\7\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{11}\begin{bmatrix}10\times 7 + 8\times 2 + 7\times -6\\10\times -2 + 8\times 1 + 7\times -3\\10\times -4 + 8\times 8 + 7\times 5\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{11}\begin{bmatrix}70 + 16 - 42\\-20 + 8 - 21\\-40 + -16 + 35\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{11}\begin{bmatrix}44\\-33\\11\end{bmatrix} = \begin{bmatrix}4\\-3\\1\end{bmatrix}$$

∴ x = 4 , y = - 3 , z = 1

40. Question

$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}$ = 10, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ = 10,

$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}$ = 13

Ans. x = $\frac{1}{2}$, y = $\frac{1}{3}$, z = $\frac{1}{5}$

Answer

To find: - x , y , z

Given set of lines are : -

$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}$ = 10

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ = 10,

$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}$ = 13

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix}2 & -3 & 3\\1 & 1 & 1\\3 & -1 & 2\end{bmatrix}\begin{bmatrix}\frac{1}{x}\\\frac{1}{y}\\\frac{1}{z}\end{bmatrix} = \begin{bmatrix}10\\10\\13\end{bmatrix}$$

$2R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & 7 & -5 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -4 \end{bmatrix}$$

$R_3 - 5R_2$

$$\begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & -1 \\ 0 & -18 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ -54 \end{bmatrix}$$

Again converting into equations we get

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{5}{y} - \frac{1}{z} = 10$$

$$-\frac{18}{y} = -54$$

$$y = \frac{1}{3}$$

$$5 \times 3 - \frac{1}{z} = 10$$

$$-\frac{1}{z} = 10 - 15$$

$Z = \frac{1}{5}$

$$\frac{2}{x} - 3 \times 3 + 3 \times 5 = 10$$

$\frac{2}{x} = 10 + 9 - 15 = 4$

$X = \frac{1}{2}$

$\therefore x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

41 VALUE. Question

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$; $\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$;

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$. $(x, y, z \neq 0)$

Answer

To find: - x , y , z

Given set of lines are : -

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z}$ = 4

$\frac{2}{x} + \frac{1}{y} - \frac{3}{z}$ = 0

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ = 2

Converting following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

R_2 - $2R_1$

R_3 - R_1

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$

Again converting into equations we get

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$$

$$\frac{3}{y} - \frac{5}{z} = -8$$

$$\frac{2}{y} = -2$$

$$y = -1$$

$$3 \times -1 - \frac{5}{z} = -8$$

$$-\frac{5}{z} = -8 + 3$$

Z = 1

$$\frac{1}{x} - 1 \times -1 + 1 \times 1 = 4$$

$\frac{1}{x} = 4$ - 1 - 1

X = $\frac{1}{2}$

∴ x = $\frac{1}{2}$, y = 1 , z = 1

42. Question

The sum of three numbers is 2. If twice the second number is added to the sum of first and third, we get 1. On adding the sum of second and third numbers to five times the first, we get 6. Find the three numbers by using matrices.

Answer

Let the three numbers be x, y and z.

According to the question,

X + y + z = 2

X + 2y + z = 1

5x + y + z = 6

Converting the following equations in matrix form,

AX = B

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$R_2 - R_1$

$R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$$

Converting back into the equations we get

X + y + z = 2

Y = - 1

4x = 5

X = $\frac{5}{4}$

$\frac{5}{4}$ – 1 + z = 2

Z = 2 - $\frac{5}{4}$ + 1

Z = $\frac{7}{4}$

∴ The numbers are $\frac{5}{4}, \frac{7}{4}, -1$.

43. Question

The cost of 4 kg potato, 3 kg wheat and 2 kg of rice is ₹ 60. The cost of 1 kg potato, 2 kg wheat and 3 kg of rice is ₹45. The cost of 6 kg potato, 2 kg wheat and 3 kg of rice is ₹70. Find the cost of each item per kg by matrix method.

Answer

Let the price of 1kg potato, wheat and rice be x, y and z respectively.

According to the question,

4x + 3y + 2z = 60

X + 2y + 3z = 45

6x + 2y + 3z = 70

Converting into matrix form

AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$4R_2 - R_1$

$2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Converting back into the equations we get

4x + 3y + 2z = 60

5y + 10z = 120

- 5y = - 40

Y = 8

5×8 + 10z = 120

10z = 120 – 40

Z = 8

4x + 3×8 + 2×8 = 60

4x = 60 - 24 - 16

4x = 20

X = 5

∴ The cost of 1 kg potatoes, wheat and rice is Rs.5, Rs.8 and Rs. 8 respectively.

44. Question

An amount of ₹ 5000 is put into three investments at 6%, 7% and 8% per annum respectively. The total annual income from these investments is ₹358. If the total annual income from first two investments is ₹70more

than the income from the third, find the amount of each investment by the matrix method.

HINT: Let these investments be ₹x, ₹y and ₹z, respectively.

