15. Integration Using Partial Fractions

Exercise 15A

1. Question

Evaluate:

$\int \frac{dx}{x(x+2)}$

Answer

Let $I = \int \frac{dx}{x(x+2)}$,

Putting $\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \ldots \ldots ..(1)$

Which implies A(x+2) + Bx = 1, putting x+2=0

Therefore x=-2,

And B = -0.5

Now put x=0, A= � ,

From equation (1), we get

$$\frac{1}{x(x+2)} = \frac{1}{2} \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x+2}$$

$$\int \frac{1}{x(x+2)} dx = \frac{1}{2} \int \frac{1}{x} dx - \frac{1}{2} \int \frac{1}{x+2} dx$$

$$= \frac{1}{2} \log|x| - \frac{1}{2} \log|x+2| + c$$

$$= \frac{1}{2} [\log|x| - \log|x+2|] + c$$

$$= \frac{1}{2} \log\left|\frac{x}{x+2}\right| + c$$

2. Question

Evaluate:

$\int \frac{(2x+1)}{(x+2)(x+3)} dx$

Answer

Let $I = \int \frac{(2x+1)}{(x+2)(x+3)} dx$,

Putting $\frac{2x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3} \ldots \ldots \ldots \ldots (1)$

Which implies 2x=1 = A(x-3) + B(x+2)

Now put x-3=0, x=3

2×3+1=A(0)+B 3+2)

So $B = \frac{7}{5}$

Now put x+2=0 ,x=-2

-4+1=A(-2-3) + B(0)

So $A = \frac{3}{5}$

From equation (1), we get ,

$$\frac{2x+1}{(x+2)(x-3)} = \frac{3}{5} \times \frac{1}{x+2} + \frac{7}{5} \times \frac{1}{x-3}$$

$$\int \frac{2x+1}{(x+2)(x-3)} dx = \frac{3}{5} \int \frac{1}{x+2} dx + \frac{7}{5} \int \frac{1}{x-3} dx$$

$$= \frac{3}{5} \log|x+2| + \frac{7}{5} \log|x-3| + c$$

3. Question

Evaluate:

$$\int \frac{x}{(x+2)(3-2x)} dx$$

Answer

Let $I = \int \frac{x}{(x+2)(3-2x)} dx,$

Putting $\frac{x}{(x+2)(3-2x)} = \frac{A}{x+2} + \frac{B}{3-2x} \dots \dots \dots (1)$

Which implies A(3-2x)+B(x+2)=x

Now put 3-2x=0

Therefore, $x = \frac{3}{2}$

$$A(0) + B\left(\frac{3}{2} + 2\right) = \frac{3}{2}$$

$$B\left(\frac{7}{2}\right) = \frac{3}{2}$$

$$B = \frac{3}{7}$$

Now put x+2=0

Therefore, x=-2

A(7)+B(0)=-2

$$A = \frac{-2}{7}$$

Now From equation (1) we get

$$\frac{x}{(x+2)(3-2x)} = \frac{-2}{7} \times \frac{1}{x+2} + \frac{3}{7} \times \frac{1}{3-2x}$$

$$\int \frac{x}{(x+2)(3-2x)} dx = \frac{-2}{7} \int \frac{1}{x+2} dx + \frac{3}{7} \int \frac{1}{3-2x} dx$$

$$= \frac{-2}{7} \log|x+2| + \frac{3}{7} \times \frac{1}{-2} \log|3-2x| + c$$

$$= \frac{-2}{7} \log|x+2| + \frac{3}{7} \times \frac{1}{-2} \log|3-2x| + c$$

$$= \frac{-2}{7} \log|x+2| - \frac{3}{14} \log|3-2x| + c$$

4. Question

Evaluate:

$\int \frac{dx}{x(x-2)(x-4)}$

Answer

Let $I = \int \frac{dx}{x(x-2)(x-4)}$,

Putting $\frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4} \dots \dots (1)$

Which implies,

A(x-2)(x-4)+Bx(x-4)+Cx(x-2)=1

Now put x-2=0

Therefore, x=2

A(0)+B×2(2-4)+C(0)=1

B×2(-2)=1

$B = -\frac{1}{4}$

Now put x-4=0

Therefore, x=4

A(0)+B×(0)+C×4(4-2)=1

C×4(2)=1

$C = \frac{1}{8}$

Now put x=0

A(0-2)(0-4)+B(0)+C(0)=1

$A = \frac{1}{8}$

Now From equation (1) we get

$$\frac{1}{x(x-2)(x-4)} = \frac{1}{8} \times \frac{1}{x} - \frac{1}{4} \times \frac{1}{x-2} + \frac{1}{8} \times \frac{1}{x-4}$$

$$\int \frac{dx}{x(x-2)(x-4)} = \frac{1}{8}\int \frac{1}{x}dx - \frac{1}{4}\int \frac{1}{x-2}dx + \frac{1}{8}\int \frac{1}{x-4}dx$$

$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$

5. Question

Evaluate:

$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)}dx$

Answer

Let $I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)}dx$

Putting $\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3} \dots \dots (1)$

Which implies,

A(x+2)(x-2)+B(x-1)(x-3)+C(x-1)(x+2)=2x-1

Now put x+2=0

Therefore, x=-2

A(0)+B(-2-1)(-2-3)+C(0)=2x-2-1

B(-3)(-5)=-5

$$B = -\frac{1}{3}$$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(2)(5)=5

$$C = \frac{1}{2}$$

Now put x-1=0

Therefore, x=1

A(3)(-2)+B(0)+C(0)=1

$$A = -\frac{1}{6}$$

Now From equation (1) we get,

$$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6} \times \frac{1}{x-1} - \frac{1}{3} \times \frac{1}{x+2} + \frac{1}{2} \times \frac{1}{x-3}$$

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx = \frac{-1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{1}{x-3} dx$$

$$= \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + c$$

6. Question

Evaluate:

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$$

Answer

Let $I = \int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$

Putting $\frac{(2x-3)}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3} \dots \dots (1)$

Which implies,

A(x+1)(2x+3)+B(x-1)(2x+3)+C(x-1)(x+1)=2x-3

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-1)(-2+3)+C(0)=-2-3

$B = -\frac{5}{2}$

Now put x-1=0

Therefore, x=1

A(2)(2+3)+B(0)+C(0)=-1

$A = -\frac{1}{10}$

Now put 2x+3=0

Therefore, $x = -\frac{3}{2}$

$A(0) + B(0) + C\left(\frac{-3}{2} - 1\right)\left(\frac{-3}{2} + 1\right) = 2\left(\frac{-3}{2}\right) - 3$

$C(\frac{-5}{2})(\frac{-1}{2}) = -3 - 3$

$C = -\frac{24}{5}$

.Now From equation (1) we get,

$$\frac{(2x-3)}{(x^2-1)(2x+3)} = \frac{-1}{10} \times \frac{1}{x-1} + \frac{5}{2} \times \frac{1}{x+1} - \frac{24}{5} \times \frac{1}{2x+3}$$

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx = \frac{-1}{10} \int \frac{1}{x-1} dx + \frac{5}{2} \int \frac{1}{x+1} dx - \frac{24}{5} \int \frac{1}{2x+3} dx$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{24}{5} \frac{\log|2x+3|}{2} + c$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{12}{5} \log|2x+3| + c$$

7. Question

Evaluate:

$$\int \frac{(2x+5)}{(x^2-x-2)} dx$$

Answer

Let $I = \int \frac{(2x+5)}{(x^2-x-2)} dx = \int \frac{(2x+5)}{(x-2)(x+1)} dx$

Putting $\frac{(2x+5)}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \ldots \ldots (1)$

Which implies,

A(x+1)+B(x-2)=2x+5

Now put x+1=0

Therefore, x=-1

A(0)+B(-1-2)=3

B=-1

Now put x-2=0

Therefore, x=2

A(2+1)+B(0)=2×2+5=9

A=3

Now From equation (1) we get,

$$\frac{(2x+5)}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{-1}{x+1}$$

$$\int \frac{(2x+5)}{(x-2)(x+1)} dx = \int \frac{3}{x-2} + \int \frac{-1}{x+1}$$

$$= 3\log|x-2| - \log|x+1| + c$$

8. Question

Evaluate:

$$\int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx$$

Answer

Let $I = \int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2+2x+1}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2}{(x^2+3x+2)} dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Which implies $I = \int dx + \int \frac{2x+1}{(x^2+3x+2)} dx$

Therefore, $I=x+I_1$

Where, $I_1=\int \frac{2x+1}{(x^2+3x+2)} dx$

Putting $\frac{(2x+1)}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \ldots \ldots (1)$

Which implies,

A(x+2)+B(x+1)=2x+1

Now put x+2=0

Therefore, x=-2

A(0)+B(-1)=-4+1

B=3

Now put x+1=0

Therefore, x=-1

A(-1+2)+B(0)=-2+1

A=-1

Now From equation (1) we get,

$$\frac{(2x+1)}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{3}{x+2}$$

$$\int \frac{(2x+1)}{(x+1)(x+2)} dx = -\int \frac{1}{x+1} dx + \int \frac{3}{x+2} dx$$

$$= -\log|x+1| + 3\log|x+2| + c$$

9. Question

Evaluate:

$$\int \frac{(x^2+1)}{(x^2-1)} dx$$

Answer

Let $I = \int \frac{x^2+1}{x^2-1} dx$

$$I = \int (1 + \frac{2}{x^2-1}) dx$$

$$I = \int dx + 2\int \frac{1}{x^2-1} dx$$

$$I = x + 2 \times \frac{1}{2} \log\left|\frac{x-1}{x+1}\right| + c$$

$$I = x + \log\left|\frac{x-1}{x+1}\right| + c$$

10. Question

Evaluate:

$$\int \frac{x^3}{(x^2-4)} dx$$

Answer

Let $I = \int \frac{x^3}{x^2-4} dx$

$$I = \int x + \frac{4x}{x^2-4} dx$$

$$I = \int x\, dx + \int \frac{4x}{x^2-4} dx$$

$$= \frac{x^2}{2} + \int \frac{4x}{(x-2)(x+2)} dx$$

Let $I_1 = \int \frac{4x}{(x-2)(x+2)} dx$

So

$$I = \frac{x^2}{2} + I_1$$

Therefore $I_1 = \int \frac{4x}{x^2-4} dx$

Putting x^2-4=t

2xdx = dt

$$I_1 = 2\int \frac{dt}{t}$$

$$I_1 = 2log|x^2-4| + c$$

Putting the value of I_1 in I,

$$I = \frac{x^2}{2} + 2log|x^2-4| + c$$

11. Question

Evaluate:

$\int \frac{(3+4x-x^2)}{(x+2)(x-1)} dx$

Answer

Let $I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$

$= \int \left(-1 + \frac{5x+1}{(x+2)(x-1)}\right) dx$

$= \int -dx + \int \frac{5x+1}{(x+2)(x-1)} dx$

$= -x + I_1$

$I_1 = \int \frac{5x+1}{(x+2)(x-1)} dx$

Put $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

A(x-1)+B(x+2)=5x+1

Now put x-1=0

Therefore, x=1

A(0)+B(1+2)=5+1=6

B=2

Now put x+2=0

Therefore, x=-2

A(-2-1)+B(0)=5×(-2)+1

A=3

Now From equation (1) we get,

$$\frac{5x+1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$$

$$\int \frac{5x+1}{(x+2)(x-1)} dx = 3\int \frac{1}{(x+2)} dx + 2\int \frac{1}{(x-1)} dx$$

$3\log|x+2| + 2\log|x-1| + c$

Therefore,

$I = -x + 3log|x+2| + 2log|x-1| + c$

12. Question

Evaluate:

$\int \frac{x^3}{(x-1)(x-2)} dx$

Answer

Let $I = \int \frac{x^3}{(x-1)(x-2)} dx$

$= \int \left\{(x+3) + \frac{7x-6}{(x-1)(x-2)}\right\} dx$

$$= \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$$

$$= \frac{x^2}{2} + 3x + I_1 \dots\dots..(1)$$

Where,

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx$$

Putting $\frac{7x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$..........(2)

A(x-2)+B(x-1)=7x-6

Now put x-2=0

Therefore, x=2

A(0)+B(2-1)=7×2-6

B=8

Now put x-1=0

Therefore, x=1

A(1-2)+B(0)=7-6=1

A=-1

Now From equation (2) we get,

$$\frac{7x-6}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{8}{x-2}$$

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx = -\int \frac{1}{x-1} dx + 8\int \frac{1}{x-2} dx$$

$$= -\log|x-1| + 8log|x-2| + c$$

Now From equation (1) we get,

$$I = \frac{x^2}{2} + 3x - \log|x-1| + 8log|x-2| + c$$

13. Question

Evaluate:

$$\int \frac{(x^3-x-2)}{(1-x^2)} dx$$

Answer

Let $I = \int \frac{(x^3-x-2)}{(1-x^2)} dx$

$$= \int \left(-x + \frac{-2}{1-x^2}\right) dx$$

$$= \int -x dx + (-2)\int \frac{1}{1-x^2} dx$$

$$= \frac{-x^2}{2} - log\left|\frac{1+x}{1-x}\right| + c$$

$$= \frac{-x^2}{2} + log\left|\frac{1-x}{1+x}\right| + c$$

14. Question

Evaluate:

$$\frac{(2x+1)}{(4-3x-x^2)}dx$$

Answer

Let $I = \int \frac{2x+1}{(4-3x-x^2)}dx$

$$= \int \frac{2x+1}{(1-x)(4+x)}dx$$

Putting $\frac{2x+1}{(1-x)(4+x)} = \frac{A}{1-x} + \frac{B}{4+x} \ldots \ldots \ldots (1)$

A(4+x)+B(1-x)=2x+1

Now put 1-x=0

Therefore, x=1

A(5)+B(0)=3

$$A = \frac{3}{5}$$

Now put 4+x=0

Therefore, x=-4

A(0)+B(5)=-8+1=-7

$$B = \frac{-7}{5}$$

Now From equation (1) we get,

$$\frac{2x+1}{(1-x)(4+x)} = \frac{3}{5} \times \frac{1}{1-x} + \frac{-7}{5} \times \frac{1}{4+x}$$

$$\int \frac{2x+1}{(1-x)(4+x)}dx = \frac{3}{5}\int \frac{1}{1-x}dx + \frac{-7}{5}\int \frac{1}{4+x}dx$$

$$= \frac{-3}{5}log|1-x| - \frac{7}{5}log|4+x| + c$$

$$= -\frac{1}{5}[3log|1-x| + 7log|4+x|] + c$$

15. Question

Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)}dx$$

Answer

Put x^2=t

2xdx=dt

$$\int \frac{dt}{(1+t)(3+t)} = \frac{1}{2}\int \left(\frac{1}{1+t} - \frac{1}{3+t}\right)dt$$

$\frac{1}{2}[log|1+t| - log|3+t|] + c = \frac{1}{2}log\left|\frac{1+t}{3+t}\right| + c$

$= \frac{1}{2}log\left|\frac{1+x^2}{3+x^2}\right| + c$

16. Question

Evaluate:

$\int \frac{\cos x}{(\cos^2 x - \cos x - 2)} dx$

Answer

Let $I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx$

Putting t=sin x

dt=cos x dx

$I = \int \frac{dt}{(1+t)(2+t)},$

Now putting, $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \dots \dots \dots . . (1)$

A(2+t)+B(1+t)=1

Now put t+1=0

Therefore, t=-1

A(2-1)+B(0)=1

A=1

Now put t+2=0

Therefore, t=-2

A(0)+B(-2+1)=1

B=-1

Now From equation (1) we get,

$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$

$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$

$= \log|1+t| - \log|t+2| + c$

$= log\left|\frac{t+1}{t+2}\right| + c$

So,

$I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx = log\left|\frac{sinx+1}{sinx+2}\right| + c$

17. Question

Evaluate:

$\int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)} dx$

Answer

Let $I = \int \frac{sec^2 x}{(2+tanx)(3+tanx)} dx$

Putting t=tanx

dt=sec^{2}xdx

$$I = \int \frac{dt}{(2+t)(3+t)},$$

Now putting, $\frac{1}{(3+t)(2+t)} = \frac{A}{2+t} + \frac{B}{3+t} \ldots\ldots\ldots..(1)$

A(3+t)+B(2+t)=1

Now put t+2=0

Therefore, t=-2

A(3-2)+B(0)=1

A=1

Now put t+3=0

Therefore, t=-3

A(0)+B(2-3)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= log\left|\frac{t+2}{t+3}\right| + c$$

So,

$$I = \int \frac{sec^2 x}{(2+tanx)(3+tanx)} dx = log\left|\frac{tanx+2}{tanx+3}\right| + c$$

18. Question

Evaluate:

$$\int \frac{\sin x \cos x}{(\cos^2 x - \cos x - 2)} dx$$

Answer

Let $I = \int \frac{sinx\ cosx}{cos^2 x - cosx - 2} dx$

Putting t=cos x

dt=-sin x dx

$$I = \int \frac{(-dt)t}{t^2 - t - 2} =, -\int \frac{tdt}{(t+1)(t-2)},$$

Now putting, $\frac{-t}{(t+1)(t-2)} = \frac{A}{t+1} + \frac{B}{t-2} \dots\dots\dots(1)$

