

12. Indefinite Integral

Exercise 12

1. Question

Evaluate :

Evaluate

i. $\int x^7 dx$

ii. $\int x^{-7} dx$

iii. $\int x^{-1} dx$

iv. $\int x^{5/3} dx$

v. $\int x^{-5/4} dx$

vi. $\int 2^x dx$

vii. $\int \sqrt[3]{x^2} dx$

viii. $\int \frac{1}{\sqrt[4]{x^3}} dx$

ix. $\int \frac{2}{x^2} dx$

Answer

i Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^7 dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

ii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c$$

$$= \frac{x^{-6}}{-6} + c$$

iii. Given:

$$\int \frac{1}{x} dx = \ln|x| + c$$

iv. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{3x^{\frac{8}{3}}}{8} + c$$

v. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{5}{4}} dx = \frac{x^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} + c$$

$$= -4x^{-\frac{1}{4}} + c$$

vi. Given:

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

vii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{2}{3}} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{3x^{\frac{5}{3}}}{5} + c$$

viii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{3}{4}} dx = \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c$$

$$= 4x^{\frac{1}{4}} + c$$

ix. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int 2x^{-2} dx = 2 \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{-2}{x} + c$$

2. Question

Evaluate :

$$\text{i. } \int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x \right) dx$$

$$\text{ii. } \int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1} \right) dx$$

$$\text{iii. } \int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax \right) dx$$

Answer

i Given:

$$\begin{aligned} & \int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x \right) dx \\ &= 6 \frac{x^{5+1}}{5+1} - 2 \frac{x^{-4+1}}{-4+1} - 7 \frac{x^2}{2} + 3 \ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c \\ &= 6 \frac{x^6}{6} - 2 \frac{x^{-3}}{-3} - 7 \frac{x^2}{2} + 3 \ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c \\ &= x^6 + \frac{2}{3} x^{-3} - \frac{7}{2} x^2 + 3 \ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c \end{aligned}$$

ii. Given:

$$\begin{aligned} & \int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1} \right) dx \\ &= 8x - \frac{x^2}{2} + 2 \frac{x^{3+1}}{3+1} - 6 \frac{x^{-3+1}}{-3+1} + 2 \frac{x^{-5+1}}{-5+1} + 5 \ln|x| + c \\ &= 8x - \frac{x^2}{2} + \frac{2}{4} x^4 + \frac{6}{2} x^2 - \frac{2}{4} x^{-4} + 5 \ln|x| + c \\ &= 8x - \frac{x^2}{2} + \frac{1}{2} x^4 + 3x^2 - \frac{1}{2} x^{-4} + 5 \ln|x| + c \end{aligned}$$

iii. Given:

$$\int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax \right) dx = \frac{1}{a} \frac{x^2}{2} + a \ln|x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\ln a} + a \frac{x^2}{2} + c$$

3. Question

Evaluate :

$$\text{i. } \int (2 - 5x)(3 + 2x)(1 - x) dx$$

$$\text{ii. } \int \sqrt{x} (ax^2 + bx + c) dx$$

$$\text{iii. } \int \left(\sqrt{x} - \sqrt[3]{x^4} + \frac{7}{\sqrt[3]{x^2}} - 6x^x + 1 \right) dx$$

Answer

i. Given:

$$\begin{aligned} & \int (2-5x)(3+2x)(1-x) dx \\ &= \int (6-11x-10x^2)(1-x) dx \\ &= \int (10x^3+x^2-17x+6) dx \\ &= \frac{10x^4}{4} + \frac{x^3}{3} - \frac{17x^2}{2} + 6x + c \\ &= \frac{5x^4}{2} + \frac{x^3}{3} - \frac{17x^2}{2} + 6x + c \end{aligned}$$

ii. Given:

$$\begin{aligned} &= \int \left(ax^{\frac{5}{2}} + bx^{\frac{3}{2}} + cx^{\frac{1}{2}} \right) dx \\ &= a \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} + b \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \\ &= \frac{2a}{7} x^{\frac{7}{2}} + \frac{2b}{5} x^{\frac{5}{2}} + \frac{2c}{3} x^{\frac{3}{2}} + C \end{aligned}$$

iii. Given:

$$\begin{aligned} & \int \left(x^{\frac{1}{2}} - x^{\frac{4}{3}} + 7x^{-\frac{2}{3}} - 6e^{\ln x^x} + 1 \right) dx \\ &= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + 7 \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} - 6e^{\ln x^x} + x + c \\ &= \frac{2x^{\frac{3}{2}}}{3} - \frac{3x^{\frac{7}{3}}}{7} - 21x^{-\frac{1}{3}} - 6x^x + x + c \end{aligned}$$

4. Question

Evaluate :

i. $\int \left(x^2 - \frac{1}{x^2} \right)^3 dx$

ii. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$

iii. $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

iv. $\int \frac{(1+2x)^3}{x^4} dx$

v. $\int \frac{(1+x)^3}{\sqrt{x}} dx$

vi. $\int \frac{2x^2 + x - 2}{(x-2)} dx$

Answer

i. Given:

$$\begin{aligned} &= \int x^6 + x^{-6} - 3x^2 - 3x^{-2} dx \\ &= \frac{x^7}{7} + \frac{x^{-5}}{-5} - x^3 + 3x^{-1} + c \end{aligned}$$

ii. Given:

$$\begin{aligned} &= \int x^{\frac{1}{2}} - x^{-\frac{1}{2}} dx \\ &= \frac{2}{3} x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c \end{aligned}$$

iii. Given:

$$\begin{aligned} &= \int \left(x + \frac{1}{x} + 2 \right) dx \\ &= \frac{x^2}{2} + \ln|x| + 2x + c \end{aligned}$$

iv. Given:

$$\begin{aligned} &= \int \frac{1 + 8x^3 + 6x + 12x^2}{x^4} dx \\ &= \int x^{-4} + \frac{8}{x} + 6x^{-3} + 12x^{-2} dx \\ &= -\frac{x^{-3}}{3} + 8\ln|x| - 3x^{-2} - 12x^{-1} + c \end{aligned}$$

v. Given:

$$\begin{aligned} &= \int \frac{1 + x^3 + 3x + 3x^2}{\sqrt{x}} dx \\ &= \int x^{-\frac{1}{2}} + x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 3x^{\frac{3}{2}} dx \\ &= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c \end{aligned}$$

vi. Given:

$$\begin{aligned} &= \int \left(\frac{2x^2}{x-2} + \frac{x-2}{x-2} \right) dx \\ &= 2 \int \left(\frac{x^2 - 4x + 4}{x-2} + \frac{4x}{x-2} - \frac{4}{x-2} \right) dx + \int dx \\ &= 2 \left[\int \frac{(x-2)^2}{x-2} dx + 4 \int \frac{x-2+2}{x-2} dx - 4 \int \frac{1}{x-2} dx \right] + x + c \\ &= 2 \left[\int (x-2) dx + 4 \left(\int dx + 2 \int \frac{1}{x-2} dx \right) - 4 \ln|x-2| \right] + x + c \\ &= 2 \left[\frac{x^2}{2} - 2x + 4x + 8 \ln|x-2| - 4 \ln|x-2| \right] + x + c \\ &= x^2 - 4x + 8x + 8 \ln|x-2| + x + c \\ &= x^2 + 5x + 8 \ln|x-2| + c \end{aligned}$$

