22. Vectors and Their Properties

Exercise 22

1. Question

Write down the magnitude of each of the following vectors:

A. $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$

B. $\vec{b} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

C. $\vec{c} = \left(\frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

D. $\vec{d} = \left(\sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k}\right)$

Answer

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{{a_x}^2 + {a_y}^2 + {a_z}^2}$

A. $\vec{a} = \hat{i} + 2\hat{j} + 5\hat{k}$

$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 5^2}$

$= \sqrt{30}$ units

B. $\vec{a} = 5\hat{i} - 4\hat{j} - 3\hat{k}$

$\therefore |\vec{a}| = \sqrt{5^2 + 4^2 + 3^2}$

$= 5\sqrt{2}$ units

C. $\vec{a} = \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore |\vec{a}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$= 1$ unit

D. $\vec{a} = \sqrt{2}\hat{i} + \sqrt{3}\hat{j} - \sqrt{5}\hat{k}$

$\therefore |\vec{a}| = \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2 + (\sqrt{5})^2}$

$= \sqrt{10}$ units

2. Question

Find a unit vector in the direction of the vector:

A. $\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right)$

B. $\left(3\hat{i} - 2\hat{j} + 6\hat{k}\right)$

C. $\left(\hat{i} + \hat{k}\right)$

D. $\left(2\hat{i}+\hat{j}+2\hat{k}\right)$

Answer

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i}+a_y\hat{j}+a_z\hat{k}}{\sqrt{a_x{}^2+a_y{}^2+a_z{}^2}}$

A. $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\therefore \hat{a} = \frac{3\hat{i} + 4\hat{j} - 5\hat{k}}{\sqrt{3^2 + 4^2 + 5^2}}$

$= \frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} - \frac{5}{5\sqrt{2}}\hat{k}$

B. $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$\therefore \hat{a} = \frac{3\hat{i} - 2\hat{j} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}}$

$= \frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

C. $\vec{a} = \hat{i} + \hat{k}$

$\therefore \hat{a} = \frac{\hat{i} + \hat{k}}{\sqrt{1^2 + 1^2}}$

$= \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$

D. $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \hat{a} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}}$

$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

3. Question

If $\vec{a} = \left(2\hat{i} - 4\hat{j} + 5\hat{k}\right)$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector.

Answer

$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\therefore \lambda\vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$

For a unit vector, its magnitude equals to 1.

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$

$\therefore |\lambda\vec{a}| = \sqrt{(2\lambda)^2 + (4\lambda)^2 + (5\lambda)^2} = 1$

⇨ $45\lambda^2 = 1$

⇨ $\lambda^2 = \frac{1}{45} = \frac{1}{\left(3\sqrt{5}\right)^2}$

$\Rightarrow \lambda = \pm\frac{1}{3\sqrt{5}}$

4. Question

If $\vec{a} = \left(-\hat{i} + \hat{j} - \hat{k}\right)$ and $\vec{b} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right)$ then find the unit vector in the direction of $\left(\vec{a} + \vec{b}\right)$.

Answer

$\vec{a} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\therefore \vec{a} + \vec{b}$

$= \left(-\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \left(2\hat{\imath} - \hat{\jmath} + 2\hat{k}\right)$

$= \hat{\imath} + \hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}}{\sqrt{a_x{}^2+a_y{}^2+a_z{}^2}}$

$\therefore \widehat{\left(\vec{a} + \vec{b}\right)}$

$= \frac{\hat{\imath} + \hat{k}}{\sqrt{1^2 + 1^2}}$

$= \frac{1}{\sqrt{2}}\left(\hat{\imath} + \hat{k}\right)$

5. Question

If $\vec{a} = \left(3\hat{i} + \hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ then find a unit vector in the direction of $\left(\vec{a} - \vec{b}\right)$.

