31. Probability Distribution

Exercise 31

1. Question

Find the mean (u), variance (σ^2) and standard deviation (σ) for each of the following probability distributions:

(i)

	0	1	2	3
()	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

(ii)

	1	2	3	4
	0.4	0.3	0.2	0.1

(iii)

	-3	-1	0	2
	0.2	0.4	0.3	0.1

(iv)

	-2	-1	0	1	2
	0.1	0.2	0.4	0.2	0.1

Answer

(i) Given :

	0	1	2	3
()	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

To find : mean (u), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4)$

Mean = E(X) = $0(\frac{1}{6}) + 1(\frac{1}{2}) + 2(\frac{3}{10}) + 3(\frac{1}{30}) = 0 + \frac{1}{2} + \frac{6}{10} + \frac{3}{30} = \frac{15+18+3}{30} = \frac{36}{30} = \frac{6}{5}$

Mean = E(X) = $\frac{6}{5}$ = 1.2

$E(X)^2 = (1.2)^2 = 1.44$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3) + (x_4)^2.P(x_4)$

$E(X^2) = (0)^2(\frac{1}{6}) + (1)^2(\frac{1}{2}) + (2)^2(\frac{3}{10}) + (3)^2(\frac{1}{30}) = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = \frac{15+36+9}{30} = \frac{60}{30}$

$E(X^2) = 2$

Variance = $E(X^2) - E(X)^2$ = 2 – 1.44 = 0.56

Variance = $E(X^2) - E(X)^2$ = 0.56

Standard deviation = $\sqrt{E(X^2) - E(X)^2} = \sqrt{0.56}$ = 0.74

Mean = 1.2

Variance = 0.56

Standard deviation = 0.74

(ii) Given :

	1	2	3	4
	0.4	0.3	0.2	0.1

To find : mean (u), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	($_1$)	($_2$)	($_3$)	($_4$)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2)$ - $E(X)^2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4)$

Mean = E(X) = 1(0.4) + 2(0.3) +3(0.2) +4(0.1) = 0.4 + 0.6 + 0.6 + 0.4 = 2

Mean = E(X) = 2

$E(X)^2 = (2)^2 = 4$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2 . P(x_1) + (x_2)^2 . P(x_2) + (x_3)^2 . P(x_3) + (x_4)^2 . P(x_4)$

$E(X^2) = (1)^2(0.4) + (2)^2 (0.3) + (3)^2 (0.2) + (4)^2 (0.1) = 0.4 + 1.2 + 1.8 + 1.6 = 5$

$E(X^2) = 5$

Variance = $E(X^2) - E(X)^2 = 5 - 4 = 1$

Variance = $E(X^2) - E(X)^2 = 1$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$ = $\sqrt{1}$ = 1

Mean = 2

Variance = 1

Standard deviation = 1

(iii) Given :

	-3	-1	0	2
	0.2	0.4	0.3	0.1

To find : mean (u), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4
()	(1)	(2)	(3)	(4)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2)$ - $E(X)^2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4)$

Mean = E(X) = -3(0.2) + (-1)(0.4) + 0(0.3) + 2(0.1)= -0.6 - 0.4 + 0 + 0.2 = -0.8

Mean = E(X) = -0.8

$E(X)^2$ = $(-0.8)^2$ = 0.64

$E(X^2)$ = $\sum_{i=1}^{i=n} (x_i)^2 . P(x_i)$ = $(x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3) + (x_4)^2.P(x_4)$

$E(X^2) = (-3)^2(0.2) + (-1)^2(0.4) + (0)^2(0.3) + (2)^2(0.1) = 1.8 + 0.4 + 0 + 0.4 = 2.6$

$E(X^2) = 2.6$

Variance = $E(X^2) - E(X)^2 = 2.6 - 0.64 = 1.96$

Variance = $E(X^2) - E(X)^2 = 1.96$

Standard deviation = $\sqrt{E(X^2) - E(X)^2} = \sqrt{1.96} = 1.4$

Mean = -0.8

Variance = 1.96

Standard deviation = 1.4

(iv) Given :

	-2	-1	0	1	2
	0.1	0.2	0.4	0.2	0.1

To find : mean (u), variance (σ^2) and standard deviation (σ)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	(1)	(2)	(3)	(4)	(5)

Mean = $E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = $E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4) + x_5P(x_5)$

Mean = E(X) = -2(0.1) + (-1)(0.2) + 0(0.4) + 1(0.2) + 2(0.1)

Mean = E(X) = -0.2 - 0.2 + 0 + 0.2 + 0.2 = 0

Mean = E(X) = 0

$E(X)^2 = (0)^2 = 0$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2 . P(x_1) + (x_2)^2 . P(x_2) + (x_3)^2 . P(x_3) + (x_4)^2 . P(x_4) + (x_5)^2 . P(x_5)$

$E(X^2) = (-2)^2(0.1) + (-1)^2 (0.2) + (0)^2 (0.4) + (1)^2 (0.2) + (2)^2(0.1)$

$E(X^2) = 0.4 + 0.2 + 0 + 0.2 + 0.4 = 1.2$

$E(X^2) = 1.2$

Variance = $E(X^2) - E(X)^2 = 1.2 - 0 = 1.2$

Variance = $E(X^2) - E(X)^2 = 1.2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2} = \sqrt{1.2} = 1.095$

Mean = 0

Variance = 1.2

Standard deviation = 1.095

2. Question

Find the mean and variance of the number of heads when two coins are tossed simultaneously.

Answer

Given : Two coins are tossed simultaneously

To find : mean (u), variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = $E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

When two coins are tossed simultaneously,

Total possible outcomes = TT , TH , HT , HH where H denotes head and T denotes tail.