5. Question

Evaluate :

$$\int \left[1 + \frac{1}{(1+x^2)} - \frac{2}{\sqrt{1-x^2}} + \frac{5}{x\sqrt{x^2-1}} + a^x \right] dx$$

Answer

Given:

$$\text{Since, } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c;$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c;$$

$$\int a^x dx = \frac{a^x}{\ln a} + c \text{ \&}$$

$$\int \frac{1}{|x|\sqrt{(x^2-1)}} dx = \sec^{-1} x + c$$

So,

$$= x + \tan^{-1} x - 2 \sin^{-1} x + 5 \sec^{-1} x + \frac{a^x}{\ln a} + c$$

6. Question

Evaluate :

i. $\int \left(\frac{x^2-1}{x^2+1} \right) dx$

ii. $\int \left(\frac{x^6-1}{x^2+1} \right) dx$

iii. $\int \left(\frac{x^4}{1+x^2} \right) dx$

iv. $\int \left(\frac{x^2}{1+x^2} \right) dx$

Answer

i. Given:

$$= \int \frac{x^2+1-2}{x^2+1} dx$$

$$= \int \frac{x^2+1}{x^2+1} - \frac{2}{x^2+1} dx$$

$$= x - 2 \tan^{-1} x + c$$

ii. Given:

$$= \int \left[\frac{x^6}{x^2+1} - \frac{1}{x^2+1} \right] dx$$

$$= \int \left[\frac{x^6+3x^2+3x^4+1-3x^2-3x^4-1}{x^2+1} - \frac{1}{x^2+1} \right] dx$$

$$\begin{aligned}
&= \int \left[\frac{(x^2 + 1)^3}{x^2 + 1} - 3 \frac{x^2}{x^2 + 1} - 3 \frac{x^4}{x^2 + 1} - \frac{1}{x^2 + 1} - \frac{1}{x^2 + 1} \right] dx \\
&= \int (x^2 + 1)^2 dx - 3 \int \left[\frac{x^2 + 1 - 1}{x^2 + 1} \right] dx - 3 \left[\int \frac{x^4 + 2x^2 + 1}{x^2 + 1} + \frac{-2x^2 - 1}{x^2 + 1} dx \right] \\
&\quad - 2 \int \frac{1}{x^2 + 1} dx \\
&= \int (x^4 + 2x^2 + 1) dx - 3 \left[\int dx - \int \frac{1}{x^2 + 1} dx \right] \\
&\quad - 3 \left[\int \frac{(x^2 + 1)^2}{x^2 + 1} dx - 2 \int \frac{x^2}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \right] - 2 \int \frac{1}{x^2 + 1} dx \\
&= \int (x^4 + 2x^2 + 1) dx - 3 \left[\int dx - \int \frac{1}{x^2 + 1} dx \right] \\
&\quad - 3 \left[\int (x^2 + 1) dx - 2 \int \frac{x^2 + 1 - 1}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \right] \\
&\quad - 2 \int \frac{x^2 + 1}{x^2 + 1} dx \\
&= \frac{x^5}{5} + \frac{2}{3} x^3 + x - 3x + 3 \tan^{-1} x - x^3 - 3x + 6x - 3 \tan^{-1} x - 2 \tan^{-1} x + c \\
&= \frac{x^5}{5} + \frac{1}{3} x^3 + x - 2 \tan^{-1} x + c
\end{aligned}$$

iii. Given:

$$\begin{aligned}
&= \int \frac{x^4 + 2x^2 + 1}{x^2 + 1} + \frac{-2x^2 - 1}{x^2 + 1} dx \\
&= \int \frac{(x^2 + 1)^2}{x^2 + 1} dx - 2 \int \frac{x^2}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \\
&= \int (x^2 + 1) dx - 2 \int \frac{x^2 + 1 - 1}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \\
&= \int (x^2 + 1) dx - 2 \int dx + 2 \int \frac{1}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx \\
&= \int (x^2 + 1) dx - 2 \int dx + \int \frac{1}{x^2 + 1} dx \\
&= \frac{1}{3} x^3 - x + \tan^{-1} x + c
\end{aligned}$$

iv. Given:

$$\begin{aligned}
&= \int \left[\frac{x^2 + 1 - 1}{x^2 + 1} \right] dx \\
&= \int dx - \int \frac{1}{x^2 + 1} dx \\
&= x - \tan^{-1} x + c
\end{aligned}$$

7. Question

Evaluate :

$$\int \left(9 \sin x - 7 \cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x \right) dx$$

Answer

Given:

$$= \int (9 \sin x - 7 \cos x - 6(\sec x)^2 + 2(\csc x)^2 + (\csc x)^2 - 1) dx$$

$$= -9 \cos x - 7 \sin x - 6 \tan x - 3 \cot x - x + c$$

8. Question

Evaluate :

$$\int \left(\frac{\cot x}{\sin x} - \tan^2 x - \frac{\tan x}{\cos x} + \frac{2}{\cos^2 x} \right) dx$$

$$\text{Ans. } -\operatorname{cosec} x + \tan x + x - \sec x + C$$

Answer

Given:

$$= \int (\cot x \csc x - (\sec x)^2 + 1 - \tan x \sec x + 2(\sec x)^2)$$

$$= -\csc x - \tan x + x - \sec x + 2 \tan x + c$$

$$= -\csc x + \tan x + x - \sec x + c$$

9. Question

Evaluate :

$$\text{i. } \int \sec x (\sec x + \tan x) dx$$

$$\text{ii. } \int \operatorname{cosec} x (\operatorname{cosec} x - \cot x) dx$$

Answer

i. Given:

$$= \int (\sec x)^2 + \sec x \tan x dx$$

$$= \tan x + \sec x + c$$

ii. Given:

$$= \int (\csc x)^2 - \cot x \csc x dx$$

$$= -\cot x + \csc x + c$$

10. Question

Evaluate :