Then, x + y + z = 5000, ...(i)

$\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}$ = 358 ⇒

6x + 7y + 8z = 35800 ...(ii)

And, $\frac{6x}{100}+\frac{7y}{100}=\frac{8z}{100}+70$

⇒6x + 7y - 8z = 7000. ...(iii)

Answer

Let these investments be ₹x, ₹y and ₹z, respectively.

Then, x + y + z = 5000

$\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}$ = 358

6x + 7y + 8z = 35800

And, $\frac{6x}{100}+\frac{7y}{100}=\frac{8z}{100}+70$

6x + 7y - 8z = 7000.

Representing in the matrix form,

AX = B

$$\begin{bmatrix}1 & 1 & 1\\6 & 7 & 8\\6 & 7 & -8\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5000\\35800\\7000\end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix}1 & 1 & 1\\6 & 7 & 8\\0 & 0 & -16\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5000\\35800\\-28800\end{bmatrix}$$

$R_2 - 6R_1$

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 2\\0 & 0 & -16\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}5000\\5800\\-28800\end{bmatrix}$$

Converting back into the equations we get

X + y + z = 5000

Y + 2z = 5800

- 16z = - 28800

Z = 1800

Y + 2×1800 = 5800

Y = 5800 - 3600

Y = 2200x + 2200 + 1800 = 5000

X = 5000 – 4000

X = 1000

Amount of 1000 , 2200 , 1800 were invested in the investments of 6% , 7%, 8% respectively.

45. Question

Two schools *A* and *B* want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school *A* wants to award ₹ x each, ₹ y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with total award money of ₹ 1,600. School *B* wants to spend ₹ 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹ 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.

HINT: By the given data, we have

$$\left.\begin{aligned}3x+2y+z&=1600\\4x+y+3z&=2300\\x+y+z&=900\end{aligned}\right\}$$

Answer

Let the amount x, y and z be considered for sincerity, truthfulness and helpfulness.

According to the questions,

3x + 2y + z = 1600

4x + y + 3z = 2300

X + y + z = 900

Converting into the matrix form

AX = B

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$R_1 - 3R_3$

$R_2 - 4R_3$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1300 \\ 900 \end{bmatrix}$$

$2R_2 - R_1$

$$\begin{bmatrix} 0 & -1 & -2 \\ 0 & -5 & 0 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1100 \\ -1500 \\ 900 \end{bmatrix}$$

Converting back into the equations we get

- y - 2z = - 1100

- 5y = - 1500

X + y + z = 900

Y = 300

- 300 - 2z = - 1100

- 2z = - 800

Z = 400

X + 300 + 400 = 900

X = 900 - 700

X = 200

₹ 200 for sincerity, ₹ 300 for truthfulness and ₹ 400 for helpfulness. One more value may be like honesty, kindness, etc.

Objective Questions

1. Question

If A and B are 2-rowed square matrices such that

(A+B)= $\begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix}$ and (A-B)= $\begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$ then AB=?

A. $\begin{pmatrix} -7 & 5 \\ 1 & -5 \end{pmatrix}$

B. $\begin{pmatrix} 7 & -5 \\ 1 & 5 \end{pmatrix}$

C. $\begin{pmatrix} 7 & -1 \\ 5 & -5 \end{pmatrix}$

D. $\begin{pmatrix} 7 & -1 \\ -5 & 5 \end{pmatrix}$

Answer

(A+B) = $\begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} ------1$

(A-B) = $\begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix} ------2$

1+2 $\Rightarrow$ 2A = $\begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} + \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$

$\Rightarrow$ 2A = $\begin{pmatrix} 2 & -4 \\ 6 & 8 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow$ A = $\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$

1-2 $\Rightarrow$ 2B = $\begin{pmatrix} 4 & -3 \\ 1 & 6 \end{pmatrix} - \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$

$\Rightarrow$ 2B = $\begin{pmatrix} 6 & -2 \\ -4 & 4 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow$ B = $\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

A x B=$\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$

= $\begin{pmatrix} 1 \times 3 + (-2) \times (-2) & (1) \times (-1) + (-2) \times (2) \\ 3 \times 3 + 4 \times (-2) & 3 \times (-1) + 4 \times 2 \end{pmatrix}$

=$\begin{pmatrix} 7 & -5 \\ 1 & 5 \end{pmatrix}$

2. Question

If $\begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix}$ + 2A = $\begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} 1 & 3 \\ -5 & 4 \end{pmatrix}$

B. $\begin{pmatrix} -1 & 5 \\ -3 & 4 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 4 \\ -6 & 2 \end{pmatrix}$

D. none of these

Answer

C

$\begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix}$ + 2A = $\begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix}$

$\Rightarrow$ 2A = $\begin{pmatrix} 5 & 6 \\ -7 & 10 \end{pmatrix} - \begin{pmatrix} 3 & -2 \\ 5 & 6 \end{pmatrix}$

$\Rightarrow$ 2A = $\begin{pmatrix} 2 & 8 \\ -12 & 4 \end{pmatrix}$

Dividing the matrix by 2

$\Rightarrow$ A = $\begin{pmatrix} 1 & 4 \\ -6 & 2 \end{pmatrix}$

3. Question

If A=$\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$and B=$\begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$are such that 4A + 3X = 5B then X=?