P(0) = $\frac{1}{4}$ (zero heads = 1 [TT])

P(1) = $\frac{2}{4}$ (one heads = 2 [HT , TH])

P(2) = $\frac{1}{4}$ (two heads = 1 [HH])

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Mean = E(X) = $0(\frac{1}{4}) + 1(\frac{2}{4}) + 2(\frac{1}{4}) = 0 + \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$

Mean = E(X) = 1

$E(X)^2 = (1)^2 = 1$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2 . P(x_1) + (x_2)^2 . P(x_2) + (x_3)^2 . P(x_3)$

$E(X^2) = (0)^2(\frac{1}{4}) + (1)^2 (\frac{2}{4}) + (2)^2 (\frac{1}{4}) = 0 + \frac{2}{4} + \frac{4}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$

$E(X^2) = 1.5$

Variance = $E(X^2) - E(X)^2 = 1.5 - 1 = 0.5$

Variance = $E(X^2) - E(X)^2 = 0.5$

Mean = 1

Variance = 0.5

3. Question

Find the mean and variance of the number of tails when three coins are tossed simultaneously.

Answer

Given : Three coins are tossed simultaneously

To find : mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4
$()$	$(_1)$	$(_2)$	$(_3)$	$(_4)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2)$ - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

When three coins are tossed simultaneously,

Total possible outcomes = TTT , TTH , THT , HTT , THH , HTH , HHT , HHH where H denotes head and T denotes tail.

P(0) = $\frac{1}{8}$ (zero tails = 1 [HHH])

P(1) = $\frac{3}{8}$ (one tail = 3 [HTH , THH , HHT])

P(2) = $\frac{3}{8}$ (two tail = 3 [HTT , THT , TTH])

P(3) = $\frac{1}{8}$ (three tails = 1 [TTT])

The probability distribution table is as follows,

	0	1	2	3
$()$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4)$

Mean = E(X) = $0(\frac{1}{8}) + 1(\frac{3}{8}) + 2(\frac{3}{8}) + 3(\frac{1}{8}) = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2}$

Mean = E(X) = $\frac{3}{2}$ = 1.5

$E(X)^2 = (1.5)^2 = 2.25$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3) + (x_4)^2.P(x_4)$

$E(X^2) = (0)^2(\frac{1}{8}) + (1)^2 (\frac{3}{8}) + (2)^2 (\frac{3}{8}) + (3)^2 (\frac{1}{8}) = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = \frac{3+12+9}{8} = \frac{24}{8} = 3$

$E(X^2) = 3$

Variance = $E(X^2) - E(X)^2$ = 3 – 2.25 = 0.75

Variance = $E(X^2) - E(X)^2$ = 0.75

Mean = 1.5

Variance = 0.75

4. Question

A die is tossed twice. 'Getting an odd number on a toss' is considered a success. Find the probability distribution of a number of successes. Also, find the mean and variance of the number of successes.

Answer

Given : A die is tossed twice and 'Getting an odd number on a toss' is considered a success.

To find : probability distribution of the number of successes and mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

'Getting an odd number on a toss' is considered a success.

P(0) = $\frac{9}{36}$ = $\frac{1}{4}$ (zero odd numbers = 9)

P(1) = $\frac{18}{36}$ = $\frac{1}{2}$ (one odd number = 18)

P(2) = $\frac{9}{36}$ = $\frac{1}{4}$ (two odd numbers = 9)

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Mean = E(X) = 0($\frac{1}{4}$) + 1($\frac{1}{2}$) +2($\frac{1}{4}$) = 0 + $\frac{2}{4}$ + $\frac{2}{4}$ = $\frac{4}{4}$ = 1

Mean = E(X) = 1

$E(X)^2$ = $(1)^2$ = 1

E(X^2) = $\sum_{i=1}^{i=n}(x_i)^2.P(x_i)$ = $(x_1)^2$.P(x_1) + $(x_2)^2$.P(x_2) + $(x_3)^2$.P(x_3)

E(X^2) = $(0)^2$($\frac{1}{4}$) + $(1)^2$ ($\frac{2}{4}$) +$(2)^2$ ($\frac{1}{4}$) = 0 + $\frac{2}{4}$ + $\frac{4}{4}$ = $\frac{6}{4}$ = $\frac{3}{2}$ = 1.5

E(X^2) = 1.5

Variance = E(X^2) - $E(X)^2$ = 1.5 – 1 = 0.5

Variance = $E(X^2) - E(X)^2 = 0.5$

The probability distribution table is as follows,

	0	1	2
()	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Mean = 1

Variance = 0.5

5. Question

A die is tossed twice. 'Getting a number greater than 4' is considered a success. Find the probability distribution of a number of successes. Also, find the mean and variance of the number of successes.

Answer

Given : A die is tossed twice and 'Getting a number greater than 4 ' is considered a success.

To find : probability distribution of the number of successes and mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

'Getting a number greater than 4' is considered a success.

P(0) = $\frac{16}{36} = \frac{4}{9}$ (zero numbers greater than 4 = 16)

P(1) = $\frac{16}{36} = \frac{4}{9}$ (one number greater than 4= 16)

P(2) = $\frac{4}{36} = \frac{1}{9}$ (two numbers greater than 4= 4)

The probability distribution table is as follows,

	0	1	2
()	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean = E(X) = $0(\frac{4}{9}) + 1(\frac{4}{9}) + 2(\frac{1}{9}) = 0 + \frac{4}{9} + \frac{2}{9} = \frac{4+2}{9} = \frac{6}{9} = \frac{2}{3}$

Mean = E(X) = $\frac{2}{3}$

$E(X)^2 = (\frac{2}{3})^2 = \frac{4}{9}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{4}{9}) + (1)^2(\frac{4}{9}) + (2)^2(\frac{1}{9}) = 0 + \frac{4}{9} + \frac{4}{9} = \frac{8}{9}$

$E(X^2) = \frac{8}{9}$

Variance = $E(X^2) - E(X)^2 = \frac{8}{9} - \frac{4}{9} = \frac{4}{9}$

Variance = E(X^2) - $E(X)^2$ = $\frac{4}{9}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Mean = $\frac{2}{3}$

Variance = $\frac{4}{9}$

6. Question

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of a number of successes, find the probability distribution of the number of successes. Also, find the mean and variance of a number of successes. [CBSE 2008]

Answer

Given : A die is tossed twice and 'Getting a number greater than 4 ' is considered a success.