Answer

$\vec{a} = 3\hat{\imath} + \hat{\jmath} - 5\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\therefore \vec{a} - \vec{b}$

$= \left(3\hat{\imath} + \hat{\jmath} - 5\hat{k}\right) - \left(\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$

$= 2\hat{\imath} - \hat{\jmath} - 4\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{k}}{\sqrt{a_x{}^2+a_y{}^2+a_z{}^2}}$

$\therefore \widehat{\left(\vec{a} - \vec{b}\right)}$

$= \frac{2\hat{\imath} - \hat{\jmath} - 4\hat{k}}{\sqrt{2^2 + 1^2 + 4^2}}$

$= \frac{1}{\sqrt{21}}\left(2\hat{\imath} - \hat{\jmath} - 4\hat{k}\right)$

6. Question

If $\vec{a} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + 4\hat{j} + 9\hat{k}\right)$ then find a unit vector parallel to $\left(\vec{a} + \vec{b}\right)$.

Answer

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = 2\hat{i} + 4\hat{j} + 9\hat{k}$

$\therefore \vec{a} + \vec{b}$

$= (\hat{i} + 2\hat{j} - 3\hat{k}) + (2\hat{i} + 4\hat{j} + 9\hat{k})$

$= 3\hat{i} + 6\hat{j} + 6\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{i}+a_y\hat{j}+a_z\hat{k}}{\sqrt{a_x{}^2+a_y{}^2+a_z{}^2}}$

$\therefore \widehat{(\vec{a} + \vec{b})}$

$= \frac{3\hat{i} + 6\hat{j} + 6\hat{k}}{\sqrt{3^2 + 6^2 + 6^2}}$

$= \pm\frac{1}{9}(3\hat{i} + 6\hat{j} + 6\hat{k})$

$= \pm\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$

7. Question

Find a vector of magnitude 9 units in the direction of the vector $\left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$.

Answer

Let λ be an arbitrary constant and the required vector is $-2\lambda\hat{i} + \lambda\hat{j} + 2\lambda\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$

$\therefore \sqrt{(2\lambda)^2 + (\lambda)^2 + (2\lambda)^2} = 9$

$\Rightarrow 3\lambda = 9$

$\Rightarrow \lambda = 3$

The required vector is $-6\hat{i} + 3\hat{j} + 6\hat{k}$

8. Question

Find a vector of magnitude 8 units in the direction of the vector $\left(5\hat{i} - \hat{j} + 2\hat{k}\right)$.

Answer

Let λ be an arbitrary constant and the required vector is $5\lambda\hat{i} - \lambda\hat{j} + 2\lambda\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$

$\therefore \sqrt{(5\lambda)^2 + (\lambda)^2 + (2\lambda)^2} = 8$

$\Rightarrow \sqrt{30}\lambda = 8$

$\Rightarrow \lambda = \frac{8}{\sqrt{30}}$

The required vector is $\frac{8}{\sqrt{30}}\left(5\hat{\imath} - \hat{\jmath} + 2\hat{k}\right)$

9. Question

Find a vector of magnitude 21 units in the direction of the vector $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$.

Answer

Let λ be an arbitrary constant and the required vector is $2\lambda\hat{\imath} - 3\lambda\hat{\jmath} + 6\lambda\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$

$\therefore \sqrt{(2\lambda)^2 + (3\lambda)^2 + (6\lambda)^2} = 21$

⇨ $7\lambda = 21$

⇨ $\lambda = 3$

The required vector is $\left(6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}\right)$

10. Question

If $\vec{a} = \left(\hat{i} - 2\hat{j}\right), \vec{b} = \left(2\hat{i} - 3\hat{j}\right)$ and $\vec{c} = \left(2\hat{i} + 3\hat{k}\right)$, find $\left(\vec{a} + \vec{b} + \vec{c}\right)$.

Answer

$\vec{a} = \hat{\imath} - 2\hat{\jmath}$

$\vec{b} = 2\hat{\imath} - 3\hat{\jmath}$

$\vec{c} = 2\hat{\imath} + 3\hat{k}$

$\therefore \vec{a} + \vec{b} + \vec{c}$

$= (\hat{\imath} - 2\hat{\jmath}) + (2\hat{\imath} - 3\hat{\jmath}) + (2\hat{\imath} + 3\hat{k})$

$= 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

11. Question

If A(-2, 1, 2) and B(2, -1, 6) are two given points, find a unit vector in the direction of $\overline{AB}$.

