14. Some Special Integrals

Exercise 14A

1. Question

Evaluate:

$$\int \frac{dx}{(1-9x)^2}$$

Answer

To find: $\int \frac{dx}{(1-9x)^2}$

Formula Used: $\int x^n = \frac{x^{n+1}}{n+1} + C$

Let y = (1 – 9x) ... (1)

Differentiating with respect to x,

$$\frac{dy}{dx} = -9$$

i.e., dy = -9 dx

Substituting in the equation to evaluate,

$$\Rightarrow \int \frac{\frac{dy}{-9}}{y^2}$$

$$\Rightarrow \frac{-1}{9} \int \frac{dy}{y^2}$$

$$\Rightarrow \frac{-1}{9} \times \int y^{-2}\ dy$$

$$\Rightarrow \frac{-1}{9} \times \frac{y^{-2+1}}{-2+1} + C$$

Simplifying and substituting the value of y from (1),

$$\Rightarrow \frac{-1}{9} \times \frac{-1}{(1-9x)} + C$$

$$\Rightarrow \frac{1}{9(1-9x)} + C$$

Therefore,

$$\int \frac{dx}{(1-9x)^2} = \frac{1}{9(1-9x)} + C$$

2. Question

Evaluate:

$$\int \frac{dx}{\left(25-4x^2\right)}$$

Answer

To find: $\int \frac{dx}{(25-4x^2)}$

Formula Used: $\frac{dx}{(a^2-x^2)} = \frac{1}{2a} \times \log\left|\frac{a+x}{a-x}\right| + C$

Given equation = $\int \frac{dx}{4(\frac{25}{4}-x^2)}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{\left(\left(\frac{5}{2}\right)^2-x^2\right)}$ … (1)

Here $a = \frac{5}{2}$

Therefore, (1) becomes

$$\Rightarrow \frac{1}{4} \times \frac{1}{5} \times \log\left|\frac{\frac{5}{2}+x}{\frac{5}{2}-x}\right| + C$$

$$\Rightarrow \frac{1}{20} \times \log\left|\frac{5+2x}{5-2x}\right| + C$$

Therefore,

$$\int \frac{dx}{(25-4x^2)} = \frac{1}{20} \times \log\left|\frac{5+2x}{5-2x}\right| + C$$

3. Question

Evaluate:

$$\int \frac{dx}{\left(x^2+16\right)}$$

Answer

To find: $\int \frac{dx}{(x^2+16)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{4^2+x^2}$$

Here a = 4

$$\Rightarrow \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2+16)} = \frac{1}{4} \times \tan^{-1}\left(\frac{x}{4}\right) + C$$

4. Question

Evaluate:

$$\int \frac{dx}{\left(4+9x^2\right)}$$

Answer

To find: $\int \frac{dx}{(4+9x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{4}{9}\right) + x^2}$$

$$\Rightarrow \frac{1}{9}\int \frac{dx}{\left(\frac{2}{3}\right)^2 + x^2}$$

Here $a = \frac{2}{3}$

$$\Rightarrow \frac{1}{9} \times \frac{3}{2} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(4+9x^2)} = \frac{1}{6} \times \tan^{-1}\left(\frac{3x}{2}\right) + C$$

5. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{\left(50+2\mathrm{x}^2\right)}$$

Answer

To find: $\int \frac{dx}{(50+2x^2)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{2}\int \frac{dx}{25 + x^2}$$

$$\Rightarrow \frac{1}{2}\int \frac{dx}{5^2 + x^2}$$

Here a = 5

$$\Rightarrow \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2+16)} = \frac{1}{10} \times \tan^{-1}\left(\frac{x}{5}\right) + C$$

6. Question

Evaluate:

$$\int \frac{dx}{(16x^2 - 25)}$$

Answer

To find: $\int \frac{dx}{(16x^2-25)}$

Formula Used: $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

Rewriting the given equation,

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{25}{16}\right)}$$

$$\Rightarrow \frac{1}{16} \int \frac{dx}{x^2 - \left(\frac{5}{4}\right)^2}$$

Here $a = \frac{5}{4}$

$$\Rightarrow \frac{1}{16} \times \frac{2}{5} \times \ln\left|\frac{x - \frac{5}{4}}{x + \frac{5}{4}}\right| + C$$

$$\Rightarrow \frac{1}{40} \times ln\left|\frac{4x-5}{4x+5}\right| + C$$

Therefore,

$$\int \frac{dx}{(16x^2 - 25)} = \frac{1}{40} \times \log\left|\frac{4x-5}{4x+5}\right| + C$$

7. Question

Evaluate:

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx$$

Answer

To find: $\int \frac{(x^2-1)}{(x^2+4)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \int \frac{(x^2 + 4 - 5)}{(x^2 + 4)}\, dx$$

$$\Rightarrow \int \frac{(x^2 + 4)}{(x^2 + 4)} dx - \int \frac{5}{(x^2 + 4)}\, dx$$

$$\Rightarrow \int dx - 5 \int \frac{1}{(x^2 + 2^2)}\, dx$$

Here a = 2,

$$\Rightarrow x - \frac{5}{2}\, tan^{-1}\frac{x}{2} + C$$

Therefore,

$$\int \frac{(x^2-1)}{(x^2+4)}\, dx = x - \frac{5}{2}\, tan^{-1}\frac{x}{2} + C$$

8. Question

Evaluate:

$$\int \frac{x^2}{(9+4x^2)} dx$$

Answer

To find: $\int \frac{x^2}{(9+4x^2)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Given equation can be rewritten as the following:

$$\Rightarrow \frac{1}{4}\int \frac{x^2}{(x^2+\frac{9}{4})}\, dx$$

$$\Rightarrow \frac{1}{4}\int \frac{x^2+\frac{9}{4}-\frac{9}{4}}{(x^2+\frac{9}{4})}\, dx$$

$$\Rightarrow \frac{1}{4}\int dx\ - \frac{9}{16}\int \frac{1}{\left(x^2+\left(\frac{3}{2}\right)^2\right)}\, dx$$

Here $a = \frac{3}{2}$,

$$\Rightarrow \frac{x}{4} - \left(\frac{9}{16} \times \frac{2}{3}\, tan^{-1}\frac{2x}{3}\right) + C$$

$$\Rightarrow \frac{x}{4} - \frac{3}{8}\, tan^{-1}\left(\frac{2x}{3}\right) + C$$

Therefore,

$$\int \frac{x^2}{(9+4x^2)}\, dx = \frac{x}{4} - \frac{3}{8}\, tan^{-1}\left(\frac{2x}{3}\right) + C$$

9. Question

Evaluate:

$$\int \frac{e^x}{(e^{2x}+1)} dx$$

Answer

To find: $\int \frac{e^x}{(e^{2x}+1)}\, dx$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Let y = e^x … (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{y^2+1}$$

⇒ $\tan^{-1} y$

From (1),

⇒ $\tan^{-1}(e^x)$

Therefore,

$$\int \frac{e^x}{(e^{2x}+1)}\, dx = \tan^{-1}(e^x) + C$$

10. Question

Evaluate:

$$\int \frac{\sin x}{(1+\cos^2 x)} dx$$

Answer

To find: $\int \frac{\sin x}{(1+\cos^2 x)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = cos x … (1)

Differentiating both sides, we get

dy = –sin x dx

Substituting in given equation,

$$\Rightarrow \int \frac{-dy}{1+y^2}$$

⇒ – $\tan^{-1} y$

From (1),

⇒ $-\tan^{-1}(\cos x)$

Therefore,

$$\int \frac{\sin x}{(1+\cos^2 x)}\, dx = -\tan^{-1}(\cos x) + C$$

11. Question

Evaluate:

$$\int \frac{\cos x}{(1+\sin^2 x)} dx$$

Answer

To find: $\int \frac{\cos x}{(1+\sin^2 x)}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = sin x ... (1)

Differentiating both sides, we get

dy = cos x dx

Substituting in given equation,

$$\Rightarrow \int \frac{dy}{1+y^2}$$

⇒ $\tan^{-1} y$

From (1),

⇒ $\tan^{-1}(\sin x)$

Therefore,

$$\int \frac{\cos x}{(1+\sin^2 x)}\, dx = \tan^{-1}(\sin x) + C$$

12. Question

Evaluate:

$$\int \frac{3x^5}{(1+x^{12})} dx$$

Answer

To find: $\int \frac{3x^5}{(1+x^{12})}\, dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = x^6 ... (1)

Differentiating both sides, we get

dy = $6x^5$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\, dy}{1+y^2}$$

$$\Rightarrow \frac{1}{2}\tan^{-1} y + C$$

From (1),

$$\Rightarrow \frac{1}{2}\tan^{-1}(x^6) + C$$

Therefore,

$$\int \frac{3x^5}{(1+x^{12})}\, dx = \frac{1}{2}\tan^{-1}(x^6) + C$$

13. Question

Evaluate:

$$\int \frac{2x^3}{(4+x^8)}dx$$

Answer

To find: $\int \frac{2x^3}{(4+x^8)}\,dx$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Let y = x^4 ... (1)

Differentiating both sides, we get

dy = $4x^3$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{4+y^2}$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{2^2+y^2}\,dy$$

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{y}{2}\right) + C$$

From (1),

$$\Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right) + C$$

Therefore,

$$\int \frac{2x^3}{(4+x^8)}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x^4}{2}\right) + C$$

14. Question

Evaluate:

$$\int \frac{dx}{(e^x+e^{-x})}$$

Answer

To find: $\int \frac{dx}{(e^x+e^{-x})}$

Formula Used: $\int \frac{dx}{1+x^2} = \tan^{-1}x$

Given equation is:

$\int \frac{dx}{(e^x+e^{-x})} = \int \frac{e^x\,dx}{(e^{2x}+1)}$ ... (1)

Let y = e^x ... (1)

Differentiating both sides, we get

dy = e^x dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{y^2+1}$$

$\Rightarrow \tan^{-1} y$

From (1),

$\Rightarrow \tan^{-1} (e^x)$

Therefore,

$$\int \frac{dx}{(e^x+e^{-x})} = \tan^{-1}(e^x) + C$$

15. Question

Evaluate:

$$\int \frac{x}{(1-x^4)} dx$$

Answer

To find: $\int \frac{x\,dx}{(1-x^4)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

Let y = x^2 ... (1)

Differentiating both sides, we get

dy = 2x dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{2}\,dy}{1-y^2}$$

Here a = 1,

$$\Rightarrow \frac{1}{2} \times \frac{1}{2} \times \log\left|\frac{1+y}{1-y}\right| + C$$

$$\Rightarrow \frac{1}{4} \log\left|\frac{1+y}{1-y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{4} \log\left|\frac{1+x^2}{1-x^2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(1-x^4)} = \frac{1}{4} \log\left|\frac{1+x^2}{1-x^2}\right| + C$$

16. Question

Evaluate:

$$\int \frac{x^2}{(a^6 - x^6)} dx$$

Answer

To find: $\int \frac{x^2\,dx}{(a^6 - x^6)}$

Formula Used: $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left|\frac{a+x}{a-x}\right| + C$

Let y = x^3 ... (1)

Differentiating both sides, we get

dy = $3x^2$ dx

Substituting in given equation,

$$\Rightarrow \int \frac{\frac{1}{3}\,dy}{a^6 - y^2}$$

$$\Rightarrow \frac{1}{3}\int \frac{1}{(a^3)^2 - y^2}\,dy$$

$$\Rightarrow \frac{1}{3} \times \frac{1}{2a^3} \times \log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + y}{a^3 - y}\right| + C$$

From (1),

$$\Rightarrow \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

Therefore,

$$\int \frac{x^2\,dx}{(a^6 - x^6)} = \frac{1}{6a^3}\log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$$

17. Question

Evaluate:

$$\int \frac{dx}{(x^2 + 4x + 8)}$$

Answer

To find: $\int \frac{dx}{(x^2 + 4x + 8)}$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{((x+2)^2 + 4)}$$

$\Rightarrow \int \frac{dx}{((x+2)^2 + 2^2)}$ ... (1)

Let y = x + 2 ... (2)

Differentiating both sides,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{dy}{(y^2 + 2^2)}$$

Here a = 2,

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{y}{2}\right) + C$$

From (2),

$$\Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$$

Therefore,

$$\int \frac{dx}{(x^2 + 4x + 8)} = \frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$$

18. Question

Evaluate:

$$\int \frac{dx}{(4x^2 - 4x + 3)}$$

Answer

To find: $\int \frac{dx}{(4x^2 - 4x + 3)}$

Formula Used: $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

Rewriting the given equation,

$\Rightarrow \int \frac{dx}{((2x-1)^2 + 2)}$ ... (1)

Let y = 2x – 1 ... (2)

Differentiating both sides,

dy = 2dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{2}dy}{\left(y^2 + (\sqrt{2})^2\right)}$$

Here a = √2,

$$\Rightarrow \frac{1}{2} \times \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right) + C$$

From (2),

$$\Rightarrow \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{dx}{(4x^2-4x+3)} = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)+C$$

19. Question

Evaluate:

$$\int \frac{dx}{(2x^2+x+3)}$$

Answer

To find: $\int \frac{dx}{(2x^2+x+3)}$

Formula Used: $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x+\frac{1}{2\sqrt{2}}\right)^2+3-\frac{1}{8}\right)}$$

$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x+\frac{1}{2\sqrt{2}}\right)^2+\frac{23}{8}\right)}$... (1)

Let $y = \sqrt{2}x + \frac{1}{2\sqrt{2}}$... (2)

Differentiating both sides,

dy = √2 dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2+\left(\frac{\sqrt{23}}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{\sqrt{23}}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{23}}\tan^{-1}\left(\frac{y \times 2\sqrt{2}}{\sqrt{23}}\right)+C$$

From (2),

$$\Rightarrow \frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$$

Therefore,

$$\int \frac{dx}{(2x^2+x+3)} = \frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$$

20. Question

Evaluate:

$$\int \frac{dx}{(2x^2-x-1)}$$

Answer

To find: $\int \frac{dx}{(2x^2-x-1)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - 1 - \left(\frac{1}{2\sqrt{2}}\right)^2\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - 1 - \frac{1}{8}\right)}$$

$$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x - \frac{1}{2\sqrt{2}}\right)^2 - \frac{9}{8}\right)}$$

$\Rightarrow \int \frac{dx}{\left(\left(\sqrt{2}x-\frac{1}{2\sqrt{2}}\right)^2-\left(\frac{3}{2\sqrt{2}}\right)^2\right)}$ ... (1)

Let $y = \sqrt{2}x - \frac{1}{2\sqrt{2}}$ ... (2)

Differentiating both sides,

dy = √2 dx

Substituting in (1),

$$\Rightarrow \int \frac{\frac{1}{\sqrt{2}}dy}{\left(y^2 - \left(\frac{3}{2\sqrt{2}}\right)^2\right)}$$

Here $a = \frac{3}{2\sqrt{2}}$

$$\Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{3} \times \log\left|\frac{\frac{3}{2\sqrt{2}} + y}{\frac{3}{2\sqrt{2}} - y}\right| + C$$

$$\Rightarrow \frac{1}{3} \times \log\left|\frac{3 + 2\sqrt{2}y}{3 - 2\sqrt{2}y}\right| + C$$