To find : probability distribution of the number of successes and mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	$(_1)$	$(_2)$	$(_3)$	$(_4)$	$(_5)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

When a die is tossed 4 times,

Total possible outcomes = 6^2 = 36

Getting a doublet is considered as a success

The possible doublets are (1,1) , (2,2) , (3,3) , (4,4) , (5,5) , (6,6)

Let p be the probability of success,

$p = \frac{6}{36} = \frac{1}{6}$

$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

$q = \frac{5}{6}$

since the die is thrown 4 times, n = 4

x can take the values of 1,2,3,4

$P(x) = {}^nC_x p^x q^{n-x}$

$P(0) = {}^4C_0(\frac{1}{6})^0(\frac{5}{6})^4 = \frac{625}{1296}$

$P(1) = {}^4C_1(\frac{1}{6})^1(\frac{5}{6})^3 = \frac{500}{1296} = \frac{125}{324}$

$P(2) = {}^4C_2(\frac{1}{6})^2(\frac{5}{6})^2 = \frac{150}{1296} = \frac{25}{216}$

$P(3) = {}^4C_3(\frac{1}{6})^3(\frac{5}{6})^1 = \frac{20}{1296} = \frac{5}{324}$

$P(4) = {}^4C_4(\frac{1}{6})^4(\frac{5}{6})^0 = \frac{1}{1296}$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4) + x_5P(x_5)$

Mean = E(X) = $0(\frac{625}{1296}) + 1(\frac{125}{324}) + 2(\frac{25}{216}) + 3(\frac{5}{324}) + 4(\frac{1}{1296})$

Mean = E(X) = 0 + $\frac{125}{324}$ + $\frac{50}{216}$ + $\frac{15}{324}$ + $\frac{4}{1296}$ = $\frac{500+300+60+4}{1296}$ = $\frac{864}{1296}$ = $\frac{2}{3}$

Mean = E(X) = $\frac{2}{3}$

$E(X)^2 = (\frac{2}{3})^2 = \frac{4}{9}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2 . P(x_i) = (x_1)^2 . P(x_1) + (x_2)^2 . P(x_2) + (x_3)^2 . P(x_3) + (x_4)^2 . P(x_4) + (x_5)^2 . P(x_5)$

$E(X^2) = (0)^2(\frac{625}{1296}) + (1)^2(\frac{125}{324}) + (2)^2(\frac{25}{216}) + (3)^2(\frac{5}{324}) + (4)^2(\frac{1}{1296})$

$E(X^2) = 0 + \frac{125}{324} + \frac{100}{216} + \frac{45}{324} + \frac{16}{1296} = \frac{500+600+180+16}{1296} = \frac{1296}{1296}$

$E(X^2) = 1$

Variance = $E(X^2) - E(X)^2 = 1 - \frac{4}{9} = \frac{5}{9}$

Variance = $E(X^2) - E(X)^2 = \frac{5}{9}$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{625}{1296}$	$\frac{125}{324}$	$\frac{25}{216}$	$\frac{5}{324}$	$\frac{1}{1296}$

Mean = $\frac{2}{3}$

Variance = $\frac{5}{9}$

7. Question

A coin is tossed 4 times. Let X denote the number of heads. Find the probability distribution of X. also, find the mean and variance of X.

Answer

Given : A coin is tossed 4 times

To find : probability distribution of X and mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3	x_4	x_5
	$(_1)$	$(_2)$	$(_3)$	$(_4)$	$(_5)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2)$ - $E(X)^2$

A coin is tossed 4 times,

Total possible outcomes = 2^4 = 16

X denotes the number of heads

Let p be the probability of getting a head,

p = $\frac{1}{2}$

q = 1 – p = 1 - $\frac{1}{2}$ = $\frac{1}{2}$

q = $\frac{1}{2}$

since the coin is tossed 4 times, n = 4

X can take the values of 1,2,3,4

P(x) = ${}^nC_x p^x q^{n-x}$

P(0) = ${}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

P(1) = ${}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 = \frac{4}{16} = \frac{1}{4}$

P(2) = ${}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$

P(3) = ${}^4C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{4}{16} = \frac{1}{4}$

P(4) = ${}^4C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4) + x_5P(x_5)$

Mean = E(X) = $0(\frac{1}{16}) + 1(\frac{1}{4}) + 2(\frac{3}{8}) + 3(\frac{1}{4}) + 4(\frac{1}{16})$

Mean = E(X) = $0 + \frac{1}{4} + \frac{6}{8} + \frac{3}{4} + \frac{4}{16} = \frac{4+12+12+4}{16} = \frac{32}{16} = 2$

Mean = E(X) = 2

$E(X)^2 = (2)^2 = 4$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3) + (x_4)^2.P(x_4) + (x_5)^2.P(x_5)$

$E(X^2) = (0)^2(\frac{1}{16}) + (1)^2(\frac{1}{4}) + (2)^2(\frac{3}{8}) + (3)^2(\frac{1}{4}) + (4)^2(\frac{1}{16})$

$E(X^2) = 0 + \frac{1}{4} + \frac{12}{8} + \frac{9}{4} + \frac{16}{16} = \frac{0+4+24+36+16}{16} = \frac{80}{16} = 5$

$E(X^2) = 5$

Variance = $E(X^2) - E(X)^2 = 5 - 4 = 1$

Variance = $E(X^2) - E(X)^2 = 1$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Mean = 2

Variance = 1

8. Question

Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice. Find the probability distribution of X. Also, find the mean, variance and standard deviation of X.