A. $\begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$

B. $\begin{pmatrix} 4 & 5 \\ -6 & -2 \end{pmatrix}$

C. $\begin{pmatrix} -4 & 5 \\ 6 & -2 \end{pmatrix}$

D. none of these

Answer

4A + 3X = 5B

$\Longrightarrow$ 4 $\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$+ 3X = 5$\begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$

$\Longrightarrow$ 3X = 5$\begin{pmatrix} 4 & -3 \\ -6 & 2 \end{pmatrix}$-4 $\begin{pmatrix} 2 & 0 \\ -3 & 1 \end{pmatrix}$

$\Longrightarrow$ 3X = $\begin{pmatrix} 20 & -15 \\ -30 & 10 \end{pmatrix}$- $\begin{pmatrix} 8 & 0 \\ -12 & 4 \end{pmatrix}$

$\Rightarrow$ 3X = $\begin{pmatrix} 12 & -15 \\ -18 & 6 \end{pmatrix}$

Dividing by 3

$\Rightarrow$ X = $\begin{pmatrix} 4 & -5 \\ -6 & 2 \end{pmatrix}$

4. Question

If (A-2B)=$\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$and (2A-3B) =$\begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$then B=?

A. $\begin{pmatrix} 6 & -4 \\ -3 & 3 \end{pmatrix}$

B. $\begin{pmatrix} -4 & 6 \\ -3 & -3 \end{pmatrix}$

C. $\begin{pmatrix} 4 & -6 \\ 3 & -3 \end{pmatrix}$

D. none of these

Answer

B

(A-2B) = $\begin{pmatrix} 1 & -2 \\ 3 & 0 \end{pmatrix}$

Multiplying equation by 2

2A-4B = $\begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$ ------------- (i)

2A-3B = $\begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$ ---------- (ii)

(ii)-(i)

B = $\begin{pmatrix} -2 & 2 \\ 3 & -3 \end{pmatrix}$ - $\begin{pmatrix} 2 & -4 \\ 6 & 0 \end{pmatrix}$

= $\begin{pmatrix} -4 & 6 \\ 3 & -3 \end{pmatrix}$

5. Question

If (2A – B) = $\begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$ and (2B + A) = $\begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} -3 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$

D. none of these

Answer

(2A – B) = $\begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$

Multiplying by 2

4A – 2B = $\begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix}$ ------- (i)

2B + A = $\begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$ -------- (ii)

(i)+ (ii)

5A = $\begin{pmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{pmatrix}$ + $\begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$

$= \begin{pmatrix} 15 & 10 & 5 \\ -10 & 5 & -5 \end{pmatrix}$

Dividing each element of the matrix by 5

$A = \begin{pmatrix} 3 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$

6. Question

If $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

A. (x=-2, y=8)

B. (x=2, y=-8)

C. (x=3, y=-6)

D. (x=-3, y=6)

Answer

$2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

To solve this problem we will use the comparison that is we will use that all the elements of L.H.S are equal to R.H.S .

$= \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix}$

Comparing with R.H.S

8+y = 0

y= -8

2x+1 = 5

2x = 4

x=2

7. Question

If $\begin{pmatrix} x-y & 2x-y \\ 2x+z & 3z+w \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 5 & 13 \end{pmatrix}$ then

A. z=3, w=4

B. z=4, w=3

C. z=1, w=2

D. z=2, w=-1

Answer

A

By comparing L.H.S and R.H.S

x - y = -1 -------- i

2x – y = 0 -------- ii

2x+ z = 5 --------- iii

3z + w =13 ------- iv

Using i in equation ii

x = -1 + y

ii becomes, -2 + 2y –y = 0

y = 2

x = 1

Putting x in iii

2 + z =5

z = 3

Putting z in iv

9 + w = 13

w = 4

8. Question

If $\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$ then

A. x=1, y=2

B. x=2, y=1

C. x=1, y=1

D. none of these

Answer

C

$\begin{pmatrix} x & y \\ 3y & x \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} x \times 1 + y \times 2 \\ 3y \times 1 + x \times 2 \end{pmatrix}$

$= \begin{pmatrix} x + 2y \\ 3y + 2x \end{pmatrix}$

Comparing with R.H.S

x + 2y = 3 ------- (i)

2x + 3y = 5 -------- (ii)

(i) x 2 – (ii)

2x + 4y – 2x + 3y = 6 - 5

y = 1

Putting y in (i)

x + 2(1) = 3

x= 1

9. Question

If the matrix A = $\begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular then x=?

A. 0

B. 1

C. -1

D. -2

Answer

When a given matrix is singular then the given matrix determinant is 0.