A(t-2)+B(t+1)=-t

Now put t-2=0

Therefore, t=2

A(0)+B(2+1)=-2

$B = \frac{-2}{3}$

Now put t+1=0

Therefore, t=-1

A(-1-2)+B(0)=1

$A = \frac{-1}{3}$

Now From equation (1) we get,

$$\frac{-t}{(t+1)(t-2)} = \frac{-1}{3} \times \frac{1}{t+1} - \frac{2}{3} \times \frac{1}{t-2}$$

$$\int \frac{-t}{(t+1)(t-2)} dt = \frac{-1}{3} \int \frac{1}{t+1} - \frac{2}{3} \int \frac{1}{t-2}$$

$$= \frac{-1}{3} \log|t+1| - \frac{2}{3} \log|t-2| + c$$

So,

$$I = \int \frac{sinx\ cosx}{cos^2x - cosx - 2} dx = \frac{-1}{3} \log|cosx + 1| - \frac{2}{3} \log|cosx - 2| + c$$

19. Question

Evaluate:

$$\int \frac{e^x}{(e^{2x} + 5e^x + 6)} dx$$

Answer

Let $I = \int \frac{e^x}{e^{2x}+5e^x+6} dx$

Putting $t=e^x$

$dt=e^x\ dx$

$$I = \int \frac{dt}{(t^2 + 5t + 6)},$$

Now putting, $\frac{1}{(t^2+5t+6)} = \frac{A}{2+t} + \frac{B}{3+t} \dots\dots\dots(1)$

A(3+t)+B(2+t)=1

Now put t+2=0

Therefore, t=-2

A(3-2)+B(0)=1

A=1

Now put t+3=0

Therefore, t=-3

A(0)+B(2-3)=1

B=-1

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= log\left|\frac{t+2}{t+3}\right| + c$$

$$= log\left|\frac{e^x+2}{e^x+3}\right| + c$$

20. Question

Evaluate:

$$\int \frac{e^x}{(e^{3x} - 3e^{2x} - e^x + 3)} dx$$

Answer

Let $I = \int \frac{e^x}{e^{3x} - 3e^{2x} - e^x + 3} dx$

Putting t=e^x

dt=e^x dx

$$I = \int \frac{dt}{(t^3 - 3t^2 - t + 3)} =, \int \frac{dt}{(t^2)(t-3) - (t-3)} = \int \frac{dt}{(t^2-1)(t-3)}$$

Now putting, $\frac{1}{(t-1)(t+1)(t-3)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{t-3} \ldots\ldots\ldots..(1)$

A(t+1)(t-3)+B(t-1)(t-3)+C(t-1)(t+1)=1

Now put t+1=0

Therefore, t=-1

A(0)+B(-1-1)(-1-3)+C(0)=1

B(-2)(-4)=1

$$B = \frac{1}{8}$$

Now put t-1=0

Therefore, t=1

A(1+1)(1-3)+B(0)+C(0)=1

$$A = \frac{-1}{4}$$

Now put t-3=0

Therefore, t=3

A(0)+B(0)+C(3-1)(3+1)=1

$C = \frac{1}{8}$

Now From equation (1) we get,

$$\frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \times \frac{1}{t-1} + \frac{1}{8} \times \frac{1}{t+1} + \frac{1}{8} \times \frac{1}{t-3}$$

$$\int \frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \int \frac{1}{t-1} + \frac{1}{8} \int \frac{1}{t+1} + \frac{1}{8} \int \frac{1}{t-3}$$

$$= \frac{-1}{4} \log|t-1| + \frac{1}{8} \log|t+1| + \frac{1}{8} \log|t-3| + c$$

$$\int \frac{e^x}{e^{3x} - 3e^{2x} - e^x + 3} dx = \frac{-1}{4} \log|e^x - 1| + \frac{1}{8} \log|e^x + 1| + \frac{1}{8} \log|e^x - 3| + c$$

21. Question

Evaluate:

$$\int \frac{2\log x}{x\left[2(\log x)^2 - \log x - 3\right]} dx$$

Answer

Let $I = \int \frac{2logx}{x[2(logx)^2 - logx - 3]} dx$

Putting t=log x

dt=dx/x

$$I = \int \frac{2tdt}{(2t^2 - t - 3)},$$

Now putting, $\frac{2t}{(2t^2-t-3)} = \frac{A}{2t-3} + \frac{B}{t+1} \ldots\ldots\ldots(1)$

A(t+1)+B(2t-3)=2t

Now put 2t-3=0

Therefore, $t = \frac{3}{2}$

$$A\left(\frac{3}{2} + 1\right) + B(0) = 2 \times \frac{3}{2} = 3$$

$A = \frac{6}{5}$

Now put t+1=0

Therefore, t=-1

A(0)+B(-2-3)=-2

$B = \frac{2}{5}$

Now From equation (1) we get,

$$\frac{2t}{(2t^2 - t - 3)} = \frac{6}{5} \times \frac{1}{2t-3} + \frac{2}{5} \times \frac{1}{t+1}$$

$$\int \frac{2t}{(2t^2-t-3)}dt = \frac{6}{5}\int \frac{1}{2t-3}dt + \frac{2}{5}\int \frac{1}{t+1}dt$$

$$= \frac{6}{5}\log\left|\frac{6}{5} \times \frac{log\ (2t-3)}{2}\right| + \frac{2}{5}\log|\log x+1| + c$$

$$\int \frac{2logx}{x[2(logx)^2 - logx - 3]}dx = \frac{3}{5}\log|2logx - 3| + \frac{2}{5}\log|logx + 1| + c$$

22. Question

Evaluate:

$$\int \frac{\operatorname{cosec}^2 x}{(1-\cot^2 x)}dx$$

Answer

Let $I = \int \frac{cosec^2 x}{(1-cot^2 x)}dx$

Putting t=cot x

dt=-cosec^{2}xdx

$$I = \int \frac{-dt}{(1-t^2)} = -\int \frac{1}{(1-t^2)}dt$$

$$= \frac{-1}{2}log\left|\frac{1+cotx}{1-cotx}\right| + c$$

23. Question

Evaluate:

$$\int \frac{\sec^2 x}{(\tan^3 x + 4\tan x)}dx$$

Answer

Let $I = \int \frac{sec^2 x}{(tan^3 x + 4tanx)}dx$

Putting t=tan x

dt=sec^{2}xdx

$$I = \int \frac{dt}{(t^3+4t)} = \int \frac{dt}{t(t^2+4)}$$

Now putting, $\frac{1}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4} \ldots \ldots \ldots (1)$

A(t^2+4)+(Bt + C)t=1

Putting t=0,

A(0+4)× B(0)=1

$$A = \frac{1}{4}$$

By equating the coefficients of t^2 and constant here,

A+B=0

$$\frac{1}{4} + B = 0$$

$B = -\frac{1}{4}, C = 0$

Now From equation (1) we get,

$$\int \frac{1}{t(t^2+4)} dt = \frac{1}{4}\int \frac{dt}{t} - \frac{1}{4}\int \frac{t}{t^2+4} dt$$

$$= \frac{1}{4} logt - \frac{1}{4} \times \frac{1}{2} \log(t^2+4) + c$$

$$= \frac{1}{4} \log tanx - \frac{1}{8} \log(tan^2 x + 4) + c$$

41. Question

$\int \frac{dx}{(x^3-1)}$

Answer

Let $I = \int \frac{dx}{x^3-1}$

Put $\frac{1}{x^3-1} = \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \ldots \ldots \ldots (1)$

$A(x^2+x+1)+(Bx+C)(x-1)=1$

Now putting x-1=0

X=1

A(1+1+1)+0=1

$A = \frac{1}{3}$

By equating the coefficient of x^2 and constant term, A+B=0

$\frac{1}{3} + B = 0$

$B = -\frac{1}{3}$

A-C=1

$\frac{1}{3} - C = 1$

$C = \frac{1}{3} - 1$

$C = \frac{-2}{3}$

From the equation(1), we get,

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{1}{3} \times \frac{1}{x-1} + \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+1}$$

$$I = \int \frac{1}{(x-1)(x^2+x+1)} dx$$

$$= \frac{1}{3}\int \frac{1}{x-1} dx - \frac{1}{3}\int \frac{x}{x^2+x+1} dx - \frac{2}{3}\int \frac{1}{x^2+x+1} dx$$

$$=\frac{1}{3}log|x-1|-\frac{1}{6}\int\frac{2x+1-1}{x^2+x+1}dx-\frac{2}{3}\int\frac{1}{x^2+x+1}dx$$

$$=\frac{1}{3}log|x-1|-\frac{1}{6}\int\frac{2x+1}{x^2+x+1}dx+\frac{1}{6}\int\frac{1}{x^2+x+1}dx-\frac{2}{3}\int\frac{1}{x^2+x+1}dx$$

Put $t=x^2+x+1$

$dt=(2x+1)dx$

$$I=\frac{1}{3}log|x-1|-\frac{1}{6}\int\frac{dt}{t}+\left(\frac{1}{6}-\frac{2}{3}\right)\int\frac{dx}{x^2+x+1}$$

$$=\frac{1}{3}log|x-1|-\frac{1}{6}\log t+\left(\frac{1-4}{6}\right)\int\frac{dx}{x^2+2\times\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$$

$$=\frac{1}{3}log|x-1|-\frac{1}{6}log|x^2+x+1|-\frac{1}{2}\times\frac{1}{\sqrt{3}/2}tan^{-1}\frac{x+1/2}{\sqrt{3}/2}+c$$

$$=\frac{1}{3}log|x-1|-\frac{1}{6}log|x^2+x+1|-\frac{1}{\sqrt{3}}tan^{-1}\frac{2x+1}{\sqrt{3}}+c$$

42. Question

$\int\frac{dx}{(x^3+1)}$

Answer

Let $I=\int\frac{dx}{x^3+1}$

Put $\frac{1}{x^3-1}=\frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\ldots\ldots\ldots.(1)$

$A(x^2-x+1)+(Bx+C)(x+1)=1$

Now putting x+1=0

X=-1

A(1+1+1)+C(0)=1

$$A=\frac{1}{3}$$

By equating the coefficient of x^2 and constant term, A+B=0

$$\frac{1}{3}+B=0$$

$$B=-\frac{1}{3}$$

A+C+=1

$$\frac{1}{3}+C=1$$

$$C=1-\frac{1}{3}$$

$$C=\frac{2}{3}$$

From the equation(1), we get,

$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3} \times \frac{1}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$$

$$I = \int \frac{1}{(x+1)(x^2-x+1)} dx$$

$$= \frac{1}{3}\int \frac{1}{x+1} dx - \frac{1}{3}\int \frac{x}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6}\int \frac{2x-1+1}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6}\int \frac{2x-1}{x^2-x+1} dx - \frac{1}{6}\int \frac{1}{x^2-x+1} dx + \frac{2}{3}\int \frac{1}{x^2-x+1} dx$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6} log|x^2-x+1| - \frac{1}{2} \times \frac{1}{\sqrt{3}/2} tan^{-1}\frac{x-1/2}{\sqrt{3}/2} + c$$

$$= \frac{1}{3} log|x+1| - \frac{1}{6} log|x^2-x+1| + \frac{1}{\sqrt{3}} tan^{-1}\frac{2x-1}{\sqrt{3}} + c$$

24. Question

Evaluate:

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$$

Answer

Let $I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$

Putting t=sin x

dt=cos x dx

$$I = \int \frac{2t}{(1+t)(2+t)} dt$$

Now putting, $\frac{2t}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \ldots\ldots\ldots..(1)$

A(2+t)+B(1+t)=2t

Now put t+2=0

Therefore, t=-2

A(0)+B(1-2)=-4

B=4

Now put t+1=0

Therefore, t=-1

A(2-)+B(0)=-2

A=-2

Now from equation (1), we get,

$$\frac{2t}{(1+t)(2+t)} = \frac{-2}{1+t} + \frac{4}{2+t}$$

$$\int \frac{2t}{(1+t)(2+t)} dt = -2\int \frac{1}{1+t} dt + 4\int \frac{1}{2+t} dt$$

$= 4\log|2+t| - 2\log|1+t| + c$

So,

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx = 4\log|2+t| - 2\log|1+t| + c$$

25. Question

Evaluate:

$$\frac{e^x}{e^x(e^x-1)}dx$$

Answer

Let $I = \int \frac{e^x}{e^x(e^x-1)} dx$

Putting $t=e^x$

$dt=e^x dx$

$$I = \int \frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \ldots\ldots(1)$

A(t-1)+Bt=1

Now put t-1=0

Therefore, t=1

A(0)+B(1) =1

B=1

Now put t=0

A(0-1)+B(0)=1

A=-1

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\int \frac{1}{t(t-1)} dt = -\int \frac{1}{t} dt + \int \frac{1}{t-1} dt$$

$$= -\log t + \log|t-1| + c$$

$$= log\left|\frac{t-1}{t}\right| + c$$

$$= log\left|\frac{e^x-1}{e^x}\right| + c$$

43. Question

$$\int \frac{dx}{(x+1)^2(x^2+1)}$$

Answer

Let $I = \int \frac{dx}{(x^2+1)(x+1)^2}$

Put $\frac{1}{(x^2+1)(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1} \dots\dots\dots..(1)$

$A(x+1)(x^2+1)+B(x^2+1)+(Cx+D)(x+1)^2=1$

Put x+1=0

X=-1

A(0)+B(1+1)+0=1

$B = \frac{1}{2}$

By equating the coefficient of x^2 and constant term, A+C=0

A+B+2C=0……(2)

$A + 2C = \frac{-1}{2} \dots\dots\dots.(3)$

A+B+D=1

Solving (2) and (3),we get,

$$\frac{1}{(x^2+1)(x+1)^2} = \frac{1}{2} \times \frac{1}{x+1} + \frac{1}{2} \times \frac{1}{(x+1)^2} + \frac{-\frac{1}{2}x + 0}{x^2+1}$$

$$\int \frac{1}{(x^2+1)(x+1)^2} dx = \frac{1}{2}\int \frac{1}{x+1} dx + \frac{1}{2}\int \frac{1}{(x+1)^2} dx - \frac{1}{2}\int \frac{x}{x^2+1} dx$$

$$= \frac{1}{2} log|x+1| - \frac{1}{2} \times \frac{1}{x+1} - \frac{1}{4}\log|x^2+1| + c$$

26. Question

Evaluate:

$\int \frac{dx}{x(x^4-1)}$

Answer

Let $I = \int \frac{dx}{x(x^4-1)} dx$

Putting $t=x^4$

$dt=4x^3dx$

$$I = \int \frac{x^3 dx}{x^4(x^4-1)} = \frac{1}{4} \times \int \frac{dt}{t(t-1)}$$

Now putting, $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \dots\dots..(1)$

A(t-1)+Bt=1

Now put t-1=0

Therefore, t=1

A(0)+B(1) =1

B=1

Now put t=0

A(0-1)+B(0)=1

A=-1

Now From equation (1) we get,

$$\frac{1}{t(t-1)}=\frac{-1}{t}+\frac{1}{t-1}$$

$$\frac{1}{4}\int\frac{1}{t(t-1)}dt=-\frac{1}{4}\int\frac{1}{t}dt+\frac{1}{4}\int\frac{1}{t-1}dt$$

$$=-\frac{1}{4}\log t+\frac{1}{4}\log|t-1|+c$$

$$=-\frac{1}{4}\log x^4+\frac{1}{4}\log|x^4-1|+c$$

$$=-\log|x|+\frac{1}{4}\log|x^4-1|+c$$

44. Question

$$\int\frac{17}{(2x+1)(x^2+4)}dx$$

Answer

Let $I=\int\frac{17}{(2x+1)(x^2+4)}dx$

Put $\frac{17}{(2x+1)(x^2+4)}=\frac{A}{2x+1}+\frac{Bx+C}{x^2+4}\ldots\ldots\ldots.(1)$

$A(x^2+4)+(Bx+C)(2x+1)=17$

Put 2x+1=0

$$x=-\frac{1}{2}$$

$$A\left(\frac{1}{4}+4\right)+0=17$$

$$A\left(\frac{17}{4}\right)=17$$

A=4

By equating the coefficient of x^2 and constant term,

A+2B=0

4+2B=0

B=-2

4A+C=17

4×4+C=17

C=1

From the equation(1), we get,

$$\frac{17}{(2x+1)(x^2+4)}=\frac{4}{2x+1}+\frac{-2x+1}{x^2+4}$$

$$\int\frac{17}{(2x+1)(x^2+4)}dx=4\int\frac{1}{2x+1}dx-2\int\frac{2x}{x^2+4}dx+\int\frac{1}{x^2+2^2}dx$$

$= \frac{4\log|2x+1|}{2} - \log|x^2+4| + \frac{1}{2}tan^{-1}\frac{x}{2} + c$

$= 2\log|2x+1| - \log|x^2+4| + \frac{1}{2}tan^{-1}\frac{x}{2} + c$

27. Question

Evaluate:

$\int \frac{(1-x^2)}{x(1-2x)} dx$

Answer

Let $I = \int \frac{(x^2-1)}{x(2x-1)} dx = \int \left(\frac{1}{2} + \frac{\left(\frac{1}{2}x-1\right)}{x(2x-1)}\right) dx = \int \frac{1}{2} dx + \int \frac{x}{x(2x-1)} dx - \int \frac{1}{x(2x-1)} dx$