Answer

Given : Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

To find : probability distribution of X ,mean (u) and variance (σ^2) and standard deviation

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Standard deviation = $\sqrt{E(X^2) - E(X)^2}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

When a die is tossed twice,

Total possible outcomes =

{(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6)

(2,1) , (2,2) , (2,3) , (2,4) , (2,5) , (2,6)

(3,1) , (3,2) , (3,3) , (3,4) , (3,5) , (3,6)

(4,1) , (4,2) , (4,3) , (4,4) , (4,5) , (4,6)

(5,1) , (5,2) , (5,3) , (5,4) , (5,5) , (5,6)

(6,1) , (6,2) , (6,3) , (6,4) , (6,5) , (6,6)}

Let X denote the number of times 'a total of 9' appears in two throws of a pair of dice

p = $\frac{4}{36} = \frac{1}{9}$

$q = 1 - \frac{1}{9} = \frac{8}{9}$

Two dice are tossed twice, hence n = 2

$P(0) = {}^2C_0(\frac{1}{9})^0(\frac{8}{9})^2 = \frac{64}{81}$

$P(1) = {}^2C_1(\frac{1}{9})^1(\frac{8}{9})^1 = \frac{16}{81}$

$P(2) = {}^2C_2(\frac{1}{9})^2(\frac{8}{9})^0 = \frac{1}{81}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Mean = $E(X) = 0(\frac{64}{81}) + 1(\frac{16}{81}) + 2(\frac{1}{81}) = 0 + \frac{16}{81} + \frac{2}{81} = \frac{16+2}{81} = \frac{18}{81} = \frac{2}{9}$

Mean = $E(X) = \frac{2}{9}$

$E(X)^2 = (\frac{2}{9})^2 = \frac{4}{81}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{64}{81}) + (1)^2(\frac{16}{81}) + (2)^2(\frac{1}{81}) = 0 + \frac{16}{81} + \frac{4}{81} = \frac{20}{81}$

$E(X^2) = \frac{20}{81}$

Variance = $E(X^2) - E(X)^2 = \frac{20}{81} - \frac{4}{81} = \frac{16}{81}$

Variance = $E(X^2) - E(X)^2 = \frac{16}{81}$

Standard deviation = $\sqrt{E(X^2) - E(X)^2} = \sqrt{\frac{16}{81}} = \frac{4}{9}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

Mean = $\frac{2}{9}$

Variance = $\frac{16}{81}$

Standard deviation = $\frac{4}{9}$

9. Question

There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn. Find the mean and variance of X.

Answer

Given : There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn.

To find : mean (u) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	$(_1)$	$(_2)$	$(_3)$	$(_4)$	$(_5)$	$(_6)$	$(_7)$	$(_8)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

There are 5 cards, numbers 1 to 5, one number on each card. Two cards are drawn at random without replacement.

X denote the sum of the numbers on two cards drawn

The minimum value of X will be 3 as the two cards drawn are 1 and 2

The maximum value of X will be 9 as the two cards drawn are 4 and 5

For X = 3 the two cards can be (1,2) and (2,1)

For X = 4 the two cards can be (1,3) and (3,1)

For X = 5 the two cards can be (1,4) , (4,1) , (2,3) and (3,2)

For X = 6 the two cards can be (1,5) , (5,1) , (2,4) and (4,2)

For X = 7 the two cards can be (3,4) , (4,3) , (2,5) and (5,2)

For X = 8 the two cards can be (5,3) and (3,5)

For X = 9 the two cards can be (4,5) and (4,5)

Total outcomes = 20

$P(3) = \frac{2}{20} = \frac{1}{10}$

$P(4) = \frac{2}{20} = \frac{1}{10}$

$P(5) = \frac{4}{20} = \frac{1}{5}$

$P(6) = \frac{4}{20} = \frac{1}{5}$

$P(7) = \frac{4}{20} = \frac{1}{5}$

$P(8) = \frac{2}{20} = \frac{1}{10}$

$P(9) = \frac{2}{20} = \frac{1}{10}$

The probability distribution table is as follows,

x_i	3	4	5	6	7	8	9
P_i	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3) + x_4P(x_4) + x_5P(x_5) + x_6P(x_6) + x_7P(x_7)$

Mean = E(X) = $3(\frac{1}{10}) + 4(\frac{1}{10}) + 5(\frac{1}{5}) + 6(\frac{1}{5}) + 7(\frac{1}{5}) + 8(\frac{1}{10}) + 9(\frac{1}{10})$

Mean = E(X) = $\frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10} = \frac{3+4+10+12+14+8+9}{10} = \frac{60}{10} = 6$

Mean = E(X) = 6

$E(X)^2 = (6)^2 = 36$

$E(X^2) = \sum_{i=1}^{i=n} (x_i)^2 . P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3) + (x_4)^2.P(x_4) + (x_5)^2.P(x_5) + (x_6)^2.P(x_6) + (x_7)^2.P(x_7)$

$E(X^2) = (3)^2 (\frac{1}{10}) + (4)^2(\frac{1}{10}) + (5)^2 (\frac{1}{5}) + (6)^2 (\frac{1}{5}) + (7)^2 (\frac{1}{5}) + (8)^2(\frac{1}{10}) + (9)^2 (\frac{1}{10})$

$E(X^2) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{10} + \frac{81}{10} = \frac{9+16+50+72+98+64+81}{10} = \frac{390}{10} = 39$

$E(X^2) = 39$

Variance = $E(X^2) - E(X)^2 = 39 - 36 = 3$

Variance = $E(X^2) - E(X)^2 = 3$

Mean = 6

Variance = 3

10. Question

Two cards are drawn from a well-shuffled pack of 52 cards. Find the probability distribution of a number of kings. Also, compute the variance for the number of kings. [CBSE 2007]

Answer

Given : Two cards are drawn from a well-shuffled pack of 52 cards.