$|A|$ = 0

Given, A = $\begin{pmatrix} 3-2x & x+1 \\ 2 & 4 \end{pmatrix}$

$|A|$ = 0

4(3-2x) – 2(x+1) = 0

12 – 8x -2x -2 =0

10 -10x= 0

10x = 0

x= 1

10. Question

If $A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ then $(A_\alpha)^2$=?

A. $\begin{pmatrix} \cos^2\alpha & \sin^2\alpha \\ -\sin^2\alpha & \cos^2\alpha \end{pmatrix}$

B. $\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}$

C. $\begin{pmatrix} 2\cos\alpha & 2\sin\alpha \\ -\sin\alpha & 2\cos\alpha \end{pmatrix}$

D. none of these

Answer

Given, $A_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

$A_\alpha^2 = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

= $\begin{pmatrix} \cos\alpha\times\cos\alpha - \sin\alpha\times\sin\alpha & \cos\alpha\times\sin\alpha + \sin\alpha\times\cos\alpha \\ -\sin\alpha\times\cos\alpha - \cos\alpha\times\sin\alpha & -\sin\alpha\times\sin\alpha + \cos\alpha\times\cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos2\alpha & \sin2\alpha \\ -\sin2\alpha & \cos2\alpha \end{pmatrix}$

11. Question

If A = $\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$ be such that A + A$'$ = I, then α =?

A. ∏

B. $\frac{\pi}{3}$

C. ∏

D. $\frac{2\pi}{3}$

Answer

L.H.S: A + A$'$ = $\begin{pmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{pmatrix} + \begin{pmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{pmatrix}$

$= \begin{pmatrix} \cos a + \cos a & \sin a - \sin a \\ -\sin a + \sin a & \cos a + \cos a \end{pmatrix}$

$= \begin{pmatrix} 2\cos a & 0 \\ 0 & 2\cos a \end{pmatrix}$

This will be equal to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

When 2cosa = 1

cosa $=\frac{1}{2}$

a = $\frac{\pi}{3}$

12. Question

If A= $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}$ is singular then k=?

A. $\frac{16}{3}$

B. $\frac{34}{3}$

C. $\frac{33}{2}$

D. none of these

Answer

When a given matrix is singular then the given matrix determinant is 0.

$|A| = 0$

Given,

$$A=\begin{pmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{pmatrix}$$

$|A| = 0$

1(-4k + 6) –k(-12 + 4) +3 (9 -2k)= 0

-4k + 6 +12k -4k + 27 -6k = 0

-2k +33 = 0

$k = \frac{33}{2}$.

13. Question

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then adj A=?

A. $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

B. $\begin{pmatrix} -d & b \\ c & -a \end{pmatrix}$

C. $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

D. $\begin{pmatrix} -d & -b \\ c & a \end{pmatrix}$

Answer

To find adj A we will first find the cofactor matrix

C_{11} = d C_{12} = -c

C_{21}= -b C_{22} = a

Cofactor matrix A = $\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

Adj A =$\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}'$

= $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

14. Question

If $A = \begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}$ and A^{-1}=$\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$ then x=?

A. 1

B. 2

C. $\frac{1}{2}$

D. -2

Answer

We know that A x A^{-1} = I

$$\begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x \times 1 + 0 \times (-1) & 2x \times 0 + 0 \times 2 \\ x \times 1 + x \times (-1) & x \times 0 + x \times 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2x & 0 \\ 0 & 2x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To satisfy the above condition 2x = 1

x = $\frac{1}{2}$

15. Question

If A and B are square matrices of the same order then (A + B)(A – B) = ?

A. (A^2- B^2)

B. A^2 + AB – BA – B^2

C. A^2– AB + BA – B^2

D. none of these

Answer

Since A and B are square matrices of same order.

(A+B)(A-B) = A^2 –AB +BA – B

16. Question

If A and B are square matrices of the same order then $(A + B)^2$ =?

A. A^2 + 2AB + B^2

B. A^2+AB+BA +B^2

C.A^2 + 2BA + B^2

D. none of these

Answer

Since A and B are square matrices of same order.

$(A + B)^2$ = (A+ B)(A + B)

= A^2 + AB + BA+ B^2

17. Question

If A and B are square matrices of the same order then $(A – B)^2$ =?

A. A^2 - 2AB + B^2

B. A^2–AB - BA + B^2

C. A^2 - 2BA + B^2

D. none of these

Answer

Since A and B are square matrices of same order.

$(A - B)^2 = (A- B)(A - B)$

$= A^2 - AB - BA+ B^2$

18. Question

If A and B are symmetric matrices of the same order then (AB – BA) is always

A. a symmetric matrix

B. a skew-symmetric matrix

C. a zero matrix

D. an identity matrix

Answer

Given A and B are symmetric matrices

A’ = A --- 1

B’ = B ---- 2

Now (AB – BA)’ = (AB)’ – (BA)’

=B’A’ – A’B’

[$\because$ (AB)’ = B’A’]

= BA – AB [Using 1 and 2]

$\therefore$ (AB – BA)’ = - (AB - BA)

AB-BA is a skew symmetric matrix.