$I = \frac{1}{2}x + \frac{1}{2} \times \frac{\log|2x-1|}{2} - I_1 \ldots \ldots (1)$

Where $I_1 = \int \frac{1}{x(2x-1)} dx$........(2)

Now putting, $\frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$

A(2x-1)+Bx=1

Putting 2x-1=0

$x = \frac{1}{2}$

$A(0) + B\left(\frac{1}{2}\right) = 1$

B=2

Putting x=0,

A(0-1)+B(0)=1

A=-1

From equation (2), we get,

$\frac{1}{x(2x-1)} = -\frac{1}{x} + \frac{2}{2x-1}$

$\int \frac{1}{x(2x-1)} dx = -\int \frac{1}{x} dx + 2\int \frac{1}{2x-1} dx$

$= -\log|x| + \frac{2\log|2x-1|}{2} + c$

$= \log|2x-1| - \log x + c$

From equation (1),

$I = \frac{1}{2}x + \frac{1}{4}\log|2x-1| - \log|2x-1| + logx + c$

$= \frac{1}{2}x - \frac{3}{4}\log|1-2x| + log|x| + c$

45. Question

$\int \frac{dx}{(x^2+2)(x^2+4)}$

Answer

Let $I = \int \frac{dx}{(x^2+2)(x^2+4)} dx$

Put $\frac{1}{(x^2+2)(x^2+4)} = \frac{1}{(t+2)(t+4)} = \frac{A}{t+2} + \frac{B}{t+4} \dots\dots\dots(1)$

A(t+4)+B(t+2) = 1

Put t+4=0

t=-4

A(0)+B(-4+2)=1

$B = -\frac{1}{2}$

Put t+2=0

t=-2

A(-2+4)+B(0)=1

$A = \frac{1}{2}$

From equation(1),we get,

$$\frac{1}{(t+2)(t+4)} = \frac{1}{2} \times \frac{1}{t+2} - \frac{1}{2} \times \frac{1}{t+4}$$

$$\int \frac{1}{(x^2+2)(x^2+4)} dx = \frac{1}{2} \int \frac{1}{x^2+2} dx - \frac{1}{2} \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{2} tan^{-1} \frac{x}{2} + c$$

$$= \frac{1}{4} tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{4} tan^{-1} \frac{x}{2} + c$$

28. Question

Evaluate:

$\int \frac{(x^2+x+1)}{(x+2)(x+1)^2} dx$

Answer

Let $I = \int \frac{x^2+x+1}{(x+2)(x+1)^2} dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \dots\dots(1)$

$A(x+1)^2+B(x+2)(x+1)+C(x+2)=x^2+x+1$

Now put x+1=0

Therefore, x=-1

A(0)+B(0)+C(-1+2) =1-1+1=1

C=1

Now put x+2=0

Therefore, x=-2

A(-2+1)^2+B(0)+C(0) =4-2+1=3

A=3

Equating the coefficient of x^2,A+B=1

3+B=1

B=-2

Form equation (1),we get,

$$\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{3}{(x+2)} - \frac{2}{(x+1)} + \frac{1}{(x+1)^2}$$

So,

$$\int \frac{x^2+x+1}{(x+2)(x+1)^2}dx = \int \frac{3}{(x+2)}dx - \int \frac{2}{(x+1)}dx + \int \frac{1}{(x+1)^2}dx$$

$$= 3\log|x+2| - 2\log|x+1| - \frac{1}{1+x} + c$$

46. Question

$$\frac{x^2+1}{(x^2+4)(x^2+25)}dx$$

Answer

Let $I = \int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$

Putting $\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{t+1}{(t+4)(t+25)} = \frac{A}{t+4} + \frac{B}{t+25} \ldots\ldots\ldots(1)$

Where t=x^2

(A+B)t+(25A+4B)=t+1

A+B=1............(1)

25A+4B=1..........(2)

Solving equation (1)and(2), we get,

$$A = \frac{-1}{7}\ and\ B = \frac{8}{7}$$

Now,

$$\frac{t+1}{(t+4)(t+25)} = \frac{-1}{7} \times \frac{1}{t+4} + \frac{8}{7} \times \frac{1}{t+25}$$

$$\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{-1}{7} \times \frac{1}{x^2+4} + \frac{8}{7} \times \frac{1}{x^2+25}$$

$$\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx = \frac{-1}{7}\int \frac{1}{x^2+2^2}dx + \frac{8}{7}\int \frac{1}{x^2+5^2}dx$$

$$= -\frac{1}{7} \times \frac{1}{2}tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{7} \times \frac{1}{5}tan^{-1}\left(\frac{x}{5}\right) + c$$

$= -\frac{1}{14}tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{35}tan^{-1}\left(\frac{x}{5}\right) + c$

29. Question

Evaluate:

$\int \frac{(2x+9)}{(x+2)(x-3)^2}dx$

Answer

Let $I = \int \frac{2x+9}{(x+2)(x-3)^2}dx$

Now putting, $\frac{2x+9}{(x+2)(x-3)^2} = \frac{A}{(x+2)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} \ldots \ldots (1)$

$A(x-3)^2+B(x+2)(x-3)+C(x+2)=2x+9$

Now put x-3=0

Therefore, x=3

A(0)+B(0)+C(3+2) =6+9=15

C=3

Now put x+2=0

Therefore, x=-2

$A(-2-3)^2+B(0)+C(0) = -4+9=5$

$A = \frac{1}{5}$

Equating the coefficient of x^2,we get,

A+B=0

$\frac{1}{5} + B = 0$

$B = -\frac{1}{5}$

From equation (1), we get,

$$\frac{2x+9}{(x+2)(x-3)^2} = \frac{1}{5} \times \frac{1}{(x+2)} - \frac{1}{5} \times \frac{1}{(x-3)} + \frac{3}{(x-3)^2}$$

$$\int \frac{2x+9}{(x+2)(x-3)^2}dx = \frac{1}{5}\int \frac{1}{(x+2)}dx - \frac{1}{5}\int \frac{1}{(x-3)}dx + 3\int \frac{1}{(x-3)^2}dx$$

$$= \frac{1}{5}\log|x+2| - \frac{1}{5}\log|x-3| - \frac{3}{x-3} + c$$

47. Question

$\int \frac{dx}{(e^x-1)^2}$

Answer

putting $t=e^x-1$

$e^x=t+1$

$dt= e^x\, dx$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

Put$\frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2} \dots \dots (1)$

A(t^2)+(Bt+C)(t+1)=1

Put t+1=0

t=-1

A=1

Equating coefficients

A+B=0

1+B=0

B=-1

C=1

From equation (1),we get,

$$\frac{1}{(1+t)t^2} = \frac{1}{t+1} + \frac{-t+1}{t^2}$$

$$\int \frac{1}{(1+t)t^2} dt = \int \frac{1}{t+1} dt - \int \frac{t}{t^2} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \int \frac{1}{t} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - log|t| - \frac{1}{t} + c$$

$$\int \frac{1}{(e^x-1)^2} dx = log\,|e^x| - \log|e^x - 1| - \frac{1}{e^x-1} + c$$

48. Question

$$\int \frac{dx}{x(x^5+1)}$$

Answer

Let $I = \int \frac{dx}{x(x^5+1)}$

Put t=x^5

dt=5x^4dx

$$\int \frac{dt}{\frac{(5x^4)}{x(t+1)}} = \frac{1}{5}\int \frac{dt}{x^5(t+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \dots \dots ..(1)$

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= logt - \log|t+1| + c$$

$$= log\left|\frac{t}{t+1}\right| + c$$

$$\int \frac{dx}{x(x^5+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)} = \frac{1}{5} log\left|\frac{x^5}{x^5+1}\right| + c$$

$$= logx - \frac{1}{5} log|x^5+1| + c$$

30. Question

Evaluate:

$$\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$$

Answer

Let $I = \int \frac{x^2+1}{(x+3)(x-1)^2} dx$

Now putting, $\frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \ldots \ldots (1)$

$A(x-1)^2+B(x+3)(x-1)+C(x+3)=x^2+1$

Now put x-1=0

Therefore, x=1

A(0)+B(0)+C(4) =2

$$C = \frac{1}{2}$$

Now put x+3=0

Therefore, x=-3

$A(-3-1)^2+B(0)+C(0) =9+1=10$

$$A = \frac{5}{8}$$

By equating the coefficient of x^2, we get, A+B=1

$$\frac{5}{8} + B = 1$$

$$B = 1 - \frac{5}{8} = \frac{3}{8}$$

From equation (1), we get,

$$\frac{x^2+1}{(x+3)(x-2)^2}=\frac{5}{8}\times\frac{1}{(x+3)}+\frac{3}{8}\times\frac{1}{(x-2)}+\frac{1}{(x-2)^2}$$

$$\int\frac{x^2+1}{(x+3)(x-2)^2}dx=\frac{5}{8}\int\frac{1}{(x+3)}dx+\frac{3}{8}\int\frac{1}{(x-2)}dx+\int\frac{1}{(x-2)^2}dx$$

$$=\frac{5}{8}\log|x+3|+\frac{3}{8}\log|x-1|-\frac{1}{2(x-1)}+c$$

31. Question

Evaluate:

$$\int\frac{(x^2+1)}{(x+3)(x-1)}dx$$

Answer

Let $I=\int\frac{x^2+1}{(x-3)(x-1)^2}dx$

Now putting, $\frac{x^2+1}{(x-3)(x-1)^2}=\frac{A}{(x-3)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}\ldots\ldots(1)$

$A(x-1)^2+B(x-3)(x-1)+C(x-3)=x^2+1$

Putting x-1=0,

X=1

A(0)+B(0)+C(1-3)=1+1

C=-1

Putting x-3=0,

X=3

$A(3-1)^2+B(0)+C(0)=9+1$

A(4)=10

$$A=\frac{5}{2}$$

Equating the coefficient of x^2

A+B=1

$$\frac{5}{2}+B=1$$

$$B=1-\frac{5}{2}=\frac{-3}{2}$$

From (i) $\int\frac{x^2+1}{(x-3)(x-1)^2}dx=\frac{5}{2}\int\frac{1}{x-3}dx-\frac{3}{2}\int\frac{1}{x-1}dx-\int\frac{1}{(x-1)^2}dx$

$$=\frac{5}{2}log|x-3|-\frac{3}{2}\log|x-1|+\frac{1}{x-1}+C$$

49. Question

$$\int\frac{dx}{x(x^6+1)}$$

Answer

Let $I=\int \frac{dx}{x(x^6+1)}$

Put $t=x^6$

$dt=6x^5dx$

$$\int \frac{dt}{\frac{(6x^5)}{x(t+1)}}=\frac{1}{6}\int \frac{dt}{x^6(t+1)}=\frac{1}{6}\int \frac{dt}{t(t+1)}$$

Putting $\frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{t+1}\ldots\ldots..(1)$

A(t+1)+Bt=1

Now put t+1=0

t=-1

A(0)+B(-1)=1

B=-1

Now put t=0

A(0+1)+B(0)=1

A=1

$$\frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)}dt=\int \frac{1}{t}dt-\int \frac{1}{t+1}dt$$

$$=logt-\log|t+1|+c$$

$$=log\left|\frac{t}{t+1}\right|+c$$

$$\int \frac{dx}{x(x^6+1)}=\frac{1}{6}\int \frac{dt}{t(t+1)}=\frac{1}{6}log\left|\frac{x^6}{x^6+1}\right|+c$$

$$=logx-\frac{1}{6}log|x^6+1|+c$$

32. Question

Evaluate:

$$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)}dx$$

Answer

Let $I=\int \frac{x^2+x+1}{(x+2)(x^2+1)}dx$

Now putting, $\frac{x^2+x+1}{(x+2)(x^2+1)}=\frac{A}{(x+2)}+\frac{Bx+C}{(x^2+1)}$

$A(x^2+1)+(Bx+C)(x+2) = x^2+x+1$

$Ax^2+A+Bx^2+Cx+2Bx+2C = x^2+x+1$

$(A+B)x^2+(C+2B)x+(A+2C) = x^2+x+1$

Equating coefficients A+B=1.......(i)

A+2C=1

A=1-2C……(ii)

2B+C=1

2B=1-C

$$B = \frac{1-C}{2} \ldots\ldots..(iii)$$

$$(1-2C) + \frac{1-C}{2} = 1$$

2-4C+1-C=2

3-5C=2

-5C=-1

$$C = \frac{1}{5}$$

And $2B = 1 - \frac{1}{5} = \frac{4}{5}$

$$B = \frac{2}{5}$$

$$A = 1 - 2 \times \frac{1}{5}$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \int \frac{A}{(x+2)} dx + \int \frac{Bx+C}{(x^2+1)} dx$$

$$= \frac{3}{5} \times \int \frac{1}{(x+2)} dx + \frac{1}{5} \times \int \frac{2x+1}{(x^2+1)} dx$$

$$= \frac{3}{5} \log|x+2| + \frac{1}{5} I_1 + C_1$$

$$I_1 = \int \frac{2x+1}{(x^2+1)} dx = \int \frac{2x}{(x^2+1)} dx + \int \frac{1}{(x^2+1)} dx$$

$$= log|x^2+1| + tan^{-1}x + C_2$$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \frac{3}{5} \log|x+2| + \frac{1}{5} log|x^2+1| + \frac{1}{5} tan^{-1}x + C$$

50. Question

$$\int \frac{dx}{\sin x\,(3+2\cos x)}$$

Answer

let $I = \int \frac{dx}{sinx\,(3+2cosx)}$

Put t=cosx

dt=-sinxdx

$$\frac{dt}{-sinx} = dx$$

$$I = \int \frac{\frac{dt}{-sinx}}{sinx(3+2t)}$$

$$= -\int \frac{dt}{sin^2x(3+2t)} = -\int \frac{dt}{(1-cos^2x)(3+2t)}$$

$$= -\int \frac{dt}{(1-t^2)(3+2t)}$$

$$\frac{1}{(1-t^2)(3+2t)} = \frac{1}{(1-t)(1+t)(3+2t)}$$

Putting $\frac{1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t} \dots \dots (1)$

A(1+t)(3+2t)+B(1-t)(3+2t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(3-2)+C(0)=1

$$B = \frac{1}{2}$$

Now Putting 1-t=0

t=1

A(2)(5)+B(0)+C(0)=1

$$A = \frac{1}{10}$$

Now Putting 3+2t=0

$$t = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$$

$$C = \frac{-4}{5}$$

$$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$$

$$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10}\int \frac{1}{1-t} dt + \frac{1}{2}\int \frac{1}{1+t} dt - \frac{4}{5}\int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} log|1-t| + \frac{1}{2} log|1+t| - \frac{4}{5} \times \frac{log|3+2t|}{2} + c$$

$$= -\frac{1}{10} log|1-cosx| + \frac{1}{2} log|1+cosx| - \frac{2}{5} log|3+2cosx| + c$$

33. Question

Evaluate:

$$\int \frac{2x}{(2x+1)^2} dx$$

Answer

Let $I = \int \frac{2x}{(2x+1)^2} dx$

Now putting, $\frac{2x}{(2x+1)^2} = \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} \dots \dots \dots \dots (1)$

A(2x+1)+B = 2x

Putting 2x+1=0,

$x = \frac{-1}{2}$

A(0)+B=-1

B=-1

By equating the coefficient of x,

2A=2

A=1

From equation (1),we get,

$$\frac{2x}{(2x+1)^2} = \frac{1}{(2x+1)} - \frac{1}{(2x+1)^2}$$

$$\int \frac{2x}{(2x+1)^2} dx = \int \frac{1}{(2x+1)} dx - \int \frac{1}{(2x+1)^2} dx$$

$$= \frac{\log|2x+1|}{2} + \frac{1}{2(2x+1)} + c$$

$$= \frac{1}{2}\left[\log|2x+1| + \frac{1}{2x+1}\right] + c$$

51. Question

$\int \frac{dx}{\cos x(5-4\sin x)}$

Answer

let $I = \int \frac{dx}{cosx\,(5-4sinx)}$

Put t=sinx

dt=cosxdx

$$I = \int \frac{dt}{(1-sin^2 x)(5-4t)} = \int \frac{dt}{(1-t^2)(5-4t)}$$

$$\frac{1}{(1-t^2)(5-4t)} = \frac{1}{(1-t)(1+t)(5-4t)}$$

Putting $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t} \dots \dots (1)$

A(1+t)(5-4t)+B(1-t)(5-4t)+C(1+t)(1-t)=1

Now Putting 1+t=0

t=-1

A(0)+B(2)(9)+C(0)=1

$B = \frac{1}{18}$

Now Putting 1-t=0

t=1

A(2) +B(0)+C(0)=1

$A = \frac{1}{2}$

Now Putting 5-4t=0

$t = \frac{5}{4}$

$A(0) + B(0) + C\left(1 - \frac{25}{16}\right) = 1$

$C = \frac{-16}{9}$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(5-4t)} = \frac{1}{2} \times \frac{1}{1-t} + \frac{1}{18} \times \frac{1}{1+t} - \frac{16}{9} \times \frac{1}{5-4t}$$

$$\int \frac{1}{(1-t)(1+t)(5-4t)} dt = \frac{1}{2}\int \frac{1}{1-t} dt + \frac{1}{18}\int \frac{1}{1+t} dt - \frac{16}{9}\int \frac{1}{5-4t} dt$$

$$= -\frac{1}{2} log|1-t| + \frac{1}{18} log|1+t| - \frac{16}{9} \times \frac{log|5-4t|}{-4} + c$$

$$= -\frac{1}{2} log|1 - sinx| + \frac{1}{18} log|1 + sinx| + \frac{4}{9} log|5 - 4sinx| + c$$