To find : probability distribution of the number of kings and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Two cards are drawn from a well-shuffled pack of 52 cards.

Let X denote the number of kings in the two cards

There are 4 king cards present in a pack of well-shuffled pack of 52 cards.

$P(0) = \frac{{}^{48}_{2}C}{{}^{52}_{2}C} = \frac{48 \times 47}{52 \times 51} = \frac{188}{221}$

$P(1) = \frac{{}^{48}_{1}C \times {}^{4}_{1}C}{{}^{52}_{2}C} = \frac{48 \times 4 \times 2}{52 \times 51} = \frac{32}{221}$

$P(2) = \frac{{}^{4}_{2}C}{{}^{52}_{2}C} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Mean = E(X) = $0(\frac{188}{221}) + 1(\frac{32}{221}) + 2(\frac{1}{221}) = 0 + \frac{32}{221} + \frac{2}{221} = \frac{32+2}{221} = \frac{34}{221}$

Mean = E(X) = $\frac{34}{221}$

$E(X)^2 = (\frac{34}{221})^2 = \frac{1156}{48841}$

E(X^2) = $\sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

E(X^2) = $(0)^2(\frac{188}{221}) + (1)^2(\frac{32}{221}) + (2)^2(\frac{1}{221}) = 0 + \frac{32}{221} + \frac{4}{221} = \frac{36}{221}$

E(X^2) = $\frac{36}{221}$

Variance = E(X^2) - $E(X)^2$ = $\frac{36}{221} - \frac{1156}{48841} = \frac{7956-1156}{48841} = \frac{6800}{48841} = \frac{400}{2873}$

Variance = $E(X^2) - E(X)^2 = \frac{400}{2873}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Variance = $\frac{400}{2873}$

11. Question

A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random from the box. Let X be the number of defective bulbs drawn. Find the mean and variance of X.

Answer

Given : A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

To find : mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

A box contains 16 bulbs, out of which 4 bulbs are defective. Three bulbs are drawn at random

Let X denote the number of defective bulbs drawn

There are 4 defective bulbs present in 16 bulbs

$P(0) = \frac{^{12}_{3}C}{^{16}_{3}C} = \frac{12 \times 11 \times 10}{16 \times 15 \times 14} = \frac{11}{28}$

$P(1) = \frac{^{12}_{2}C \times ^{4}_{1}C}{^{16}_{3}C} = \frac{12 \times 11 \times 4 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{33}{70}$

$P(2) = \frac{^{12}_{1}C \times ^{4}_{2}C}{^{16}_{3}C} = \frac{12 \times 4 \times 3 \times 3 \times 2}{16 \times 15 \times 14 \times 2} = \frac{9}{70}$

$P(3) = \frac{^{4}_{3}C}{^{16}_{3}C} = \frac{4 \times 3 \times 2}{16 \times 15 \times 14} = \frac{1}{140}$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{11}{28}$	$\frac{33}{70}$	$\frac{9}{70}$	$\frac{1}{140}$

Mean = E(X) = $0(\frac{11}{28}) + 1(\frac{33}{70}) + 2(\frac{9}{70}) + 3(\frac{1}{140}) = 0 + \frac{33}{70} + \frac{18}{70} + \frac{3}{140} = \frac{66+36+3}{140}$

Mean = E(X) = $\frac{105}{140} = \frac{3}{4}$

$E(X)^2 = (\frac{3}{4})^2 = \frac{9}{16}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{11}{28}) + (1)^2(\frac{33}{70}) + (2)^2(\frac{9}{70}) + (3)^2(\frac{1}{140}) = 0 + \frac{33}{70} + \frac{36}{70} + \frac{9}{140} = \frac{66+72+9}{140}$

$E(X^2) = \frac{147}{140}$

Variance = $E(X^2) - E(X)^2 = \frac{147}{140} - \frac{9}{16} = \frac{588-315}{560} = \frac{273}{560} = \frac{39}{80}$

Variance = $E(X^2) - E(X)^2 = \frac{39}{80}$

Mean = E(X) = $\frac{3}{4}$

Variance = $\frac{39}{80}$

12. Question

20% of the bulbs produced by a machine are defective. Find the probability distribution of the number of defective bulbs in a sample of 4 bulbs chosen at random. [CBSE 2004C]

Answer

Given : 20% of the bulbs produced by a machine are defective.

To find probability distribution of a number of defective bulbs in a sample of 4 bulbs chosen at random.

Formula used :

The probability distribution table is given by ,

	x_1	x_2	x_3	x_4	x_5
	$(_1)$	$(_2)$	$(_3)$	$(_4)$	$(_5)$

Where P(x) = ${}^nC_x p^x q^{n-x}$

Here p is the probability of getting a defective bulb.

q = 1 − p

Let the total number of bulbs produced by a machine be x

20% of the bulbs produced by a machine are defective.

Number of defective bulbs produced by a machine = $\frac{20}{100} \times (x) = \frac{x}{5}$

X denotes the number of defective bulbs in a sample of 4 bulbs chosen at random.