19. Question

Matrices A and B are inverse of each other only when

A. AB=BA

B. AB=BA=0

C. AB=0, BA=I

D. AB=BA=I

Answer

$A = B^{-1}$

$B=A^{-1}$

We know that

$AA^{-1}= I$

(Given $B=A^{-1}$)

AB= I ------ 1

We know that

$BB^{-1}= I$

(Given $A=B^{-1}$)

$BA= I$ ------ 2

From 1 and 2

$AB= BA = I$

20. Question

For square matrices A and B of the same order, we have adj(AB)=?

A. (adj A)(adj B)

B. (adj B)(adj A)

C. $|AB|$

D. none of these

Answer

We know that $(AB)^{-1} = adj(AB)/ |AB|$

adj $(AB)= (AB)^{-1}. |AB|$

We also know that $(AB)^{-1} = B^{-1}. A^{-1}$

$|AB| = |A|\ |B|$

Putting them in 1

Adj $(AB) = B^{-1}. A^{-1}. |\mathbf{A}|. |\mathbf{B}|$

$= (A^{-1}. |\mathbf{A}|) (B^{-1}|\mathbf{B}|)$

= adj(A) adj(B)

Since, adj $(A)= (A)^{-1}. |\mathbf{A}|$

adj $(B)= (B)^{-1}. |\mathbf{B}|$

21. Question

If A is a 3-rowed square matrix and $|A|=4$ then adj(adj A)=?

A. 4A

B. 16A

C. 64A

D. none of these

Answer

The property states that

adj(adj A) = $|\mathbf{A}|^{n-2}$. A

Here n=2

adj(adj A) = $|\mathbf{4}|^{3-2}$. A

= 4A

22. Question

If A is a 3-rowed square matrix and $|A|=5$ then $|adj\ A|=?$

A. 5

B. 25

C. 125

D. none of these

Answer

The property states that $|\text{adj } A| = |A|^{n-1}$

Here n= 3 and $|A|=5$

$|\text{adj } A| = |5|^{3-1}$

$= |5|^2$

= 25.

23. Question

For any two matrices A and B,

A. AB=BA is always true

B. AB=BA is never true

C. sometimes AB=BA and sometimes AB≠BA

D. whenever AB exists, then BA exists

Answer

If the two matrices A and B are of same order it is not necessary that in every situation AB= BA

AB= BA = I only when $A = B^{-1}$

$B=A^{-1}$

Other time AB≠BA

24. Question

If $A\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

D. none of these

Answer

The matrix on the R.H.S of the given matrix is of order 2 x 2 and the one given on left side is 2 x 2 . Therefore A has to be a 2 x 2 matrix.

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

$\begin{pmatrix} 3a+b & 2a-b \\ 3c+d & 2c-d \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$

3a+b = 4 ----- 1

2a-b = 1 ------ 2

3c+d= 2 ------ 3

2c-d=3 ------- 4

Using 1 and 2

a=1

b=1

Using 3 and 4

c=1

d = -1

So A becomes $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

25. Question

If A is an invertible square matrix then $|A^{-1}|$=?

A. $|A|$

B. $\frac{1}{|A|}$

C. 1

D.0

Answer

B

We know that $AA^{-1} = I$

Taking determinant both sides

$|AA^{-1}| = |I|$

$|A||A^{-1}| = |I|$ ($|AB|=|A||B|$)

$|A||A^{-1}| = 1$ ($|I| = 1$)

$|A^{-1}| = \frac{1}{|A|}$

26. Question

If A and B are invertible matrices of the same order then $(AB)^{-1}$=?

A. (A^{-1} x B^{-1})

B. (A x B^{-1})

C. (A^{-1} x B)

D. (B^{-1} x A^{-1})

Answer

$(AB)(AB)^{-1} = I$

$A^{-1}(AB)(AB)^{-1} = IA^{-1}$

$(A^{-1}A)B\ (AB)^{-1}=A^{-1}$

$IB(AB)^{-1} = A^{-1}$

$B(AB)^{-1} = A^{-1}$

$B^{-1}B(AB)^{-1} = B^{-1}\ A^{-1}$

$I\ (AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

27. Question

If A and B are two nonzero square matrices of the same order such that AB=0 then

A. $|A|=0$ or $|B|=0$

B. $|A|=0$ and $|B|=0$

C. $|A|\neq 0$ and $|B|\neq 0$

D.None of these

Answer

s AB is a 0 matrix its determinant has to be 0.

So $|AB|=|A||B|=0$

So $|A|=|B|=0$

28. Question

If A is a square matrix such that $|A|\neq 0$ and $A^2 - A + 2I = 0$ then A^{-1}=?

A. (I-A)

B. (I+A)

C. $\frac{1}{2}(I-A)$

D. $\frac{1}{2}(I+A)$

Answer

$^2 - A + 2I = 0$

Multiplying by A^{-1}

$A^{-1}A^2 - A^{-1}A + 2I\ A^{-1} = 0$

$A- I + 2\ A^{-1} = 0$

$A^{-1}= \frac{1}{2}(I-A)$

29. Question

If $A=\begin{pmatrix} 1 & \lambda & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$ is not invertible then λ=?