34. Question

Evaluate:

$$\int \frac{3x+1}{(x+2)(x-2)^2} dx$$

Answer

Let $I = \int \frac{3x+1}{(x+2)(x-2)^2} dx$

Now putting, $\frac{3x+1}{(x+2)(x-2)^2} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \dots \dots (1)$

$A(x-2)^2+B(x+2)(x-2)+C(x+2)=3x+1$

Putting x-2=0,

X=2

A(0)+B(0)+C(2+1)=3×2+1

$C = \frac{7}{4}$

Putting x+2=0,

X=-2

$A(-4)^2$+B(0)+C(0)=-6+1=-5

$A = \frac{-5}{16}$

By equation the coefficient of x^2, we get, A+B=0

$\frac{-5}{16}+B=0$

$B=\frac{5}{16}$

$I=-\frac{5}{16}log|x+2|+\frac{5}{16}log|x-2|-\frac{7}{4(x-2)}+c$

52. Question

$\int\frac{dx}{\sin x\,\cos^2 x}$

Answer

Let $I=\int\frac{1}{sinx\times cos^2x}dx=\int\frac{sin^2x+cos^2x}{sinx\times cos^2x}dx=\int\frac{sin^2x}{sinx\times cos^2x}dx+\int\frac{cos^2x}{sinx\times cos^2x}dx$

$=\int\frac{sinx}{cos^2x}dx+\int\frac{1}{sinx}dx$

$=\int(\tan x\sec x+cosec\,x)dx$

$=\sec x-\frac{1}{2}logcot^2\frac{x}{2}=\sec x-\frac{1}{2}\log\left(\frac{1+cosx}{1-cosx}\right)+c$

53. Question

$\int\frac{\tan x}{(1-\sin x)}dx$

Answer

let $I=\int\frac{\tan x}{(1-sinx)}$dx $=\int\frac{\sin x}{\cos x(1-sinx)}$dx

Put t=sinx

dt=cosxdx

$I=\int\frac{\sin x\times cosx}{cos^2x\,(1-sinx)}dx=\int\frac{tdt}{(1-sin^2x)(1-t)}=\int\frac{tdt}{(1-t^2)(1-t)}$

Putting $\frac{t}{(1-t)(1+t)(1-t)}=\frac{A}{1+t}+\frac{B}{1-t}+\frac{C}{(1-t)^2}\dots\dots(1)$

$A(1+t)^2+B(1-t)(1+t)+C(1+t)=t$

Now Putting 1-t=0

t=1

A(0)+B(0)+C(1+1)=1

$C=\frac{1}{2}$

Now Putting 1+t=0

t=-1

$A(2)^2+B(0)+C(0)=-1$

$A=-\frac{1}{4}$

By equating the coefficient of t^2,we get,A-B=0

$$\frac{-1}{4} - B = 0$$

$$B = -\frac{1}{4}$$

From equation(1),we get,

$$\frac{t}{(1-t)(1+t)(1-t)} = \frac{-1}{4} \times \frac{1}{1+t} - \frac{1}{4} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{(1-t)^2}$$

$$\int \frac{t}{(1-t)(1+t)(1-t)} dt = \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= -\frac{1}{4} log|1+t| - \frac{1}{4} log|1-t| - \frac{1}{2} \times \frac{1}{1-t} + c$$

$$= -\frac{1}{4} log|1+sinx| - \frac{1}{4} log|1-sinx| - \frac{1}{2} \times \frac{1}{1-sinx} + c$$

35. Question

Evaluate:

$$\int \frac{(5x+8)}{x^2(3x+8)} dx$$

Answer

Let $I = \int \frac{5x+8}{x^2(3x+8)} dx$

Now putting, $\frac{5x+8}{x^2(3x+8)} = \frac{A}{(3x+8)} + \frac{Bx+C}{x^2} \ldots \ldots (1)$

$Ax^2 + (Bx + C)(3x+8) = 5x+8$

Putting 3x+8=0,

$$x = -\frac{8}{3}$$

$$A\left(\frac{64}{9}\right) + B(0) = 5\left(-\frac{8}{3}\right) + 8$$

$$A\left(\frac{64}{9}\right) = \frac{-40+24}{3}$$

$$A\left(\frac{64}{9}\right) = \frac{-16}{3}$$

$$A = \frac{-3}{4}$$

By equating the coefficient of x^2 and constant term,

A+3B=0

$$\frac{-3}{4} + 3B = 0$$

$$3B = \frac{3}{4}$$

$B=\frac{1}{4}$

8C=8

C=1

From equation (1), we get,

$$\int\frac{5x+8}{x^2(3x+8)}dx=\frac{-3}{4}\times\int\frac{1}{(3x+8)}dx+\frac{1}{4}\times\int\frac{x+1}{x^2}dx$$

$$=\frac{-3}{4}\times\frac{\log(3x+8)}{3}+\frac{1}{4}\int\frac{x}{x^2}dx+\int\frac{1}{x^2}dx$$

$$=-\frac{1}{4}\log|3x+8|+\frac{1}{4}logx-\frac{1}{x}+c$$

Putting x+2=0,

X=-2

A(-4)^2+B(0)+C(0)=-6+1=-5

$A=\frac{-5}{16}$

54. Question

$\int\frac{dx}{(\sin x+\sin 2x)}$

Answer

let $I=\int\frac{dx}{(sinx+sin2x)}=\int\frac{dx}{(sinx+2sinxcosx)}$

Put t=cosx

dt=-sinxdx

$\frac{-dt}{sinx}=dx$

$$I=\int\frac{-dt}{sin^2x(1+2t)}=\int\frac{dt}{(1-cos^2x)(1+2t)}=\int\frac{dt}{(1-t^2)(1+2t)}$$

Putting $\frac{t}{(1-t)(1+t)(1+2t)}=\frac{A}{1-t}+\frac{B}{1+t}+\frac{C}{1+2t}\ldots\ldots(1)$

A(1+t)(1+2t)+B(1-t)(1+2t)+C(1-t^2)=1

Putting 1+t=0

t=-1

A(0)+B(2)(1-2)+C(0)=1

$B=-\frac{1}{2}$

Putting 1-t=0

t=1

A(2)(3)+B(0)+C(0)=1

$A=\frac{1}{6}$

Putting 1+2t=0

$$t = -\frac{1}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{1}{4}\right) = 1$$

$$C = \frac{4}{3}$$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} \times \frac{1}{1-t} - \frac{1}{2} \times \frac{1}{1+t} + \frac{4}{3} \times \frac{1}{1+2t}$$

$$\int \frac{1}{(1-t)(1+t)(1+2t)} dt = \frac{1}{6}\int \frac{1}{1-t} dt - \frac{1}{2}\int \frac{1}{1+t} dt + \frac{4}{3}\int \frac{1}{1+2t} dt$$

$$= \frac{1}{6} log|1-t| - \frac{1}{2} log|1+t| + \frac{2}{3} log|1+2t| + c$$

$$= \frac{1}{6} log|1-cosx| - \frac{1}{2} log|1+cosx| + \frac{2}{3} log|1+2cosx| + c$$

36. Question

Evaluate:

$$\int \frac{(5x^2 - 18x + 17)}{(x-1)^2 (2x-3)} dx$$

Answer

Let $I = \int \frac{5x^2 18x+17}{(x-1)^2(2x-3)} dx$

Now putting, $\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \ldots.(1)$

$A(x-1)^2+B(2x-3)(x-1)+C(2x-3) = 5x^2-18x+17$

Putting x-1=0,

X=1

A(0)+B(0)+C(2-3)=5-18+17

C(-1)=4

Putting 2x-3=0,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right) + 0 = 5 \times \frac{9}{4} - 27 + 17$$

$$A\left(\frac{1}{4}\right) = \frac{45}{4} - 10 = \frac{5}{4}$$

A=5

By equating the coefficient of x^2, we get ,

A+2B=5

5+2B=5

2B=0

B=0

From equation (1), we get,

$$\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = 5\times\frac{1}{(2x-3)} + 0 - 4\times\frac{1}{(x-1)^2}$$

$$\int \frac{5x^2 18x+17}{(x-1)^2(2x-3)}dx = \frac{5}{2}\log(2x-3) + \frac{4}{x-1} + c$$

37. Question

Evaluate:

$$\int \frac{8}{(x+2)(x^2+4)}dx$$

Answer

Let $I = \int \frac{8}{(x+2)(x^2+4)}dx$

Now putting, $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+4)} \ldots\ldots(1)$

A(x^2+4)+(Bx+C)(x+2)= 8

Putting x+2=0,

X=-2

A(4+4)+0=8

A=1

By equating the coefficient of x^2 and constant term, A+B=0

1+B=0

B=-1

4A+2C=8

4×1+2C=8

2C=4

C=2

From equation (1),we get,

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{(x^2+4)}$$

$$\int \frac{8}{(x+2)(x^2+4)}dx = \int \frac{1}{x+2}dx - \int \frac{x}{(x^2+4)}dx + 2\int \frac{1}{(x^2+4)}dx$$

$$= log|x+2| - \frac{1}{2}\log(x^2+4) + 2\times\frac{1}{2}\times tan^{-1}\frac{x}{2} + c$$

$$= log|x+2| - \frac{1}{2}\log|x^2+4| + tan^{-1}\frac{x}{2} + c$$

55. Question

$\int \frac{x^2}{(x^4-x^2-12)}dx$

Answer

Let $I=\int \frac{x^2}{(x^4-x^2-12)}dx$

Putting $\frac{x^2}{(x^4-x^2-12)}=\frac{t}{t^2-t-12}=\frac{t}{(t-4)(t+3)}=\frac{A}{t-4}+\frac{B}{t+3}\ldots\ldots\ldots.(1)$

Where $t=x^2$

A(t+3)+B(t-4)=t

Now put t+3=0

t=-3

A(0)+B(-7)=-3

$B=\frac{3}{7}$

Now put t-4=0

t=4

A(4+3)+B(0)=4

$A=\frac{4}{7}$

From equation(1)

$$\frac{t}{(t-4)(t+3)}=\frac{4}{7}\times\frac{1}{t-4}+\frac{3}{7}\times\frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)}=\frac{4}{7}\times\frac{1}{x^2-2^2}+\frac{3}{7}\times\frac{1}{x^2+(\sqrt{3})^2}$$

$$\int\frac{x^2}{(x^2-4)(x^2+3)}dx=\frac{4}{7}\int\frac{1}{x^2-2^2}dx+\frac{3}{7}\int\frac{1}{x^2+\left(\sqrt{3}\right)^2}dx$$

$$=\frac{4}{7}\times\frac{1}{2}\times\frac{1}{2}log\left|\frac{x-2}{x+2}\right|+\frac{3}{7}\times\frac{1}{\sqrt{3}}tan^{-1}\frac{x}{\sqrt{3}}+c$$

$$=\frac{1}{7}log\left|\frac{x-2}{x+2}\right|+\frac{\sqrt{3}}{7}tan^{-1}\frac{x}{\sqrt{3}}+c$$

56. Question

$\int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)}dx$

Answer

Let $I=\int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)}dx$

Putting $\frac{(x^2)^2}{(x^2+1)(x^2+9)(x^2+16)}=\frac{t^2}{(t+1)(t+9)(t+16)}=\frac{A}{t+1}+\frac{B}{t+9}+\frac{C}{t+16}\ldots\ldots\ldots.(1)$

Where $t=x^2$

t^2=A(t+9)(t+16)+B(t+1)(t+16)+C(t+1)(t+9)

Now put t+1=0

t=-1

A(8)(15)+B(0)+C(0)=1

$A = \frac{1}{120}$

Now put t+9=0

t=-9

A(-9+9)(-9+16)+B(-9+1)(-9+16)+C(-9+1)(-9+9)=$(-9)^2$

A(0)+B(-56)+C(0)=81

$B = -\frac{81}{56}$

Now put t+16=0

t=-16

A(0)+B(0)+C(-15)(-7)=$(-16)^2$

A(0)+B(0)+C(105)=256

$C = \frac{256}{105}$

From equation(1)

$$\frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16}$$

$$\int \frac{t^2}{(t+1)(t+9)(t+16)} dt = \int \left[\frac{\frac{1}{120}}{t+1} - \frac{\frac{81}{56}}{t+9} + \frac{\frac{256}{105}}{t+16}\right] dt$$

$$= \frac{1}{120}\int \frac{1}{t+1} dt - \frac{81}{56}\int \frac{1}{t+9} dt + \frac{256}{105}\int \frac{1}{t+16} dt$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+9} dx + \frac{256}{105}\int \frac{1}{x^2+16} dx$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+(3)^2} dx + \frac{256}{105}\int \frac{1}{x^2+(4)^2} dx$$

$$= \frac{1}{120} tan^{-1} x - \frac{81}{56} \times \frac{1}{3} tan^{-1}\left(\frac{x}{3}\right) + \frac{256}{105} \times \frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$$

$$= \frac{1}{120} tan^{-1} x - \frac{27}{56} tan^{-1}\left(\frac{x}{3}\right) + \frac{64}{105} tan^{-1}\left(\frac{x}{4}\right) + c$$

38. Question

Evaluate:

$$\int \frac{(3x+5)}{(x^3 - x^2 + x - 1)} dx$$

Answer

Let $I = \int \frac{3x+5}{(x^3 - x^2 + x - 1)} dx$

Now putting, $\frac{3x+5}{(x^3 - x^2 + x - 1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)} \ldots\ldots(1)$

A(x^2+1)+(Bx+C)(x-1)=3x+5

Putting x-1=0,

X=1

A(2)+B(0)=3+5=8

A=4

By equating the coefficient of x^2 and constant term, A+B=0

4+B=0

B=-4

A-C=5

4-C=5

C=-1

From equation (1),we get,

$$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{(x^2+1)}$$

$$\int \frac{3x+5}{(x-1)(x^2+1)} dx = 4\int \frac{1}{x-1} dx - 4\int \frac{1}{(x^2+1)} dx - \int \frac{1}{(x^2+1)} dx$$

$$= 4\log(x-1) - \frac{4}{2}\log(x^2+1) - tan^{-1}x + c$$

$$= 4\log(x-1) - 2\log(x^2+1) - tan^{-1}x + c$$

57. Question

$$\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$$

Answer

let $I = \int \frac{sin2x}{(1-cos2x)(2-cos2x)} dx$

Put t=cos2x

dt=-2sin2xdx

$$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2}\int \frac{dt}{(t-2)(1-t)}$$

Putting $\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t} \dots\dots..(1)$

A(1-t)+B(t-2)=1

Putting 1-t=0

t=1

A(0)+B(1-2) =1

B=-1

Putting t-2=0

t=2

A(1-2)+B(0) =1

A=-1

From equation (1), we get,

$$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$$

$$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$$

$$= -log|2-t| + log|t-1| + c$$

$$= log|t-1| - log|2-t| + c$$

$$= log|cos2x - 1| - log|2 - cos2x| + c$$

39. Question

Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx$$

Answer

Let $I = \int \frac{2x}{(x^2+1)(x^2+3)} dx$

Put t=x^2

dt=2xdx

Now putting, $\frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3} \ldots. \ldots.(1)$

A(t+3) +B(t+1) = 1

Putting t+3=0,

X=-3

A(0) +B(-3+1)=1

$$B = -\frac{1}{2}$$

Putting t+1=0,

X=-1

A(-1+3)+B(0)=1

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+1)(t+3)} = \frac{1}{2} \times \frac{1}{t+1} - \frac{1}{2} \times \frac{1}{t+3}$$

$$\int \frac{1}{(t+1)(t+3)} dt = \frac{1}{2} \int \frac{1}{t+1} dt - \frac{1}{2} \int \frac{1}{t+3} dt$$

$$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$$

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{1}{2} \log|x^2+1| - \frac{1}{2} \log|x^2+3| + c$$

58. Question

$$\int \frac{2}{(1-x)(1+x^2)} dx$$

Answer

Let $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Put $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \dots\dots\dots..(1)$

$A(1+x^2)+Bx(1-x)+C(1-x) = 2$

Put x=1

2=2A+0+0

A=1

Put x=0

2=A+C

C=2-A

C=2-1=1

Putting x=2

We have 2=5A-2B-C

2=5×1-2B-1

2B=2

B=1

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$$

$$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$-log|1-x| + \frac{1}{2} log|1+x^2| + tan^{-1}x + c$$

40. Question

Evaluate:

$\int \frac{x^2}{(x^4-1)} dx$

Answer

Let $I = \int \frac{x^2}{(x^4-1)} dx$

Put $t=x^2$

dt=2xdx

Now putting, $\frac{x^2}{(x^4-1)} = \frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \dots\dots(1)$