$$\text{i. } \int (\tan x + \cot x)^2 dx$$

$$\text{ii. } \int \left(\frac{1 + 2 \sin x}{\cos^2 x} \right) dx$$

$$\text{iii. } \int \left(\frac{3 \cos x + 4}{\sin^2 x} \right) dx$$

Answer

i. Given:

$$\begin{aligned}
&= \int ((\tan x)^2 + (\cot x)^2 + 2) dx \\
&= \int ((\sec x)^2 - 1 + (\csc x)^2 - 1 + 2) dx \\
&= \int ((\sec x)^2 + (\csc x)^2) dx \\
&= \tan x - \cot x + c
\end{aligned}$$

ii. Given:

$$\begin{aligned}
&= \int \left(\frac{1}{(\cos x)^2} + 2 \frac{\sin x}{(\cos x)^2} \right) dx \\
&= \int ((\sec x)^2 + 2 \tan x \sec x) dx \\
&= \tan x + 2 \sec x + c
\end{aligned}$$

iii. Given:

$$\begin{aligned}
&= \int (2 \cot x \csc x + 4(\csc x)^2) dx \\
&= -2 \csc x - 4 \cot x + c
\end{aligned}$$

11. Question

Evaluate :

i. $\int \frac{1}{(1 - \cos x)} dx$

ii. $\int \frac{1}{(1 - \sin x)} dx$

Answer

i. Given:

Multiply and divide by $(1 + \cos x)$

$$\begin{aligned}
&= \int \frac{1 + \cos x}{1 - (\cos x)^2} dx \\
&= \int \frac{1 + \cos x}{(\sin x)^2} dx \\
&= \int ((\csc x)^2 + \csc x \cot x) dx \\
&= -\cot x - \csc x + c
\end{aligned}$$

ii. Given:

Multiply and divide by $(1 + \sin x)$

$$\begin{aligned}
&= \int \frac{1 + \sin x}{1 - (\sin x)^2} dx \\
&= \int \frac{1 + \sin x}{(\cos x)^2} dx \\
&= \int ((\sec x)^2 + \sec x \tan x) dx
\end{aligned}$$

$$= \tan x + \sec x + c$$

12. Question

Evaluate :

$$\text{i. } \int \frac{\tan x}{(\sec x + \tan x)} dx$$

$$\text{ii. } \int \frac{\operatorname{cosec} x}{(\operatorname{cosec} x - \cot x)} dx$$

Answer

i. Given:

Multiply and divide by $(\sec x - \tan x)$

$$= \int \frac{\tan x \sec x - (\tan x)^2}{(\sec x)^2 - (\tan x)^2} dx$$

$$= \int \tan x \sec x - (\tan x)^2 dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

$$= \sec x - \tan x + x + c$$

ii. Given:

Multiply and divide by $(\csc x + \cot x)$

$$= \int \frac{(\csc x)^2 + \csc x \cot x}{(\csc x)^2 - (\cot x)^2} dx$$

$$= \int (\csc x)^2 + \csc x \cot x dx$$

$$= -\cot x - \csc x + c$$

13. Question

Evaluate :

$$\text{i. } \int \frac{\cos x}{1 + \cos x} dx$$

$$\text{ii. } \int \frac{\sin x}{(1 - \sin x)} dx$$

Answer

i. Given:

Multiply and divide by $(1 - \cos x)$

$$= \int \frac{\cos x - (\cos x)^2}{1 - (\cos x)^2} dx$$

$$= \int \frac{\cos x - (\cos x)^2}{(\sin x)^2} dx$$

$$= \int (\cot x \csc x - (\cot x)^2) dx$$

$$= \int (\cot x \csc x - (\csc x)^2 + 1) dx$$

$$= -\csc x + \cot x + x + c$$

ii. Given:

Multiply and divide by $(1 + \sin x)$

$$= \int \frac{\sin x + (\sin x)^2}{1 - (\sin x)^2} dx$$

$$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$$

$$= \int (\tan x \sec x + (\tan x)^2) dx$$

$$= \int (\tan x \sec x + (\sec x)^2 - 1) dx$$

$$= \sec x + \tan x - x + c$$

14. Question

Evaluate :

i. $\int \sqrt{1 + \cos 2x} dx$

ii. $\int \sqrt{1 - \cos 2x} dx$

Answer

i. Given:

$$= \int \sqrt{2(\cos x)^2} dx$$

$$= \sqrt{2} \int \cos x dx$$

$$= \sqrt{2} \sin x + c$$

ii. Given:

$$= \int \sqrt{2(\sin x)^2} dx$$

$$= \sqrt{2} \int \sin x dx$$

$$= -\sqrt{2} \cos x + c$$

15. Question

Evaluate :

i. $\int \frac{1}{(1 + \cos 2x)} dx$

ii. $\int \frac{1}{(1 - \cos 2x)} dx$

Answer

i. Given:

$$= \int \frac{1}{2(\cos x)^2} dx$$

$$= \frac{1}{2} \int (\sec x)^2 dx$$

$$= \frac{1}{2} \tan x + c$$

ii. Given:

$$= \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2} \int (\csc x)^2 dx$$

$$= -\frac{1}{2} \cot x + c$$

16. Question

Evaluate :

$$\int \sqrt{1 + \sin 2x} dx$$

Answer

Given:

$$= \int \sqrt{\left(1 + \frac{2 \tan x}{1 + (\tan x)^2}\right)} dx$$

$$= \int \sqrt{\left(\frac{(1 + \tan x)^2}{(\sec x)^2}\right)} dx$$

$$= \int \left(\frac{1 + \tan x}{\sec x}\right) dx$$

$$= \int (\cos x + \sin x) dx$$

$$= \sin x - \cos x + c$$

17. Question

Evaluate :

$$\int \left(\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\right) dx$$

Ans. $\sec x - \operatorname{cosec} x + C$

Answer

Given:

$$= \int \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2} + \frac{(\cos x)^3}{(\sin x)^2(\cos x)^2} dx$$

$$= \int (\tan x \sec x + \csc x \cot x) dx$$

$$= \sec x - \csc x + c$$

18. Question

Evaluate :

$$\int \tan^{-1} \left(\frac{\sin 2x}{1 + \cos 2x} \right) dx$$

$$\text{Ans. } \frac{x^2}{2} + C$$

Answer

Given:

$$= \int \tan^{-1} \left(\frac{2 \sin x \cos x}{2(\cos x)^2} \right) dx$$

$$= \int \tan^{-1} (\tan x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

19. Question

Evaluate :