A. 2

B. 1

C. -1

D. 0

Answer

$$=\begin{pmatrix} 1 & \lambda & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{pmatrix}$$

$|A|=0$

$1(2 \times 1 - 5 \times 1) - \lambda(1 \times 1 - 5 \times 2) + 2(1 \times 1 - 2 \times 2) = 0$

$-3+9\lambda-6 = 0$

$9\lambda = 9$

$\lambda = 1$

30. Question

If $A=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ then A^{-1}=?

A. A

B. –A

C. Adj A

D. –adj A

Answer

$A=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$|A| = \cos^2\theta - (-\sin^2\theta)$

$= \cos^2\theta + (\sin^2\theta)$

= 1 ---------- (I)

We know that $A^{-1} = \frac{1}{|A|}$ adj A

= adj A [From I]

31. Question

The matrix $A=\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$ is

A. idempotent

B. Orthogonal

C. Nilpotent

D. None of these

Answer

Matrix A is said to be nilpotent since there exist a positive integer k=1 such that Ak is zero matrix.

32. Question

The matrix A= $\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ is

A. Nonsingular

B. Idempotent

C. Nilpotent

D. Orthogonal

Answer

Here the diagonal value is 2+3-3= 1

So the given matrix is idempotent.

33. Question

If A is singular then A(adjA)=?

A. A unit matrix

B.A null matrix

C.A symmetric matrix

D. None of these

Answer

A(adjA)= A($|A|$ x A^{-1})

Since determinant of singular matrix is always 0

A(adjA)= 0

So, it is a null matrix.

34. Question

For any 2-rowed square matrix A, if A(adjA) = $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ then the value of $|A|$ is

A. 0

B.8

C.64

D.4

Answer

(adjA) = $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$

= 8 $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

= $|A|$I

|A|= 8.

35. Question

If A $=\begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$ then $|A^{-1}|$=?

A. -5

B. $\frac{-1}{5}$

C. $\frac{1}{25}$

D. 25

Answer

A $=\begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}$

|A|= -2-3 = -5

We know that $|A^{-1}|=\frac{1}{|A|}$

$=\frac{1}{-5}$

36. Question

If A $=\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$ and $A^2 + xI = yA$ then the values of x and y are

A. X=6, y=6

B. X=8, y=8

C. X=5, y=8

D. X=6, y=8

Answer

$^2 + xI = yA$

$\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix} + x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$\begin{pmatrix} 16 & 8 \\ 56 & 32 \end{pmatrix}+x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

$8\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}+x\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = y\begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$

Comparing L.H.S and R.H.S

x=8 y=8

37. Question

If matrices A and B anticommute then

A. AB=BA

B. AB=-BA

C. $(AB)=(BA^{-1})$

D. None of these

Answer

If A and B anticommute then AB= -BA

38. Question

If A = $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ then adj A=?

A. $\begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

B. $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

C. $\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$

D.None of these

Answer

To find adj A we will first find the cofactor matrix

$C_{11} = 3$ $C_{12} = -1$

$C_{21} = -5$ $C_{22} = 2$

Cofactor matrix A = $\begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{-5} & \mathbf{2} \end{pmatrix}$

Adj A = $\begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{-5} & \mathbf{2} \end{pmatrix}'$

= $\begin{pmatrix} \mathbf{3} & \mathbf{-5} \\ \mathbf{-1} & \mathbf{2} \end{pmatrix}$

39. Question

If A = $\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$ and B is a square matrix of order 2 such that AB=I then B=?

A. $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

B. $\begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}$

C. $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

D.None of these

Answer

B=I

$B = A^{-1} I$ ---------1

$A^{-1} = \frac{1}{|A|}$ adj A --------- 2

$|A| = 3 \times 2 - (-4) \times (-1)$

$= 2$

$C_{11} = 2$ $C_{12} = 1$

$C_{21} = 4$ $C_{22} = 3$

Cofactor matrix A = $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

Adj A = $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}'$

$= \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

Putting in 2

$A^{-1} = \frac{1}{|2|} \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

Putting in 1

$B = A^{-1} I$

$= A^{-1}$

$= \begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$

40. Question

If A and B are invertible square matrices of the same order then $(AB)^{-1}$=?

A. AB^{-1}

B. $A^{-1}B$

C. $A^{-1}B^{-1}$

D. $B^{-1}A^{-1}$

Answer

$(AB)(AB)^{-1} = I$

$A^{-1}(AB)(AB)^{-1} = IA^{-1}$

$(A^{-1}A)B\,(AB)^{-1} = A^{-1}$

$IB(AB)^{-1} = A^{-1}$

$B(AB)^{-1} = A^{-1}$

$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$

$I\,(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

41. Question

If A=$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$, then A^{-1}=?