A(t+1)+B(t-1) = t

Putting t+1=0,

t=-1

A(0)+B(-1-1)=-1

$$B = \frac{1}{2}$$

Putting t-1=0,

t=1

A(1+1)+B(0)=1

$A = \frac{1}{2}$

From equation(1),we get,

$$\frac{t}{(t-1)(t+1)} = \frac{1}{2} \times \frac{1}{t-1} + \frac{1}{2} \times \frac{1}{t+1}$$

$$\int \frac{x^2}{(x^4-1)} dt = \frac{1}{2}\int \frac{1}{x^2-1} dt + \frac{1}{2}\int \frac{1}{x^2+1} dt$$

$$= \frac{1}{2} \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2} tan^{-1}x + c$$

$$= \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| + \frac{1}{2} tan^{-1}x + c$$

59. Question

$$\int \frac{2x^2+1}{x^2(x^2+4)} dx$$

Answer

Let $I = \int \frac{2x^2+1}{x^2(x^2+4)} dx$

Again let x^2=t

$$\frac{2t+1}{t(t+4)} = \frac{A}{t} + \frac{B}{(t+4)} \dots\dots(1)$$

2t+1=A(t+4)+B(t)

Putting t=-4

2(-4)+1=A(-4+4)+B(-4)

-8+1=0-4B

-7=-4B

$B = \frac{7}{4}$

Putting t=0

2(0)+1=A(0+4)+B(0)

1=4A

$A = \frac{1}{4}$

$$\frac{2t+1}{t(t+4)} = \frac{\frac{1}{4}}{t} + \frac{\frac{7}{4}}{(t+4)}$$

$$\int \frac{2t+1}{t(t+4)} dt = \int \frac{2x^2+1}{x^2(x^2+4)} dx = \frac{1}{4}\int \frac{1}{x^2} dx + \frac{7}{4}\int \frac{1}{(x^2+2^2)} dx$$

$$= \frac{1}{4} \times \frac{(-1)}{x} + \frac{7}{4} \times \frac{1}{2} tan^{-1}\left(\frac{x}{2}\right) + c$$

$I = \frac{-1}{4x} + \frac{7}{8}tan^{-1}\left(\frac{x}{2}\right) + c$

Exercise 15B

1. Question

Evaluate:

$$\int x^{-6} dx$$

Answer

$\int x^{-6}dx = \frac{x^{-6+1}}{-6+1} + c$

$\because \left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$

$= \frac{x^{-5}}{-5} + c$

$\int x^{-6}dx = -\frac{1}{5x^5} + c$

2. Question

Evaluate:

$$\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$$

Answer

$\int (\sqrt{x} + 1/\sqrt{x})dx \;=\; \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$

$\left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx \;=\; \int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\,dx$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx \;=\; \int \frac{2}{3}x^{\frac{3}{2}} + 2\sqrt{x} + c$

3. Question

Evaluate:

$$\int \sin\, 3x\, dx$$

Answer

$\int sin\, 3x\, dx = \frac{-1}{3}cos\, 3x + c$

$\left\{\int \sin ax\, dx = \frac{-1}{a}\cos ax\right\}$

4. Question

Evaluate:

$$\int \frac{x^2}{\left(1+x^3\right)} dx$$

Answer

Let $x^3 + 1 = t$

$3x^2 dx = dt$

$$\frac{1}{3}\int \frac{dt}{t} = \frac{1}{3} ln\, t + c$$

$$\int \frac{x^2}{1+x^3} dx = \frac{1}{3} ln(x^3+1) + c$$

5. Question

Evaluate:

$$\int \frac{2\cos x}{3\sin^2 x} dx$$

Answer

Let sin x=t

cos x dx=dt

$$\int \frac{2\cos x}{} dx = \int \frac{2}{} dt = -\frac{2}{} + c$$

$$\int \frac{2\cos x}{} dx = \frac{-2}{} cs\,c\,x + c$$

6. Question

Evaluate:

$$\int \frac{\left(3\sin\phi - 2\right)\cos\phi}{\left(5-\cos^2\phi - 4\sin\phi\right)} d\phi$$

Answer

$$\frac{(3\sin\emptyset - 6 + 4)\cos\emptyset}{(4+1-cos^2\emptyset - 4\sin\emptyset)} = \frac{3(\sin\emptyset - 2)\cos\emptyset + 4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$= \frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$\int \left(\frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}\right) d\,\emptyset$$

Let (sin ∅-2)= t

cos ∅ d∅=dt

$$\int \frac{3dt}{t} + \frac{4dt}{t^2} = 3\ln t - \frac{4}{t} + c$$

$$\int \frac{(3si\,n\,\phi - 2)co\,s\,\phi}{(5 - cos^2\phi - 4si\,n\,\phi)} d\phi = 3l\,n|si\,n\,\phi - 2| - \frac{4}{(si\,n\,\phi - 2)} + c$$

7. Question

Evaluate:

$$\int \sin^2 x\, dx$$

Answer

$\int sin^2 x\, dx \;=\; \int \frac{1}{2} - \frac{\cos 2x}{2}\, dx$

{1-cos 2x=2 $\sin^2$ x}

$$\int sin^2 x\, dx \;=\; \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\left\{\int \cos ax\, dx = \frac{1}{a} \sin ax\right\}$$

8. Question

Evaluate:

$$\int \frac{(\log x)^2}{x} dx$$

Answer

Let log x = t

$$\frac{1}{x} dx \;=\; dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(log\, x)^2}{x} dx \;=\; \frac{(log\, x)^3}{3} + c$$

9. Question

Evaluate:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

$\int \frac{(x+1)(x+logx)]^2}{x} \;=\; \int \left(1 + \frac{1}{x}\right)(x + logx)^2 dx$

Let $x + logx = t$

$$\left(1 + \frac{1}{x}\right) dx \;=\; dt$$

$$\int t^2 dt \;=\; \frac{t^3}{3} + c$$

$$\int \frac{(x+1)(x + logx)^2}{x} = \frac{(x + logx)^3}{3} + c$$

10. Question

Evaluate:

$$\int \frac{\sin x}{(1+\cos x)} dx$$

Answer

Let 1+cosx=t

-sin x dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{\sin x}{(1+\cos x)} dx = -\ln|1+\cos x| + c$$

11. Question

Evaluate:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx$$

Answer

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

-(sin x + cos x)dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{1+\tan x}{1-\tan x} dx = -\ln|\cos x - \sin x| + c$$

12. Question

Evaluate:

$$\int \frac{(1-\cot x)}{(1+\cot x)} dx$$

Answer

$$\frac{1-\cot x}{1+\cot x} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let sin x + cos x=t

(cos x-sin x)dx=dt

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{-dt}{t} = -\ln|\sin x + \cos x| + c$$

$$\int \frac{1-\cot x}{1+\cot x} dx = -\ln|\sin x + \cos x| + c$$

13. Question

Evaluate:

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} dx$$

Answer

Let (x + log (sin x))=t

(1+cot x) dx=dt

$$\int \frac{dt}{t} = ln\, t + c$$

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} = \ln|x+\log(\sin x)| + c$$

14. Question

Evaluate:

$$\int \frac{(1-\sin 2x)}{\left(x+\cos^2 x\right)} dx$$

Answer

Let (x + $\cos^2$ x)=t

(1-sin 2x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{1-\sin 2x}{x+\cos^2 x} = ln(|x+\cos^2 x|) + c$$

15. Question

Evaluate:

$$\int \frac{\sec^2 (\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t dt = \tan t + c$$

$$\int \frac{\sec^2 (\log x)}{x} dx = \tan(\log x) + c$$

16. Question

Evaluate:

$$\int \frac{\sin\left(2 \tan^{-1} x\right)}{\left(1+x^2\right)} dx$$

Answer

Let $\tan^{-1} x = t$

$\frac{1}{1+x^2} dx = dt$

$\int \sin 2t = -\frac{\cos 2t}{2} + c$

$\int \frac{\sin(2\tan^{-1} x)}{(1+x^2)} dx = \frac{-1}{2} \cos(2\tan^{-1} x) + c$

17. Question

Evaluate:

$$\int \frac{\tan x \sec^2 x}{\left(1-\tan^2 x\right)} dx$$

Answer

Let 1-tan^2 x=t

-2 tan x. sec^2 x dx=dt

$\frac{-1}{2} \int \frac{dt}{t} = \frac{-1}{2} log\, t + c$

$\int \frac{\tan x \sec^2 x}{} dx = \frac{-1}{2} log\, |1 - tan^2 x| + c$

18. Question

Evaluate:

$$\int \frac{\left(x^4+1\right)}{\left(x^2+1\right)} dx$$

Answer

$\frac{x^4+1}{x^2+1} = \frac{x^4-1+2}{x^2+1}$

$= x^2 - 1 + \frac{2}{x^2+1}$

$\int \left(x^2 - 1 + \frac{2}{x^2+1}\right) dx = \frac{x^3}{3} - x + 2\tan^{-1} x + c$

19. Question

Evaluate:

$$\int \tan^{-1} \sqrt{\frac{1-\sin x}{1+\sin x}} dx$$

Answer

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$

$$tan^{-1}\sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}} = tan^{-1}\sqrt{\frac{2sin^2(\frac{\pi}{4}-\frac{x}{2})}{2cos^2(\frac{\pi}{4}-\frac{x}{2})}}$$

$$= tan^{-1}((tan\left(\frac{\pi}{4}-\frac{x}{2}\right))$$

$$\int\left(\frac{\pi}{4}-\frac{x}{2}\right)dx = \frac{\pi}{4}x-\frac{x^2}{4}+c$$

20. Question

Evaluate:

$$\int \log\left(1+x^2\right)dx$$

Answer

Using Integration by Parts

$$\int u_{II}v_I dx = u\int vdx - \int u'\int vdx\, dx + c$$

Here 1 is the first function and $\log(x^2+1)$ is second function

$$\int log(1+x^2)dx = (log(1+x^2))x - \int \frac{2x}{1+x^2}xdx$$

$$= (log(1+x^2))x - 2\int \frac{x^2+1-1}{x^2+1}dx$$

$$= (log(1+x\hat{}2))x - 2x + 2 \ 〖tan〗\hat{}(-1)x + c$$

21. Question

Evaluate:

$$\int \cos x \cos 3x \, dx$$

Answer

$$\frac{1}{2}\int 2\cos x\cos 3x\, dx$$

{2 cos A cos B=cos(A+B)+cos(A-B)}

$$\frac{1}{2}\int(\cos 4x+\cos 2x)\,dx = \frac{\sin 4x}{8}+\frac{\sin 2x}{4}+c$$

22. Question

Evaluate:Evaluate $\int \sin 3x \sin x \, dx$

Answer

$$\frac{1}{2}\int 2\sin 3x\sin x\, dx$$

{ 2 sin A sin B=cos(A-B)-cos(A+B) }

$$\frac{1}{2}\int(cos\,2x - cos\,4x)dx = \frac{sin\,2x}{4}-\frac{sin\,4x}{8}+c$$

23. Question

Evaluate:

$$\int \frac{xe^x}{(x+1)^2}dx$$

Answer

$$\frac{e^x(x+1-1)}{(x+1)^2} = e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)$$

$$\{\int \left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c\}$$

$$\int \frac{xe^x}{(x+1)^2}dx \;=\; \frac{e^x}{x+1} + c$$

24. Question

Evaluate:

$$\int e^x \{\tan x - \log \cos x\} dx$$

Answer

$$\int \left(e^x(f(x)+f'(x)\right)dx = e^x f(x) + c$$

Here f(x)=-log cos x

∫e^x (tan x- log cos x)dx=-e^x (log cos x)+c

25. Question

Evaluate:

$$\int \frac{dx}{(1-\sin x)}$$

Answer

Multiplying Num^r and Den^r with (1+sinx)

$$\int \frac{1+si\,n\,x}{cos^2 x}dx \;=\; \int \sec^2 x \;+ \sec x \tan x \;\, dx$$

= tan x + sec x + c

26. Question

Evaluate:

$$\int c \cos x^2 dx$$

Answer

Let x^2 =t

2xdx=dt

$$\frac{1}{2}\int \cos t\, dt = \frac{1}{2}\sin t + c$$

$$\int x \cos x^2 \;\, dx = \frac{1}{2}\sin x^2 + c$$

27. Question

Evaluate:

$$\int \frac{\cot x}{\sqrt{\sin x}} dx$$

Answer

$$\frac{\cot x}{\sqrt{\sin x}} = \frac{\cos x}{(\sin x)^{3/2}}$$

Let sin x=t

cos x dx=dt

$$\int \frac{dt}{t^{3/2}} = \frac{-2}{\sqrt{t}} + c$$

$$\int \frac{\cot x}{\sqrt{\sin x}} dx = \frac{-2}{\sqrt{\sin x}} + c$$

28. Question

Evaluate:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

Answer

$$\frac{sec^2 x}{cosec^2 x} = tan^2 x$$

$$\int tan^2 x\, dx = \int (sec^2 x - 1)\, dx$$

$$\int \frac{sec^2 x}{cosec^2 x} dx = \tan x - x + c$$

29. Question

Evaluate:

$$\int \sin^{-1}(\cos x) dx$$

Answer

$$\int sin^{-1}(cos\, x)\, dx = \int \left(\frac{\pi}{2} - \cos^{-1}(cosx)\right) dx$$

$$\int (\frac{\pi}{2} - x)\, dx = \frac{\pi}{2} x - \frac{x^2}{2} + c$$

30. Question

Evaluate:

$$\int \frac{dx}{\left(\sqrt{x+2} + \sqrt{x+1}\right)}$$

Answer

On rationalizing

$$\int \frac{dx}{(\sqrt{x+2} + \sqrt{x+1})} = \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(\sqrt{x+2} + \sqrt{x+1})\sqrt{x+2} - \sqrt{x+1}} dx$$

$$= \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(x+2-x-1)} dx$$

$$\int \frac{\sqrt{x+2} - \sqrt{x+1}}{1} dx \; = \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

31. Question

Evaluate:

$$\int 2^x \, dx$$

Answer

We know that,

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

.

32. Question

Evaluate:

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} dx$$

Answer

Let (x + log (sec x))=t

(1+tan x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} = \ln|x + \log(\sec x)| + c$$

33. Question

Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t dt = \tan t + c$$

$$\int \frac{\sec^2(\log x)}{x} dx = \tan(\log x) + c$$

34. Question

Evaluate:

$$\int (2x+1)\left(\sqrt{x^2+x+1}\right)dx$$

Answer

Let $x^2+x+1=t$

(2x+1)dx=dt

$$\int \sqrt{t}dt = \frac{2}{3}t^{3/2}+c = \frac{2}{3}(x^2+x+1)^{3/2}+c$$

35. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2+16}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2+b^2}} = \frac{1}{a}\log\left|ax+\sqrt{(ax)^2+b^2}\right|+c$$

$$\int \frac{dx}{\sqrt{(3x)^2+4^2}} = \frac{1}{3}\log\left|3x+\sqrt{9x^2+16}\right|+c$$

36. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4-9x^2}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{b^2-(ax)^2}} = \frac{1}{a}\sin^{-1}\frac{ax}{b}+c$$

$$\int \frac{dx}{\sqrt{2^2-(3x)^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{2}+c$$

37. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2-25}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2-b^2}} = \frac{1}{a}\log\left|ax+\sqrt{(ax)^2-b^2}\right|+c$$

$$\int \frac{dx}{\sqrt{(2x)^2-5^2}} = \frac{1}{2}\log\left|2x+\sqrt{4x^2-25}\right|+c$$

38. Question

Evaluate:

$$\int \sqrt{4-x^2}\, dx$$

Answer

We know that,

$$\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{2^2-x^2}\, dx = \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + c$$

39. Question

Evaluate:

$$\int \sqrt{9+x^2}\, dx$$

Answer

We know that,

$$\int \sqrt{a^2+x^2}\, dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right| + c$$

$$\int \sqrt{3^2+x^2}\, dx = \frac{x}{2}\sqrt{9+x^2} + \frac{9}{2}\log\left|x+\sqrt{9+x^2}\right| + c$$

40. Question

Evaluate:

$$\int \sqrt{x^2-16}\, dx$$

Answer

We know that,

$$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{x^2-4^2}\, dx = \frac{x}{2}\sqrt{x^2-16} - 8\log\left|x+\sqrt{x^2-16}\right| + c$$

Objective Questions I

1. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9+x^2\right)} = ?$$

A. $\tan^{-1}\frac{x}{3} + C$

B. $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$

C. $3\tan^{-1}\frac{x}{3} + C$

D. none of these

Answer

$$= \int \frac{dx}{x^2 + 3^2}$$

We know, $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{3}\tan^{-1}\frac{x}{3} + c$$

2. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4 + 16x^2\right)} = ?$$

A. $\frac{1}{32}\tan^{-1} 4x + C$

B. $\frac{1}{16}\tan^{-1}\frac{x}{2} + C$

C. $\frac{1}{8}\tan^{-1} 2x + C$

D. $\frac{1}{4}\tan^{-1}\frac{x}{2} + C$

Answer

$$= \int \frac{dx}{(4x)^2 + 2^2}$$

4x=t

4dx=dt

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{8}\tan^{-1}\frac{t}{2} + c$$

put t=4x

$$= \frac{1}{8}\tan^{-1}\frac{4x}{2} + c$$

$$= \frac{1}{8}\tan^{-1} 2x + c$$

3. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9+4x^2\right)} dx = ?$$

A. $\frac{1}{2}\tan^{-1}\frac{2x}{3} + C$

B. $\frac{1}{6}\tan^{-1}\frac{2x}{3} + C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2} + C$

D. none of these

Answer

$$\int \frac{dx}{(2x)^2 + 3^2}$$

2x=t

2dx=dt

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + 3^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{6}\tan^{-1}\frac{t}{3} + c$$

put t=2x

$$= \frac{1}{6}\tan^{-1}\frac{2x}{3} + c$$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\left(1+\cos^2 x\right)} dx = ?$$