$$\int \cos^{-1} \left(\frac{1 - \tan^2 x}{1 + \tan^2 x} \right) dx$$

$$\text{Ans. } x^2 + C$$

Answer

Given:

$$= \int \cos^{-1}(\cos 2x) dx$$

$$= \int 2x dx$$

$$= x^2 + c$$

20. Question

Evaluate :

$$\int \cos^{-1}(\sin x) dx$$

$$\text{Ans. } \left(\frac{\pi x}{2} - \frac{x^2}{2} + C \right)$$

Answer

Given:

$$\sin^{-1}(\sin x) + \cos^{-1}(\sin x) = \frac{\pi}{2}$$

$$= \int \left[\frac{\pi}{2} - \sin^{-1}(\sin x) \right] dx$$

$$= \int \left[\frac{\pi}{2} - x \right] dx$$

$$= \frac{\pi}{2}x - \frac{x^2}{2} + c$$

21. Question

Evaluate :

$$\int \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}} dx$$

$$\text{Ans. } \frac{\pi x}{4} - \frac{x^2}{4} + C$$

Answer

Given:

$$\begin{aligned} &= \int \tan^{-1} \sqrt{\frac{(1 - \sin x)^2}{1 - (\sin x)^2}} dx \\ &= \int \tan^{-1} \left(\frac{1 - \sin x}{\cos x} \right) dx \\ &= \int \tan^{-1} \left(\frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} \right) dx \\ &= \int \tan^{-1} \left(\frac{2 \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2 \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) \cos\left(\frac{\pi}{4} - \frac{x}{2}\right)} \right) dx \\ &= \int \tan^{-1} \left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \right) dx \\ &= \int \left(\frac{\pi}{4} - \frac{x}{2} \right) dx \\ &= \frac{\pi}{4}x - \frac{x^2}{4} + c \end{aligned}$$

22. Question

Evaluate :

$$\int (3 \cot x - 2 \tan x)^2 dx$$

$$\text{Ans. } 4 \tan x - 9 \cos x - 25x + C$$

Answer

Given:

$$\begin{aligned} &= \int (9(\cot x)^2 + 4(\tan x)^2 - 12) dx \\ &= \int (9(\csc x)^2 + 4(\sec x)^2 - 25) dx \\ &= -9 \cot x + 4 \tan x - 25x + c \end{aligned}$$

23. Question

Evaluate :

$$\int (3 \sin x + 4 \operatorname{cosec} x)^2 dx$$

$$\text{Ans. } \frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + C$$

Answer

Given:

$$\begin{aligned} &= \int (9(\sin x)^2 + 16(\operatorname{csc} x)^2 + 24) dx \\ &= \int \left(\frac{9}{2}(1 - \cos 2x) + 16(\operatorname{csc} x)^2 + 24 \right) dx \\ &= \frac{9}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + 24x + c \\ &= \frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + c \end{aligned}$$

24. Question

Evaluate :

$$\int \frac{dx}{(\sqrt{x+1} + \sqrt{x+2})}$$

$$\text{Ans. } \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2} + C$$

Answer

Given:

Multiply and divide by $\sqrt{x+1} - \sqrt{x+2}$

$$\begin{aligned} &= \int \frac{(\sqrt{x+1} - \sqrt{x+2})}{x+1 - x-2} dx \\ &= - \int \sqrt{x+3} + \sqrt{x+2} dx \\ &= \frac{-2}{3}(x+1)^{3/2} + \frac{2}{3}(x+2)^{3/2} + c \end{aligned}$$

25. Question

Evaluate :

$$\int \frac{dx}{(\sqrt{x+3} - \sqrt{x+2})}$$

$$\text{Ans. } \frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+2)^{3/2} + C$$

Answer

Given:

Multiply and divide by $\sqrt{x+3} + \sqrt{x+2}$

$$\begin{aligned}
&= \int \frac{(\sqrt{x+3} + \sqrt{x+2})}{x+3-x-2} dx \\
&= \int \sqrt{x+3} + \sqrt{x+2} dx \\
&= \frac{2}{3}(x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c
\end{aligned}$$

26. Question

Evaluate :

$$\int \left(\frac{1 + \cos x}{1 - \cos x} \right) dx$$

$$\text{Ans. } -2 \cot \frac{x}{2} - x + C$$

Answer

Given:

Multiply and divide by $(1 + \cos x)$

$$\begin{aligned}
&= \int \left(\frac{(1 + \cos x)^2}{1 - (\cos x)^2} \right) dx \\
&= \int \frac{1 + (\cos x)^2 + 2 \cos x}{(\sin x)^2} dx \\
&= \int (\csc x)^2 + (\cot x)^2 + 2 \cot x \csc x dx \\
&= \int (\csc x)^2 + (\csc x)^2 - 1 + 2 \cot x \csc x dx \\
&= -2 \cot x - 2 \csc x - x + c \\
&= -2(\csc x + \cot x) - x + c \\
&= -2 \left(\frac{1 + \cos x}{\sin x} \right) - x + c \\
&= -2 \left(\frac{2 \cos \frac{x}{2} \cos \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right) - x + c \\
&= -2 \cot \frac{x}{2} - x + c
\end{aligned}$$

27. Question

Evaluate :

$$\int \frac{(1 + \tan x)}{(1 - \tan x)} dx$$

$$\text{Ans. } -\log |\cos x - \sin x| + C$$

Answer

Given:

$$= \int \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let $\cos x - \sin x = t$

$$(-\sin x - \cos x) dx = dt$$

$$-(\sin x + \cos x) dx = dt$$

$$dx = -dt$$

$$\text{So, } I = - \int \frac{dt}{t}$$

$$= -\ln|t| + c$$

$$= -\ln|\cos x - \sin x| + c$$

28. Question

Evaluate :

$$\int \frac{\cos(x+a)}{\sin(x+b)} dx$$

$$\text{Ans. } \cos(a-b) \log |\sin(x+b)| - x \sin(a-b) + C$$

Answer

Given:

$$= \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx$$

$$= \int \frac{\cos(x+b) \cos(a-b) - \sin(x+b) \sin(a-b)}{\sin(x+b)} dx$$

$$= \cos(a-b) \int \cot(x+b) dx - \int \sin(a-b) dx$$

$$= \cos(a-b) \ln|\sin(x+b)| - x \sin(a-b) + c$$

29. Question

Evaluate :