A. $\begin{pmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$

B. $\begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{-1}{7} & \frac{2}{7} \end{pmatrix}$

C. $\begin{pmatrix} \frac{1}{3} & \frac{1}{7} \\ \frac{1}{7} & \frac{2}{7} \end{pmatrix}$

D.None of these

Answer

$^{-1}= \frac{\mathbf{1}}{|\mathbf{A}|}$ adj A --------- 1

$|\mathbf{A}|$ = 3 x 2 – (1) x (-1)

= 7

C_{11} = 3 C_{12} = -1

C_{21}= 1 C_{22} = 2

Cofactor matrix A = $\begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{4} & \mathbf{3} \end{pmatrix}$

Adj A =$\begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}'$

= $\begin{pmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} \end{pmatrix}$

Putting in 1

A^{-1}= $\frac{\mathbf{1}}{|\mathbf{7}|} \begin{pmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} \end{pmatrix}$

= $\begin{pmatrix} \frac{\mathbf{3}}{\mathbf{7}} & \frac{\mathbf{1}}{\mathbf{7}} \\ \frac{\mathbf{-1}}{\mathbf{7}} & \frac{\mathbf{2}}{\mathbf{7}} \end{pmatrix}$

42. Question

If $|A|$=3 and A^{-1}=$\begin{pmatrix} 3 & -1 \\ \frac{-5}{3} & \frac{2}{3} \end{pmatrix}$ then adj A=?

A. $\begin{pmatrix} 9 & 3 \\ -5 & -2 \end{pmatrix}$

B. $\begin{pmatrix} 9 & -3 \\ -5 & 2 \end{pmatrix}$

C. $\begin{pmatrix} -9 & 3 \\ 5 & -2 \end{pmatrix}$

D. $\begin{pmatrix} 9 & -3 \\ 5 & -2 \end{pmatrix}$

Answer

$^{-1} = \frac{1}{|\mathbf{A}|}$ adj A

adj A = $|\mathbf{A}|$ x A^{-1}

= 3 x $\begin{pmatrix} \mathbf{3} & \mathbf{-1} \\ \frac{\mathbf{-5}}{\mathbf{3}} & \frac{\mathbf{2}}{\mathbf{3}} \end{pmatrix}$

= $\begin{pmatrix} \mathbf{9} & \mathbf{-3} \\ \mathbf{-5} & \mathbf{2} \end{pmatrix}$

43. Question

If A is an invertible matrix and $A^{-1} = \begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$ then A=?

A. $\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

B. $\begin{pmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{5} & \frac{1}{6} \end{pmatrix}$

C. $\begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$

D.None of these

Answer

y property of inverse

$(A^{-1})^{-1} = A$

$(A^{-1})^{-1} = \begin{pmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{5} & \mathbf{6} \end{pmatrix}^{-1}$

A $= \begin{pmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{5} & \mathbf{6} \end{pmatrix}^{-1}$ ------------ 1

$|\mathbf{A}|^{-1}$ = 3 x 6 – 4 x 5

= -2

$C_{11} = 6$ $C_{12} = -5$

$C_{21} = -4$ $C_{22} = 3$

Cofactor matrix A = $\begin{pmatrix} 6 & -5 \\ -4 & 3 \end{pmatrix}$

Adj A = $\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

$\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}^{-1} = \frac{1}{-2}\begin{pmatrix} 6 & -4 \\ -5 & 3 \end{pmatrix}$

$= \begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$

Putting in 1

A = $\begin{pmatrix} -3 & 2 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$

44. Question

If A=$\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x)=2x^2 - 4x + 5$ then $f(A)$=?

A. $\begin{pmatrix} 19 & -32 \\ -16 & 51 \end{pmatrix}$

B. $\begin{pmatrix} 19 & -16 \\ -32 & 51 \end{pmatrix}$

C. $\begin{pmatrix} 19 & -11 \\ -27 & 51 \end{pmatrix}$

D. None of these

Answer

$f(A) = 2A^2 - 4A + 5$

$A^2 = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$

$= \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$

$f(A) = 2A^2 - 4A + 5I$

$= 2\begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix} - 4\begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix} + 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 18 & -8 \\ -16 & 34 \end{pmatrix} - \begin{pmatrix} 4 & 8 \\ 16 & -12 \end{pmatrix} + \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= \begin{pmatrix} 19 & -16 \\ -32 & 51 \end{pmatrix}$

45. Question

If A=$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ then $A^2 - 4A$ = ?

A. I

B. 5I

C. 3I

D. 0

Answer

$A^2 = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix}$

$A^2 - 4A = \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - 4\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 9 & 16 \\ 8 & 17 \end{pmatrix} - \begin{pmatrix} 4 & 16 \\ 8 & 12 \end{pmatrix}$

$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

$= 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

= 5I

46. Question

If A is a 2-rowed square matrix and |A|=6 then $A \cdot \mathrm{adj}A$ =?

A. $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

D. None of these

Answer

(adj A) = |A|I

$= 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

47. Question

If A is an invertible square matrix and k is a non-negative real number then $(KA)^{-1}$=?