A. $-\tan^{-1}\left(\cos x\right) + C$

B. $\cot^{-1}\left(\cos x\right) + C$

C. $-\cot^{-1}\left(\cos x\right) + C$

D. $\tan^{-1}\left(\cos x\right) + C$

Answer

$$\int \frac{\sin x}{(\cos x)^2 + 1^2}\, dx$$

cos x=t

-sin x dx=dt

$$= -\int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= -\tan^{-1}\frac{t}{1} + c$$

put t=cos x

=-tan^{-1} (cos x)+c

5. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\left(1+\sin^2 x\right)} dx = ?$$

A. $-\tan^{-1}(\sin x) + C$

B. $\tan^{-1}(\cos x) + C$

C. $\tan^{-1}(\sin x) + C$

D. $-\tan^{-1}(\cos x) + C$

Answer

$$\int \frac{\cos x}{(\sin x)^2 + 1^2}\, dx$$

sin x=t

cos x dx=dt

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \tan^{-1}\frac{t}{1} + c$$

put t=sin x

= tan^{-1} (sin x)+c

6. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x}{\left(e^{2x}+1\right)} dx = ?$$

A. $\cot^{-1}\left(e^x\right) + C$

B. $\tan^{-1}\left(e^x\right) + C$

C. $2\tan^{-1}\left(e^{x}\right)+C$

D. none of these

Answer

$$=\int \frac{e^x}{(e^x)^2+1^2}\,dx$$

$e^x = t$

$e^x\,dx = dt$

$$=\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\tan^{-1}\frac{t}{1}+c$$

put $t=e^x$

$\tan^{-1} e^x + c$

7. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^5}{\left(1+x^{12}\right)}dx = ?$$

A. $\tan^{-1}x^6+C$

B. $\frac{1}{4}\tan^{-1}x^6+C$

C. $\frac{1}{2}\tan^{-1}x^6+C$

D. none of these

Answer

$$=\int \frac{3x^5}{(x^6)^2+1^2}\,dx$$

Let $x^6 = t$

$6x^5\,dx = dt$

$$3x^5\,dx = \frac{dt}{2}$$

$$=\frac{1}{2}\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{2}\tan^{-1}\frac{t}{1}+c$$

put $t=x^6$

$= \frac{1}{2} \tan^{-1} \frac{x^6}{1} + c$

$= \frac{1}{2} \tan^{-1} x^6 + c$

8. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2x^3}{\left(4 + x^8\right)} dx = ?$$

A. $\frac{1}{2} \tan^{-1} \frac{x^4}{2} + C$

B. $\frac{1}{4} \tan^{-1} \frac{x^4}{2} + C$

C. $\frac{1}{2} \tan^{-1} x^4 + C$

D. none of these

Answer

$= \int \frac{2x^3}{(x^4)^2 + 2^2} dx$

Let $x^4 = t$

$4x^3 dx = dt$

$2x^3 dx = \frac{dt}{2}$

$= \frac{1}{2} \int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$= \frac{1}{4} \tan^{-1} \frac{t}{2} + c$

put $t = x^4$

$= \frac{1}{4} \tan^{-1} \frac{x^4}{2} + c$

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(x^2 + 4x + 8\right)} = ?$$

A. $\frac{1}{2} \tan^{-1} \left(\frac{x + 1}{2} \right) + C$

B. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

C. $\frac{1}{2}\tan^{-1}(x+2)+C$

D. $\tan^{-1}\left(\frac{x+2}{2}\right)+C$

Answer

$$=\int \frac{dx}{x^2+4x+8}$$

Completing the square

$x^2 +4x+8 = x^2+4x+8\ (+4-4)$

$=x^2+4x+4+4$

$=(x+2)^2 +2^2$

$$=\int \frac{dx}{(x+2)^2+2^2}$$

Let x+2=t

dx=dt

$$=\int \frac{dt}{t^2+2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{2}\tan^{-1}\frac{t}{2}+c$$

put t=x+2

$$=\frac{1}{2}\tan^{-1}\frac{x+2}{2}+c$$

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(2x^2+x+3)} = ?$$

A. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

B. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{x+1}{\sqrt{23}}\right)+C$

C. $\frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

D. none of these

Answer

$= \int \frac{dx}{2x^2 + x + 3}$

Completing the square

$\Rightarrow 2x^2 + x + 3 \ = 2x^2 + \frac{1}{2}x + \frac{3}{2})$

$= 2(x^2 + \frac{1}{2}x + \frac{3}{2} + \frac{1}{16} - \frac{1}{16})$

$= 2(\ (x + \frac{1}{4})^2 + \frac{23}{16})$

$= \frac{1}{2}\int \frac{dx}{(\ (x+\frac{1}{4})^2 + \frac{23}{16})}$

$Let\ x + \frac{1}{4} = t$

dx=dt

$= \int \frac{dt}{t^2 + \frac{\sqrt{23}}{4}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{4}{2\sqrt{23}}\tan^{-1}\frac{t}{\frac{\sqrt{23}}{4}} + c$

$put\ t = x + \frac{1}{4}$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{x+\frac{1}{4}}{\frac{\sqrt{23}}{4}} + c$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{4x+1}{\sqrt{23}} + c$

11. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(e^x + e^{-x}\right)} = ?$

A. $\tan^{-1}\left(e^x\right) + C$

B. $\tan^{-1}\left(e^{-x}\right) + C$

C. $-\tan^{-1}\left(e^{-x}\right) + C$

D. none of these

Answer

$= \int \frac{1}{e^x + e^{-x}}\ dx$

$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$

e^x =t e^x

e^x dx=dt

$= \int \frac{dt}{t^2 + 1^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \tan^{-1}\frac{t}{1} + c$

put t= e^x

= $\tan^{-1} e^x$ +c

12. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{(9+4x^2)} = ?$

A. $\frac{x}{4} - \frac{1}{8}\tan^{-1}\frac{x}{3} + C$

B. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{x}{3} + C$

C. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + C$

D. none of these

Answer

$\int \frac{x^2}{4x^2+9} = \frac{1}{4}\int \frac{4x^2+9-9}{4x^2+9}\, dx$

$= \frac{1}{4}\int 1 + \frac{1}{4}\int \frac{-9}{4x^2+9}\, dx$

$= \frac{x}{4} - \frac{9}{4}\int \frac{1}{(2x)^2 + 3^2}\, dx$

Let 2x=t

2 dx=dt

$= \frac{x}{4} - \frac{9}{8}\int \frac{1}{(t)^2 + 3^2}\, dx$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{x}{4} - \frac{9}{4.2.3}\tan^{-1}\frac{t}{3} + c$

put t=2x

$= \frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + c$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\left(x^2-1\right)}{\left(x^2+4\right)} dx = ?$$

A. $x - 5\tan^{-1}\frac{x}{2} + C$

B. $x - \frac{5}{2}\tan^{-1}\frac{x}{2} + C$

C. $x - \frac{5}{2}\tan^{-1}\frac{5x}{2} + C$

D. none of these

Answer

$$\int \frac{x^2-1}{x^2+4} = \int \frac{x^2}{x^2+4} - \int \frac{1}{x^2+4}$$

$$= \int \frac{x^2}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int \frac{x^2+4-4}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int (1 - \frac{4}{x^2+4}) - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= x - 2\tan^{-1}\frac{x}{2} - \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= x - \frac{5}{2}\tan^{-1}\frac{x}{2} + c$$

14. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4+9x^2\right)} = ?$$

A. $\frac{2}{3}\tan^{-1}\frac{3x}{2} + C$

B. $\frac{1}{6}\tan^{-1}3x + C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2} + C$

D. none of these

Answer

Consider $\int \frac{dx}{(3x)^2+2^2}$,

3x=t

3dx=dt

$dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{6}\tan^{-1}\frac{t}{2} + c$

put t=3x

$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + c$

15. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{(4x^2 - 4x + 3)} = ?$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

B. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

C. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

D. none of these

Answer

Consider $\int \frac{dx}{4x^2-4x+3}$,

Completing the square

$4x^2 - 4x + 3 = 4(x^2 - x + \frac{3}{4})$

$= 4(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4})$

$= 4(\,(x - \frac{1}{2})^2 + \frac{1}{2})$

$= \frac{1}{4}\int \frac{dx}{(\,(x-\frac{1}{2})^2 + \frac{1}{2})}$

$Let\ x - \frac{1}{2} = t$

dx=dt

$= \frac{1}{4}\int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}} + c$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\sqrt{2}t + c$$

put t=x- �

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{2x-1}{\sqrt{2}} + c$$

16. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^4 x + \cos^4 x\right)} = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$

B. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\tan x}\right) + C$

C. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}\tan x}\right) + C$

D. None of these

Answer

$$\int \frac{dx}{sin^4 x + cos^4 x} = \int \frac{1}{cos^4 x(tan^4 x + 1)}\,dx$$

$$= \int \frac{sec^4 x}{tan^4 x + 1}\,dx$$

$$= \int \frac{sec^2 x\, sec^2 x}{tan^4 x + 1}\,dx$$

$$= \int \frac{sec^2 x\,(1 + tan^2 x)}{tan^4 x + 1}\,dx$$

tan x=t

sec^2 x dx=dt

$$= \int \frac{1 + t^2}{t^4 + 1}\,dt$$

$$= \int \frac{t^2 + 1}{t^4 + 1}\,dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2}}dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2} + 2 - 2}dt$$

$= \int \frac{1+t^{-2}}{(t-t^{-1})^2+2} dt$

Let t-t^{-1} =u

1+x^{-2} dt=du

$= \int \frac{du}{(u)^2+\sqrt{2}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c$

put u=t-t^{-1}

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t-t^{-1}}{\sqrt{2}} + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t^2-1}{\sqrt{2}t} + c$

put t=tan x

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{tan^2 x-1}{\sqrt{2}\tan x} + c$

17. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = ?$

A. $\tan^{-1}\frac{(x^2-1)}{\sqrt{3}} + C$

B. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{(x^2-1)}{\sqrt{3}} + C$

C. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{(x^2-1)}{\sqrt{3}x} + C$

D. none of these

Answer

$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = \int \frac{1+x^{-2}}{x^2+1+x^{-2}} dx$

$= \int \frac{1+x^{-2}}{x^2+1+x^{-2}+2-2} dx$

$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+3} dx$

Let x-x^{-1} =t

$1+x^{-2}\,dx=dt$

$$=\int \frac{dt}{(t)^2+\sqrt{3}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}} + c$$

put $t=x-x^{-1}$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{3}} + c$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2-1}{\sqrt{3}x} + c$$

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin\ 2x}{\left(\sin^4 x + \cos^4 x\right)} dx = ?$$

A. $\tan^{-1}(\tan^2 x) + C$

B. $x^2 + C$

C. - tan-1 $(\tan^2 x)$ +C

D. none of these

Answer

$$\int \frac{\sin 2x}{sin^4 x + cos^4 x} dx = \int \frac{2\sin x \cos x}{cos^4 x(tan^4 x + 1)} dx$$

$$= \int \frac{2\tan x\ sec^2 x}{(tan^2 x)^2 + 1} dx$$

$$= \int \frac{2\ \tan x\ sec^2 x}{(sec^2 x - 1)^2 + 1} dx$$

Let $\sec^2 x-1=t$

2 sec x sec x tan x dx=dt

$$= \int \frac{dt}{(t)^2 + 1}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$=\tan^{-1} t + c$

put $t=\sec^2 x-1$

$= \tan^{-1} \sec^2 x - 1 + c$

$= \tan^{-1} \tan^2 x + c$

19. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1-9x^2\right)} = ?$$

A. $\frac{1}{3}\log\left|\frac{1+3x}{1-3x}\right| + C$

B. $\frac{1}{3}\log\left|\frac{1-3x}{1+3x}\right| + C$

C. $\frac{1}{6}\log\left|\frac{1+3x}{1-3x}\right| + C$

D. $\frac{1}{6}\log\left|\frac{1-3x}{1+3x}\right| + C$

Answer

Consider $\int \frac{dx}{(1)^2-(3x)^2}$

3x=t

3dx=dt

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3}\int \frac{dt}{1^2-(t)^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$= \frac{1}{6}\log\frac{1+t}{1-t} + c$$

put t=3x

$$\frac{1}{6}\tan^{-1}\frac{1+3x}{1-3x} + c$$

20. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(16-4x^2\right)} = ?$$

A. $\frac{1}{8}\log\left|\frac{2-x}{2+x}\right| + C$

B. $\frac{1}{16}\log\left|\frac{2-x}{2+x}\right| + C$

C. $\frac{1}{8}\log\left|\frac{2+x}{2-x}\right| + C$

D. $\frac{1}{16}\log\left|\frac{2+x}{2-x}\right| + C$

Answer

Consider$\int \frac{dx}{(4)^2-(2x)^2}$

2x=t

2dx=dt

$dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{4^2-(t)^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{16}\log\frac{4+t}{4-t} + c$

put t=2x

$= \frac{1}{16}\tan^{-1}\frac{4+2x}{4-2x} + c$

$= \frac{1}{16}\tan^{-1}\frac{2+x}{2-x} + c$

21. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{(1-x^6)}dx = ?$

A. $\frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$

B. $\frac{1}{6}\log\left|\frac{1-x^3}{1+x^3}\right| + C$

C. $\frac{1}{3}\log\left|\frac{1-x^3}{1+x^3}\right| + C$

D. none of these

Answer

$= \int \frac{x^2}{(1)^2-(x^3)^2}\,dx$

Let x^3 =t

$3x^2$ dx=dt

$x^2\,dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{1^2-t^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{6} \log \frac{1+t}{1-t} + c$

put t=x^3

$= \frac{1}{6} \log \frac{1+x^3}{1-x^3} + c$

22. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x}{\left(1-x^4\right)} dx = ?$

A. $\frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$

B. $\frac{1}{4} \log \left| \frac{1-x^2}{1+x^2} \right| + C$

C. $\frac{1}{2} \log \left| \frac{1+x^2}{1-x^2} \right| + C$

D. none of these

Answer

$= \int \frac{x}{(1)^2 - (x^2)^2} dx$

Let x^2 =t

2x dx=dt

$x\, dx = \frac{dt}{2}$

$= \frac{1}{2} \int \frac{dt}{1^2 - t^2}$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$= \frac{1}{4} \log \frac{1+t}{1-t} + c$

put t=x^2

$= \frac{1}{4} \log \frac{1+x^2}{1-x^2} + c$

23. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{\left(a^6 - x^6\right)} dx = ?$

A. $\frac{1}{3a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right|+C$

B. $\frac{1}{6a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right|+C$

C. $\frac{1}{6a^3}\log\left|\frac{a^3-x^3}{a^3+x^3}\right|+C$

D. none of these

Answer

$$=\int \frac{x^2}{(a^3)^2-(x^3)^2}\,dx$$

Let $x^3 = t$

$3x^2\,dx = dt$

$$x^2\,dx = \frac{dt}{3}$$

$$=\frac{1}{3}\int \frac{dt}{(a^3)^2-t^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$$=\frac{1}{6a^3}\log\frac{a^3+t}{a^3-t}+c$$

put $t = x^3$

$$=\frac{1}{6a^3}\log\frac{a^3+x^3}{a^3-x^3}+c$$

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(3-2x-x^2\right)} = ?$$

A. $\frac{1}{4}\log\left|\frac{3+x}{3-x}\right|+C$

B. $\frac{1}{4}\log\left|\frac{1+x}{1-x}\right|+C$

C. $\frac{1}{4}\log\left|\frac{3+x}{1-x}\right|+C$

D. none of these

Answer

$$=-\int \frac{dx}{x^2+2x-3}$$

Completing the square

$x^2 + 2x - 3 = x^2 + 2x - 3 + 1 - 1$

$(x+1)^2 - 4$

$$= -\int \frac{dx}{(x+1)^2 - 4}$$

Let x+1=t

dx=dt

$$= -\int \frac{dt}{t^2 - 2^2}$$

$$= -\int \frac{dt}{2^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{4} \log \frac{2+t}{2-t} + c$$

put t=x+1

$$= \frac{1}{4} \log \frac{2+x+1}{2-x-1} + c$$

$$= \frac{1}{4} \log \frac{x+3}{1-x} + c$$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\cos^2 x - 3\sin^2 x\right)} = ?$$