Let p be the probability of getting a defective bulb,

$p = \frac{\frac{x}{5}}{x} = \frac{1}{5}$

$p = \frac{1}{5}$

$q = 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

$q = \frac{4}{5}$

since 4 bulbs are chosen at random, n = 4

X can take the values of 0,1,2,3,4

P(x) = $^{n}C_{x} p^{x} q^{n-x}$

P(0) = $^{4}C_{0}\left(\frac{1}{5}\right)^{0}\left(\frac{4}{5}\right)^{4} = \frac{256}{625}$

P(1) = $^{4}C_{1}\left(\frac{1}{5}\right)^{1}\left(\frac{4}{5}\right)^{3} = \frac{256}{625}$

P(2) = $^{4}C_{2}\left(\frac{1}{5}\right)^{2}\left(\frac{4}{5}\right)^{2} = \frac{96}{625}$

P(3) = $^{4}C_{3}\left(\frac{1}{5}\right)^{3}\left(\frac{4}{5}\right)^{1} = \frac{16}{625}$

P(4) = $^{4}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{4}{5}\right)^{0} = \frac{1}{625}$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

13. Question

Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement. Let X be the number of bad eggs drawn. Find the mean and variance of X.

Answer

Given : Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

To find : mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Four bad eggs are mixed with 10 good ones. Three eggs are drawn one by one without replacement.

Let X denote the number of bad eggs drawn

There are 4 bad eggs present in 14 eggs

$P(0) = \frac{{}^{10}_{3}C}{{}^{14}_{3}C} = \frac{10 \times 9 \times 8}{14 \times 13 \times 12} = \frac{30}{91}$

$P(1) = \frac{{}^{10}_{2}C \times {}^{4}_{1}C}{{}^{14}_{3}C} = \frac{10 \times 9 \times 4 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{45}{91}$

$P(2) = \frac{{}^{10}_{1}C \times {}^{4}_{2}C}{{}^{14}_{3}C} = \frac{10 \times 4 \times 3 \times 3 \times 2}{14 \times 13 \times 12 \times 2} = \frac{15}{91}$

$P(3) = \frac{{}^{4}_{3}C}{{}^{14}_{3}C} = \frac{4 \times 3 \times 2}{14 \times 13 \times 12} = \frac{1}{91}$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{30}{91}$	$\frac{45}{91}$	$\frac{15}{91}$	$\frac{1}{91}$

Mean = E(X) = $0(\frac{30}{91}) + 1(\frac{45}{91}) + 2(\frac{15}{91}) + 3(\frac{1}{91}) = 0 + \frac{45}{91} + \frac{30}{91} + \frac{3}{91} = \frac{45 + 30 + 3}{91}$

Mean = E(X) = $\frac{78}{91} = \frac{6}{7}$

$E(X)^2 = (\frac{6}{7})^2 = \frac{36}{49}$

E(X^2) = $\sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

E(X^2) = $(0)^2(\frac{30}{91}) + (1)^2(\frac{45}{91}) + (2)^2(\frac{15}{91}) + (3)^2(\frac{1}{91}) = 0 + \frac{45}{91} + \frac{60}{91} + \frac{9}{91} = \frac{45+60+9}{91}$

E(X^2) = $\frac{114}{91}$

Variance = E(X^2) - $E(X)^2 = \frac{114}{91} - \frac{36}{49} = \frac{798-468}{637} = \frac{330}{637}$

Variance = E(X^2) - $E(X)^2 = \frac{330}{637}$

Mean = E(X) = $\frac{6}{7}$

Variance = $\frac{330}{637}$

14. Question

Four rotten oranges are accidentally mixed with 16 good ones. Three oranges are drawn at random from the mixed lot. Let X be the number of rotten oranges drawn. Find the mean and variance of X.

Answer

Given : Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

To find : mean (u) and variance (σ^2)

Formula used :

	x_1	x_2	x_3
()	($_1$)	($_2$)	($_3$)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Four rotten oranges are mixed with 16 good ones. Three oranges are drawn one by one without replacement.

Let X denote the number of rotten oranges drawn

There are 4 rotten oranges present in 20 oranges

$$P(0) = \frac{{}^{16}_{3}C}{{}^{20}_{3}C} = \frac{16 \times 15 \times 14}{20 \times 19 \times 18} = \frac{28}{57}$$

$$P(1) = \frac{{}^{16}_{2}C \times {}^{4}_{1}C}{{}^{20}_{3}C} = \frac{16 \times 15 \times 4 \times 3 \times 2}{20 \times 19 \times 18 \times 2} = \frac{8}{19}$$

$$P(2) = \frac{{}^{16}_{1}C \times {}^{4}_{2}C}{{}^{20}_{3}C} = \frac{16 \times 4 \times 3 \times 3 \times 2}{20 \times 19 \times 18 \times 2} = \frac{8}{95}$$

$$P(3) = \frac{{}^{4}_{3}C}{{}^{20}_{3}C} = \frac{4 \times 3 \times 2}{20 \times 19 \times 18} = \frac{1}{285}$$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{28}{57}$	$\frac{8}{19}$	$\frac{8}{95}$	$\frac{1}{285}$

$$\text{Mean} = E(X) = 0\left(\frac{28}{57}\right) + 1\left(\frac{8}{19}\right) + 2\left(\frac{8}{95}\right) + 3\left(\frac{1}{285}\right) = 0 + \frac{8}{19} + \frac{16}{95} + \frac{3}{285} = \frac{120 + 48 + 3}{285}$$

$$\text{Mean} = E(X) = \frac{171}{285} = \frac{3}{5}$$

$$E(X)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$$

$$E(X^2) = (0)^2\left(\frac{28}{57}\right) + (1)^2\left(\frac{8}{19}\right) + (2)^2\left(\frac{8}{95}\right) + (3)^2\left(\frac{1}{285}\right) = 0 + \frac{8}{19} + \frac{32}{95} + \frac{9}{285} = \frac{120 + 96 + 9}{285}$$

$$E(X^2) = \frac{225}{285} = \frac{15}{19}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{15}{19} - \frac{9}{25} = \frac{375 - 171}{475} = \frac{204}{475}$$

$$\text{Variance} = E(X^2) - E(X)^2 = \frac{204}{475}$$

Mean = E(X) = $\frac{3}{5}$

Variance = $\frac{204}{475}$

15. Question

Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls. Let X be the number of red balls drawn. Find the mean and variance of X.