A. $k \cdot A^{-1}$

B. $\frac{1}{k} \cdot A^{-1}$

C. $-k \cdot A^{-1}$

D. None of these

Answer

y the property of inverse

$(AB)^{-1} = B^{-1}A^{-1}$

$(KA)^{-1} = A^{-1}K^{-1}$

$= \frac{1}{K} A^{-1}$

48. Question

If $A = \begin{pmatrix} 3 & 4 & 1 \\ 1 & 0 & -2 \\ -2 & -1 & 2 \end{pmatrix}$ then A^{-1}=?

A. $\begin{pmatrix} 2 & 9 & -8 \\ -2 & 8 & 7 \\ -1 & 5 & -4 \end{pmatrix}$

B. $\begin{pmatrix} -2 & 9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & 4 \end{pmatrix}$

C. $\begin{pmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{pmatrix}$

D. None of these

Answer

$|A|$ = 3 x (0 - 2) – 4 x (2-4)+1 x (-1)

= -6+8-1

= 1

$C_{11} = -2$ $C_{12} = 2$ $C_{13} = -1$

$C_{21} = -9$ $C_{22} = 8$ $C_{23} = -5$

$C_{31} = -8$ $C_{32} = 7$ $C_{33} = -4$

Cofactor (A)= $\begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}$

Adj A = $\begin{bmatrix} -2 & 2 & -1 \\ -9 & 8 & -5 \\ -8 & 7 & -4 \end{bmatrix}'$

$= \begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adj A

$= \frac{1}{1} \begin{bmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{bmatrix}$

$$\begin{pmatrix} -2 & -9 & -8 \\ 2 & 8 & 7 \\ -1 & -5 & -4 \end{pmatrix}$$

49. Question

If A is a square matrix then $(A + A')$ is

A. A null matrix

B. An identity matrix

C. A symmetric matrix

D. A skew-symmetric matrix

Answer

Let X = A+A’

X’ = (A+A’)’

= A’ + (A’)’

=A + A’

= X

Therefore (A+A’) is symmetric matrix.

50. Question

If A is a square matrix then $(A-A')$ is

A. A null matrix

B. An identity matrix

C. A symmetric matrix

D. A skew-symmetric matrix

Answer

Let X = A-A’

X’ = (A-A’)’

= A’ - (A’)’

=A’ – A

= -(A – A’)

= -X

Therefore (A-A’) is skew symmetric matrix.

51. Question

If A is a 3-rowed square matrix and $|3A|=k\ |A|$ then k =?

A. 3 B.9

C. 27 D.1

Answer

Since the matrix is of order 3 so 3 will be taken common from each row or column.

So, k= 27

Tagging

52. Question

Which one of the following is a scalar matrix?

A. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

B. $\begin{pmatrix} 6 & 0 \\ 0 & 3 \end{pmatrix}$

C. $\begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix}$

D. None of these

Answer

$= \begin{pmatrix} \mathbf{-8} & \mathbf{0} \\ \mathbf{0} & \mathbf{-8} \end{pmatrix}$

$= -8\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

Since -8 could be taken common from each row or column. Hence C is a scalar matrix.

53. Question

If A=$\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$and B= $\begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and

$(A + B)^2 = (A^2 + B^2)$ then

A. a = 2, b = -3

B. a = -2, b = 3

C. a = 1, b = 4

D. none of these

Answer

$=\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}$ B=$\begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

A+B = $\begin{pmatrix} \mathbf{1+a} & \mathbf{0} \\ \mathbf{2+b} & \mathbf{-2} \end{pmatrix}$

$(A+B)^2 = \begin{pmatrix} \mathbf{1+a} & \mathbf{0} \\ \mathbf{2+b} & \mathbf{-2} \end{pmatrix}\begin{pmatrix} \mathbf{1+a} & \mathbf{0} \\ \mathbf{2+b} & \mathbf{-2} \end{pmatrix}$

$= \begin{pmatrix} \mathbf{(1+a)^2} & \mathbf{0} \\ \mathbf{(2+b)(1+a)-4-2b} & \mathbf{-4} \end{pmatrix}$

$= \begin{pmatrix} \mathbf{(1+a)^2} & \mathbf{0} \\ \mathbf{2+2a+b+ab-4-2b} & \mathbf{4} \end{pmatrix}$

$= \begin{pmatrix} \mathbf{(1+a)^2} & \mathbf{0} \\ \mathbf{2a+ab-b-2} & \mathbf{4} \end{pmatrix}$

$A^2=\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}\begin{pmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} \end{pmatrix}$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$B^2 = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix} \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

$$= \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$$

$(A + B)^2 = (A^2 + B^2)$

$$\begin{pmatrix} (1 + a)^2 & 0 \\ 2a + ab - b - 2 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} a^2 + b & a - 1 \\ ab - b & b + 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 + a^2 + b & a - 1 \\ ab - b & b \end{pmatrix}$$

By comparison,

a-1 = 0

a=1

b=4