A. $\frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$

B. $\frac{1}{\sqrt{3}} \log \left| \frac{1 - \sqrt{3} \tan x}{1 + \sqrt{3} \tan x} \right| + C$

C. $\frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$

D. none of these

Answer

$$\int \frac{1}{cos^2 x - 3sin^2 x} dx = \int \frac{1}{cos^2 x(1 - 3tan^2 x)} dx$$

$$= \int \frac{sec^2 x}{(1 - (\sqrt{3} \tan x)^2)} dx$$

Let √3 tan x=t

√3 sec^2 x dx=dt

$= \frac{1}{\sqrt{3}} \int \frac{dt}{1^2 - t^2}$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$= \frac{1}{2\sqrt{3}} \log \frac{1+t}{1-t} + c$

put t=√3 tan x

$= \frac{1}{2\sqrt{3}} \log \frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x} + c$

26. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\operatorname{cosec}^2 x}{\left(1 - \cot^2 x\right)} dx = ?$

A. $\frac{1}{2} \log \left| \frac{1 + \cot x}{1 - \cot x} \right| + C$

B. $-\frac{1}{2} \log \left| \frac{1 + \cot x}{1 - \cot x} \right| + C$

C. $\frac{1}{2} \log \left| \frac{1 - \cot x}{1 + \cot x} \right| + C$

D. none of these

Answer

$\int \frac{cosec^2 x}{1 - cot^2 x} dx$

Let cot x=t

-cosec^2 x dx=dt

$= -\int \frac{dt}{1^2 - t^2}$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$= \frac{-1}{2} \log \frac{1+t}{1-t} + c$

put t=cot x

$= \frac{-1}{2} \log \frac{1+\cot x}{1-\cot x} + c$

27. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(4x^2 - 1\right)} = ?$

A. $\frac{1}{2}\log\left|\frac{2x-1}{2x+1}\right|+C$

B. $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right|+C$

C. $\frac{1}{4}\log\left|\frac{2x-1}{2x+1}\right|+C$

D. none of these

Answer

Consider

$$\int\frac{dx}{(2x)^2-1^2}$$

2x=t

2dx=dt

$$dx=\frac{dt}{2}$$

$$=\frac{1}{2}\int\frac{dt}{t^2-1^2}$$

We know, $\int\frac{1}{x^2-a^2}=\frac{1}{2a}\log\frac{x-a}{x+a}+c$

$$=\frac{1}{4}\log\frac{t-1}{t+1}+c$$

put t=2x

$$=\frac{1}{4}\log\frac{2x-1}{2x+1}+c$$

28. Question

Mark (√) against the correct answer in each of the following:

$$\int\frac{x}{\left(x^4-16\right)}dx=?$$

A. $\frac{1}{4}\log\left|\frac{x^2+4}{x^2-4}\right|+C$

B. $\frac{1}{16}\log\left|\frac{x^2+4}{x^2-4}\right|+C$

C. $\frac{1}{16}\log\left|\frac{x^2-4}{x^2+4}\right|+C$

D. none of these

Answer

$$=\int\frac{x}{(x^2)^2-(4)^2}\,dx$$

Let x^2 =t

2x dx=dt

$$x\,dx = \frac{dt}{2}$$

$$= \frac{1}{2}\int \frac{1}{(t)^2-(4)^2}\,dt$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$$= \frac{1}{16}\log\frac{t-4}{t+4} + c$$

put t=x^2

$$= \frac{1}{16}\log\frac{x^2-4}{x^2+4} + c$$

29. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^2 x - 4\cos^2 x\right)} = ?$$

A. $\frac{1}{4}\log\left|\frac{\tan x-2}{\tan x+2}\right| + C$

B. $\frac{1}{4}\log\left|\frac{\tan x+2}{\tan x-2}\right| + C$

C. $\frac{1}{4}\log\left|\frac{1-\tan x}{1+\tan x}\right| + C$

D. none of these

Answer

$$\int \frac{1}{sin^2x - 4cos^2x}dx = \int \frac{1}{cos^2x(tan^2x-4)}dx$$

$$= \int \frac{sec^2x}{((\tan x)^2-2^2)}dx$$

Let tan x=t

sec^2 x dx=dt

$$= \int \frac{dt}{t^2-2^2}$$

We know, $\int \frac{1}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c$

$$= \frac{1}{4}\log\frac{t-2}{t+2} + c$$

put t=tan x

$$= \frac{1}{4}\log\frac{\tan x - 2}{\tan x + 2} + c$$

30. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4\sin^2 x + 5\cos^2 x\right)} = ?$$

A. $\frac{1}{2}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + C$

B. $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + C$

C. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + C$

D. none of these

Answer

$$\int \frac{1}{4sin^2x + 5cos^2x}dx = \int \frac{1}{cos^2x(4tan^2x + 5)}dx$$

$$\int \frac{sec^2x}{((2\tan x)^2 + \sqrt{5}^2)}dx$$

Let 2 tan x=t

2 sec^2 x dx=dt

$$= \frac{1}{2}\int \frac{dt}{t^2 + \sqrt{5}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}} + c$$

put t=2 tan x

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{2\tan x}{\sqrt{5}} + c$$

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin 3x}dx = ?$$

A. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\sin x}{\sqrt{3}-\sin x}\right| + C$

B. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\cos x}{\sqrt{3}-\cos x}\right| + C$

C. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+C$

D. none of these

Answer

$\int \frac{\sin x}{\sin 3x}dx = \int \frac{\sin x}{3\sin x - 4sin^3 x}dx$

$= \int \frac{1}{3-4sin^2 x}dx$

$= \int \frac{1}{cos^2 x(3sec^2 x - 4tan^2 x)}dx$

$= \int \frac{sec^2 x}{3(1+tan^2 x) - 4tan^2 x}dx$

$= \int \frac{sec^2 x}{3 - tan^2 x}dx$

Let tan x=t

sec^2 x dx=dt

$= \int \frac{dt}{\sqrt{3}^2 - t^2}$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a}\log\frac{a+x}{a-x} + c$

$= \frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+t}{\sqrt{3}-t} + c$

put t= tan x

$= \frac{1}{2\sqrt{3}}\log\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} + c$

32. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{(x^2+1)}{(x^4+1)}dx = ?$

A. $\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+C$

B. $\frac{1}{2}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right)+C$

C. $\frac{1}{\sqrt{2}}\log\left(\frac{x^2+1}{x^2-1}\right)+C$

D. none of these

Answer

$$\int \frac{(x^2+1)}{(x^4+1)}dx = \int \frac{1+x^{-2}}{x^2+x^{-2}}dx$$

$$= \int \frac{1+x^{-2}}{x^2+x^{-2}+2-2}dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+2}dx$$

Let $x-x^{-1}=t$

$1+x^{-2}\ dx=dt$

$$= \int \frac{dt}{(t)^2+\sqrt{2}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}+c$$

put $t=x-x^{-1}$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{2}}+c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+c$$

Objective Questions II

1. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4-9x^2}} = ?$$

A. $\frac{1}{3}\sin^{-1}\frac{x}{3}+C$

B. $\frac{2}{3}\sin^{-1}\left(\frac{2x}{3}\right)+C$

C. $\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{1}{3}\frac{dx}{\sqrt{\frac{4}{9}-x^2}}$$

$$= \int \frac{1}{3}\frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2-x^2}}$$

$$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{2}{3}}+c$$

$$= \frac{1}{3}\sin^{-1}\frac{3x}{2} + c.$$

2. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16 - 4x^2}} = ?$$

A. $\frac{1}{2}\sin^{-1}\frac{x}{2} + C$

B. $\frac{1}{4}\sin^{-1}\frac{x}{2} + C$

C. $\frac{1}{2}\sin^{-1}\frac{x}{4} + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{16 - 4x^2}} = \int \frac{1}{2}\frac{dx}{\sqrt{\frac{16}{4} - x^2}}$$

$$= \int \frac{1}{2}\frac{dx}{\sqrt{(2)^2 - x^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{x}{2} + c$$

3. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{4 - \sin^2 x}} = ?$$

A. $\sin^{-1}\frac{x}{2} + C$

B. $\sin^{-1}\left(\frac{1}{2}\cos x\right) + C$

C. $\sin^{-1}(2\sin x) + C$

D. $\sin^{-1}\left(\frac{1}{2}\sin x\right) + C$

Answer

Put sin x =t

⇒ cos x dx = dt

∴ The given equation becomes

$$\int \frac{dt}{\sqrt{4 - t^2}}$$

$= \sin^{-1}\frac{t}{2} + c$

But t = sin x

$= \sin^{-1}\left(\frac{\sin x}{2}\right) + c$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{2^x}{\sqrt{1-4^x}} dx = ?$$

A. $\sin^{-1}(2^x) \log 2 + C$

B. $\frac{\sin^{-1}(2^x)}{\log 2} + C$

C. $\sin^{-1}(2^x) + C$

D. none of these

Answer

⇒ Let $t=2^x$

$dt = \log 2.\ 2^x.dx$

$\Rightarrow \frac{dt}{log 2} = 2^x.dx$

$= \int \frac{dt}{\log 2\sqrt{1-t^2}}$

$= \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\log 2}\sin^{-1} t$

But $t = 2^x$

$= \frac{1}{\log 2}\sin^{-1}(2^x)$

5. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x-x^2}} = ?$$

A. $\sin^{-1}(x + 1) + C$

B. $\sin^{-1}(x - 2) + C$

C. $\sin^{-1}(x - 1) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+1-1}}$$

$$= \int \frac{dx}{\sqrt{-x^2+2x-1+1}}$$

$$= \int \frac{dx}{\sqrt{1-(x-1)^2}}$$

$=\sin^{-1}(x-1)+c$

6. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{x(1-2x)} = ?$$

A. $\frac{1}{\sqrt{2}}\sin^{-1}(2x-1)+C$

B. $\frac{1}{\sqrt{2}}\sin^{-1}(2x+1)+C$

C. $\frac{1}{\sqrt{2}}\sin^{-1}(4x+1)+C$

D. $\frac{1}{\sqrt{2}}\sin^{-1}(4x-1)+C$

Answer

$$\int \frac{dx}{\sqrt{x-2x^2}} = \int \frac{dx}{\sqrt{2}\sqrt{-x^2+\frac{1}{2}x}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x\right)}+\frac{1}{16}-\frac{1}{16}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{-\left(x^2-\frac{1}{2}x+\frac{1}{16}\right)}+\frac{1}{16}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\frac{1}{16}-\left(x-\frac{1}{4}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{2}\sqrt{\left(\frac{1}{4}\right)^2-\left(\frac{4x-1}{4}\right)^2}}$$

$$= \frac{1}{\sqrt{2}}(\sin^{-1}\left(\frac{\frac{4x-1}{4}}{\frac{1}{4}}\right)$$

$= \frac{1}{\sqrt{2}} \sin^{-1}(4x - 1)$

7. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$$

A. $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{3}\right)+C$

B. $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+C$

C. $4\sin^{-1}\left(\frac{x^3}{4}\right)+C$

D. none of these

Answer

$\Rightarrow \int \frac{3x^2 dx}{\sqrt{9-16x^6}}$

Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^6 = t^2$

$\Rightarrow \int \frac{1}{4} \frac{dt}{\sqrt{\frac{9}{16}-t^2}}$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4t}{3}\right)+c$

But $t = x^3$

$\Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right)+c$

8. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2+2x-x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right)+C$

B. $\sin^{-1}\left(\frac{x-1}{\sqrt{2}}\right)+C$

C. $\sin^{-1}\sqrt{3}(x-1)+C$

D. none of these

Answer

$\Rightarrow \int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+2+3-3}}$

$\Rightarrow \int \frac{dx}{\sqrt{-((x^2-2x+1)-3)}}$

$\Rightarrow \int \frac{dx}{\sqrt{3-(x-1)^2}}$

$\Rightarrow \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c.$

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16-6x-x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-3}{5}\right)+C$

B. $\sin^{-1}\left(\frac{x+3}{5}\right)+C$

C. $\frac{1}{5}\sin^{-1}(x+3)+C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{16-6x-x^2}} = \int \frac{dx}{\sqrt{-x^2-6x-9+16+9}}$

$= \int \frac{dx}{\sqrt{25-(x+3)^2}}$

$= \sin^{-1}\left(\frac{x+3}{5}\right) + c.$

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x-x^2}} = ?$$

A. $\sin^{-1}(x-1)+C$

B. $\sin^{-1}(x+1)+C$

C. $\sin^{-1}(2x-1)+C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-x^2+x-}}$

$$= \int \frac{dx}{\sqrt{-(x^2 - x) + \frac{1}{4} - \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{-\left(x^2 - x + \frac{1}{4}\right) + \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{\frac{2x-1}{2}}{\frac{1}{2}}\right) + c$$

$= \sin^{-1}(2x-1)+c$

11. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = ?$$

A. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{3x-1}{2}\right) + C$

B. $\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

C. $\frac{1}{\sqrt{3}} \sin^{-1}\left(\frac{2x-1}{3}\right) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = \int \frac{dx}{\sqrt{3}\sqrt{-x^2 + \frac{2}{3}x + \frac{1}{3}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right)}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x - \frac{1}{3}\right) + \frac{1}{9} - \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{-\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + \frac{1}{3} + \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\frac{4}{9} - \left(x - \frac{1}{3}\right)^2}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\left(\frac{2}{3}\right)^2 - \left(\frac{3x-1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}}(\sin^{-1}\left(\frac{\frac{3x-1}{3}}{\frac{2}{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{3x-1}{2}\right)$$

12. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 16}} = ?$$

A. $\sin^{-1}\left(\frac{x}{4}\right) + C$

B. $\log\left|x + \sqrt{x^2 - 16}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 16}\right| + C$

D. none of these

Answer

We know

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right|$$

$$\int \frac{dx}{\sqrt{x^2 - 4^2}} = \log\left|x + \sqrt{x^2 - 16}\right|$$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4x^2 - 9}} = ?$$

A. $\frac{1}{2}\log\left|2x + \sqrt{4x^2 - 9}\right| + C$

B. $\frac{1}{4}\log\left|x + \sqrt{4x^2 - 9}\right| + C$

C. $\log\left|2x + \sqrt{4x^2 - 9}\right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{(2x)^2 - (3)^2}}$$

Put t =2x

dt =2 dx

$\Rightarrow dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{\sqrt{t^2-9}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$

$= \frac{1}{2}\log|t+\sqrt{t^2-9}$

But t = 2x

$= \frac{1}{2}\log|2x+\sqrt{4x^2-9}|$

14. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{x^6-1}dx = ?$

A. $\frac{1}{2}\log\left|x^3+\sqrt{x^6-1}\right|+C$

B. $\frac{1}{3}\log\left|x^3+\sqrt{x^6-1}\right|+C$

C. $\frac{1}{3}\log\left|x^3-\sqrt{x^6-1}\right|+C$

D. none of these

Answer

$\Rightarrow \int \frac{x^2\,dx}{\sqrt{(x^3)^2-(1)^2}}$

Put t =x^3

dt =$3x^2$dx

$\Rightarrow dx = \frac{dt}{3x^2}$

$\Rightarrow \frac{1}{3}\int \frac{1}{x^2}\frac{x^2dt}{\sqrt{t^2-1}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$

$= \frac{1}{3}\log|t+\sqrt{t^2-1}$

But t = x^3

$= \frac{1}{3}\log|x^3+\sqrt{x^6-1}|$

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} = ?$$

A. $-\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

B. $-\frac{1}{3}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + C$

C. $-\frac{1}{6}\log\left|2\cos x + \sqrt{2\cos^2 x - 1}\right| + C$

D. none of these

Answer

$\Rightarrow \int \frac{\sin x dx}{\sqrt{(2\cos x)^2 - (1)^2}}$

Put t =2cos x

dt =-2sinxdx

$\Rightarrow\ dx = -\frac{dt}{2\sin x}$

$= -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$

$\Rightarrow\ \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$= -\frac{1}{2}\log|t + \sqrt{t^2 - 1}$

But t = 2cosx

$\Rightarrow -\frac{1}{2}\log|2\cos x + \sqrt{4\cos^2 x - 1}|$

16. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 4}} dx = ?$$

A. $\log\left|\tan x - \sqrt{\tan^2 x - 4}\right| + C$

B. $\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

C. $\frac{1}{2}\log\left|\tan x + \sqrt{\tan^2 x - 4}\right| + C$

D. none of these

Answer

$$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2 - (1)^2}}$$

Put t =tanx

dt = sec^{2}x

$$\Rightarrow\ dx = -\frac{dt}{sec^2 x}$$

$$= \int \frac{1}{sec^2 x} \frac{sec^2 x\, dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$$

$$= \log|t + \sqrt{t^2 - 1}\,|$$

But t = tanx

$$= \log|\tan x + \sqrt{4\tan^2 x - 1}|$$

17. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(1 - e^{2x}\right)} = ?$$

A. $\log\left|e^{x} + \sqrt{e^{2x} - 1}\right| + C$

B. $\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$

C. $-\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$

D. none of these

Answer

Differentiating both side with respect to t

$$-2e^{2x}\frac{dx}{dt} = 1 \Rightarrow dx = \ -\frac{1}{2}\frac{dt}{1 - t}$$

$$y = \ -\frac{1}{2}\int \frac{1}{(1 - t)t}dt$$

$$y = -\frac{1}{2}\int \frac{t + (1 - t)}{(1 - t)t}dt$$

$$y = \ -\frac{1}{2}\int \frac{1}{(1 - t)} + \frac{1}{t}dt$$

$$y = -\frac{1}{2}(-\log(1 - t) + \log t) + c$$

Again put, t = 1 – e^{2x}

$$y = \ -\frac{1}{2}(-\log e^{2x} + \log(1 - e^{2x})) + c$$

$y = -\log\sqrt{\frac{1-e^{2x}}{e^{2x}}} + c$

$y = \ -\log\sqrt{e^{-2x} - 1} + c$

18. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = ?$

A. $\log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right| + C$

B. $\log\left|x + \sqrt{x^2 - 3x + 2}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 3x + 2}\right| + C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = \int \frac{dx}{\sqrt{x^2 - 3x + 2 + \frac{9}{4} - \frac{9}{4}}}$