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$$

$$\text{Ans. } x \cos 2\alpha - \sin 2\alpha \cdot \log |\sin(x+\alpha)| + C$$

Answer

Given:

$$= \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \frac{\sin(x+\alpha) \cos(2\alpha) - \sin(2\alpha) \cos(x+\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \cos 2\alpha dx - \sin 2\alpha \int \cot(x+\alpha) dx$$

$$= x \cos 2\alpha - \sin 2\alpha \ln|\sin(x+\alpha)| + c$$

30. Question

Evaluate :

$$\int (1-x)\sqrt{x} \, dx$$

$$\text{Ans. } \frac{2}{15} x\sqrt{x} (5-3x) + C$$

Answer

Given:

$$= \int \left(x^{\frac{1}{2}} - x^{\frac{3}{2}} \right) dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + c$$

$$= \frac{2}{15} x^{\frac{3}{2}} (5-3x) + c$$

31. Question

Evaluate :

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

$$\text{Ans. } \tan x - x + C$$

Answer

Given:

$$= \int (\tan x)^2 dx$$

$$= \int ((\sec x)^2 - 1) dx$$

$$= \tan x - x + c$$

32. Question

Evaluate :

$$\int \left\{ \frac{2-3\sin x}{\cos^2 x} \right\} dx$$

$$\text{Ans. } 2 \tan x - 3 \sec x + C$$

Answer

Given:

$$= \int \frac{2}{(\cos x)^2} dx - \int \frac{3 \sin x}{(\cos x)^2} dx$$

$$= 2 \int (\sec x)^2 dx - 3 \int \tan x \sec x dx$$

$$= 2 \tan x - 3 \sec x + c$$

Objective Questions

1. Question

Mark (✓) against the correct answer in each of the following:

$$\int x^6 dx = ?$$

A. $7x^7 + C$

B. $\frac{x^7}{7} + C$

C. $6x^5 + C$

D. $6x^7 + C$

Answer

Given:

$$\int x^6 dx,$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^6 dx = \frac{x^{6+1}}{6+1} + c$$

$$= \frac{x^7}{7} + c$$

2. Question

Mark (✓) against the correct answer in each of the following:

$$\int x^{5/3} dx = ?$$

A. $\frac{3}{5} x^{2/3} + C$

B. $\frac{8}{3} x^{8/3} + C$

C. $\frac{3}{8} x^{8/3} + C$

D. $\frac{5}{3} x^{8/3} + C$

Answer

Given:

$$\int x^{5/3} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{5/3} dx = \frac{x^{5/3+1}}{5/3+1} + c$$

$$= \frac{x^{8/3}}{8/3} + c$$

$$= \frac{3}{8}x^{\frac{8}{3}} + c$$

3. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{x^3} dx = ?$$

A. $\frac{-3}{x^2} + C$

B. $\frac{-1}{2x^2} + C$

C. $\frac{-1}{3x^2} + C$

D. $\frac{x^{-2}}{2} + C$

Answer

Given:

$$\int \frac{1}{x^3} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x^3} dx = \frac{x^{-3+1}}{-3+1} + c$$

$$= -\frac{x^{-2}}{2} + c$$

$$= -\frac{1}{2x^2} + c$$

4. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt[3]{x} dx = ?$$

A. $\frac{3}{4}x^{\frac{3}{4}} + C$

B. $\frac{4}{3}x^{\frac{3}{4}} + C$

C. $\frac{3}{4}x^{\frac{4}{3}} + C$

D. $\frac{4}{3}x^{\frac{4}{3}} + C$

Answer

Given:

$$\int \sqrt[3]{x} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$= \frac{3}{4} x^{\frac{4}{3}} + c$$

5. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{\sqrt[3]{x}} dx = ?$$

A. $\frac{3}{2} x^{\frac{2}{3}} + C$

B. $\frac{3}{2x^{\frac{2}{3}}} + C$

C. $\frac{2}{3x^{\frac{2}{3}}} + C$

D. $\frac{2}{3} x^{\frac{3}{2}} + C$

Answer

Given:

$$\int \frac{1}{\sqrt[3]{x}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$$

$$= \frac{3}{2} x^{\frac{2}{3}} + c$$

6. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt[3]{x^2} dx = ?$$

A. $\frac{5}{3}x^{5/3} + C$

B. $\frac{3}{5}x^{5/3} + C$

C. $\frac{5}{3}x^{3/5} + C$

D. $\frac{3}{5}x^{3/5} + C$

Answer

Given:

$$\int \sqrt[3]{x^2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x^2} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} + c$$

7. Question

Mark (✓) against the correct answer in each of the following:

$$\int 3^x dx = ?$$

A. $3^x (\log 3) + C$

B. $3^x + C$

C. $\frac{3^x}{\log 3} + C$

D. $\frac{\log 3}{3^x} + C$

Answer

Given:

$$\int 3^x dx$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 3^x dx = \frac{3^x}{\ln 3} + c$$

8. Question

Mark (✓) against the correct answer in each of the following:

$$\int 2^{\log x} dx = ?$$

A. $\frac{2^{\log x+1}}{(\log x + 1)} + C$

B. $\frac{x^{(\log 2+1)}}{(\log 2 + 1)} + C$

C. $\frac{2^{\log x}}{\log 2} + C$ 2adc

D. $\frac{2^{\log x}}{2} + C$

Answer

Given:

$$\int 2^{\log x} dx$$

$$\text{As } 2^{\log x} = x^{\log 2}$$

$$I = \int x^{\log 2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\log 2} dx = \frac{x^{\log 2+1}}{\log 2 + 1} + c$$

9. Question

Mark (✓) against the correct answer in each of the following:

$$\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx = ?$$

A. $\cot x - \operatorname{cosec} x + C$

B. $-\cot x + \operatorname{cosec} x + C$

C. $\cot x + \operatorname{cosec} x + C$

D. $-\cot x - \operatorname{cosec} x + C$

Answer

Given:

$$\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx = \int (\operatorname{csc} x)^2 + \cot x \operatorname{csc} x dx$$

$$= -\cot x - \operatorname{csc} x + C$$

10. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec x}{(\sec x + \tan x)} dx = ?$$

- A. $\tan x + \sec x + C$
- B. $\tan x - \sec x + C$
- C. $-\tan x + \sec x + C$
- D. $-\tan x - \sec x + C$

Answer

Given:

$$\int \frac{\sec x}{(\sec x + \tan x)} dx$$

Multiply and divide by $(\sec x - \tan x)$

$$= \int \frac{(\sec x)^2 - \tan x \sec x}{(\sec x)^2 - (\tan x)^2} dx$$

$$= \int (\sec x)^2 - \tan x \sec x dx$$

$$= \tan x - \sec x + C$$

11. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

- A. $\tan x + x + C$
- B. $\tan x - x + C$
- C. $-\tan x + x + C$
- D. $-\tan x - x + C$