From (2),

$$\Rightarrow \frac{1}{3} \times \log\left|\frac{3 + 4x - 1}{3 - 4x + 1}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{1 + 2x}{2(1-x)}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{2(x-1)}{2x+1}\right| + C$$

Therefore,

$$\int \frac{dx}{(2x^2 - x - 1)} = \frac{1}{3}\log\left|\frac{2(x-1)}{2x+1}\right| + C$$

21. Question

Evaluate:

$$\int \frac{dx}{(3-2x-x^2)}$$

Answer

To find: $\int \frac{dx}{(3-2x-x^2)}$

Formula Used: $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

Rewriting the given equation,

$$\Rightarrow \int \frac{-\,dx}{(x^2+2x-3)}$$

$$\Rightarrow \int \frac{-dx}{(x+1)^2-4}$$

$\Rightarrow \int \frac{-dx}{(x+1)^2-2^2}$ ... (1)

Let y = x + 1 ... (2)

Differentiating both sides wrt x,

dy = dx

Substituting in (1),

$$\Rightarrow \int \frac{-dy}{y^2-2^2}$$

$$\Rightarrow \int \frac{dy}{2^2-y^2}$$

Here a = 2,

$$\Rightarrow \frac{1}{4}\log\left|\frac{2+y}{2-y}\right| + C$$

From (2),

$$\Rightarrow \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$$

Therefore,

$$\int \frac{dx}{(3-2x-x^2)} = \frac{1}{4}\log\left|\frac{x+3}{1-x}\right| + C$$

22. Question

Evaluate:

$$\int \frac{x}{(x^2+3x+2)}dx$$

Answer

To find: $\int \frac{x\,dx}{(x^2+3x+2)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Using partial fractions,

$$x = A\left(\frac{d}{dx}(x^2+3x+2)\right) + B$$

x = A (2x + 3) + B

Equating the coefficients of x,

1 = 2A

$$A = \frac{1}{2}$$

Also, 0 = 3A + B

$$B = \frac{-3}{2}$$

Therefore, the given equation becomes,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+3) - \frac{3}{2}}{(x^2+3x+2)}\,dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 + 2 - \left(\frac{3}{2}\right)^2\right)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\int \frac{1}{\left(\left(x+\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right)}dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\times\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right| + C$$

$$\Rightarrow \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

Therefore,

$$\int \frac{x\,dx}{(x^2+3x+2)} = \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C$$

23. Question

Evaluate:

$$\int \frac{(x-3)}{(x^2+2x-4)}dx$$

Answer

To find: $\int \frac{(x-3)\,dx}{(x^2+2x-4)}$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$

Using partial fractions,

$$(x-3) = A\left(\frac{d}{dx}(x^2+2x-4)\right) + B$$

x – 3 = A (2x + 2) + B

Equating the coefficients of x,

1 = 2A

$$\Rightarrow A = \frac{1}{2}$$

Also, -3 = 2A + B

⇒ B = -4

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(2x+2) - 4}{(x^2+2x-4)} dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - 4\int \frac{1}{(x+1)^2 - \left(\sqrt{5}\right)^2}\, dx$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \left(4 \times \frac{1}{2\sqrt{5}} \times \log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|\right) + C$$

$$\Rightarrow \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

Therefore,

$$\int \frac{(x-3)\, dx}{(x^2+2x\ -\ 4)} = \frac{1}{2}\log|x^2+2x-4| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right| + C$$

24. Question

Evaluate:

$$\int \frac{(2x-3)}{(x^2+3x-18)} dx$$

Answer

To find: $\int \frac{(2x-3)}{(x^2+3x-18)} dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-3) = A\left(\frac{d}{dx}(x^2+3x-18)\right) + B$$

2x – 3 = A(2x + 3) + B

Equating the coefficients of x,

2 = 2A

A = 1

Also, -3 = 3A + B

⇒ B = -6

Substituting in the given equation,

$$\Rightarrow \int \frac{(2x+3)-6}{(x^2+3x-18)} dx$$

$$\Rightarrow \log|x^2+3x-18| + C_1 - 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx \text{ ... (1)}$$

Let I = $6\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - 18 - \left(\frac{3}{2}\right)^2} dx$

$$\Rightarrow 6\int \frac{1}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} dx$$

Here a = $\frac{9}{2}$

$$\Rightarrow \frac{6}{9} \times \log\left|\frac{x+\frac{3}{2}-\frac{9}{2}}{x+\frac{3}{2}+\frac{9}{2}}\right| + C_2$$

$$\Rightarrow \frac{2}{3} \times \log\left|\frac{x-3}{x+6}\right| + C_2 \text{ ... (2)}$$

Substituting (2) in (1),

$$\Rightarrow \log|x^2+3x-18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

Therefore,

$$\int \frac{(2x-3)}{(x^2+3x-18)} dx = \log|x^2+3x-18| - \frac{2}{3}\log\left|\frac{x-3}{x+6}\right| + C$$

25. Question

Evaluate:

$$\int \frac{x^2}{(x^2+6x-3)} dx$$

Answer

To find: $\int \frac{x^2}{(x^2+6x-3)} dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$

Given equation can be rewritten as following:

$$\Rightarrow \int \frac{x^2 + (6x - 3) - (6x - 3)}{(x^2 + 6x\ - 3)}\, dx$$

$$\Rightarrow \int \frac{(x^2 + 6x - 3) - (6x - 3)}{(x^2 + 6x - 3)}\ dx$$

$$\Rightarrow x - \int \frac{6x - 3}{x^2 + 6x - 3}\, dx$$

Let I = $\int \frac{6x-3}{x^2+6x-3}\, dx$ … (2)

Using partial fractions,

$$(6x - 3) = A\left(\frac{d}{dx}(x^2 + 6x - 3)\right) + B$$

6x – 3 = A (2x + 6) + B

Equating the coefficients of x,

6 = 2A

A = 3

Also, -3 = 6A + B

⇒ B = -21

Substituting in (1),

$$\Rightarrow \int \frac{3(2x + 6) - 21}{(x^2 + 6x - 3)}\, dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \int \frac{1}{(x + 3)^2 - \left(\sqrt{12}\right)^2}\, dx$$

$$\Rightarrow 3 \times \log|x^2 + 6x - 3| + C_1 - 21 \times \frac{1}{2\sqrt{12}} \times \log\left|\frac{x + 3 - \sqrt{12}}{x + 3 + \sqrt{12}}\right| + C_2$$

I $= 3\log|x^2 + 6x - 3| - \frac{7\sqrt{3}}{4} \times \log\left|\frac{x+3-2\sqrt{3}}{x+3+2\sqrt{3}}\right| + C$

Therefore,

$$\int \frac{x^2}{(x^2 + 6x\ - 3)}\, dx = x - \ 3\log|x^2 + 6x - 3| + \frac{7\sqrt{3}}{4} \times \log\left|\frac{x + 3 - 2\sqrt{3}}{x + 3 + 2\sqrt{3}}\right| + C$$

26. Question

Evaluate:

$$\int \frac{(2x - 1)}{(2x^2 + 2x + 1)}\, dx$$

Answer

To find: $\int \frac{2x-1}{(2x^2+2x+1)}\, dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + C$

Using partial fractions,

$$(2x-1) = A\left(\frac{d}{dx}(2x^2+2x+1)\right) + B$$

2x – 1 = A (4x + 2) + B

Equating the coefficients of x,

2 = 4A

A = $\frac{1}{2}$

Also, -1 = 2A + B

⇒ B = -2

Substituting in the given equation,

$$\Rightarrow \int \frac{\frac{1}{2}(4x+2)-2}{(2x^2+2x+1)}dx$$

$$\Rightarrow \frac{1}{2}\log|2x^2+2x+1| - 2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}\, dx$$

Let I = $2\int \frac{1}{2\left(x^2+x+\frac{1}{2}\right)}\, dx$ … (1)

$$\Rightarrow \int \frac{1}{\left(x^2+x+\frac{1}{2}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\left(\frac{1}{2}\right)^2\right)}\, dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\frac{1}{2}-\frac{1}{4}\right)}\, dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2\right)}\, dx$$

Here $a = \frac{1}{2}$

$$\Rightarrow 2\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{1}{2}}\right) + C$$

⇒ $2\tan^{-1}(2x + 1) + C$

Substituting in (1) and combining with original equation,

$$\Rightarrow \frac{1}{2}\log|2x^2+2x+1| - 2\tan^{-1}(2x+1) + C$$

Therefore,

$$\int \frac{2x-1}{(2x^2+2x+1)}dx = \frac{1}{2}\log|2x^2+2x+1| - 2\tan^{-1}(2x+1) + C$$

27. Question

Evaluate:

$$\int \frac{(1-3x)}{(3x^2+4x+2)}dx$$

Answer

To find: $\int \frac{1-3x}{(3x^2+4x+2)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow -\int \frac{3x-1}{(3x^2+4x+2)}\,dx$$

Using partial fractions,

$$(3x-1) = A\left(\frac{d}{dx}(3x^2+4x+2)\right) + B$$

3x – 1 = A (6x + 4) + B

Equating the coefficients of x,

3 = 6A

A = $\frac{1}{2}$

Also, -1 = 4A + B

⇒ B = -3

Substituting in the original equation,

$$\Rightarrow -\int \frac{\frac{1}{2}(6x+4)-3}{(3x^2+4x+2)}\,dx$$

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2| + 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$$

Let I $= 3\int \frac{1}{3\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$

$$\Rightarrow \int \frac{1}{\left(x^2+\frac{4}{3}x+\frac{2}{3}\right)}\,dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\frac{2}{3}-\frac{4}{9}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right)} dx$$

Here $a = \frac{\sqrt{2}}{3}$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right) + C$$

$$\Rightarrow \frac{3}{\sqrt{2}} \tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

Substituting in (1) and combining with original equation,

$$\Rightarrow -\frac{1}{2}\log|3x^2+4x+2| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1-3x}{(3x^2+4x+2)} dx = -\frac{1}{2}\log|3x^2+4x+2| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right) + C$$

28. Question

Evaluate:

$$\int \frac{2x}{(2+x-x^2)} dx$$

Answer

To find: $\int \frac{2x}{(2+x-x^2)} dx$

Formula Used:

1. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

2. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

Rewriting the given equation,

$$\Rightarrow -2\int \frac{x}{(x^2-x-2)} dx$$

Using partial fractions,

$$x = A\left(\frac{d}{dx}(x^2-x-2)\right) + B$$

x = A (2x – 1) + B

Equating the coefficients of x,

1 = 2A

A = $\frac{1}{2}$

Also, 0 = -A + B

$B = \frac{1}{2}$

Substituting in the original equation,

$$\Rightarrow -2\int \frac{\frac{1}{2}(2x-1)+\frac{1}{2}}{(x^2-x-2)}\,dx$$

$$\Rightarrow -\log|x^2-x-2| - \int \frac{1}{(x^2-x-2)}\,dx$$

Let $I = \int \frac{1}{(x^2-x-2)}dx$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-2-\frac{1}{4}\right)}dx$$

$$\Rightarrow \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

Here $a = \frac{3}{2}$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right| + C$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

Substituting for I and combining with the original equation,

$$\Rightarrow -\log|x^2-x-2| + \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

Therefore,

$$\int \frac{2x}{(2+x-x^2)}dx = -\log|x^2-x-2| + \frac{1}{3}\log\left|\frac{x-2}{x+1}\right| + C$$

or

$$\int \frac{2x}{(2+x-x^2)}dx = -\log|2+x-x^2| + \frac{1}{3}\log\left|\frac{1+x}{2-x}\right| + C$$

29. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(1+\cos^2 \mathrm{x})}$$

Answer

To find: $\int \frac{1}{(1+\cos^2 x)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x \, dx}{1+\sec^2 x}$ … (1)

Let y = tan x

dy = $\sec^2$ x dx … (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ … (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{1+y^2+1}$$

$$\Rightarrow \int \frac{dy}{y^2+(\sqrt{2})^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y}{\sqrt{2}}\right)+C$$

Since y = tan x,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$$

Therefore,

$$\int \frac{1}{(1+\cos^2 x)}dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+C$$

30. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(2+\sin^2 \mathrm{x})}$$

Answer

To find: $\int \frac{1}{(2+\sin^2 x)}dx$

Formula Used:

1. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

2. $\sec^2 x = 1 + \tan^2 x$

Dividing the given equation by $\cos^2 x$ in the numerator and denominator gives us,

$\Rightarrow \int \frac{\sec^2 x \, dx}{2\sec^2 x + \tan^2 x}$ … (1)

Let y = tan x

dy = $\sec^2$ x dx … (2)

Also, $y^2 = \tan^2 x$

i.e., $y^2 = \sec^2 x - 1$

$\sec^2 x = y^2 + 1$ … (3)

Substituting (2) and (3) in (1),

$$\Rightarrow \int \frac{dy}{2y^2 + 2 + y^2}$$

$$\Rightarrow \int \frac{dy}{3y^2 + 2}$$

$$\Rightarrow \frac{1}{3} \int \frac{dy}{y^2 + \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}} \tan^{-1}\left(\frac{y\sqrt{3}}{\sqrt{2}}\right) + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

Therefore,

$$\int \frac{1}{(2 + \sin^2 x)} dx = \frac{1}{\sqrt{6}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{2}}\right) + C$$

31. Question

Evaluate:

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$$

Answer

To find: $\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)}$

Formula Used:

1. $\text{Sec}^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{a^2 + b^2 \tan^2 x}$$

Let y = tan x

$dy = \sec^2 x \, dx$

Therefore,

$$\Rightarrow \int \frac{dy}{a^2 + b^2 y^2}$$

$$\Rightarrow \frac{1}{b^2} \int \frac{dy}{\left(\frac{a}{b}\right)^2 + y^2}$$

$\Rightarrow \frac{1}{b^2} \times \frac{b}{a} \tan^{-1} \frac{yb}{a} + C$

Since y = tan x,

$\Rightarrow \frac{1}{ab} \tan^{-1}\left(\frac{b \tan x}{a}\right) + C$

Therefore,

$$\int \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)} = \frac{1}{a^2} \tan^{-1}\left(\frac{b}{a} \tan x\right) + C$$

32. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(\cos^2 \mathrm{x} - 3\sin^2 \mathrm{x})}$$

Answer

To find: $\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$\Rightarrow \int \frac{\sec^2 x \ dx}{1 - 3\tan^2 x}$

Let y = tan x

$dy = \sec^2 x \, dx$

Therefore,

$\Rightarrow \int \frac{dy}{1 - 3y^2}$

$\Rightarrow \frac{1}{3} \int \frac{dy}{\left(\frac{1}{\sqrt{3}}\right)^2 - y^2}$

$\Rightarrow \frac{1}{3} \times \frac{\sqrt{3}}{2} \log \left| \frac{\frac{1}{\sqrt{3}} + y}{\frac{1}{\sqrt{3}} - y} \right| + C$

$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + y\sqrt{3}}{1 - y\sqrt{3}} \right| + C$

Since y = tan x,

$\Rightarrow \frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$

Therefore,

$$\int \frac{dx}{(\cos^2 x - 3\sin^2 x)} = \frac{1}{2\sqrt{3}} \log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + C$$

33. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(\sin^2 \mathrm{x} - 4\cos^2 \mathrm{x})}$$

Answer

To find: $\int \frac{dx}{(\sin^2 x - 4\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$