Answer

A = (-2, 1, 2)

B = (2, -1, 6)

$\therefore \overrightarrow{AB}$

$= \{2 - (-2)\}\hat{\imath} + \{(-1) - 1\}\hat{\jmath} + \{6 - 2\}\hat{k}$

$= 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}}{\sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}}$

$\therefore \widehat{AB}$

$= \frac{4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}}{\sqrt{4^2 + 2^2 + 4^2}}$

$= \frac{4}{6}\hat{i} - \frac{2}{6}\hat{j} + \frac{4}{6}\hat{k}$

$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

12. Question

Find the direction ratios and direction cosines of the vector $\vec{a} = \left(5\hat{i} - 3\hat{j} + 4\hat{k}\right)$.

Answer

$\vec{a} = 5\hat{i} - 3\hat{j} + 4\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the direction ratios are represented as (a_x , a_y ,a_z) and the direction cosines are given by $\frac{a_x}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}, \frac{a_y}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}, \frac{a_z}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}$

The direction ratios are (5,-3, 4)

The direction cosines are $\frac{5}{\sqrt{5^2+3^2+4^2}}, \frac{-3}{\sqrt{5^2+3^2+4^2}}, \frac{4}{\sqrt{5^2+3^2+4^2}}$

$= \frac{5}{5\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$

$= \frac{1}{\sqrt{2}}, \frac{-3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}$

13. Question

Find the direction ratios and the direction cosines of the vector joining the points A(2, 1, -2) and B(3, 5, -4).

Answer

A = (2,1,-2)

B = (3,5,-4)

$\therefore \overrightarrow{AB}$

$= \{3-2\}\hat{i} + \{5-1\}\hat{j} + \{(-4)-(-2)\}\hat{k}$

$= \hat{i} + 4\hat{j} - 2\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the direction ratios are represented as (a_x , a_y ,a_z) and the direction cosines are given by $\frac{a_x}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}, \frac{a_y}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}, \frac{a_z}{\sqrt{{a_x}^2+{a_y}^2+{a_z}^2}}$

The direction ratios are (1,4, -2)

The direction cosines are $\frac{1}{\sqrt{1^2+4^2+2^2}}, \frac{4}{\sqrt{1^2+4^2+2^2}}, \frac{-2}{\sqrt{1^2+4^2+2^2}}$

$= \frac{1}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}$

14. Question

Show that the points A, B and C having position vectors $\left(\hat{i} + 2\hat{j} + 7\hat{k}\right), \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$ and $\left(3\hat{i} + 10\hat{j} - 3\hat{k}\right)$ respectively, are collinear.

Answer

$A = \hat{i} + 2\hat{j} + 7\hat{k}$

$B = 2\hat{i} + 6\hat{j} + 2\hat{k}$

$C = 3\hat{i} + 10\hat{j} - 3\hat{k}$

$\therefore \overrightarrow{AB}$

$= (2\hat{i} + 6\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= \hat{i} + 4\hat{j} - 5\hat{k}$

$\therefore \overrightarrow{BC}$

$= (3\hat{i} + 10\hat{j} - 3\hat{k}) - (2\hat{i} + 6\hat{j} + 2\hat{k})$

$= \hat{i} + 4\hat{j} - 5\hat{k}$

$\overrightarrow{AB} = \overrightarrow{BC}$

So, the points A, B and C are collinear.

15. Question

The position vectors of the points A, B and C are $(2\hat{i} + \hat{j} - \hat{k}), (3\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 4\hat{j} - 3\hat{k})$ respectively. Show that the points A, B and C are collinear.

Answer

$A = 2\hat{i} + \hat{j} - \hat{k}$

$B = 3\hat{i} - 2\hat{j} + \hat{k}$

$C = \hat{i} + 4\hat{j} - 3\hat{k}$

$\therefore \overrightarrow{AB}$

$= (3\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$

$= \hat{i} - 3\hat{j} + 2\hat{k}$

$\therefore \overrightarrow{BC}$

$= (\hat{i} + 4\hat{j} - 3\hat{k}) - (3\hat{i} - 2\hat{j} + \hat{k})$

$= -2\hat{i} + 6\hat{j} - 4\hat{k}$

$(-3)\overrightarrow{AB} = \overrightarrow{BC}$

So, the points A, B and C are collinear.

16. Question

If the position vectors of the vertices A, B and C of a ΔABC be $(\hat{i} + 2\hat{j} + 3\hat{k})$, $(2\hat{i} + 3\hat{j} + \hat{k})$ and $(3\hat{i} + \hat{j} + 2\hat{k})$ respectively, prove that ΔABC is equilateral.