Answer

Given : Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

To find : mean (u) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	($_1$)	($_2$)	($_3$)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2)$ - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls.

Let X be the number of red balls drawn.

P(0) = $\frac{{}^5_3C}{{}^9_3C} = \frac{5 \times 4}{9 \times 8 \times 7} = \frac{5}{126}$

P(1) = $\frac{{}^5_2C \times {}^4_1C}{{}^9_3C} = \frac{5 \times 4 \times 4 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{10}{21}$

P(2) = $\frac{{}^5_1C \times {}^4_2C}{{}^9_3C} = \frac{5 \times 4 \times 3 \times 3 \times 2}{9 \times 8 \times 7 \times 2} = \frac{5}{14}$

P(3) = $\frac{{}^4_3C}{{}^9_3C} = \frac{4 \times 3 \times 2}{9 \times 8 \times 7} = \frac{1}{21}$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{5}{126}$	$\frac{10}{21}$	$\frac{5}{14}$	$\frac{1}{21}$

Mean = E(X) = $0(\frac{5}{126}) + 1(\frac{10}{21}) + 2(\frac{5}{14}) + 3(\frac{1}{21}) = 0 + \frac{10}{21} + \frac{10}{14} + \frac{3}{21} = \frac{20+30+6}{42}$

Mean = E(X) = $\frac{56}{42} = \frac{4}{3}$

$E(X)^2 = (\frac{4}{3})^2 = \frac{16}{9}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{5}{126}) + (1)^2(\frac{10}{21}) + (2)^2(\frac{5}{14}) + (3)^2(\frac{1}{21}) = 0 + \frac{10}{21} + \frac{20}{14} + \frac{9}{21} = \frac{20+60+18}{42}$

$E(X^2) = \frac{98}{42} = \frac{7}{3}$

Variance = $E(X^2) - E(X)^2 = \frac{7}{3} - \frac{16}{9} = \frac{21-16}{9} = \frac{5}{9}$

Variance = $E(X^2) - E(X)^2 = \frac{5}{9}$

Mean = E(X) = $\frac{4}{3}$

Variance = $\frac{5}{9}$

16. Question

Two cards are drawn without replacement from a well-shuffled deck of 52 cards. Let X be the number of face cards drawn. Find the mean and variance of X.

Answer

Given : Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

To find : mean (u) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	($_1$)	($_2$)	($_3$)

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

Let X denote the number of face cards drawn

There are 12 face cards present in 52 cards

P(0) = $\frac{{}^{40}_{2}C}{{}^{52}_{2}C} = \frac{40 \times 39}{52 \times 51} = \frac{10}{17}$

P(1) = $\frac{{}^{40}_{1}C \times {}^{12}_{1}C}{{}^{52}_{2}C} = \frac{40 \times 12 \times 2}{52 \times 51} = \frac{80}{221}$

P(2) = $\frac{{}^{12}_{2}C}{{}^{52}_{2}C} = \frac{12 \times 11}{52 \times 51} = \frac{11}{221}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{10}{17}$	$\frac{80}{221}$	$\frac{11}{221}$

Mean = E(X) = $0(\frac{10}{17}) + 1(\frac{80}{221}) + 2(\frac{11}{221}) = 0 + \frac{80}{221} + \frac{22}{221} = \frac{80+22}{221} = \frac{102}{221} = \frac{6}{13}$

Mean = E(X) = $\frac{6}{13}$

$E(X)^2 = (\frac{6}{13})^2 = \frac{36}{169}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{10}{17}) + (1)^2 (\frac{80}{221}) + (2)^2 (\frac{11}{221}) = 0 + \frac{80}{221} + \frac{44}{221} = \frac{80+44}{221}$

$E(X^2) = \frac{124}{221}$

Variance = $E(X^2) - E(X)^2 = \frac{124}{221} - \frac{36}{169} = \frac{1612-612}{2873} = \frac{1000}{2873}$

Variance = $E(X^2) - E(X)^2 = \frac{1000}{2873}$

Mean = $E(X) = \frac{6}{13}$

Variance = $\frac{1000}{2873}$

17. Question

Two cards are drawn one by one with replacement from a well-shuffled deck of 52 cars. Find the mean and variance of the number of aces.

Answer

Given : Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

To find : mean (u) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	($_1$)	($_2$)	($_3$)

Mean = $E(X) = \sum_{i=1}^{i=n} x_i P(x_i)$

Variance = $E(X^2) - E(X)^2$

Mean = $E(X) = \sum_{i=1}^{i=n} x_i P(x_i) = x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Two cards are drawn with replacement from a well-shuffled deck of 52 cards.

Let X denote the number of ace cards drawn

There are 4 face cards present in 52 cards

X can take the value of 0,1,2.