$= \int \frac{dx}{\sqrt{x^2 - 3x + \frac{9}{4} - \frac{1}{4}}}$

$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$= \log|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}.$

19. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}} dx = ?$

A. $\log\left|\sin x + \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

B. $\log\left|(\sin x - 1) + \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

C. $\log\left|(\sin x - 1) - \sqrt{\sin^2 x - 2\sin x - 3}\right| + C$

D. none of these

Answer

$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx$

Let t =sin x

dt =cos x dx

$\Rightarrow\ dx = \frac{dt}{cos\, x}$

$= \frac{cos\, x\, dt}{cos\, x\, \sqrt{t^2 - 2t - 3 + 2 - 2}}$

$= \frac{dt}{\sqrt{(t^2 - 2t + 2) - 5}}$

$= \frac{dt}{\sqrt{(t-1)^2 - 5}}$

$\Rightarrow\ \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$\Rightarrow\ \int \frac{dt}{\sqrt{(t-1)^2 - 5}} = log\,|t - 1 + \sqrt{t^2 - 2t - 3}|$

But t =sin x

$\therefore\ log\,|\,sin\, x - 1 + \sqrt{sin^2\, x - 2\, sin\, x - 3}|$

20. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\sqrt{2 - 4x + x^2}} = ?$

A. $\log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

B. $\log\left|x + \sqrt{x^2 - 4x + 2}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 4x + 2}\right| + C$

D. none of these

Answer

$\int \frac{dx}{\sqrt{x^2 - 4x + 2}} = \int \frac{dx}{\sqrt{x^2 - 4x + 2 + 4 - 4}}$

$= \int \frac{dx}{\sqrt{(x-2)^2 - 2}}$

$\Rightarrow\ \int \frac{dx}{\sqrt{x^2 - a^2}} = log\left|x + \sqrt{x^2 - a^2}\right|$

$\Rightarrow\ \int \frac{dx}{\sqrt{(x-2)^2 - 2}} = log\left|x - 2 + \sqrt{x^2 - 4x + 2}\right|$

21. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = ?$$

A. $\log\left|x+\sqrt{x^2+6x+5}\right|+C$

B. $\log\left|x-\sqrt{x^2+6x+5}\right|+C$

C. $\log\left|(x+3)+\sqrt{x^2+6x+5}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2+6x+5}} = \int \frac{dx}{\sqrt{x^2+6x+5+9-9}}$$

$$= \int \frac{dx}{\sqrt{(x+3)^2-4}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x+3)^2-4}} = log\left|x+3+\sqrt{x^2+6x+5}\right|$$

22. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{(x-3)^2-1}} = ?$$

A. $\log\left|(x-3)+\sqrt{x^2-6x+8}\right|+C$

B. $\log\left|x+\sqrt{x^2-6x+8}\right|+C$

C. $\log\left|(x-3)-\sqrt{x^2-6x+8}\right|+C$

D. none of these

Answer

$$\Rightarrow \int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x+\sqrt{x^2-a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+9-1}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2-1}} = log\left|x-3+\sqrt{x^2-6x+8}\right|$$

23. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}} = ?$$

A. $\log\left|x + \sqrt{x^2 - 6x + 10}\right| + C$

B. $\log\left|(x-3) + \sqrt{x^2 - 6x + 10}\right| + C$

C. $\log\left|x - \sqrt{x^2 - 6x + 10}\right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}} = \int \frac{dx}{\sqrt{x^2 - 6x + 10 + 9 - 9}}$$

$$= \int \frac{dx}{\sqrt{(x-3)^2 + 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = log\left|x + \sqrt{x^2 + a^2}\right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-3)^2 + 1}} = log\left|x + 3 + \sqrt{x^2 - 6x + 10}\right|$$

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{x^2 dx}{\sqrt{x^6 + a^6}} dx = ?$$

A. $\frac{1}{3}\log\left|x^6 + a^6\right| + C$

B. $\frac{1}{3}\tan^{-1}\left(\frac{x^3}{a^3}\right) + C$

C. $\frac{1}{3}\log\left|x^3 + \sqrt{x^6 + a^6}\right| + C$

D. none of these

Answer

$$\int \frac{x^2 dx}{\sqrt{(x^3)^2 + (a)^6}}$$

Put t $= x^3$

dt $= 3x^2$dx

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$= \frac{1}{3}\int \frac{1}{x^2}\frac{x^2 dt}{\sqrt{t^2+a^6}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$

$= \frac{1}{3}\log|t+\sqrt{t^2+a^6}$

But t = x^3

$= \frac{1}{3}log|x^3+\sqrt{x^6+a^6}|+c.$

25. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{16+\tan^2 x}}dx = ?$$

A. $\log\left|\tan x+\sqrt{\tan^2 x+16}\right|+C$

B. $\log\left|x+\sqrt{\tan^2 x+16}\right|+C$

C. $\log\left|\tan x-\sqrt{\tan^2 x+16}\right|+C$

D. none of these

Answer

$\int \frac{\sec^2 x\, dx}{\sqrt{(\tan x)^2+(4)^2}}$

Put t =tan x

dt = $\sec^2 x$

$\Rightarrow\ dx = \frac{dt}{sec^2 x}$

$= \int \frac{1}{\sec^2 x}\frac{\sec^2 x\, dt}{\sqrt{t^2+16}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$

$= \log|t+\sqrt{t^2+16}$

But t = tan x

$= \log|\tan x+\sqrt{\tan^2 x+16}|$

26. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{3x^2+6x+12}} = ?$$

A. $\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

B. $\frac{1}{3}\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

C. $\frac{1}{\sqrt{3}}\log\left|(x+1)+\sqrt{x^2+2x+4}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{3x^2+6x+12}} = \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2+2x+4}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2+2x+3+1}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{(x+1)^2+3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$$\Rightarrow \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{(x+1)^2+3}} = log\left|x+1+\sqrt{x^2+2x+4}\right|$$

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = ?$$

A. $\frac{1}{2}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

B. $\frac{1}{\sqrt{2}}\log\left|(x+1)+\sqrt{x^2+2x+3}\right|+C$

C. $\frac{1}{\sqrt{2}}\log\left|x+\sqrt{x^2+2x+3}\right|+C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{2x^2+4x+6}} = \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2+2x+3}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2+2x+1+2}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{(x+1)^2+2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$$

$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{dx}{\sqrt{(x+1)^2+2}} = log\left|x+1+\sqrt{x^2+2x+3}\right|$

28. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{\sqrt{x^6+2x^3+3}} dx = ?$

A. $\frac{1}{3}\log\left|\left(x^3+1\right)+\sqrt{x^6+2x^3+3}\right|+C$

B. $\log\left|x^3+\sqrt{x^6+2x^3+3}\right|+C$

C. $\frac{1}{3}\log\left|\left(x^3+1\right)-\sqrt{x^6+2x^3+3}\right|+C$

D. none of these

Answer

$\int \frac{x^2dx}{\sqrt{x^6+2x^3+3}}$

Let x^3 = t

⇒ $3x^2dx$ = dt

$\Rightarrow \frac{dt}{3x^2} = dx$

$\int \frac{x^2dt}{3x^2\sqrt{t^2+2t+3}} = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+2t+3}}$

$= \int \frac{1}{3}\frac{dx}{\sqrt{t^2+2t+1+2}}$

$= \int \frac{1}{3}\frac{dx}{\sqrt{(t+1)^2+2}}$

$\Rightarrow \int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x+\sqrt{x^2+a^2}\right|$

$\Rightarrow \frac{1}{3}\int \frac{dx}{\sqrt{(t+1)^2+2}} = log\left|t+1+\sqrt{t^2+2t+3}\right|$

But t $=x^3$

$= \log\left|x^3+1+\sqrt{x^6+2x^3+3}\right|$

29. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{4-x^2}\, dx = ?$

A. $\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

B. $x\sqrt{4-x^2}+\sin^{-1}\frac{x}{2}+C$

C. $\frac{1}{2}x\sqrt{4-x^2}-2\sin^{-1}\frac{x}{2}+C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \int \sqrt{2^2-x^2} = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\left(\frac{x}{2}\right)+C$$

$$\Rightarrow \int \sqrt{4-x^2} = \frac{x}{2}\sqrt{4-x^2}+2\,sin^{-1}\left(\frac{x}{2}\right)+C$$

30. Question

Mark (√) against the correct answer in each of the following:

$\int\sqrt{1-9x^2}\,dx = ?$

A. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{18}\sin^{-1}3x+C$

B. $\frac{3x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

C. $\frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}\sin^{-1}3x+C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}sin^{-1}\left(\frac{x}{a}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = 3\sqrt{\frac{1}{9}-x^2}$$

$$\Rightarrow 3\sqrt{\frac{1}{9}-x^2} = \frac{3x}{2}\sqrt{\frac{1}{9}-x^2}+\frac{\frac{1}{9}}{2}sin^{-1}\left(\frac{x}{\frac{1}{3}}\right)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{3}{18}sin^{-1}(3x)+C$$

$$\Rightarrow \sqrt{1^2-(3x)^2} = \frac{x}{2}\sqrt{1-9x^2}+\frac{1}{6}sin^{-1}(3x)+C$$

31. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{9-4x^2}\,dx = ?$

A. $\frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

B. $x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3} + C$

C. $\frac{x}{2}\sqrt{9-4x^2} - \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

D. none of these

Answer

We know

$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} sin^{-1}\left(\frac{x}{a}\right) + C$

$\Rightarrow \sqrt{3^2-(2x)^2} = 2\sqrt{\frac{9}{4}-x^2}$

$\Rightarrow 2\sqrt{\frac{9}{4}-x^2}. = \frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{\frac{9}{4}}{2} sin^{-1}\left(\frac{x}{\frac{3}{2}}\right) + C$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{2.9}{8} sin^{-1}(2x) + C$

$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4} sin^{-1}(2x) + C$

32. Question

Mark (√) against the correct answer in each of the following:

$\int \cos x\sqrt{9-\sin^2 x}\,dx = ?$

A. $\frac{1}{2}\sin x\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

B. $\frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

C. $\frac{1}{2}\cos x\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

D. none of these

Answer

Given: $\int \cos x\sqrt{9-\sin^2 x}\,dx$

Let sin x =t

cos x dx =dt

$\Rightarrow \frac{dt}{cos\, x} = dx$

$= \frac{dt}{\cos x}\sqrt{9 - \sin^2 x}\cos x$

$= \sqrt{9 - t^2}\, dt$

$\Rightarrow \int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

$\Rightarrow \int \sqrt{3^2 - t^2} = \frac{t}{2}\sqrt{9 - t^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + C$

But t =sin x

$\Rightarrow \int \cos x\sqrt{9 - \sin^2 x} = \frac{\sin x}{2}\sqrt{9 - \sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

33. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{x^2 - 16}\, dx = ?$

A. $x\sqrt{x^2 - 16} - 4\log\left|x + \sqrt{x^2 - 16}\right| + C$

B. $\frac{x}{2}\sqrt{x^2 - 16} - 8\log\left|x + \sqrt{x^2 - 16}\right| + C$

C. $\frac{x}{2}\sqrt{x^2 - 16} + 8\log\left|x + \sqrt{x^2 - 16}\right| + C$

D. none of these

Answer

We know

$\Rightarrow \int \sqrt{x^2 - a^2} = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$

$\Rightarrow \int \sqrt{x^2 - 4^2} = \frac{x}{2}\sqrt{x^2 - 4^2} - \frac{4^2}{2}\log\left|x + \sqrt{x^2 - 4^2}\right| + C$

$\Rightarrow \int \sqrt{x^2 - 16} = \frac{x}{2}\sqrt{x^2 - 16} - 8\log\left|x + \sqrt{x^2 - 16}\right| + C$

34. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{x^2 - 4x + 2}\, dx = ?$

A. $\frac{1}{2}(x-2)\sqrt{x^2 - 4x + 2} + \log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

B. $(x-2)\sqrt{x^2 - 4x + 2} + \frac{1}{2}\log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

C. $\frac{1}{2}(x-2)\sqrt{x^2 - 4x + 2} - \log\left|(x-2) + \sqrt{x^2 - 4x + 2}\right| + C$

D. none of these

Answer

$\sqrt{x^2 - 4x + 2}dx$

It can be written as

$\Rightarrow \sqrt{x^2 - 4x + 2 + 2 - 2} = \sqrt{x^2 - 4x + 4 - 2}$

$= \sqrt{(x-2)^2 - 2}$

We know

$$\Rightarrow \int \sqrt{x^2 - a^2} = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + C$$

$$\Rightarrow \int \sqrt{(x-2)^2 - 2} = \frac{(x-2)}{2}\sqrt{(x-2)^2 - 2} - \frac{(\sqrt{2})^2}{2} log\left|\sqrt{(x-2)^2 - 2}\right| + C$$

$$\Rightarrow \int \sqrt{x^2 - 4x + 2} = \frac{x-2}{2}\sqrt{x^2 - 4x + 2} - log|x^2 - 4x + 2| + C$$

35. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt{9x^2 + 16}\, dx = ?$

A. $\frac{x}{2}\sqrt{9x^2 + 16} + \frac{8}{3}\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

B. $\frac{x}{2}\sqrt{9x^2 + 16} - \frac{8}{3}\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

C. $x\sqrt{9x^2 + 16} + 24\log\left|3x + \sqrt{9x^2 + 16}\right| + C$

D. none of these

Answer

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + C$$

$$\Rightarrow 3\int \sqrt{x^2 + \left(\frac{4}{3}\right)^2} = 3\left(\frac{x}{2}\sqrt{x^2 + \left(\frac{4}{3}\right)^2} + \frac{\frac{16}{9}}{2} log\left|x + \sqrt{x^2 + \left(\frac{4}{3}\right)^2}\right|\right)$$

$$\Rightarrow \int \sqrt{9x^2 + 16}dx = \frac{x}{2}\sqrt{9x^2 + 16} + \frac{8}{3} log\left|3x + \sqrt{9x^2 + 16}\right|$$

36. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \sqrt{e^{2x} + 4}\, dx = ?$

A. $\frac{1}{2}e^x\sqrt{e^{2x} + 4} - 2\log\left|e^x + \sqrt{e^{2x} + 4}\right| + C$

B. $\frac{1}{2}e^x\sqrt{e^{2x} + 4} + 2\log\left|e^x + \sqrt{e^{2x} + 4}\right| + C$

C. $e^x\sqrt{e^{2x}+4}+\frac{1}{2}\log\left|e^x+\sqrt{e^{2x}+4}\right|+C$

D. none of these

Answer

$$\int e^x\sqrt{e^{2x}+4}dx$$

Let $e^x = t$

e^x dx =dt

$$=\int\sqrt{t^2+2^2}dt$$

$$\Rightarrow\int\sqrt{x^2+a^2}=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\Rightarrow\int\sqrt{t^2+2^2}=\frac{t}{2}\sqrt{t^2+2^2}+\frac{2^2}{2}log\left|t+\sqrt{t^2+2^2}\right|+C$$

But t $=e^x$

$$\Rightarrow\int e^x\sqrt{e^{2x}+4}dx=\frac{e^x}{2}\sqrt{e^{2x}+4}+2\,log\left|e^x+\sqrt{e^{2x}+4}\right|+C$$

37. Question

Mark (√) against the correct answer in each of the following:

$$\int\frac{\sqrt{16+(\log x)^2}}{x}dx=?$$

A. $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+8\ \log\ \left|\log x+\sqrt{16+(\log x)^2}\right|+C$

B. $\frac{1}{2}\log x\cdot\sqrt{16+(\log x)^2}+4\ \log\ \left|\log x+\sqrt{16+(\log x)^2}\right|+C$

C. $\log x\cdot\sqrt{16+(\log x)^2}+16\ \log\ \left|\log x+\sqrt{16+(\log x)^2}\right|+C$

D. none of these

Answer

$$\int\frac{\sqrt{16+(\log x)^2}}{x}dx$$

Let log x =t

$$\Rightarrow\frac{1}{x}dx=dt$$

$$=\int\sqrt{t^2+4^2}\,dt$$

$$\Rightarrow\int\sqrt{x^2+a^2}=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\Rightarrow \int \sqrt{t^2 + 4^2 dt} = \frac{t}{2}\sqrt{t^2 + 4^2} + \frac{4^2}{2} log\left|t + \sqrt{t^2 + 4^2}\right| + C$$

But t =log x

$$\Rightarrow \int \frac{\sqrt{16 + (log\, x)^2}}{x} dx$$
$$= \frac{log\, x}{2}\sqrt{log^2 x + 16} + 8\, log\left|log\, x + \sqrt{log^2 x + 16}\right| + C$$