Dividing by $\cos^2 x$ in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \; dx}{\tan^2 x - 4}$$

Let y = tan x

dy = $\sec^2 x$ dx

Therefore,

$$\Rightarrow \int \frac{dy}{y^2 - 2^2}$$

$$\Rightarrow \frac{1}{4} \log\left|\frac{y-2}{y+2}\right| + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{4} \log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin^2 x - 4\cos^2 x)} = \frac{1}{4} \log\left|\frac{\tan x - 2}{\tan x + 2}\right| + C$$

34. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(\sin \mathrm{x} \cos \mathrm{x} + 2\cos^2 \mathrm{x})}$$

Answer

To find: $\int \frac{dx}{(\sin x \cos x + 2\cos^2 x)}$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{x} dx = \log x + C$

Dividing by $\cos^2$ x in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x \ dx}{\tan x + 2}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$$\Rightarrow \int \frac{dy}{y+2}$$

⇒ log |y + 2| + C

Since y = tan x,

⇒ log |tan x + 2| + C

Therefore,

$$\int \frac{dx}{(\sin x \cos x + 2\cos^2 x)} = \log|\tan x + 2| + C$$

35. Question

Evaluate:

$$\int \frac{\sin 2\mathrm{x}}{(\sin^4 \mathrm{x} + \cos^4 \mathrm{x}} \mathrm{dx}$$

Answer

To find: $\int \frac{\sin 2x \, dx}{(\sin^4 x + \cos^4 x)}$

Formula Used:

1. $\sec^2$ x = 1 + $\tan^2$ x

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. sin 2x = 2 sin x cos x

Rewriting the given equation,

$$\Rightarrow \int \frac{2 \sin x \cos x}{\sin^4 x + \cos^4 x} \ dx$$

Dividing by $\cos^4$ x in the numerator and denominator,

$$\Rightarrow \int \frac{2 \tan x \sec^2 x \ dx}{\tan^4 x + 1}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$$\Rightarrow \int \frac{2y}{y^4 + 1} \ dy$$

Let z = y^2

dz = 2y dy

$\Rightarrow \int \frac{dz}{1+z^2}$

$\Rightarrow \tan^{-1} z + C$

Since $z = y^2$,

$\Rightarrow \tan^{-1}(y^2) + C$

Since $y = \tan x$,

$\Rightarrow \tan^{-1}(\tan^2 x) + C$

Therefore,

$$\int \frac{\sin 2x \; dx}{(\sin^4 x + \cos^4 x)} = \tan^{-1}(\tan^2 x) + C$$

36. Question

Evaluate:

$$\int \frac{(2\sin 2\phi - \cos\phi)}{(6 - \cos^2\phi - 4\sin\phi)} d\phi$$

Answer

To find: $\int \frac{(2\sin 2\phi - \cos\phi)}{(6-\cos^2\phi - 4\sin\phi)} \, d\phi$

Formula Used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

3. $\sin 2x = 2 \sin x \cos x$

Rewriting the given equation,

$$\Rightarrow \int \frac{4\sin\phi\cos\phi - \cos\phi}{6 - \cos^2\phi - 4\sin\phi} \, d\phi$$

$$\Rightarrow \int \frac{\cos\phi \; (4\sin\phi - 1)}{6 - (1 - \sin^2\phi) - 4\sin\phi} \, d\phi$$

$$\Rightarrow \int \frac{\cos\phi \; (4\sin\phi - 1)}{5 + \sin^2\phi - 4\sin\phi} \, d\phi$$

Let $y = \sin\phi$

$dy = \cos\phi \, d\phi$

Substituting in the original equation,

$\Rightarrow \int \frac{4y-1}{y^2-4y+5} \, dy$ ... (1)

Using partial fraction,

$$4y - 1 = A\left(\frac{d}{dy}(y^2 - 4y + 5)\right) + B$$

$4y - 1 = A(2y - 4) + B$

Equating the coefficients of y,

4 = 2A

A = 2

Also, -1 = -4A + B

B = 7

Substituting in (1),

$$\Rightarrow \int \frac{2(2y-4)+7}{y^2-4y+5}\, dy$$

$$\Rightarrow 2\log|y^2-4y+5| + 7\int \frac{1}{((y-2)^2+1)}\, dy$$

⇒ 2 log $|y^2$ – 4y + 5| + 7 $\tan^{-1}$(y – 2) + C

But y = sin ϕ

⇒ 2 log $|\sin^2 \phi$ – 4 sin ϕ + 5| + 7 $\tan^{-1}$(sin ϕ – 2) + C

Therefore,

$$\int \frac{(2\sin 2\phi - cos\phi)}{(6-\cos^2\phi - 4\sin\phi)}\, d\phi = 2\log|\sin^2\phi - 4\sin\phi + 5| + 7\tan^{-1}(\sin\phi - 2) + C$$

37. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{(\sin \mathrm{x} - 2\cos \mathrm{x})(2\sin \mathrm{x} + \cos \mathrm{x})}$$

Answer

To find: $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$

Formula Used:

1. $\sec^2$ x = 1 + $\tan^2$ x

2. $\int \frac{1}{x} dx = \log x + C$

Dividing by $\cos^2$ x in the numerator and denominator,

$$\Rightarrow \int \frac{\sec^2 x\ dx}{(\tan x - 2)(2\tan x + 1)}$$

Let y = tan x

dy = $\sec^2$ x dx

Therefore,

$\Rightarrow \int \frac{dy}{(y-2)(2y+1)}$ … (1)

Let

$$\frac{1}{(y-2)(2y+1)} = \frac{A}{(y-2)} + \frac{B}{(2y+1)}$$

1 = A (2y + 1) + B(y – 2)

When y = 0,

1 = A – 2B … (2)

When y = 1,

1 = 3A – B ⇒ 2 = 6A – 2B … (3)

Solving (2) and (3),

1 = 5A

A $= \frac{1}{5}$

So, B $= \frac{-2}{5}$

(1) becomes,

$$\Rightarrow \int \frac{\frac{1}{5}}{(y-2)} + \frac{\frac{-2}{5}}{(2y+1)}$$

$$\Rightarrow \frac{1}{5}\log|y-2| - \frac{2}{5}\log|2y+1| \times \frac{1}{2} + C$$

Since y = tan x,

$$\Rightarrow \frac{1}{5}\log|\tan x - 2| - \frac{1}{5}\log|2\tan x + 1| + C$$

$$\Rightarrow \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

Therefore,

$$\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)} = \frac{1}{5}\log\left|\frac{\tan x - 2}{2\tan x + 1}\right| + C$$

38. Question

Evaluate:

$$\int \frac{\left(1-x^2\right)}{\left(1+x^4\right)} dx$$

Answer

To find: $\int \frac{(1-x^2)}{(1+x^4)}\, dx$

Formula used: $\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2} dx$$

$$\Rightarrow \int \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2} + x^2 + 2 - 2} dx$$

$$\Rightarrow \int \frac{-\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2}\,dx$$

Let $y = x + \frac{1}{x}$

Differentiating wrt x,

$$dy\ = \left(1-\frac{1}{x^2}\right)dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{-dy}{y^2-\left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{-1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right|+C$$

Substituting for $y = x + \frac{1}{x}$ and taking reciprocal of the value within logarithm, we get

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{x+\frac{1}{x}+\sqrt{2}}{x+\frac{1}{x}-\sqrt{2}}\right|+C$$

$$\Rightarrow \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+x^2+1}{\sqrt{2}x-x^2+1}\right|+C$$

Therefore,

$$\int \frac{(1-x^2)}{(1+x^4)}\,dx = \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}x+x^2+1}{\sqrt{2}x-x^2+1}\right|+C$$

39. Question

Evaluate:

$$\int \frac{\left(x^2+1\right)}{\left(x^4+x^2+1\right)}dx$$

Answer

To find: $\int \frac{(x^2+1)}{(x^4+x^2+1)}\,dx$

Formula used: $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$

On dividing by x^2 in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

$$\Rightarrow \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $y = x - \frac{1}{x}$

Differentiating wrt x,

$$dy = \left(1 + \frac{1}{x^2}\right) dx$$

Substituting in the original equation,

$$\Rightarrow \int \frac{dy}{y^2 + (\sqrt{3})^2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{\sqrt{3}} + C$$

Substituting for $y = x - \frac{1}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x - \frac{1}{x}}{\sqrt{3}} \right) + C$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

Therefore,

$$\int \frac{(x^2 + 1)}{(x^4 + x^2 + 1)} \, dx = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + C$$

40. Question

Evaluate:

$$\int \frac{dx}{\left(\sin^4 x + \cos^4 x\right)}$$

Answer

To find: $\int \frac{dx}{(\sin^4 x + \cos^4 x)}$

Formula used:

1. $\sec^2 x = 1 + \tan^2 x$

2. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

Dividing by $\cos^4 x$ in the numerator and denominator of the given equation,

$$\Rightarrow \int \frac{\sec^4 x}{(\tan^4 x + 1)} dx$$

$$\Rightarrow \int \frac{\sec^2 x \,(1 + \tan^2 x)}{(1 + \tan^4 x)} dx$$

Let y = tan x

dy = $\sec^2$ x dx

Substituting in the original equation,

$$\Rightarrow \int \frac{1 + y^2}{1 + y^4} dy$$

Dividing by y^2 in the numerator and denominator,

$$\Rightarrow \int \frac{y^{-2}+1}{y^{-2}+y^2} dy$$

$$\Rightarrow \int \frac{1+y^{-2}}{y^2+y^{-2}-2+2}\ dy$$

$$\Rightarrow \int \frac{1+y^{-2}}{\left(y-\frac{1}{y}\right)^2+2} dy$$

Let $z = y - \frac{1}{y}$

$$dz = \left(1+\frac{1}{y^2}\right)\ dy$$

Therefore,

$$\Rightarrow \int \frac{dz}{z^2+\left(\sqrt{2}\right)^2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right)+C$$

Substituting for z,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y-\frac{1}{y}}{\sqrt{2}}\right)+C$$

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{y^2-1}{y\sqrt{2}}\right)+C$$

Substituting for y = tan x,

$$\Rightarrow \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{2}\tan x}\right)+C$$

Therefore,

$$\int \frac{dx}{(\sin^4 x+\cos^4 x)} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x-1}{\sqrt{2}\tan x}\right)+C$$

Exercise 14B

1. Question

Evaluate:

$$\int \frac{\mathrm{dx}}{\sqrt{16-\mathrm{x}^2}}$$

Answer

Formula to be used - $\int \frac{\mathrm{dx}}{\sqrt{\mathrm{a}^2-\mathrm{x}^2}} = \sin^{-1}\frac{\mathrm{x}}{\mathrm{a}} + \mathrm{c}$ where c is the integrating constant

$$\therefore \int \frac{\mathrm{dx}}{\sqrt{16-\mathrm{x}^2}}$$

$$= \int \frac{dx}{\sqrt{4^2 - x^2}}$$

$= \sin^{-1}\frac{x}{4} + c$, c being the integrating constant

2. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1-9x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - 9x^2}}$$

$$= \int \frac{dx}{\sqrt{9\left\{\left(\frac{1}{9}\right) - x^2\right\}}}$$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{1^2 - \left(\frac{x}{3}\right)^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{1}{3}} + c$$

$= \frac{1}{3}\sin^{-1} 3x + c$, c being the integrating constant

3. Question

Evaluate:

$$\int \frac{dx}{\sqrt{15-8x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{15 - 8x^2}}$$

$$= \int \frac{dx}{\sqrt{15\left\{1 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2\right\}}}$$

$$= \frac{1}{\sqrt{15}}\int \frac{dx}{\sqrt{1^2 - \left(\frac{\sqrt{8}}{\sqrt{15}}x\right)^2}}$$

$$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{x}{\left(\frac{\sqrt{15}}{\sqrt{8}}\right)} + c$$

$= \frac{1}{\sqrt{15}}\sin^{-1}\frac{\sqrt{8}}{\sqrt{15}}x + c$, c being the integrating constant

4. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 4}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 4}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2^2}}$$

$= \log|x + \sqrt{x^2 - 4}| + c$, c being the integrating constant

5. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2 - 1}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{4x^2 - 1}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 - 1^2}}$$

$= \frac{1}{2}\log|2x + \sqrt{4x^2 - 1}| + c$, c being the integrating constant

6. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2 - 7}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9x^2 - 7}}$$

$$= \int \frac{dx}{\sqrt{(3x)^2 - \sqrt{7}^2}}$$

$= \log|3x + \sqrt{9x^2 - 7}| + c$, c being the integrating constant

7. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 9}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 9}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3^2}}$$

$= \log|x + \sqrt{x^2 - 9}| + c$, c being the integrating constant

8. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 4x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 1^2}}$$

$= \frac{1}{2}\log|2x + \sqrt{4x^2 + 1}| + c$, c being the integrating constant

9. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9 + 4x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{9 + 4x^2}}$$

$$= \int \frac{dx}{\sqrt{(2x)^2 + 3^2}}$$

$= \frac{1}{2}\log|2x + \sqrt{4x^2 + 9}| + c$, c being the integrating constant

10. Question

Evaluate:

$$\int \frac{x}{\sqrt{9-x^4}} dx$$

Answer

Tip – $d(x^2) = 2xdx$ i.e. $xdx = (1/2)\times d(x^2)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{xdx}{\sqrt{9-x^4}}$$

$$= \frac{1}{2}\int \frac{d(x^2)}{\sqrt{3^2-(x^2)^2}}$$

$= \frac{1}{2}\sin^{-1}\frac{x^2}{3} + c$, c being the integrating constant

11. Question

Evaluate:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx$$

Answer

Tip – $d(x^3) = 3x^2dx$ so, $d(4x^3) = 4\times 3x^2dx$ i.e $3x^2dx = (1/4)d(2x^3)$

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{3x^2dx}{\sqrt{9-16x^6}}$$

$$= \frac{1}{4}\int \frac{d(2x^3)}{\sqrt{3^2-(4x^3)^2}}$$

$= \frac{1}{4}\sin^{-1}\frac{4x^3}{3} + c$, c being the integrating constant

12. Question

Evaluate:

$$\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}} dx$$

Answer

Tip – $d(\tan x) = \sec^2 x dx$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\sec^2 x dx}{\sqrt{16 + \tan^2 x}}$$

$$= \int \frac{d(\tan x)}{\sqrt{4^2 + (\tan x)^2}}$$

$= \log|\tan x + \sqrt{16 + \tan^2 x}| + c$, c being the integrating constant

13. Question

Evaluate:

$$\int \frac{\sin x}{\sqrt{4 + \cos^2 x}} dx$$

Answer

Tip – d(cosx) = - sinxdx i.e. sinxdx = - d(cosx)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\sin x dx}{\sqrt{4 + \cos^2 x}}$$

$$= \int \frac{-d(\cos x)}{\sqrt{(\cos x)^2 + 2^2}}$$

$= -\log|\cos x + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

14. Question

Evaluate:

$$\int \frac{\cos x}{\sqrt{9 \sin^2 x - 1}} dx$$

Answer

Tip – d(sinx) = cosxdx so, d(3sinx) = 3cosxdx i.e. cosxdx = (1/3)d(3sinx)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{\cos x dx}{\sqrt{9 \sin^2 x - 1}}$$

$$= \frac{1}{3} \int \frac{d(3 \sin x)}{\sqrt{(3 \sin x)^2 - 1^2}}$$

$= \frac{1}{3} \log|\cos x + \sqrt{4 + \cos^2 x}| + c$, c being the integrating constant