Answer

$A = \hat{i} + 2\hat{j} + 3\hat{k}$

$B = 2\hat{i} + 3\hat{j} + \hat{k}$

$C = 3\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \overrightarrow{AB}$

$= (2\hat{i} + 3\hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} + \hat{j} - 2\hat{k}$

$\therefore \overrightarrow{BC}$

$= (3\hat{i} + \hat{j} + 2\hat{k}) - (2\hat{i} + 3\hat{j} + \hat{k})$

$= \hat{i} - 2\hat{j} + \hat{k}$

$\therefore \overrightarrow{CA}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (3\hat{i} + \hat{j} + 2\hat{k})$

$= -2\hat{i} + \hat{j} + \hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x{}^2 + a_y{}^2 + a_z{}^2}$

$\therefore |\overrightarrow{AB}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$\therefore |\overrightarrow{BC}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\therefore |\overrightarrow{CA}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

$\therefore |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}|$

The three sides of the triangle are equal in magnitude, so the triangle is equilateral.

17. Question

Show that the points A, B and C having position vectors $(3\hat{i} - 4\hat{j} - 4\hat{k}), (2\hat{i} - \hat{j} + \hat{k})$ and $(\hat{i} - 3\hat{j} - 5\hat{k})$ respectively, form the vertices of a right-angled triangle.

Answer

$A = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$B = 2\hat{i} - \hat{j} + \hat{k}$

$C = \hat{i} - 3\hat{j} - 5\hat{k}$

$\therefore \overrightarrow{AB}$

$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$\therefore \overrightarrow{BC}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$\therefore \overrightarrow{CA}$

$= (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= 2\hat{i} - \hat{j} + \hat{k}$

Tip – For any 2 perpendicular vectors $\vec{a}$ & $\vec{b}$, $\vec{a}.\vec{b} = 0$

$\therefore \overrightarrow{AB}.\overrightarrow{CA}$

$= (-\hat{i} + 3\hat{j} + 5\hat{k}).(2\hat{i} - \hat{j} + \hat{k})$

$= -2 - 3 + 5$

$= 0$

The triangle is right-angled.

18. Question

Using vector method, show that the points A(1, -1, 0), B(4, -3, 1) and C(2, -4, 5) are the vertices of a right-angled triangle.

Answer

A = (1,-1,0)

B = (4,-3,1)

C = (2,-4,5)

$\therefore \overrightarrow{AB}$

$= (4-1)\hat{i} + (-3+1)\hat{j} + (1-0)\hat{k}$

$= 3\hat{i} - 2\hat{j} + \hat{k}$

$\therefore \overrightarrow{BC}$

$= (2-4)\hat{i} + (-4+3)\hat{j} + (5-1)\hat{k}$

$= -2\hat{i} - \hat{j} + 4\hat{k}$

$\therefore \overrightarrow{CA}$

$= (1-2)\hat{i} + (-1+4)\hat{j} + (0-5)\hat{k}$

$= -\hat{i} + 3\hat{j} - 5\hat{k}$

Tip – For any 2 perpendicular vectors $\vec{a}$ & $\vec{b}$, $\vec{a}.\vec{b} = 0$

$\therefore \overrightarrow{AB}.\overrightarrow{BC}$

$= (3\hat{i} - 2\hat{j} + \hat{k}).(-2\hat{i} - \hat{j} + 4\hat{k})$

$= -6 + 2 + 4$

$= 0$

The triangle is right-angled.

19. Question

Find the position vector of the point which divides the join of the points $\left(2\vec{a} - 3\vec{b}\right)$ and $\left(3\vec{a} - 2\vec{b}\right)$ (i) internally and (ii) externally in the ratio 2 : 3.

Answer

$\vec{A} = 2\hat{a} - 3\hat{b}$

$\vec{B} = 3\hat{a} - 2\hat{b}$

Formula to be used – The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{mb \pm na}{m+b}$ respectively.