Answer

Given:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = \int \frac{(2(\sin x)^2)}{(2(\cos x)^2)} dx$$

$$= \int (\tan x)^2 dx$$

$$= \int ((\sec x)^2 - 1) dx$$

$$= \tan x - x + C$$

12. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$$

- A. $\tan x + \cot x + C$
- B. $-\tan x + \cot x + C$

- C. $\tan x - \cot x + C$
 D. none of these

Answer

Given:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx$$

As we know $\sin^2 x + \cos^2 x = 1$

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{(\sin x)^2 + (\cos x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\cos x)^2} + \frac{1}{(\sec x)^2} dx$$

$$= \int (\sec x)^2 + (\csc x)^2 dx$$

$$= \tan x - \cot x + C$$

13. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

- A. $-\cot x - \tan x + C$
 B. $-\cot x + \tan x + C$
 C. $\cot x - \tan x + C$
 D. $\cot x + \tan x + C$

Answer

Given:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2} dx$$

$$= \int (\csc x)^2 - (\sec x)^2 dx$$

$$= -\cot x - \tan x + C$$

14. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

- A. $2 \sin x + 2x \cos \alpha + C$
 B. $2 \sin x - 2x \cos \alpha + C$
 C. $-2 \sin x + 2x \cos \alpha + C$
 D. $-2 \sin x - 2x \cos \alpha + C$

Answer

Given:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = \int \frac{2 \sin(x + \alpha) \sin(x - \alpha)}{2 \sin\left(\frac{x + \alpha}{2}\right) \sin\left(\frac{x - \alpha}{2}\right)} dx$$

$$= \int 4 \cos\left(\frac{x + \alpha}{2}\right) \cos\left(\frac{x - \alpha}{2}\right) dx$$

$$= 2 \int \cos(x + \alpha) dx$$

$$= 2 \sin x + 2x \cos \alpha + c$$

15. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 + \cos 2x} dx = ?$$

A. $\sqrt{2} \cos x + C$

B. $\sqrt{2} \sin x + C$

C. $-\sqrt{2} \cos x + C$

D. $-\sqrt{2} \sin x + C$

Answer

Given:

$$\int \sqrt{1 + \cos 2x} dx = \int \sqrt{2(\cos x)^2} dx$$

$$= \sqrt{2} \int \cos x dx$$

$$= \sqrt{2} \sin x + c$$

16. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1 + \sin 2x} dx = ?$$

A. $\sin x + \cos x + C$

B. $-\sin x + \cos x + C$

C. $\sin x - \cos x + C$

D. $-\sin x - \cos x + C$

Answer

Given:

$$\int \sqrt{1 + \sin 2x} dx = \int \sqrt{1 + \frac{2 \tan x}{1 + (\tan x)^2}} dx$$

$$= \int \sqrt{\frac{(1 + \tan x)^2}{(\sec x)^2}} dx$$

$$= \int \frac{1 + \tan x}{\sec x} dx$$

$$= \int \cos x + \sin x \, dx$$

$$= \sin x - \cos x + C$$

17. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} \, dx = ?$$

A. $\cot x + \tan x + C$

B. $-\cot x + \tan x + C$

C. $\cot x - \tan x + C$

D. $-\cot x - \tan x + C$

Answer

Given:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} \, dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} \, dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2} \, dx$$

$$= \int (\csc x)^2 - (\sec x)^2 \, dx$$

$$= -\cot x - \tan x + C$$

18. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - \cos 2x)} = ?$$

A. $\frac{1}{2} \cot x + C$

B. $2 \cot x + C$

C. $-\frac{1}{2} \cot x + C$

D. $-2 \cot x + C$

Answer

Given:

$$\int \frac{dx}{(1 - \cos 2x)} = \int \frac{1}{2(\sin x)^2} \, dx$$

$$= \frac{1}{2} \int (\csc x)^2 \, dx$$

$$= -\frac{1}{2} \cot x + C$$

19. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin 2x}{\sin x} dx = ?$$

- A. $2 \sin x + C$
- B. $\frac{1}{2} \sin x + C$
- C. $2 \cos x + C$
- D. $\frac{1}{2} \cos x + C$

Answer

Given:

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx$$

$$= 2 \int \cos x dx$$

$$= 2 \sin x + c$$

20. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 - \sin x)}{\cos^2 x} dx = ?$$

- A. $\tan x + \sec x + C$
- B. $\tan x - \sec x + C$
- C. $-\tan x + \sec x + C$
- D. $-\tan x - \sec x + C$

Answer

Given:

$$\int \frac{(1 - \sin x)}{\cos^2 x} dx = \int \frac{1}{(\cos x)^2} - \frac{\sin x}{(\cos x)^2} dx$$

$$= \int (\sec x)^2 - \tan x \sec x dx$$

$$= \tan x - \sec x + c$$

21. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cot^2 x dx = ?$$

- A. $-\cot x - x + C$
- B. $\cot x - x + C$
- C. $-\cot x + x + C$
- D. $\cot x + x + C$

Answer

Given:

$$\int \cot^2 x \, dx = \int ((\csc x)^2 - 1) \, dx$$

$$= -\cot x - x + C$$

22. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sec x (\sec x + \tan x) \, dx = ?$$

A. $\tan x - \sec x + C$

B. $-\tan x + \sec x + C$

C. $\tan x + \sec x + C$

D. $-\tan x - \sec x + C$

Answer

Given:

$$\int \sec x (\sec x + \tan x) \, dx = \int (\sec x)^2 + \sec x \tan x \, dx$$

$$= \tan x + \sec x + C$$

23. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} \, dx = ?$$

A. $\tan x + x + C$

B. $\tan x - x + C$

C. $-\tan x + x + C$

D. $-\tan x - x + C$

Answer

Given:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} \, dx = \int \frac{(\sin x)^2}{(\cos x)^2} \, dx$$

$$= \int (\tan x)^2 \, dx$$

$$= \int (\sec x)^2 - 1 \, dx$$

$$= \tan x - x + C$$

24. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin^2 x}{(1 + \cos x)} \, dx = ?$$

A. $x + \sin x + C$

B. $x - \sin x + C$

C. $\sin x - x + C$

D. $-\sin x - x + C$

Answer

$$\begin{aligned} \text{Given: } \int \frac{\sin^2 x}{(1 + \cos x)} dx &= \int \frac{1 - (\cos x)^2}{(1 + \cos x)} dx \\ &= \int \frac{(1 + \cos x)(1 - \cos x)}{(1 + \cos x)} dx \\ &= \int (1 - \cos x) dx \\ &= x - \sin x + c \end{aligned}$$

25. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cot x}{(\operatorname{cosec} x - \cot x)} dx = ?$$

- A. $-\operatorname{cosec} x - \cot x - x + C$
- B. $\operatorname{cosec} x - \cot x - x + C$
- C. $-\operatorname{cosec} x + \cot x - x + C$
- D. $\operatorname{cosec} x + \cot x - x + C$

Answer

Given:

$$\begin{aligned} \int \frac{\cot x}{(\operatorname{cosec} x - \cot x)} dx &= \int \frac{\cot x (\csc x + \cot x)}{((\csc x)^2 - (\cot x)^2)} dx \\ &= \int \cot x \csc x + (\csc x)^2 dx \\ &= -\csc x - \cot x + c \end{aligned}$$

26. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1 + \sin x)} dx = ?$$

- A. $\sec x + \tan x + x + C$
- B. $\sec x - \tan x + x + C$
- C. $-\sec x + \tan x + x + C$
- D. None of these

Answer

Given:

$$\begin{aligned} \int \frac{\sin x}{(1 + \sin x)} dx \\ \text{Multiply and divide by } (1 - \sin x) \\ &= \int \frac{\sin x - (\sin x)^2}{1 - (\sin x)^2} dx \\ &= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx \end{aligned}$$

$$= \int (\tan x \sec x - (\tan x)^2) dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

$$= \sec x - \tan x + x + c$$

27. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx = ?$$

A. $2 \tan x + 2 \sec x + x + C$

B. $2 \tan x + 2 \sec x - x + C$

C. $\tan x + \sec x - x + C$

D. None of these

Answer

Given:

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx$$

Multiply and divide with $(1 + \sin x)$ to get,

$$\int \frac{(1 + \sin x)}{(1 - \sin x)} dx$$

$$= \int \frac{1 + (\sin x)^2 + 2 \sin x}{1 - (\sin x)^2} dx$$

$$= \int \frac{1 + (\sin x)^2 + 2 \sin x}{(\cos x)^2} dx$$

$$= \int (\sec x)^2 + (\tan x)^2 + 2 \tan x \sec x dx$$

$$= \int 2(\sec x)^2 - 1 + 2 \tan x \sec x dx$$

$$= 2 \tan x - x + 2 \sec x + c$$

28. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{1}{(1 + \cos x)} dx = ?$$

A. $-\cot x + \operatorname{cosec} x + C$

B. $\cot x - \operatorname{cosec} x + C$

C. $\cot x + \operatorname{cosec} x + C$

D. None of these

Answer

Given:

$$\int \frac{1}{(1 + \cos x)} dx$$

Multiply and divide by $(1 - \cos x)$

$$\int \frac{1}{(1 + \cos x)} dx = \int \frac{1 - \cos x}{1 - (\cos x)^2} dx$$

$$= \int \frac{1 - \cos x}{(\sin x)^2} dx$$

$$= \int (\csc x)^2 - \cot x \csc x dx$$

$$= -\cot x + \csc x + c$$

29. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1}(\cos x) dx = ?$$

A. $\operatorname{cosec} x + C$

B. $\frac{\pi x}{2} + \frac{x^2}{2} + C$

C. $\frac{\pi x}{2} - \frac{x^2}{2} + C$

D. $\frac{x^2}{2} - \frac{\pi x}{2} + C$

Answer

Given:

$$\int \sin^{-1}(\cos x) dx$$

$$\sin^{-1}(\cos x) + \cos^{-1}(\cos x) = \frac{\pi}{2}$$

$$= \int \frac{\pi}{2} - \cos^{-1}(\cos x) dx$$

$$= \int \frac{\pi}{2} - x dx$$

$$= \frac{\pi}{2} x - \frac{x^2}{2} + c$$

30. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1} \left\{ \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \right\} dx = ?$$

A. $\frac{-1}{(1+x^2)} + C$

B. $\frac{1}{\sqrt{1+x^2}} + C$

$$C. \frac{1}{\sqrt{1-x^2}} + C$$

$$D. \frac{x^2}{2} + C$$

Answer

Given:

$$\int \tan^{-1} \left\{ \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} \right\} dx = \int \tan^{-1} \sqrt{\frac{2(\sin x)^2}{2(\cos x)^2}} dx$$

$$= \int \tan^{-1}(\tan x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

31. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cot^{-1} \left(\frac{\sin 2x}{1-\cos 2x} \right) dx = ?$$

$$A. \frac{-1}{(1+x^2)} + C$$

$$B. \frac{-1}{(1-x^2)} + C$$

$$C. \frac{x^2}{2} + C$$

$$D. 2x^2 + C$$

Answer

Given:

$$\int \cot^{-1} \left(\frac{\sin 2x}{1-\cos 2x} \right) dx = \int \cot^{-1} \left(\frac{2 \sin x \cos x}{1-1+2(\sin x)^2} \right) dx$$

$$= \int \cot^{-1}(\cot x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

32. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin^{-1} \left(\frac{2 \tan x}{1+\tan^2 x} \right) dx = ?$$

A. $-x^2 + C$

B. $x^2 + C$

C. $\frac{x^2}{2} + C$

D. $2x^2 + C$

Answer

Given:

$$\int \sin^{-1}\left(\frac{2 \tan x}{1 + \tan^2 x}\right) dx = \int \sin^{-1}(\sin 2x) dx$$

$$= \int 2x dx$$

$$= x^2 + c$$

33. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos^{-1}\left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right) dx = ?$$

A. $x^2 + C$

B. $-x^2 + C$

C. $\frac{1}{\sqrt{1+x^2}} + C$

D. $\frac{1}{\sqrt{1-x^2}} + C$

Answer

Given:

$$\int \cos^{-1}\left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right) dx = \int \cos^{-1}(\cos 2x) dx$$

$$= \int 2x dx$$

$$= x^2 + c$$

34. Question

Mark (✓) against the correct answer in each of the following:

$$\int \tan^{-1}(\operatorname{cosec} x - \cot x) dx = ?$$

A. $\frac{x^2}{4} + C$

B. $\frac{-x^2}{4} + C$

C. $\frac{x^2}{2} + C$

D. $\frac{-x^2}{2} + C$

Answer

Given:

$$\begin{aligned} \int \tan^{-1}(\operatorname{cosec} x - \cot x) dx &= \int \tan^{-1}\left(\frac{1 - \cos x}{\sin x}\right) dx \\ &= \int \tan^{-1}\left(\frac{2 \sin \frac{x}{2} \sin \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}}\right) dx \\ &= \int \tan^{-1}\left(\tan \frac{x}{2}\right) dx \\ &= \int \frac{x}{2} dx \\ &= \frac{x^2}{4} + c \end{aligned}$$