15. Question

Evaluate:

$$\int \frac{e^x}{\sqrt{4 + e^{2x}}} dx$$

Answer

Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{e^x dx}{\sqrt{4 + e^{2x}}}$$

$$= \int \frac{d(e^x)}{\sqrt{2^2 + (e^x)^2}}$$

$= \log|e^x + \sqrt{4 + e^{2x}}| + c$, c being the integrating constant

16. Question

Evaluate:

$$\int \frac{2e^x}{\sqrt{4 - e^{2x}}} dx$$

Answer

Tip – $d(e^x) = e^x dx$

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{2e^x dx}{\sqrt{4 - e^{2x}}}$$

$$= 2\int \frac{d(e^x)}{\sqrt{2^2 - (e^x)^2}}$$

$= 2\sin^{-1}\left(\frac{e^x}{2}\right) + c$, c being the integrating constant

17. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 - e^x}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 - e^x}}$$

$$= \int \frac{dx}{\sqrt{e^x(e^{-x} - 1)}}$$

$$= \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{e^{-x} - 1}}$$

$$= \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

Tip – Assuming $e^{-(x/2)} = a$, $-(1/2)\, e^{-(x/2)}dx = da$ i.e. $e^{-(x/2)}dx = -2da$

$$\therefore \int \frac{e^{-\frac{x}{2}} dx}{\sqrt{\left(e^{-\frac{x}{2}}\right)^2 - 1^2}}$$

$$= \int \frac{-2da}{\sqrt{a^2 - 1^2}}$$

$= -2\log|a + \sqrt{a^2 - 1}| + c$

$= -2\log|e^{-\frac{x}{2}} + \sqrt{e^{-x} - 1}| + c$, c being the integrating constant

18. Question

Evaluate:

$$\int \sqrt{\frac{a - x}{a + x}} dx$$

Answer

Tip – Taking $x = a\cos 2\theta$,

$dx = -2a\sin 2\theta\, d\theta$ and $\theta = \frac{1}{2}\cos^{-1}\frac{x}{a}$

$x = a\cos 2\theta$ i.e $\cos 2\theta = \frac{x}{a}$

$$\therefore \sin 2\theta = \sqrt{1 - \frac{x^2}{a^2}}$$

$$\therefore \int \sqrt{\frac{a - x}{a + x}} dx$$

$$= \int \sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{a(1 - \cos 2\theta)}{a(1 + \cos 2\theta)}} \times (-2a\sin 2\theta\, d\theta)$$

Formula to be used - $\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\therefore \int \sqrt{\frac{1 - \cos 2\theta}{1 + \cos 2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} \times (-2a\sin 2\theta\, d\theta)$$

$$= \int \frac{\sin\theta}{\cos\theta} \times (-2a \times 2\sin\theta\cos\theta\, d\theta)$$

$$= -2a\int 2\sin^2\theta\, d\theta$$

$$= -2a\int 1 - \cos 2\theta\, d\theta$$

$$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= -2a\left[\theta - \frac{\sin 2\theta}{2}\right] + c$$

$$= -2a\left[\frac{1}{2}\cos^{-1}\frac{x}{a} - \frac{\sqrt{1-\frac{x^2}{a^2}}}{2}\right] + c$$

$$= -a\cos^{-1}\frac{x}{a} + a\sqrt{1-\frac{x^2}{a^2}} + c$$

$= a\sin^{-1}\frac{x}{a} + \sqrt{a^2 - x^2} + c$, c being the integrating constant

19. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 + 6x + 5}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 + 6x + 5}}$$

$$= \int \frac{dx}{\sqrt{(x^2 + 2 \times x \times 3 + 3^2) + 5 - 3^2}}$$

$$= \int \frac{dx}{\sqrt{(x + 3)^2 - 2^2}}$$

$= \log|(x + 3) + \sqrt{x^2 + 6x + 5}| + c$, c being the integrating constant

20. Question

Evaluate:

$$\int \frac{dx}{\sqrt{(2-x)^2 + 1}}$$

Answer

Tip – d(2 - x) = - dx i.e. dx = - d(2 - x)

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(2-x)^2 + 1}}$$

$$= \int \frac{-d(2-x)}{\sqrt{(2-x)^2 + 1}}$$

$$= -\log|(2-x) + \sqrt{(2-x)^2 + 1}| + c$$

$= -\log|(2-x) + \sqrt{x^2 - 4x + 5}| + c$, c being the integrating constant

21. Question

Evaluate:

$$\int \frac{dx}{\sqrt{(x-3)^2+1}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

22. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2-6x+10}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 6x + 10}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$= \log|(x-3) + \sqrt{(x-3)^2 + 1}| + c$$

$= \log|(x-3) + \sqrt{x^2 - 6x + 10}| + c$, c being the integrating constant

23. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2+2x-x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{3 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{3}\right)^2 - (x-1)^2}}$$

$= \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c$, c being the integrating constant

24. Question

Evaluate:

$$\int \frac{dx}{\sqrt{8-4x-2x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{8-4x-2x^2}}$$

$$= \int \frac{dx}{\sqrt{10-2(x^2+2x+1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{10}\right)^2-2(x+1)^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2-(x+1)^2}}$$

$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c$, c being the integrating constant

25. Question

Evaluate:

$$\int \frac{dx}{\sqrt{16-6x-x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{16-6x-x^2}}$$

$$= \int \frac{dx}{\sqrt{25-(x^2+6x+9)}}$$

$$= \int \frac{dx}{\sqrt{(5)^2-(x+3)^2}}$$

$= \sin^{-1}\left(\frac{x+3}{5}\right) + c$, c being the integrating constant

26. Question

Evaluate:

$$\int \frac{dx}{\sqrt{7-6x-x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{7 - 6x - x^2}}$$

$$= \int \frac{dx}{\sqrt{16 - (x^2 + 6x + 9)}}$$

$$= \int \frac{dx}{\sqrt{(4)^2 - (x + 3)^2}}$$

$= \sin^{-1}\left(\frac{x+3}{4}\right) + c$, c being the integrating constant

27. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x - x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x - x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x^2 - 2 \times x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - (x - \frac{1}{2})^2}}$$

$$= \sin^{-1}\left(\frac{x - \frac{1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x - 1) + c$, c being the integrating constant

28. Question

Evaluate:

$$\int \frac{dx}{\sqrt{8 + 2x - x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{8 + 2x - x^2}}$$

$$= \int \frac{dx}{\sqrt{9 - (x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{(3)^2 - (x-1)^2}}$$

$= \sin^{-1}\left(\frac{x-1}{3}\right) + c$, c being the integrating constant

29. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{x^2 - 3x + 2}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 2 \times x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2}}$$

$$= \int \frac{dx}{\sqrt{(x - \frac{3}{2})^2 - \frac{1}{4}}}$$

$= \log|(x - \frac{3}{2}) + \sqrt{x^2 - 3x + 2}| + c$, c being the integrating constant

30. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 3x - 2}}$$

$$= \int \frac{dx}{\sqrt{2\left(x^2 + 2 \times x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2\right) - \frac{7}{8}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + \frac{3}{4})^2 - \left(\frac{\sqrt{7}}{4}\right)^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + \frac{3}{4}) + \sqrt{2x^2 + 3x - 2}| + c$, c being the integrating constant

31. Question

Evaluate:

$$\int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{2x^2 + 4x + 6}}$$

$$= \int \frac{dx}{\sqrt{2(x^2 + 2x + 1) + 4}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{\sqrt{2}} \log|(x + 1) + \sqrt{2x^2 + 4x + 6}| + c$, c being the integrating constant

32. Question

Evaluate:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{1 + 2x - 3x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(1 - \frac{1}{3}\right) - 3\left(x^2 - 2 \times x \times \frac{1}{3} + \left(\frac{1}{3}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - 3\left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{\left(\frac{\sqrt{2}}{3}\right)^2 - \left(x - \frac{1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{x - \frac{1}{3}}{\frac{\sqrt{2}}{3}}\right) + c$$

$= \frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{3x-1}{\sqrt{2}}\right) + c$, c being the integrating constant

33. Question

Evaluate:

$$\int \frac{dx}{\sqrt{x}\sqrt{5-x}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{5x - x^2}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x^2 - 2 \times x \times \frac{5}{2} + \left(\frac{5}{2}\right)^2\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{5}{2}\right)^2 - \left(x - \frac{5}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x - \frac{5}{2}}{\frac{5}{2}}\right) + c$$

$= \sin^{-1}\left(\frac{2x-5}{5}\right) + c$, c being the integrating constant

34. Question

Evaluate:

$$\int \frac{dx}{\sqrt{3 + 4x - 2x^2}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{dx}{\sqrt{3 + 4x - 2x^2}}$$

$$= \int \frac{dx}{\sqrt{5 - 2(x^2 - 2x + 1)}}$$

$$= \int \frac{dx}{\sqrt{\left(\sqrt{5}\right)^2 - 2(x - 1)^2}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\left(\sqrt{\frac{5}{2}}\right)^2 - (x - 1)^2}}$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x - 1}{\sqrt{\frac{5}{2}}}\right) + c$$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{\sqrt{2}(x-5)}{\sqrt{5}}\right) + c$, c being the integrating constant

35. Question

Evaluate:

$$\int \frac{x^2}{\sqrt{x^6+2x^3+3}}dx$$

Answer

Tip – $d(x^3) = 3x^2dx$ i.e. $x^2dx = (1/3)d(x^3)$

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{x^2 dx}{\sqrt{x^6 + 2x^3 + 3}}$$

$$= \int \frac{\frac{1}{3}d(x^3)}{\sqrt{(x^3)^2 + 2x^3 + 3}}$$

$$= \frac{1}{3}\int \frac{d(x^3)}{\sqrt{(x^3 + 1)^2 + (\sqrt{2})^2}}$$

$= \frac{1}{3}\log|(x^3 + 1) + \sqrt{x^6 + 2x^3 + 3}| + c$, c being the integrating constant

36. Question

Evaluate:

$$\int \frac{(2x+3)}{\sqrt{x^2+x+1}}dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(2x + 3)}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{(2x + 1) + 2}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}}dx + \int \frac{2}{\sqrt{x^2 + x + 1}}dx$$

Tip – Assuming $x^2 + x + 1 = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}}dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{x^2 + x + 1} + c_1$$

$$\therefore \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

$$= 2 \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

$$= 2 \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c_2$$

$$\therefore \int \frac{(2x + 1)}{\sqrt{x^2 + x + 1}} dx + \int \frac{2}{\sqrt{x^2 + x + 1}} dx$$

$= 2\sqrt{x^2 + x + 1} + 2\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + 1}\right| + c$, c is the integrating constant

37. Question

Evaluate:

$$\int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}} dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(5x + 3)}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \int \frac{\frac{5}{2} \times (2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

Tip – Assuming $x^2 + 4x + 10 = a^2$, $(2x + 4)dx = 2ada$

$$\therefore \frac{5}{2} \int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}} dx$$

$$= \frac{5}{2} \int \frac{2ada}{a}$$

$$= \frac{5}{2} \int 2da$$

$$= 5a + c_1$$

$$= 5\sqrt{x^2 + 4x + 10} + c_1$$

$$\therefore \int \frac{7}{\sqrt{x^2 + 4x + 10}} dx$$

$$= 7 \int \frac{dx}{\sqrt{(x + 2)^2 + (\sqrt{6})^2}}$$

$$= 7 \log\left|(x + 2) + \sqrt{x^2 + 4x + 10}\right| + c_2$$

$$\therefore \frac{5}{2}\int \frac{(2x + 4)}{\sqrt{x^2 + 4x + 10}}dx - \int \frac{7}{\sqrt{x^2 + 4x + 10}}dx$$

$= 5\sqrt{x^2 + 4x + 10} - 7\log\left|(x + 2) + \sqrt{x^2 + 4x + 10}\right| + c$, c is the integrating constant

38. Question

Evaluate:

$$\int \frac{(4x+3)}{\sqrt{2x^2+2x-3}}$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(4x + 3)}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{(4x + 2) + 1}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}}dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}}dx$$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{2ada}{a}$$

$$= \int 2da$$

$$= 2a + c_1$$

$$= 2\sqrt{2x^2 + 2x - 3} + c_1$$

$$\therefore \int \frac{1}{\sqrt{2x^2 + 2x - 3}}dx$$

$$= \int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c_2$$

$$\therefore \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}}dx + \int \frac{1}{\sqrt{2x^2 + 2x - 3}}dx$$

$= 2\sqrt{2x^2 + 2x - 3} + \frac{1}{\sqrt{2}}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c$, c is the integrating constant

39. Question

Evaluate:

$$\int \frac{(3-2x)}{\sqrt{2+x-x^2}} dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3-2x)}{\sqrt{2 + x - x^2}} dx$$

$$= \int \frac{(1-2x) + 2}{\sqrt{2 + x - x^2}} dx$$

$$= \int \frac{(1-2x)}{\sqrt{2 + x - x^2}} dx + \int \frac{2}{\sqrt{2 + x - x^2}} dx$$

Tip – Assuming $2 + x - x^2 = a^2$, $(1 - 2x)dx = 2ada$

$$\therefore \int \frac{(1-2x)}{\sqrt{2 + x - x^2}} dx$$

$$= \int \frac{2ada}{a}$$

$$= 2a + c_1$$

$$= 2\sqrt{2 + x - x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{2 + x - x^2}} dx$$

$$= 2\int \frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$$

$$= 2\sin^{-1}\frac{\left(x - \frac{1}{2}\right)}{\left(\frac{3}{2}\right)} + c_2$$

$$= 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c_2$$

$$\therefore \int \frac{(1-2x)}{\sqrt{2 + x - x^2}} dx + \int \frac{2}{\sqrt{2 + x - x^2}} dx$$

$= 2\sqrt{2 + x - x^2} + 2\sin^{-1}\left(\frac{2x-1}{3}\right) + c$, c is the integrating constant

40. Question

Evaluate:

$$\int \frac{(x+2)}{\sqrt{2x^2+2x-3}} dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\therefore \int \frac{(x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \int \frac{\frac{1}{4} \times (4x + 2) + \frac{3}{2}}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \frac{1}{4} \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \frac{3}{2} \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

Tip – Assuming $2x^2 + 2x - 3 = a^2$, $(4x + 2)dx = 2ada$

$$\therefore \frac{1}{4} \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \frac{1}{4} \int \frac{2ada}{a}$$

$$= \frac{1}{2} \int da$$

$$= \frac{a}{2} + c_1$$

$$= \frac{\sqrt{2x^2 + 2x - 3}}{2} + c_1$$

$$\therefore \frac{3}{2} \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$$= \frac{3}{2} \int \frac{dx}{\sqrt{2\left(x + \frac{1}{2}\right)^2 - \left(\sqrt{\frac{7}{2}}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}} \int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{7}}{2}\right)^2}}$$

$$= \frac{3}{2\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c_2$$

$$\therefore \frac{1}{4} \int \frac{(4x + 2)}{\sqrt{2x^2 + 2x - 3}} dx + \frac{3}{2} \int \frac{1}{\sqrt{2x^2 + 2x - 3}} dx$$