The position vector of the point dividing the line internally

$= \frac{2 \times (3\hat{a} - 2\hat{b}) + 3 \times (2\hat{a} - 3\hat{b})}{2+3}$

$= \frac{12}{5}\hat{a} - \frac{13}{5}\hat{b}$

The position vector of the point dividing the line externally

$= \frac{2 \times (3\hat{a} - 2\hat{b}) - 3 \times (2\hat{a} - 3\hat{b})}{2 - 3}$

$= -5\hat{b}$

20. Question

The position vectors of two points A and B are $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ respectively. Find the position vector of a point C which divides AB externally in the ratio 1 : 2. Also, show that A is the mid-point of the line segment CB.

Answer

$\vec{A} = 2\hat{a} + \hat{b}$

$\vec{B} = \hat{a} - 3\hat{b}$

Formula to be used – The point dividing a line joining points a and b in a ratio m:n externally is given by $\frac{mb-na}{m-b}$ respectively.

The position vector of the point C dividing the line externally

$= \frac{1 \times (\hat{a} - 3\hat{b}) - 2 \times (2\hat{a} + \hat{b})}{2 - 3}$

$= 3\hat{a} + 5\hat{b}$

The midpoint of B and C may be given by

$\frac{(\hat{a} - 3\hat{b}) + (3\hat{a} + 5\hat{b})}{2}$

$= 2\hat{a} + \hat{b}$ i.e. point A

A is the midpoint of B and C.

21. Question

Find the position vector of a point R which divides the line joining A(-2, 1, 3) and B(3, 5, -2) in the ratio 2 : 1 (i) internally (ii) externally.

Answer

A = (-2,1,3)

B = (3,5,-2)

$\therefore \overrightarrow{OA} = -2\hat{i} + \hat{j} + 3\hat{k}$

$\therefore \overrightarrow{OB} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

Formula to be used – The point dividing a line joining points a and b in a ratio m:n internally or externally is given by $\frac{mb \pm na}{m \pm b}$ respectively.

The position vector of the point dividing the line internally

$= \frac{2 \times (-2\hat{i} + \hat{j} + 3\hat{k}) + 1 \times (3\hat{i} + 5\hat{j} - 2\hat{k})}{2 + 1}$

$= \frac{4}{3}\hat{i} + \frac{11}{3}\hat{j} - \frac{1}{3}\hat{k}$

The position vector of the point dividing the line externally

$$= \frac{2 \times \left(-2\hat{i} + \hat{j} + 3\hat{k}\right) - 1 \times \left(3\hat{i} + 5\hat{j} - 2\hat{k}\right)}{2 - 1}$$

$$= 8\hat{i} + 9\hat{j} - 7\hat{k}$$

22. Question

Find the position vector of the mid-point of the vector joining the points $A\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and $B\left(\hat{i} + 4\hat{j} - 2\hat{k}\right)$.

Answer

$\overrightarrow{OA} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\overrightarrow{OB} = \hat{i} + 4\hat{j} - 2\hat{k}$

Formula to be used – The midpoint of a line joining points a and b is given by $\frac{a+b}{2}$.

The position vector of the midpoint

$$= \frac{\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) + \left(\hat{i} + 4\hat{j} - 2\hat{k}\right)}{2}$$

$$= 2\hat{i} + 3\hat{j} + 2\hat{k}$$

23. Question

If $\overrightarrow{AB} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ and A(1, 2, -1) is the given point, find the coordinates of B.

Answer

A = (1,2,-1)

Let the co-ordinates of point B be (b_1,b_2,b_3)

$\overrightarrow{AB} = 2\hat{i} + \hat{j} - 3\hat{k}$

⇨ $\left[(b_1 - 1)\hat{i} + (b_2 - 2)\hat{j} + (b_3 + 1)\hat{k}\right] = 2\hat{i} + \hat{j} - 3\hat{k}$

Comparing the respective co-efficient,

b_1-1 = 2 i.e. b_1 = 3

b_2-2 = 1 i.e. b_2 = 3

b_3+1 = -3 i.e. b_3 = -4

The required co-ordinates of B are (3,3,-4)

24. Question

Write a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 0) and (4, 5, 6) respectively.

Answer

P = (1,3,0)

Q = (4,5,6)

∴ $\overrightarrow{PQ}$

$$= (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k}$$

$= 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

Tip – For any vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ the unit vector is represented as $\hat{a} = \frac{a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$

$\therefore \widehat{PQ}$

$= \frac{3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}}{\sqrt{3^2 + 2^2 + 6^2}}$

$= \frac{1}{7}\left(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}\right)$