35. Question

Mark (✓) against the correct answer in each of the following:

$$\int \left(\frac{(x^4 + 1)}{(x^2 + 1)} \right) dx = ?$$

A. $\frac{x^3}{3} + x - \tan^{-1} x + C$

B. $\frac{x^3}{3} - x - 2 \tan^{-1} x + C$

C. $\frac{x^3}{3} + x - 2 \tan^{-1} x + C$

D. None of these

Answer

Given:

$$\begin{aligned} \int \left(\frac{(x^4 + 1)}{(x^2 + 1)} \right) dx &= \int \frac{(x^4 + 2x^2 + 1)}{(x^2 + 1)} - \frac{2x^2}{(x^2 + 1)} dx \\ &= \int \frac{(x^2 + 1)^2}{(x^2 + 1)} dx - 2 \left\{ \int \frac{(x^2 + 1)}{(x^2 + 1)} - \frac{1}{(x^2 + 1)} dx \right\} \\ &= \int (x^2 + 1) dx - 2\{x - \tan^{-1} x\} + c \\ &= \frac{x^3}{3} - x - 2 \tan^{-1} x + c \end{aligned}$$

36. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(ax + b)}{(cx + d)} dx = ?$$

A. $\frac{ax}{c} + \log|cx + d| + C$

B. $\frac{a}{c} + \log|cx + d| + C$

C. $\frac{ax}{c} + \frac{(bc - ad)}{c^2} \log|cx + d| + C$

D. None of these

Answer

Given:

$$\begin{aligned} \int \frac{(ax + b)}{(cx + d)} dx &= \int \frac{ax}{cx + d} + \frac{b}{cx + d} dx \\ &= a \int \frac{x}{cx + d} \times \frac{c}{c} dx + b \int \frac{1}{cx + d} dx \\ &= \frac{a}{c} \left(\int \frac{cx + d}{cx + d} dx - \frac{d}{cx + d} \right) + b \ln|cx + d| + c \\ &= \frac{a}{c} \left(x - \frac{d}{c} \ln|cx + d| \right) + \frac{b}{c} \ln|cx + d| + c \\ &= \frac{a}{c} x + \frac{(bc - ad)}{c^2} \ln|cx + d| + c \end{aligned}$$

37. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(\sin^3 x + \cos^3 x)}{\sin^2 x \cos^2 x} dx = ?$$

A. $\sin x - \cos x + C$

B. $\tan x - \cos x + C$

C. $\sec x - \operatorname{cosec} x + C$

D. None of these

Answer

Given:

$$\begin{aligned} \int \frac{(\sin^3 x + \cos^3 x)}{\sin^2 x \cos^2 x} dx &= \int \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} + \frac{(\cos x)^3}{(\sin x)^2 (\cos x)^2} dx \\ &= \int (\tan x \sec x + \operatorname{csc} x \cot x) dx \\ &= \sec x - \operatorname{csc} x + C \end{aligned}$$

38. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx = ?$$

- A. $x \cos \alpha + (\sin \alpha) \log |\sin(x-\alpha)| + C$
 B. $x \sin \alpha + (\sin \alpha) \log |\sin(x-\alpha)| + C$
 C. $x \cos \alpha - (\sin \alpha) \log |\sin(x-\alpha)| + C$
 D. $x \sin \alpha - (\sin \alpha) \log |\sin(x-\alpha)| + C$

Answer

Given:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx$$

Let $x-\alpha=t$

$$dx=dt$$

$$I = \int \frac{\sin(t+\alpha)}{\sin t} dx$$

$$= \int \frac{\sin t \cos \alpha + \cos t \sin \alpha}{\sin t} dt$$

$$= \int \cos \alpha + \sin \alpha \cot t dt$$

$$= t \cos \alpha + \sin \alpha \ln |\sin t| + c$$

$$= (x-\alpha) \cos \alpha + (\sin \alpha) \ln |\sin(x-\alpha)| + c$$

$$= x \cos \alpha + (\sin \alpha) \ln |\sin(x-\alpha)| + c$$

39. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sin 3x \sin 2x dx = ?$$

A. $-\frac{1}{5} \cos 5x + C$

B. $\frac{1}{2} \sin x + \frac{1}{10} \sin 5x - C$

C. $\frac{1}{2} \sin x - \frac{1}{10} \sin 5x - C$

D. $-\frac{1}{3} \cos 3x - \frac{1}{2} \sin 2x + C$

Answer

Given:

$$\int \sin 3x \sin 2x dx = \frac{1}{2} \int 2 \sin 3x \sin 2x dx$$

$$= \frac{1}{2} \int \cos x - \cos 5x dx$$

$$= \frac{1}{2} \left\{ \frac{\sin x}{1} - \frac{\sin 5x}{5} \right\} + c$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

40. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos 3x \sin 2x \, dx = ?$$

- A. $\frac{1}{2} \cos x - \frac{1}{10} \cos 5x + C$
- B. $-\frac{1}{2} \sin x + \frac{1}{10} \sin 5x + C$
- C. $-\frac{1}{2} \cos x + \frac{1}{10} \cos 5x + C$
- D. None of these

Answer

Given:

$$\int \cos 3x \sin 2x \, dx = \frac{1}{2} \int 2 \cos 3x \sin 2x \, dx$$

$$= \frac{1}{2} \int \sin 5x + \cos x \, dx$$

$$= \frac{1}{2} \left\{ \frac{-\cos 5x}{5} + \frac{\sin x}{1} \right\} + c$$

$$= -\frac{\cos 5x}{10} + \frac{\sin x}{2} + c$$

41. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos 4x \cos x \, dx = ?$$

- A. $\frac{1}{5} \sin 5x + \frac{1}{3} \sin 3x + C$
- B. $\frac{1}{5} \cos 5x - \frac{1}{3} \cos 3x + C$
- C. $\frac{1}{10} \sin 5x + \frac{1}{6} \sin 3x + C$
- D. None of these

Answer

Given:

$$\int \cos 4x \cos x \, dx = \frac{1}{2} \int 2 \cos 4x \cos x \, dx$$

$$= \frac{1}{2} \int \cos 5x + \cos 3x \, dx$$

$$= \frac{1}{2} \left\{ \frac{\sin 5x}{5} + \frac{\sin 3x}{3} \right\} + c$$

$$= \frac{\sin 5x}{10} + \frac{\sin 3x}{6} + c$$