$= \frac{\sqrt{2x^2 + 2x - 3}}{2} + \frac{3}{2\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x - \frac{3}{2}}\right| + c$, c is the integrating constant

41. Question

Evaluate:

$$\int \frac{(3x + 1)}{\sqrt{5 - 2x - x^2}} dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(3x + 1)}{\sqrt{5-2x-x^2}}dx$$

$$= \int \frac{3(x + 1) - 2}{\sqrt{5-2x-x^2}}dx$$

$$= \int \frac{3(x + 1)}{\sqrt{5-2x-x^2}}dx - \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

Tip – Assuming 5 - 2x - x^2 = a^2, (- 2 - 2x)dx = 2ada i.e. (x + 1)dx = - ada

$$\therefore \int \frac{3(x + 1)}{\sqrt{5-2x-x^2}}dx$$

$$= -3\int \frac{ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{5-2x-x^2} + c_1$$

$$\therefore \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

$$= 2\int \frac{dx}{\sqrt{\left(\sqrt{6}\right)^2 - (x + 1)^2}}$$

$$= 2\sin^{-1}\frac{(x + 1)}{\sqrt{6}} + c_2$$

$$\therefore \int \frac{3(x + 1)}{\sqrt{5-2x-x^2}}dx - \int \frac{2}{\sqrt{5-2x-x^2}}dx$$

$= -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$, c is the integrating constant

42. Question

Evaluate:

$$\int \frac{(6x+5)}{\sqrt{6+x-2x^2}}dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ where c is the integrating constant

$$\therefore \int \frac{(6x + 5)}{\sqrt{6 + x - 2x^2}}dx$$

$$= \int \frac{\frac{6}{4}(4x-1) + \frac{13}{2}}{\sqrt{6 + x - 2x^2}}dx$$

$$= \frac{3}{2}\int \frac{(4x-1)}{\sqrt{6 + x - 2x^2}}dx + \frac{13}{2}\int \frac{1}{\sqrt{6 + x - 2x^2}}dx$$

Tip – Assuming $6 + x - 2x^2 = a^2$, $(1 - 4x)dx = 2ada$ i.e. $(4x - 1)dx = -2ada$

$$\therefore \frac{3}{2}\int \frac{(4x-1)}{\sqrt{6+x-2x^2}}dx$$

$$= -\frac{3}{2}\int \frac{2ada}{a}$$

$$= -3a + c_1$$

$$= -3\sqrt{6+x-2x^2} + c_1$$

$$\therefore \frac{13}{2}\int \frac{1}{\sqrt{6+x-2x^2}}dx$$

$$= \frac{13}{2}\int \frac{dx}{\sqrt{\left(\frac{7}{2\sqrt{2}}\right)^2 - 2\left(x-\frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}}\int \frac{dx}{\sqrt{\left(\frac{7}{4}\right)^2 - \left(x-\frac{1}{4}\right)^2}}$$

$$= \frac{13}{2\sqrt{2}}\sin^{-1}\frac{\left(x-\frac{1}{4}\right)}{\left(\frac{7}{4}\right)} + c_2$$

$$= \frac{13}{2\sqrt{2}}\sin^{-1}\left(\frac{4x-1}{7}\right) + c_2$$

$$\therefore \frac{3}{2}\int \frac{(4x-1)}{\sqrt{6+x-2x^2}}dx + \frac{13}{2}\int \frac{1}{\sqrt{6+x-2x^2}}dx$$

$= -3\sqrt{6+x-2x^2} + \frac{13}{2\sqrt{2}}\sin^{-1}\left(\frac{4x-1}{7}\right) + c$, c is the integrating constant

43. Question

Evaluate:

$$\int\sqrt{\frac{1+x}{x}}dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int\sqrt{\frac{1+x}{x}}dx$$

$$= \int\sqrt{\frac{(1+x)^2}{x(1+x)}}dx$$

$$= \int \frac{1+x}{\sqrt{x^2+x}}dx$$

$$= \int \frac{\frac{1}{2}(2x+1) + \frac{1}{2}}{\sqrt{x^2+x}}dx$$

$$= \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

Tip – Taking $x^2 + x = a^2$, $(2x + 1)dx = 2ada$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx$$

$$= \frac{1}{2}\int \frac{2ada}{a}$$

$$= a + c_1$$

$$= \sqrt{x^2 + x} + c_1$$

$$\therefore \frac{1}{2}\int \frac{1}{\sqrt{x^2 + x}}dx$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c_2$$

$$\therefore \frac{1}{2}\int \frac{2x + 1}{\sqrt{x^2 + x}}dx + \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + x}}$$

$= \sqrt{x^2 + x} + \frac{1}{2}\log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x}\right| + c$, c is the integrating constant

44. Question

Evaluate:

$$\int \frac{(x+2)}{\sqrt{x^2+5x+6}}dx$$

Answer

Formula to be used - $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}) + c$ where c is the integrating constant

$$\int \frac{(x + 2)}{\sqrt{x^2 + 5x + 6}}dx$$

$$= \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}}dx$$

$$= \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$$

Tip – Taking $x^2 + 5x + 6 = a^2$, $(2x + 5)dx = 2ada$

$$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx$$

$$= \frac{1}{2}\int \frac{2ada}{a}$$

$$= a + c_1$$

$= \sqrt{x^2 + 5x + 6} + c_1$

$\therefore -\frac{1}{2}\int \frac{1}{\sqrt{x^2 + 5x + 6}}dx$

$= -\frac{1}{2}\int \frac{dx}{\sqrt{\left(x + \frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$= -\frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c_2$

$\therefore \frac{1}{2}\int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}dx - \frac{1}{2}\int \frac{dx}{\sqrt{x^2 + 5x + 6}}$

$= \sqrt{x^2 + 5x + 6} - \frac{1}{2}\log\left|\left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6}\right| + c$, c is the integrating constant

Exercise 14C

1. Question

Evaluate the following integrals:

$\int \sqrt{4 - x^2}\,dx$

Answer

To Find : $\int \sqrt{4 - x^2}\,dx$

Now, $\int \sqrt{4 - x^2}\,dx$ can be written as $\int \sqrt{2^2 - x^2}\,dx$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{2^2 - x^2}\,dx$ is of the form $\int \sqrt{a^2 - x^2}\,dx$,

Hence, $\int \sqrt{2^2 - x^2}\,dx = \frac{1}{2}x\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{1}{2}x\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{1}{2}x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$

Therefore, $\int \sqrt{4 - x^2}\,dx = \frac{1}{2}x\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$

2. Question

Evaluate the following integrals:

$\int \sqrt{4 - 9x^2}\,dx$

Answer

To Find : $\int \sqrt{4 - 9x^2}\,dx$

Now, $\int \sqrt{4 - 9x^2}\,dx$ can be written as $\int \sqrt{2^2 - (3x)^2}\,dx$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Since $\int \sqrt{2^2 - (3x)^2}\,dx$ is of the form $\int \sqrt{a^2 - x^2}\,dx$, Hence, $\int \sqrt{2^2-(3x)^2}\,dx = \frac{1}{2}(3x)\sqrt{2^2-(3x)^2} + \frac{2^2}{2}\sin^{-1}\frac{3x}{2}$ + C

$= \frac{x}{2}\sqrt{4-9x^2} + \frac{4}{6}\sin^{-1}\frac{3x}{2}$ + C

$= \frac{x}{2}\sqrt{4-9x^2} + \frac{2}{3}\sin^{-1}\frac{3x}{2}$ + C

Therefore, $\int \sqrt{4-9x^2}\,dx = \frac{x}{2}\sqrt{4-9x^2} + \frac{2}{3}\sin^{-1}\frac{3x}{2}$ + C

3. Question

Evaluate the following integrals:

$$\int\sqrt{x^2-2}dx$$

Answer

To Find : $\int \sqrt{x^2-2}\,dx$

Now, $\int \sqrt{x^2-2}\,dx$ can be written as $\int \sqrt{x^2-(\sqrt{2})^2}\,dx$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2-a^2}$|+ C

Since $\int \sqrt{x^2-(\sqrt{2})^2}\,dx$ is of the form $\int \sqrt{x^2-a^2}\,dx$,

Hence, $\int \sqrt{x^2-(\sqrt{2})^2}\,dx = \frac{x}{2}\sqrt{x^2-(\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2}$ log |x + $\sqrt{x^2-(\sqrt{2})^2}$|+ C

$= \frac{x}{2}\sqrt{x^2-2} - \frac{2}{2}$ log |x + $\sqrt{x^2-2}$|+ C

$= \frac{x}{2}\sqrt{x^2-2}$ − log |x + $\sqrt{x^2-2}$|+ C

Therefore, $\int \sqrt{x^2-2}\,dx = \frac{x}{2}\sqrt{x^2-2}$ − log |x + $\sqrt{x^2-2}$|+ C

4. Question

Evaluate the following integrals:

$$\int\sqrt{2x^2-3}dx$$

Answer

To Find : $\int\sqrt{2x^2-3}dx$

Now, $\int \sqrt{2x^2-3}\,dx$ can be written as $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2-a^2}$|+ C

Since $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx$ is of the form $\int \sqrt{x^2-a^2}\,dx$,

Hence, $\int \sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}\,dx = \frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2} - \frac{(\sqrt{3})^2}{2}$ log |$\sqrt{2}$x + $\sqrt{(\sqrt{2}x)^2-(\sqrt{3})^2}$ |+C

$= \frac{\sqrt{2}x}{2}\sqrt{2x^2-3} - \frac{3}{2}$ log |$\sqrt{2}$x + $\sqrt{2x^2-3}$|+ C

$= \frac{x}{2}\sqrt{2x^2-3} - \frac{3}{2\sqrt{2}}$ log $|\sqrt{2}x + \sqrt{2x^2-3}|$+ C

Therefore, $\int \sqrt{2x^2-3}\,dx = \frac{x}{2}\sqrt{2x^2-3} - \frac{3}{2\sqrt{2}}$ log $|\sqrt{2}x + \sqrt{2x^2-3}|$+ C

5. Question

Evaluate the following integrals:

$$\int\sqrt{x^2+5}dx$$

Answer

To Find : $\int \sqrt{x^2+5}\,dx$

Now, $\int \sqrt{x^2+5}\,dx$ can be written as $\int \sqrt{x^2+(\sqrt{5})^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}$ log $|x + \sqrt{x^2+a^2}|$+ C

Since $\int \sqrt{x^2+(\sqrt{5})^2}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$,

Hence, $\int \sqrt{x^2+(\sqrt{5})^2}\,dx = \frac{x}{2}\sqrt{x^2+(\sqrt{5})^2} + \frac{(\sqrt{5})^2}{2}$ log $|x + \sqrt{x^2+(\sqrt{5})^2}|$+ C

$= \frac{x}{2}\sqrt{x^2+5} + \frac{5}{2}$ log $|x + \sqrt{x^2+5}|$+ C

Therefore, $\int \sqrt{x^2+5}\,dx = \frac{x}{2}\sqrt{x^2+5} + \frac{5}{2}$ log $|x + \sqrt{x^2+5}|$+ C

6. Question

Evaluate the following integrals:

$$\int\sqrt{4x^2+9}dx$$

Answer

To Find : $\int \sqrt{4x^2+9}\,dx$

Now, $\int \sqrt{4x^2+9}\,dx$ can be written as $\int \sqrt{(2x)^2+3^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}$ log $|x + \sqrt{x^2+a^2}|$+ C

Since $\int \sqrt{(2x)^2+3^2}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$,

Hence, $\int \sqrt{(2x)^2+3^2}\,dx = \frac{2x}{2}\sqrt{(2x)^2+3^2} + \frac{3^2}{2}$ log $|2x + \sqrt{(2x)^2+3^2}|$+ C

$= \frac{2x}{2}\sqrt{4x^2+9} + \frac{9}{2}$ log $|2x + \sqrt{4x^2+9}\,|$ + C

$= \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}$ log $|2x + \sqrt{4x^2+9}|$+ C

Therefore, $\int \sqrt{4x^2+9}\,dx = \frac{x}{2}\sqrt{4x^2+9} + \frac{9}{4}$ log $|2x + \sqrt{4x^2+9}|$+ C

7. Question

Evaluate the following integrals:

$$\int \sqrt{3x^2+4}dx$$

Answer

To Find : $\int \sqrt{3x^2+4}\, dx$

Now, $\int \sqrt{3x^2+4}\, dx$ can be written as $\int \sqrt{(\sqrt{3}x)^2+2^2}\, dx$

Formula Used: $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}$ log |x + $\sqrt{x^2+a^2}$|+ C

Since $\int \sqrt{(\sqrt{3}x)^2+2^2}\, dx$ is of the form $\int \sqrt{x^2+a^2}\, dx$,

Hence, $\int \sqrt{(\sqrt{3}x)^2+2^2}\, dx = \frac{\sqrt{3}x}{2}\sqrt{(\sqrt{3}x)^2+2^2} + \frac{2^2}{2}$ log |$\sqrt{3}$x + $\sqrt{(\sqrt{3}x)^2+2^2}$|+ C

$= \frac{\sqrt{3}x}{2}\sqrt{3x^2+4} + \frac{4}{2}$ log |$\sqrt{3}$x + $\sqrt{3x^2+4}$ | + C

$= \frac{x}{2}\sqrt{3x^2+4} + \frac{2}{\sqrt{3}}$ log |$\sqrt{3}$x + $\sqrt{3x^2+4}$|+ C

Therefore, $\int \sqrt{3x^2+4}\, dx = \frac{x}{2}\sqrt{3x^2+4} + \frac{2}{\sqrt{3}}$ log |$\sqrt{3}$x + $\sqrt{3x^2+4}$|+ C

8. Question

Evaluate the following integrals:

$$\int \cos x\sqrt{9-\sin^2 x}dx$$

Answer

To Find : $\int cosx\sqrt{9-\sin^2 x}\, dx$

Now, let sin x = t

⇒cosx dx = dt

Therefore, $\int cosx\sqrt{9-\sin^2 x}\, dx$ can be written as $\int \sqrt{3^2-t^2}\, dt$

Formula Used: $\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since , $\int \sqrt{3^2-t^2}\, dt$ is in the form of $\int \sqrt{a^2-x^2}\, dx$ with t as a variable instead of x .

⇒ $\int \sqrt{3^2-t^2}\, dt = \frac{1}{2}t\sqrt{3^2-t^2} + \frac{3^2}{2}\sin^{-1}\frac{t}{3}$ + C

$= \frac{t}{2}\sqrt{9-t^2} + \frac{9}{2}\sin^{-1}\frac{t}{3}$ + C

Now since sin x = t and cosx dx = dt

⇒ $\int cosx\sqrt{9-\sin^2 x}\, dx = \frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}(\frac{\sin x}{3})$ + C

9. Question

Evaluate the following integrals:

$$\int \sqrt{x^2-4x+2}dx$$

Answer

To Find : $\int \sqrt{x^2 - 4x + 2}\, dx$

Now, $\int \sqrt{x^2 - 4x + 2}\, dx$ can be written as $\int \sqrt{x^2 - 4x + 2^2 - 2^2 + 2}\, dx$

i.e., $\int \sqrt{(x-2)^2 - 2}\, dx$

Here , let x – 2 = y ⇒ dx = dy

Therefore, $\int \sqrt{(x-2)^2 - 2}\, dx$ can be written as $\int \sqrt{y^2 - (\sqrt{2})^2}\, dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2 - a^2}$|+ C

Since $\int \sqrt{y^2 - (\sqrt{2})^2}\, dy$ is of the form $\int \sqrt{x^2 - a^2}\, dx$ with change in variable.