$P(0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169}$

$P(1) = {}^2_1C \times \frac{4}{52} \times \frac{48}{52} = \frac{2 \times 4 \times 48}{52 \times 52} = \frac{24}{169}$

$P(2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

The probability distribution table is as follows,

	0	1	2
()	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

Mean $= E(X) = 0(\frac{144}{169}) + 1(\frac{24}{169}) + 2(\frac{1}{169}) = 0 + \frac{24}{169} + \frac{2}{169} = \frac{24+2}{169} = \frac{26}{169} = \frac{2}{13}$

Mean $= E(X) = \frac{2}{13}$

$E(X)^2 = (\frac{2}{13})^2 = \frac{4}{169}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{144}{169}) + (1)^2(\frac{24}{169}) + (2)^2(\frac{1}{169}) = 0 + \frac{24}{169} + \frac{4}{169} = \frac{28}{169}$

$E(X^2) = \frac{28}{169}$

Variance $= E(X^2) - E(X)^2 = \frac{28}{169} - \frac{4}{169} = \frac{24}{169}$

Variance $= E(X^2) - E(X)^2 = \frac{24}{169}$

Mean $= E(X) = \frac{2}{13}$

Variance $= \frac{24}{169}$

18. Question

Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards. A random variable X denotes the number of hearts in the three cards drawn. Find the mean and variance of X.

Answer

Given : Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards.

To find : mean (u) and variance (σ^2) of X

Formula used :

	x_1	x_2	x_3
()	$(_1)$	$(_2)$	$(_3)$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$

Variance = E(X^2) - $E(X)^2$

Mean = E(X) = $\sum_{i=1}^{i=n} x_i P(x_i)$ = $x_1P(x_1) + x_2P(x_2) + x_3P(x_3)$

Three cards are drawn successively with replacement from a well – shuffled deck of 52 cards.

Let X be the number of hearts drawn.

Number of hearts in 52 cards is 13

P(0) = $\frac{39}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$

P(1) = ${}^3_1C \times \frac{13}{52} \times \frac{39}{52} \times \frac{39}{52} = \frac{27}{64}$

P(2) = ${}^3_2C \times \frac{13}{52} \times \frac{13}{52} \times \frac{39}{52} = \frac{9}{64}$

P(3) = $\frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} = \frac{1}{64}$

The probability distribution table is as follows,

	0	1	2	3
()	$\frac{27}{64}$	$\frac{27}{64}$	$\frac{9}{64}$	$\frac{1}{64}$

Mean = E(X) = $0(\frac{27}{64}) + 1(\frac{27}{64}) + 2(\frac{9}{64}) + 3(\frac{1}{64}) = 0 + \frac{27}{64} + \frac{18}{64} + \frac{3}{64} = \frac{48}{64} = \frac{3}{4}$

Mean = E(X) = $\frac{3}{4}$

$E(X)^2 = (\frac{3}{4})^2 = \frac{9}{16}$

$E(X^2) = \sum_{i=1}^{i=n}(x_i)^2.P(x_i) = (x_1)^2.P(x_1) + (x_2)^2.P(x_2) + (x_3)^2.P(x_3)$

$E(X^2) = (0)^2(\frac{27}{64}) + (1)^2(\frac{27}{64}) + (2)^2(\frac{9}{64}) + (3)^2(\frac{1}{64}) = 0 + \frac{27}{64} + \frac{36}{64} + \frac{9}{64} = \frac{72}{64} = \frac{9}{8}$

$E(X^2) = \frac{9}{8}$

Variance = $E(X^2) - E(X)^2 = \frac{9}{8} - \frac{9}{16} = \frac{18-9}{16} = \frac{9}{16}$

Variance = $E(X^2) - E(X)^2 = \frac{9}{16}$

Mean = E(X) = $\frac{3}{4}$

Variance = $\frac{9}{16}$

19. Question

Five defective bulbs are accidently mixed with 20 good ones. It is not possible to just look at a bulb and tell whether or not it is defective. Find the probability distribution from this lot.

Answer

Given : Five defective bulbs are accidently mixed with 20 good ones.

To find : probability distribution from this lot

Formula used :

	x_1	x_2	x_3	x_4	x_5
	$(_1)$	$(_2)$	$(_3)$	$(_4)$	$(_5)$

Five defective bulbs are accidently mixed with 20 good ones.

Total number of bulbs = 25

X denote the number of defective bulbs drawn

X can draw the value 0 , 1 , 2 , 3 , 4.

since the number of bulbs drawn is 4, n = 4

P(0) = P(getting a no defective bulb) = $\frac{{}^{20}_{4}C}{{}^{25}_{4}C} = \frac{20 \times 19 \times 18 \times 17}{25 \times 24 \times 23 \times 22} = \frac{969}{2530}$

P(1) = P(getting 1 defective bulb and 3 good ones) = $\frac{{}^{5}_{1}C \times {}^{20}_{3}C}{{}^{25}_{4}C} = \frac{5 \times 20 \times 19 \times 18 \times 4}{25 \times 24 \times 23 \times 22}$

P(1) = $\frac{1140}{2530} = \frac{114}{253}$

P(2) = P(getting 2 defective bulbs and 2 good one) = $\frac{{}^{5}_{2}C \times {}^{20}_{2}C}{{}^{25}_{4}C}$

P(2) = $\frac{5 \times 4 \times 20 \times 19 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2 \times 2} = \frac{380}{2530} = \frac{38}{253}$

P(3) = P(getting 3 defective bulbs and 1 good one) = $\frac{{}^{5}_{3}C \times {}^{20}_{1}C}{{}^{25}_{4}C} \frac{5 \times 4 \times 20 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22 \times 2}$

P(3) = $\frac{40}{2530} = \frac{4}{253}$

P(4) = P(getting all defective bulbs) = $\frac{{}^{5}_{4}C}{{}^{25}_{4}C} = \frac{5 \times 4 \times 3 \times 2}{25 \times 24 \times 23 \times 22} = \frac{1}{2530}$

P(4) = $\frac{1}{2530}$

The probability distribution table is as follows,

	0	1	2	3	4
	$\frac{969}{2530}$	$\frac{114}{253}$	$\frac{38}{253}$	$\frac{4}{253}$	$\frac{1}{2530}$