⇒ $\int \sqrt{y^2 - (\sqrt{2})^2}\, dy = \frac{y}{2}\sqrt{y^2 - (\sqrt{2})^2} - \frac{(\sqrt{2})^2}{2}$ log |y + $\sqrt{y^2 - (\sqrt{2})^2}$|+ C

$= \frac{y}{2}\sqrt{y^2 - 2} - \frac{4}{2}$ log |y + $\sqrt{y^2 - 2}$|+ C

$= \frac{y}{2}\sqrt{y^2 - 2} -$ 2 log |y + $\sqrt{y^2 - 2}$|+ C

Since , x – 2 = y and dx = dy

⇒ $\int \sqrt{(x-2)^2 - 2}\, dx = \frac{(x-2)}{2}\sqrt{(x-2)^2 - 2} -$ 2 log |(x-2) + $\sqrt{(x-2)^2 - 2}$|+C Therefore,

$\int \sqrt{x^2 - 4x + 2}\, dx = \frac{(x-2)}{2}\sqrt{x^2 - 4x + 2} -$ 2 log |(x - 2) + $\sqrt{x^2 - 4x + 2}$|+ C

10. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + 6x - 4}\, dx$$

Answer

To Find : $\int \sqrt{x^2 + 6x - 4}\, dx$

Now, $\int \sqrt{x^2 + 6x - 4}\, dx$ can be written as $\int \sqrt{x^2 + 6x + 3^2 - 3^2 - 4}\, dx$

i.e, $\int \sqrt{(x+3)^2 - 13}\, dx$

Here , let x + 3 = y ⇒ dx = dy

Therefore, $\int \sqrt{(x+3)^2 - 13}\, dx$ can be written as $\int \sqrt{y^2 - (\sqrt{13})^2}\, dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2 - a^2}$|+ C

Since $\int \sqrt{y^2 - (\sqrt{13})^2}\, dy$ is of the form $\int \sqrt{x^2 - a^2}\, dx$ with change in variable.

⇒ $\int \sqrt{y^2 - (\sqrt{13})^2}\, dy = \frac{y}{2}\sqrt{y^2 - (\sqrt{13})^2} - \frac{(\sqrt{13})^2}{2}$ log |y + $\sqrt{y^2 - (\sqrt{13})^2}$|+ C

$= \frac{y}{2}\sqrt{y^2 - 13} - \frac{13}{2}$ log |y + $\sqrt{y^2 - 13}$|+ C

Since , x + 3 = y and dx = dy

$\Rightarrow \int \sqrt{(x+3)^2 - 13}\, dx = \frac{(x+3)}{2}\sqrt{(x+3)^2 - 13} - \frac{13}{2}$ log |(x + 3) + $\sqrt{(x+3)^2 - 13}$|+C

Therefore,

$\int \sqrt{x^2 + 6x - 4}\, dx = \frac{(x+3)}{2}\sqrt{x^2 + 6x - 4} - \frac{13}{2}$ log |(x + 3) + $\sqrt{x^2 + 6x - 4}$|+C

11. Question

Evaluate the following integrals:

$$\int \sqrt{2x - x^2} dx$$

Answer

To Find : $\int \sqrt{2x - x^2}$ dx

Now, $\int \sqrt{2x - x^2}\, dx$ can be written as $\int \sqrt{2x - x^2 - 1^2 + 1^2}\, dx$

i.e, $\int \sqrt{1 - (x-1)^2}\, dx$

Let x – 1 = y ⇒ dx = dy

Therefore , $\int \sqrt{1 - (x-1)^2}\, dx$ becomes $\int \sqrt{1^2 - y^2}\, dy$

Formula Used: $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{1^2 - y^2}\, dy$ is of the form $\int \sqrt{a^2 - x^2}\, dx$ with change in variable,

Hence $\int \sqrt{1^2 - y^2}\, dy = \frac{1}{2}y\sqrt{1^2 - y^2} + \frac{1^2}{2}\sin^{-1}\frac{y}{1}$ + C

$= \frac{y}{2}\sqrt{1 - y^2} + \frac{1}{2}\sin^{-1}\frac{y}{1}$ + C

Here we have x – 1 = y and dx = dy

$\Rightarrow \int \sqrt{1 - (x-1)^2}\, dx = \frac{(x-1)}{2}\sqrt{1 - (x-1)^2} + \frac{1}{2}\sin^{-1}\frac{(x-1)}{1}$ + C

Therefore , $\int \sqrt{2x - x^2}\, dx = \frac{(x-1)}{2}\sqrt{2x - x^2} + \frac{1}{2}\sin^{-1}(x-1)$ + C

12. Question

Evaluate the following integrals:

$$\int \sqrt{1 - 4x - x^2} dx$$

Answer

To Find : $\int \sqrt{1 - 4x - x^2}$ dx

Now, $\int \sqrt{1 - 4x - x^2}\, dx$ can be written as $\int \sqrt{1 - 4x - x^2 - 2^2 + 2^2}\, dx$

i.e, $\int \sqrt{5 - (x+2)^2}\, dx$

Let x + 2 = y ⇒ dx = dy

Therefore , $\int \sqrt{5 - (x+2)^2}\, dx$ becomes $\int \sqrt{(\sqrt{5})^2 - y^2}\, dy$

Formula Used: $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{(\sqrt{5})^2 - y^2}\,dy$ is of the form $\int \sqrt{a^2 - x^2}\,dx$ with change in variable,

Hence $\int \sqrt{(\sqrt{5})^2 - y^2}\,dy = \frac{1}{2}y\sqrt{(\sqrt{5})^2 - y^2} + \frac{(\sqrt{5})^2}{2}\sin^{-1}\frac{y}{\sqrt{5}}$ + C

$= \frac{y}{2}\sqrt{5 - y^2} + \frac{5}{2}\sin^{-1}\frac{y}{\sqrt{5}}$ + C

Here we have x + 2 = y and dx = dy

$\Rightarrow\ \int \sqrt{5 - (x+2)^2}\,dx = \frac{(x+2)}{2}\sqrt{5 - (x+2)^2} + \frac{5}{2}\sin^{-1}(\frac{x+2}{\sqrt{5}})$ + C

Therefore , $\int \sqrt{1 - 4x - x^2}\,dx = \frac{(x+2)}{2}\sqrt{1 - 4x - x^2} + \frac{5}{2}\sin^{-1}(\frac{x+2}{\sqrt{5}})$ + C

13. Question

Evaluate the following integrals:

$$\int\sqrt{2ax - x^2}\,dx$$

Answer

To Find : $\int \sqrt{2ax - x^2}$ dx

Now, $\int \sqrt{2ax - x^2}\,dx$ can be written as $\int \sqrt{2ax - x^2 - a^2 + a^2}\,dx$

i.e, $\int \sqrt{a^2 - (x-a)^2}\,dx$

Let x - a = y $\Rightarrow$ dx = dy

Therefore , $\int \sqrt{a^2 - (x-a)^2}\,dx$ becomes $\int \sqrt{a^2 - y^2}\,dy$

Formula Used: $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{a^2 - y^2}\,dy$ is of the form $\int \sqrt{a^2 - x^2}\,dx$ with change in variable,

Hence $\int \sqrt{a^2 - y^2}\,dy = \frac{1}{2}y\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}$ + C

$= \frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}$ + C

Here we have x - a = y and dx = dy

$\Rightarrow\ \int \sqrt{a^2 - (x-a)^2}\,dx = \frac{(x-a)}{2}\sqrt{a^2 - (x-a)^2} + \frac{a^2}{2}\sin^{-1}(\frac{x-a}{a})$ + C

Therefore , $\int \sqrt{2ax - x^2}\,dx = \frac{(x-a)}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}(\frac{x-a}{a})$ + C

14. Question

Evaluate the following integrals:

$$\int\sqrt{2x^2 + 3x + 4dx}$$

Answer

To Find : $\int \sqrt{2x^2 + 3x + 4dx}$

Now , consider $\int \sqrt{2x^2 + 3x + 4dx} = \int \sqrt{2[x^2 + \frac{3}{2}x + 2]dx}$

$= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + 2}dx$

$= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + (\frac{3}{4})^2 - \left(\frac{3}{4}\right)^2 + 2}dx$

$= \sqrt{2} \int \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}dx$

Let x + $\frac{3}{4}$ = y ⇒dx = dy

Hence $\sqrt{2} \int \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}dx$ becomes $\sqrt{2} \int \sqrt{y^2 + (\frac{\sqrt{23}}{4})^2}dy$

Formula Used: $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}$ log |x + $\sqrt{x^2 + a^2}$|+ C

Now consider $\int \sqrt{y^2 + (\frac{\sqrt{23}}{4})^2}dy$ which is in the form of $\int \sqrt{x^2 + a^2}\, dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2 + (\frac{\sqrt{23}}{4})^2}dy = \frac{y}{2}\sqrt{y^2 + (\frac{\sqrt{23}}{4})^2} + \frac{(\frac{\sqrt{23}}{4})^2}{2}$ log |y + $\sqrt{y^2 + (\frac{\sqrt{23}}{4})^2}$|+ C

$= \frac{y}{2}\sqrt{y^2 + \frac{23}{16}} + \frac{23}{32}$ log |y + $\sqrt{y^2 + \frac{23}{16}}$|+ C

Since x + $\frac{3}{4}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}dx = \frac{1}{8}(4x + 3)\sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}} + \frac{23}{32}$ log $|x + \frac{3}{4} + \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}|$+ C

Now , $\sqrt{2} \int \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}dx = \frac{\sqrt{2}}{8}(4x + 3)\sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}} + \frac{23\sqrt{2}}{32}$ log $|x + \frac{3}{4} + \sqrt{(x + \frac{3}{4})^2 + \frac{23}{16}}|$+ C

Therefore,

$\int \sqrt{2x^2 + 3x + 4}dx = \frac{1}{8}(4x + 3)\sqrt{2x^2 + 3x + 4} + \frac{23}{32}$ log |($x + \frac{3}{4}$) + $\sqrt{2x^2 + 3x + 4}$|+ C

15. Question

Evaluate the following integrals:

$$\int \sqrt{x^2 + x}dx$$

Answer

To Find : $\int \sqrt{x^2 + x}\, dx$

Now, $\int \sqrt{x^2 + x}\, dx$ can be written as $\int \sqrt{x^2 + x + (\frac{1}{2})^2 - (\frac{1}{2})^2}dx$

i.e, $\int \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}}dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int \sqrt{(x + \frac{1}{2})^2 - \frac{1}{4}}dx$ can be written as $\int \sqrt{y^2 - (\frac{1}{2})^2}dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2 - a^2}$|+ C

Since $\int \sqrt{y^2-(\frac{1}{2})^2}\,dy$ is of the form $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2-(\frac{1}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2}$ log $|y + \sqrt{y^2-(\frac{1}{2})^2}|$+ C

$= \frac{y}{2}\sqrt{y^2-\frac{1}{4}} - \frac{1}{8}$ log $|y + \sqrt{y^2-\frac{1}{4}}|$+ C

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx = \frac{1}{4}(2x+1)\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}} - \frac{1}{8}$ log $|(x + \frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}|$+C

Therefore,

$\int \sqrt{x^2+x}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x} - \frac{1}{8}$ log $|x + \frac{1}{2} + \sqrt{x^2+x}|$+C

16. Question

Evaluate the following integrals:

$$\int\sqrt{x^2+x+1}dx$$

Answer

To Find : $\int \sqrt{x^2+x+1}\,dx$

Now, $\int \sqrt{x^2+x+1}\,dx$ can be written as $\int \sqrt{x^2+x+(\frac{1}{2})^2-\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$ can be written as $\int \sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}$ log $|x + \sqrt{x^2+a^2}|$+ C

Since $\int \sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$ is of the form $\int \sqrt{x^2+a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2+(\frac{\sqrt{3}}{2})^2} + \frac{(\frac{\sqrt{3}}{2})^2}{2}$ log $|y + \sqrt{y^2+(\frac{\sqrt{3}}{2})^2}|$+ C

$= \frac{y}{2}\sqrt{y^2+\frac{3}{4}} + \frac{3}{8}$ log $|y + \sqrt{y^2+\frac{3}{4}}|$+ C

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx = \frac{1}{4}(2x+1)\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{1}{8}$ log $|(x + \frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}|$+C

Therefore,

$\int \sqrt{x^2+x+1}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x+1} + \frac{3}{8}$ log $|x + \frac{1}{2} + \sqrt{x^2+x+1}|$+C

17. Question

Evaluate the following integrals:

$$\int (2x-5)\sqrt{x^2-4x+3}dx$$

Answer

To Find : $\int (2x-5)\sqrt{x^2-4x+3}\,dx$

Now, let $2x-5$ be written as (2x – 4) -1 and split

Therefore ,

$\int (2x-5)\sqrt{x^2-4x+3}\,dx = \int \{(2x-4)\sqrt{x^2-4x+3} - 1\sqrt{x^2-4x+3}\}dx$

$=\int (2x-4)\sqrt{x^2-4x+3}\,dx - \int \sqrt{x^2-4x+3}dx$

Now solving, $\int (2x-4)\sqrt{x^2-4x+3}\,dx$

Let x^2-4x+3 = u ⇒ dx = $\frac{du}{(2x-4)}$

Thus, $\int (2x-4)\sqrt{x^2-4x+3}\,dx$ becomes $\int \sqrt{u}\,du$

Now , $\int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{2}{3}u^{\frac{3}{2}}$

$=\frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

Now solving, $\int \sqrt{x^2-4x+3}dx$

$\int \sqrt{x^2-4x+3}dx = \int \sqrt{x^2-4x+2^2-2^2+3}dx$

$=\int \sqrt{(x-2)^2-1}dx$

Let x – 2 = y ⇒ dx = dy

Then $\int \sqrt{(x-2)^2-1}dx$ becomes $\int \sqrt{y^2-1^2}dy$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2-a^2}$|+ C

Since $\int \sqrt{y^2-1^2}dy$ is in the form of $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

Hence $\int \sqrt{y^2-1^2}dy = \frac{y}{2}\sqrt{y^2-1^2} - \frac{1^2}{2}$ log |y + $\sqrt{y^2-1^2}$|+ C

$= \frac{y}{2}\sqrt{y^2-1} - \frac{1}{2}$ log |y + $\sqrt{y^2-1}$|+ C

Now, since x – 2 = y and dx = dy

$\int \sqrt{(x-2)^2-1}dx = \frac{(x-2)}{2}\sqrt{(x-2)^2-1} - \frac{1}{2}$ log |(x-2) + $\sqrt{(x-2)^2-1}$|+ C

Hence $\int \sqrt{x^2-4x+3}dx = \frac{(x-2)}{2}\sqrt{x^2-4x+3} - \frac{1}{2}$ log |(x-2) + $\sqrt{x^2-4x+3}$|+ C

Therefore , $\int (2x-4)\sqrt{x^2-4x+3}\,dx - \int \sqrt{x^2-4x+3}dx = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

$-\frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2}$ log |(x-2) + $\sqrt{x^2-4x+3}$|+ C

$i.e, \int (2x-5)\sqrt{x^2-4x+3}\,dx = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}}$

$-\frac{(x-2)}{2}\sqrt{x^2-4x+3}+\frac{1}{2}$ log |x - 2 + $\sqrt{x^2-4x+3}$|+ C

18. Question

Evaluate the following integrals:

$$\int (x+2)\sqrt{x^2+x+1}dx$$

Answer

To Find : $\int (x+2)\sqrt{x^2+x+1}\,dx$

Now, let x + 2 be written as $\frac{1}{2}$(2x + 1) + $\frac{3}{2}$ and split

Therefore ,

$\int (x+2)\sqrt{x^2+x+1}\,dx = \int\{\frac{(2x+1)\sqrt{x^2+x+1}}{2}+\frac{3}{2}\sqrt{x^2+x+1}\}dx$

$=\frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx+\frac{3}{2}\int\sqrt{x^2+x+1}dx$

Now solving, $\frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx$

Let x^2+x+1 = u ⇒ dx = $\frac{du}{(2x+1)}$

Thus, $\frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx$ becomes $\frac{1}{2}\int\sqrt{u}\,du$

Now , $\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = \frac{1}{3}u^{\frac{3}{2}}$

$=\frac{1}{3}(x^2+x+1)^{\frac{3}{2}}$

Now solving , $\int\sqrt{x^2+x+1}dx$

Now, $\int\sqrt{x^2+x+1}\,dx$ can be written as $\int\sqrt{x^2+x+(\frac{1}{2})^2-\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx$ can be written as $\int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$

Formula Used: $\int\sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}$ log |x + $\sqrt{x^2+a^2}$|+ C

Since $\int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy$ is of the form $\int\sqrt{x^2+a^2}\,dx$ with change in variable.

⇒ $\int\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}+\frac{(\frac{\sqrt{3}}{2})^2}{2}$ log |y + $\sqrt{y^2+(\frac{\sqrt{3}}{2})^2}$|+ C

$=\frac{y}{2}\sqrt{y^2+\frac{3}{4}}+\frac{3}{8}$ log |y + $\sqrt{y^2+\frac{3}{4}}$|+ C

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx = \frac{1}{4}(2x+1)\sqrt{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{1}{8}$ log $|(x+\frac{1}{2}) + \sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}|$+C

Therefore,

$\int \sqrt{x^2+x+1}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x+1} + \frac{3}{8}$ log $|x+\frac{1}{2}+\sqrt{x^2+x+1}|$+C

Hence ,

$\frac{1}{2}\int(2x\ +\ 1)\sqrt{x^2+x+1}dx + \frac{3}{2}\int\sqrt{x^2+x+1}dx = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{3}{8}(2x+1)\sqrt{x^2+x+1} + \frac{9}{16}$ log $|(x+\frac{1}{2})+$ $\sqrt{x^2+x+1}|$+C

Therefore , $\int(x+2)\sqrt{x^2+x+1}\,dx = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{3}{8}(2x+1)\sqrt{x^2+x+1} + \frac{9}{16}$ log $|(x+\frac{1}{2})+\sqrt{x^2+x+1}$ |+C

19. Question

Evaluate the following integrals:

$$\int(x-5)\sqrt{x^2+x}dx$$

Answer

To Find : $\int(x-5)\sqrt{x^2+x}\,dx$

Now, let x - 5 be written as $\frac{1}{2}$(2x + 1) - $\frac{11}{2}$ and split

Therefore ,

$\int(x-5)\sqrt{x^2+x}\,dx = \int\{\frac{(2x+1)\sqrt{x^2+x}}{2} - \frac{11}{2}\sqrt{x^2+x}\}dx$

$= \frac{1}{2}\int(2x\ +\ 1)\sqrt{x^2+x}\,dx - \frac{11}{2}\int\sqrt{x^2+x}\,dx$

Now solving, $\frac{1}{2}\int(2x\ +\ 1)\sqrt{x^2+x}\,dx$

Letx^2+x = u ⇒ dx = $\frac{du}{(2x+1)}$

Thus, $\frac{1}{2}\int(2x\ +\ 1)\sqrt{x^2+x}\,dx$ becomes $\frac{1}{2}\int\sqrt{u}\,du$

Now , $\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = \frac{1}{3}u^{\frac{3}{2}}$

$=\frac{1}{3}(x^2+x)^{\frac{3}{2}}$

Now solving, $\int\sqrt{x^2+x}\,dx$

Now, $\int\sqrt{x^2+x}\,dx$ can be written as $\int\sqrt{x^2+x+(\frac{1}{2})^2-(\frac{1}{2})^2}\,dx$

i.e, $\int\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx$

Here , let x + $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx$ can be written as $\int\sqrt{y^2-(\frac{1}{2})^2}\,dy$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2-a^2}$|+ C

Since $\int \sqrt{y^2-(\frac{1}{2})^2}\,dy$ is of the form $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2-(\frac{1}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\frac{1}{2})^2} - \frac{(\frac{1}{2})^2}{2}$ log |y + $\sqrt{y^2-(\frac{1}{2})^2}$|+ C

$= \frac{y}{2}\sqrt{y^2-\frac{1}{4}} - \frac{1}{8}$ log |y + $\sqrt{y^2-\frac{1}{4}}$|+ C

Since , x + $\frac{1}{2}$ = y and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}\,dx = \frac{1}{4}(2x+1)\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}} - \frac{1}{8}$ log |(x + $\frac{1}{2}$) + $\sqrt{(x+\frac{1}{2})^2-\frac{1}{4}}$|+C

Therefore,

$\int \sqrt{x^2+x}\,dx = \frac{1}{4}(2x+1)\sqrt{x^2+x} - \frac{1}{8}$ log |x + $\frac{1}{2}$ + $\sqrt{x^2+x}$|+C

Now ,

$\frac{1}{2}\int(2x+1)\sqrt{x^2+x}dx - \frac{11}{2}\int\sqrt{x^2+x}dx = \frac{1}{3}(x^2+x)^{\frac{3}{2}}$ - $\frac{11}{8}(2x+1)\sqrt{x^2+x} + \frac{11}{16}$ log |x + $\frac{1}{2}$ + $\sqrt{x^2+x}$|+C

Therefore ,

$\int(x-5)\sqrt{x^2+x}\,dx = \frac{1}{3}(x^2+x)^{\frac{3}{2}}$ - $\frac{11}{8}(2x+1)\sqrt{x^2+x} + \frac{11}{16}$ log |x + $\frac{1}{2}$ + $\sqrt{x^2+x}$|+C

20. Question

Evaluate the following integrals:

$$\int(4x+1)\sqrt{x^2-x-2}dx$$

Answer

To Find : $\int(4x+1)\sqrt{x^2-x-2}\,dx$

Now, let 4x + 1 be written as 2(2x - 1) + 3 and split

Therefore ,

$\int(4x+1)\sqrt{x^2-x-2}\,dx = \int\{2(2x-1)\sqrt{x^2-x-2} + 3\sqrt{x^2-x-2}\}\,dx$

$= 2\int(2x-1)\sqrt{x^2-x-2}dx + 3\int\sqrt{x^2-x-2}dx$

Now solving, $2\int(2x-1)\sqrt{x^2-x-2}dx$

Let x^2-x-2 = u ⇒ dx = $\frac{du}{(2x-1)}$

Thus, $2\int(2x-1)\sqrt{x^2-x-2}dx$ becomes $2\int\sqrt{u}\,du$

Now , $2\int\sqrt{u}\,du = 2\int u^{\frac{1}{2}}\,du = 2(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = \frac{4}{3}u^{\frac{3}{2}}$

$=\frac{4}{3}(x^2-x-2)^{\frac{3}{2}}$

Now solving, $\int\sqrt{x^2-x-2}dx$

Now, $\int \sqrt{x^2 - x - 2}\, dx$ can be written as $\int \sqrt{x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 2}\, dx$

i.e, $\int \sqrt{(x - \frac{1}{2})^2 - \frac{9}{4}}\, dx$

Here , let x - $\frac{1}{2}$ = y ⇒dx = dy

Therefore, $\int \sqrt{(x - \frac{1}{2})^2 - \frac{9}{4}}\, dx$ can be written as $\int \sqrt{y^2 - (\frac{3}{2})^2}\, dy$

Formula Used: $\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}$ log |x + $\sqrt{x^2 - a^2}$|+ C

Since $\int \sqrt{y^2 - (\frac{3}{2})^2}\, dy$ is of the form $\int \sqrt{x^2 - a^2}\, dx$ with change in variable.

⇒ $\int \sqrt{y^2 - (\frac{3}{2})^2}\, dy = \frac{y}{2}\sqrt{y^2 - (\frac{3}{2})^2} - \frac{(\frac{3}{2})^2}{2}$ log |y + $\sqrt{y^2 - (\frac{3}{2})^2}$|+ C

$= \frac{y}{2}\sqrt{y^2 - \frac{9}{4}} - \frac{9}{8}$ log |y + $\sqrt{y^2 - \frac{9}{4}}$|+ C

Since , x - $\frac{1}{2}$ = y and dx = dy

⇒ $\int \sqrt{(x - \frac{1}{2})^2 - \frac{9}{4}}\, dx = \frac{1}{4}(2x - 1)\sqrt{(x - \frac{1}{2})^2 - \frac{9}{4}} - \frac{9}{8}$ log |(x - $\frac{1}{2}$) + $\sqrt{(x - \frac{1}{2})^2 - \frac{9}{4}}$|+C

Therefore,

$\int \sqrt{x^2 - x - 2}\, dx = \frac{1}{4}(2x - 1)\sqrt{x^2 - x - 2} - \frac{9}{8}$ log |x - $\frac{1}{2}$ + $\sqrt{x^2 - x - 2}$|+C

Hence ,

$2\int (2x - 1)\sqrt{x^2 - x - 2}dx + 3\int \sqrt{x^2 - x - 2}dx = \frac{4}{3}(x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4}(2x - 1)\sqrt{x^2 - x - 2} - \frac{27}{8}$ log |x - $\frac{1}{2}$ + $\sqrt{x^2 - x - 2}$|+C

Therefore ,

$\int (4x + 1)\sqrt{x^2 - x - 2}\, dx = \frac{4}{3}(x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4}(2x - 1)\sqrt{x^2 - x - 2} - \frac{27}{8}$ log |x - $\frac{1}{2}$ + $\sqrt{x^2 - x - 2}$|+C

21. Question

Evaluate the following integrals:

$$\int (x + 1)\sqrt{2x^2 + 3}dx$$

Answer

To Find : $\int (x + 1)\sqrt{2x^2 + 3}\, dx$

Now, $\int (x + 1)\sqrt{2x^2 + 3}\, dx$ can be written as

$\int (x + 1)\sqrt{2x^2 + 3}\, dx = \int \{x\sqrt{2x^2 + 3} + \sqrt{2x^2 + 3}\}\, dx$

$= \int x\sqrt{2x^2 + 3}dx + \int \sqrt{2x^2 + 3}dx$

Now solving, $\int x\sqrt{2x^2 + 3}dx$

Let $2x^2 + 3$ = u ⇒ dx = $\frac{1du}{4x}$

Thus, $\int x\sqrt{2x^2+3}dx$ becomes $\frac{1}{4}\int \sqrt{u}\,du$

Now , $\frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\int u^{\frac{1}{2}}\,du = \frac{1}{4}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = \frac{1}{6}u^{\frac{3}{2}}$

$=\frac{1}{6}(2x^2+3)^{\frac{3}{2}}$

Now solving, $\int \sqrt{2x^2+3}dx$

Now, $\int \sqrt{2x^2+3}\,dx$ can be written as $\int \sqrt{(\sqrt{2x})^2+(\sqrt{3})^2}\,dx$

Formula Used: $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x + \sqrt{x^2+a^2}| + C$

Since $\int \sqrt{2x^2+3}\,dx$ is of the form $\int \sqrt{x^2+a^2}\,dx$.

$\Rightarrow \int \sqrt{2x^2+3}\,dx = \frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2x})^2+(\sqrt{3})^2} + \frac{(\sqrt{3})^2}{2}\log|\sqrt{2}x + \sqrt{(\sqrt{2x})^2+(\sqrt{3})^2}| + C$

$= \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$

Therefore,

$\int x\sqrt{2x^2+3}dx + \int \sqrt{2x^2+3}dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$

Hence ,

$\int (x+1)\sqrt{2x^2+3}\,dx = \frac{1}{6}(2x^2+3)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log|\sqrt{2}x + \sqrt{2x^2+3}| + C$

22. Question

Evaluate the following integrals:

$$\int x\sqrt{1+x-x^2}\,dx$$

Answer

To Find : $\int x\sqrt{1+x-x^2}\,dx$

Now, let x be written as $\frac{1}{2} - \frac{1}{2}(1 - 2x)$ and split

Therefore ,

$\int x\sqrt{1+x-x^2}\,dx = \int\{\frac{\sqrt{-x^2+x+1}}{2} - \frac{(1-2x)\sqrt{-x^2+x+1}}{2}\}dx$

$= \frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx + \frac{1}{2}\int \sqrt{-x^2+x+1}\,dx$

Now solving, $\frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx$

Let $-x^2+x+1$= u $\Rightarrow$ dx = $\frac{du}{(1-2x)}$

Thus, $\frac{1}{2}\int(2x-1)\sqrt{-x^2+x+1}\,dx$becomes $-\frac{1}{2}\int \sqrt{u}\,du$

Now , $-\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\int u^{\frac{1}{2}}\,du = -\frac{1}{2}\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = -\frac{1}{3}u^{\frac{3}{2}}$

$=-\frac{1}{3}(-x^2+x+1)^{\frac{3}{2}}$

Now solving, $\int \sqrt{-x^2+x+1}\,dx$

$\int \sqrt{-x^2+x+1}\,dx$ can be written as $\int \sqrt{-x^2+x-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int \sqrt{\frac{5}{4}-(x-\frac{1}{2})^2}\,dx = \frac{1}{2}\int \sqrt{5-(2x-1)^2}\,dx$

let 2x – 1 = y ⇒ dx = $\frac{1dy}{2}$

Therefore , $\frac{1}{4}\int \sqrt{5-(2x-1)^2}\,dx$ becomes $\frac{1}{4}\int \sqrt{(\sqrt{5})^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{(\sqrt{5})^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int \sqrt{(\sqrt{5})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\sqrt{5})^2-y^2}+\frac{(\sqrt{5})^2}{2}\sin^{-1}\frac{y}{\sqrt{5}}$ + C

$= \frac{1}{2}y\sqrt{5-y^2}+\frac{5}{2}\sin^{-1}\frac{y}{\sqrt{5}}$ + C

Since , 2x – 1 = y and dx = $\frac{1dy}{2}$

Therefore,

$\frac{1}{4}\int \sqrt{5-(2x-1)^2}\,dx = \frac{1}{8}(2x-1)\sqrt{5-(2x-1)^2}+\frac{5}{8}\sin^{-1}\frac{(2x-1)}{\sqrt{5}}$ + C

i.e, $\int \sqrt{-x^2+x+1}\,dx = \frac{1}{8}(2x-1)\sqrt{-x^2+x+1}+\frac{5}{8}\sin^{-1}\frac{(2x-1)}{\sqrt{5}}$ + C

hence , $\int x\sqrt{1+x-x^2}\,dx = \frac{1}{2}\int(\,2\mathrm{x}-1)\sqrt{-x^2+x+1}\,dx+\frac{1}{2}\int\sqrt{-x^2+x+1}\,dx = =-\frac{1}{3}(-x^2+x+1)^{\frac{3}{2}}+$ $\frac{1}{16}(2x-1)\sqrt{-x^2+x+1}+\frac{5}{16}\sin^{-1}(\frac{2x-1}{\sqrt{5}})$ + C

23. Question

Evaluate the following integrals:

Answer

To Find : $\int(2x-5)\sqrt{2+3x-x^2}\,dx \int (2\mathrm{x}-5)\sqrt{2+3\mathrm{x}-\mathrm{x}^2}\,\mathrm{dx}$

Now, let 2x - 5 be written as (2x – 3) -2 and split

Therefore ,

$\int(2x-5)\sqrt{2+3x-x^2}\,dx = \int\{(2x-3)\sqrt{-x^2+3x+2}-2\sqrt{-x^2+3x+2}\}\,dx$

$= \int(\,2\mathrm{x}-3)\sqrt{-x^2+3x+2}\,dx$ - 2 $\int\sqrt{-x^2+3x+2}\,dx$

Now solving, $\int(\,2\mathrm{x}-3)\sqrt{-x^2+3x+2}\,dx$

Let $-x^2+3x+2$= u ⇒ dx = $\frac{du}{(3-2x)}$

Thus,$\int(\,2\mathrm{x}-3)\sqrt{-x^2+3x+2}\,dx$ becomes $-\int\sqrt{u}\,du$

Now , $-\int \sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du = -\left(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) = -\frac{2}{3}u^{\frac{3}{2}}$

$= -\frac{2}{3}(-x^2+3x+2)^{\frac{3}{2}}$

Now solving, $\int \sqrt{-x^2+3x+2}\,dx$

$\int \sqrt{-x^2+3x+2}\,dx$ can be written as $\int \sqrt{-x^2+3x-\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2+2}\,dx$

i.e, $\int \sqrt{\frac{17}{4}-(x-\frac{3}{2})^2}\,dx$

let x - $\frac{3}{2}$ = y ⇒ dx = dy

Therefore , $\int \sqrt{\frac{17}{4}-(x-\frac{3}{2})^2}\,dx$ becomes $\int \sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int \sqrt{(\frac{\sqrt{17}}{2})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\frac{\sqrt{17}}{2})^2-y^2} + \frac{(\frac{\sqrt{17}}{2})^2}{2}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}}$ + C

$= \frac{1}{2}y\sqrt{\frac{17}{4}-y^2} + \frac{17}{8}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}}$ + C

Since , x - $\frac{3}{2}$ = y and dx = dy

Therefore,

$\int \sqrt{\frac{17}{4}-(x-\frac{3}{2})^2}\,dx = \frac{1}{4}(2x-3)\sqrt{\frac{17}{4}-(x-\frac{3}{2})^2} + \frac{17}{8}\sin^{-1}(\frac{2x-3}{\sqrt{17}})$ + C

i.e, $\int \sqrt{-x^2+3x+2}\,dx = \frac{1}{4}(2x-3)\sqrt{-x^2+3x+2} + \frac{17}{8}\sin^{-1}(\frac{2x-3}{\sqrt{17}})$ + C

hence ,

$\int (2x-5)\sqrt{2+3x-x^2}\,dx = \int (\ 2x-3)\sqrt{-x^2+3x+2}\,dx$ - 2 $\int \sqrt{-x^2+3x+2}\,dx = -\frac{2}{3}(-x^2+3x+2)^{\frac{3}{2}}$ - $\frac{1}{2}(2x-3)\sqrt{-x^2+3x+2}$ - $\frac{17}{4}\sin^{-1}(\frac{2x-3}{\sqrt{17}})$ + C

24. Question

Evaluate the following integrals:

$$\int (6x+5)\sqrt{6+x-2x^2}\,dx$$

Answer

To Find : $\int (6x+5)\sqrt{6+x-2x^2}\,dx$

Now, let 6x + 5 be written as $\frac{13}{2}$ - $\frac{3}{2}$(1 - 4x)and split

Therefore ,

$\int (6x+5)\sqrt{6+x-2x^2}\,dx = \int \{\frac{13\sqrt{-2x^2+x+6}}{2} - \frac{3(1-4x)\sqrt{-2x^2+x+6}}{2}\}\,dx$

$= \frac{3}{2}\int(4x-1)\sqrt{-2x^2+x+6}\,dx + \frac{13}{2}\int\sqrt{-2x^2+x+6}\,dx$

Now solving, $\int(4x-1)\sqrt{-2x^2+x+6}dx$

Let $-2x^2+x+6$ = u ⇒ dx = $\frac{du}{(1-4x)}$

Thus,$\int(4x-1)\sqrt{-2x^2+x+6}\,dx$ becomes $-\int\sqrt{u}\,du$

Now , $-\int\sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du = -(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = -\frac{2}{3}u^{\frac{3}{2}}$

$=-\frac{2}{3}(-2x^2+x+6)^{\frac{3}{2}}$

Now solving, $\int\sqrt{-2x^2+x+6}\,dx$

$\int\sqrt{-2x^2+x+6}\,dx$ can be written as $\int\sqrt{-(\sqrt{2}x)^2+x-\left(\frac{1}{2\sqrt{2}}\right)^2+\left(\frac{1}{2\sqrt{2}}\right)^2+6}\,dx$

i.e, $\int\sqrt{\frac{49}{8}-(\sqrt{2}x-\frac{1}{2\sqrt{2}})^2}\,dx$

let $\sqrt{2}x-\frac{1}{2\sqrt{2}}$= y ⇒ dx = $\frac{dy}{\sqrt{2}}$

Therefore , $\int\sqrt{\frac{49}{8}-(\sqrt{2}x-\frac{1}{2\sqrt{2}})^2}\,dx$ becomes $\int\sqrt{(\frac{7}{2\sqrt{2}})^2-y^2}\,dy$

Formula Used: $\int\sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int\sqrt{(\frac{7}{2\sqrt{2}})^2-y^2}\,dy$ is of the form $\int\sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int\sqrt{(\frac{7}{2\sqrt{2}})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\frac{7}{2\sqrt{2}})^2-y^2}+\frac{(\frac{7}{2\sqrt{2}})^2}{2}\sin^{-1}\frac{y}{\frac{7}{2\sqrt{2}}}$ + C

$= \frac{1}{2}y\sqrt{\frac{49}{8}-y^2}+\frac{7}{16}\sin^{-1}\frac{y}{\frac{\sqrt{17}}{2}}$ + C

Since , $\sqrt{2}x-\frac{1}{2\sqrt{2}}$= y and dx = $\frac{dy}{\sqrt{2}}$

Therefore,

$\int\sqrt{\frac{49}{8}-(\sqrt{2}x-\frac{1}{2\sqrt{2}})^2}\,dx = \frac{1}{4\sqrt{2}}(4x-1)\sqrt{\frac{49}{8}-(\sqrt{2}x-\frac{1}{2\sqrt{2}})^2}+\frac{49}{16}\sin^{-1}(\frac{4x-1}{7})$ + C

i.e, $\int\sqrt{-2x^2+x+6}\,dx = \frac{1}{4\sqrt{2}}(4x-1)\sqrt{-2x^2+x+6}+\frac{49}{16}\sin^{-1}(\frac{4x-1}{7})$ + C

hence ,

$\int(6x+5)\sqrt{6+x-2x^2}\,dx = \frac{3}{2}\int(4x-1)\sqrt{-2x^2+x+6}\,dx+\frac{13}{2}\int\sqrt{-2x^2+x+6}\,dx = -(-2x^2+x+6)^{\frac{3}{2}}+\frac{13}{16}(4x-1)\sqrt{-2x^2+x+6}+\frac{637}{32\sqrt{2}}\sin^{-1}(\frac{4x-1}{7})$ + C

25. Question

Evaluate the following integrals:

$$\int(x+1)\sqrt{1-x-x^2}dx$$

Answer

To Find : $\int (x+1)\sqrt{1-x-x^2}\,dx$

Now, let x + 1 be written as $\frac{1}{2}$ - $\frac{1}{2}$(-2x - 1) and split

Therefore ,

$$\int (x+1)\sqrt{1-x-x^2}\,dx = \int \{\frac{\sqrt{-x^2-x+1}}{2} - \frac{(-2x-1)\sqrt{-x^2-x+1}}{2}\}\,dx$$

$$= \frac{1}{2}\int (2x-1)\sqrt{-x^2-x+1}\,dx + \frac{1}{2}\int \sqrt{-x^2-x+1}\,dx$$

Now solving, $\int (2x-1)\sqrt{-x^2-x+1}dx$

Let $-x^2-x+1$ = u ⇒ dx = $\frac{du}{-2x-1}$

Thus, $\int (2x-1)\sqrt{-x^2-x+1}\,dx$ becomes $-\int \sqrt{u}\,du$

Now , $-\int \sqrt{u}\,du = -\int u^{\frac{1}{2}}\,du = -(\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = -\frac{2}{3}u^{\frac{3}{2}}$

$$= -\frac{2}{3}(-x^2-x+1)^{\frac{3}{2}}$$

Now solving, $\int \sqrt{-x^2-x+1}\,dx$

$\int \sqrt{-x^2-x+1}\,dx$ can be written as $\int \sqrt{-x^2-x-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+1}\,dx$

i.e, $\int \sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}\,dx$

let $x+\frac{1}{2}$= y ⇒ dx = dy

Therefore , $\int \sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}\,dx$ becomes $\int \sqrt{(\frac{\sqrt{5}}{2})^2-y^2}\,dy$

Formula Used: $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$ + C

Since $\int \sqrt{(\frac{\sqrt{5}}{2})^2-y^2}\,dy$ is of the form $\int \sqrt{a^2-x^2}\,dx$ with change in variable .

Hence, $\int \sqrt{(\frac{\sqrt{5}}{2})^2-y^2}\,dy = \frac{1}{2}y\sqrt{(\frac{\sqrt{5}}{2})^2-y^2} + \frac{(\frac{\sqrt{5}}{2})^2}{2}\sin^{-1}\frac{y}{\frac{\sqrt{5}}{2}}$ + C

$$= \frac{1}{2}y\sqrt{\frac{5}{4}-y^2} + \frac{5}{8}\sin^{-1}\frac{y}{\frac{\sqrt{5}}{2}} + C$$

Since , $x+\frac{1}{2}$= y and dx = dy

Therefore,

$$\int \sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}\,dx = \frac{1}{4}(2x+1)\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2} + \frac{5}{8}\sin^{-1}(\frac{2x+1}{\sqrt{5}}) + C$$

i.e, $\int \sqrt{-x^2-x+1}\,dx = \frac{1}{4}(2x+1)\sqrt{-x^2-x+1} + \frac{5}{8}\sin^{-1}(\frac{2x+1}{\sqrt{5}})$ + C

hence ,

$$\int (x+1)\sqrt{1-x-x^2}\,dx = \frac{1}{2}\int (2x-1)\sqrt{-x^2-x+1}\,dx + \frac{1}{2}\int \sqrt{-x^2-x+1}\,dx = -\frac{1}{3}(-x^2-x+1)^{\frac{3}{2}} +$$

$\frac{1}{8}(2x+1)\sqrt{-x^2-x+1} + \frac{5}{16}\sin^{-1}(\frac{2x+1}{\sqrt{5}}) + C$

26. Question

Evaluate the following integrals:

$$\int (x-3)\sqrt{x^2+3x-18}dx$$

Answer

To Find : $\int (x-3)\sqrt{x^2+3x-18}\,dx$

Now, let x - 3 be written as $\frac{1}{2}(2x+3) - \frac{9}{2}$ and split

Therefore ,

$\int (x-3)\sqrt{x^2+3x-18}\,dx = \int \{\frac{(2x+3)\sqrt{x^2+3x-18}}{2} - \frac{9\sqrt{x^2+3x-18}}{2}\}\,dx$

$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\,dx - \frac{9}{2}\int \sqrt{x^2+3x-18}\,dx$

Now solving, $\int (2x+3)\sqrt{x^2+3x-18}dx$

Let $x^2+3x-18 = u \Rightarrow dx = \frac{du}{2x+3}$

Thus, $\int (2x+3)\sqrt{x^2+3x-18}\,dx$ becomes $\int \sqrt{u}\,du$

Now , $\int \sqrt{u}\,du = \int u^{\frac{1}{2}}\,du = (\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}) = \frac{2}{3}u^{\frac{3}{2}}$

$= \frac{2}{3}(x^2+3x-18)^{\frac{3}{2}}$

Now solving, $\int \sqrt{x^2+3x-18}\,dx$

$\int \sqrt{x^2+3x-18}\,dx$ can be written as $\int \sqrt{x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2-18}\,dx$

i.e, $\int \sqrt{(x+\frac{3}{2})^2-\frac{81}{4}}\,dx$

let $x+\frac{3}{2} = y \Rightarrow dx = dy$

Therefore, $\int \sqrt{(x+\frac{3}{2})^2-\frac{81}{4}}\,dx$ can be written as $\int \sqrt{y^2-(\frac{9}{2})^2}\,dy$

Formula Used: $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + C$

Since $\int \sqrt{y^2-(\frac{9}{2})^2}\,dy$ is of the form $\int \sqrt{x^2-a^2}\,dx$ with change in variable.

$\Rightarrow \int \sqrt{y^2-(\frac{9}{2})^2}\,dy = \frac{y}{2}\sqrt{y^2-(\frac{9}{2})^2} - \frac{(\frac{9}{2})^2}{2}\log|y+\sqrt{y^2-(\frac{9}{2})^2}| + C$

$= \frac{y}{2}\sqrt{y^2-\frac{81}{4}} - \frac{81}{8}\log|y+\sqrt{y^2-\frac{81}{4}}| + C$

Since , $x+\frac{3}{2} = y$ and dx = dy

$\Rightarrow \int \sqrt{(x+\frac{3}{2})^2-\frac{81}{4}}\,dx = \frac{1}{4}(2x+3)\sqrt{(x+\frac{3}{2})^2-\frac{81}{4}} - \frac{81}{8}\log|(x+\frac{3}{2}) + \sqrt{(x+\frac{3}{2})^2-\frac{81}{4}}|+C$

Therefore,

$\int \sqrt{x^2+3x-18}\,dx = \frac{1}{4}(2x+3)\sqrt{x^2+3x-18} - \frac{81}{8}\log|x+\frac{3}{2}+\sqrt{x^2+3x-18}|+C$

Hence ,

$\int (x-3)\sqrt{x^2+3x-18}\,dx = \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\,dx - \frac{9}{2}\int \sqrt{x^2+3x-18}\,dx =$
$\frac{1}{3}(x^2+3x-18)^{\frac{3}{2}} - \frac{9}{8}(2x+3)\sqrt{x^2+3x-18} + \frac{726}{16}\log|x+\frac{3}{2}+\sqrt{x^2+3x-18}|+C$