Sets

Exercise 1A

Q. 1 A. Which of the following are sets? Justify your answer.

The collection of all whole numbers less than 10.

Answer : Whole numbers are 0, 1, 2, 3, …

Whole numbers less than 10 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

As the collection of all whole numbers, less than 10 is known and can be counted, i.e. well – defined.

∴, this is a set.

Q. 1 B. Which of the following are sets? Justify your answer.

The collection of good hockey players in India.

Answer : As a collection of good hockey players in India may vary from person to person.

So, it is not well – defined.

∴, this is not a set.

Q. 1 C. Which of the following are sets? Justify your answer.

The collection of all the questions in this chapter.

Answer : As the collection of all questions in this chapter is known and can be counted .i.e. well – defined.

∴, this is a set.

Q. 1 D. Which of the following are sets? Justify your answer.

The collection of all the difficult chapters in this book.

Answer : As the collection of all difficult chapters in this book may vary from person to person.

∴, this is not a set.

Q. 1 E. Which of the following are sets? Justify your answer.

A collection of Hindi novels written by Munshi Prem Chand.

Answer : As the collection of Hindi novels written by Munshi Prem Chand is known and can be counted, i.e. well – defined.

∴, this is a set.

Q. 1 F. Which of the following are sets? Justify your answer.

A team of 11 best cricket players of India.

Answer : As a collection of 11 best cricket players of India may vary from person to person.

So, it is not well – defined.

∴, this is not a set.

Q. 1 G. Which of the following are sets? Justify your answer.

The collection of all the months of the year whose names begin with the letter M.

Answer : Months of the Year = Jan, Feb, March, April, May, June, July, Aug, Sep, Oct, Nov, Dec

Months of the year whose names begin with the letter M are:

• March

• May

As, the collection of all the months of the year whose names begin with the letter M is known and can be counted .i.e. well – defined.

∴, this is a set.

Q. 1 H. Which of the following are sets? Justify your answer.

The collection of all interesting books.

Answer : As the collection of all interesting books may vary to person to person.

∴, this is not a set.

Q. 1 I. Which of the following are sets? Justify your answer.

The collection of all short boys of your class.

Answer : As the collection of all short boys of your class may vary to person to person. Maybe someone consider short boys of height less than 120 cm and maybe someone consider short boys of height less than 90cm. Here, the set is not well – defined.

∴, this is not a set.

Q. 1 J. Which of the following are sets? Justify your answer.

The collection of all those students of your class whose ages exceed 15 years.

Answer : As the collection of all those students of your class whose ages exceed 15 years is known and can be counted, i.e. well – defined.

∴, this is a set

Q. 1 K. Which of the following are sets? Justify your answer.

The collection of all rich persons of Kolkata.

Answer : As the collection of all rich persons of Kolkata may vary from person to person. Someone considers a person whose income is Rs 1 lakh per annum as a rich person, and someone considers a person whose income is Rs 1 crore per annum as a rich person. Here, the set is not well – defined.

∴, this is not a set

Q. 1 L. Which of the following are sets? Justify your answer.

The collection of all persons of Kolkata whose assessed annual incomes exceed (say) Rs 20 lakh in the 4 financial years 2016-17.

Answer : As the collection of all persons of Kolkata whose assessed annual incomes exceed (say) Rs 20 lakh in the 4 financial years 2016-17 is known and well – defined.

∴, this is a set.

Q. 1 M. Which of the following are sets? Justify your answer.

The collection of all interesting dramas written by Shakespeare.

Answer : As the collection of all interesting dramas written by Shakespeare is not well - defined because it depends on person interest.

∴, this is not a set.

Q. 2. Let A be the set of all even whole numbers less than 10.

(i) Write A in the roster from.

(ii) Fill in the blanks with the approximate symbol ∉ or ϵ :

(a) 0 …. A

(b) 10 …. A

(c) 3 …. A

(d) 6 …. A

Answer : (i) Whole numbers are 0, 1, 2, 3, …

Even whole numbers less than 10 are 0, 2, 4, 6, 8, 9

So, A = {0, 2, 4, 6, 8}

(ii) (a) Here, A = {0, 2, 4, 6, 8}

∈ - belongs to
∉ - does not belongs to

As 0 is in set A.

Hence, 0 ∈ A

(b) Here, A = {0, 2, 4, 6, 8}

∈ - belongs to
∉ - does not belongs to

As 10 is not in a set A

Hence, 10 ∉ A

∈ - belongs to
∉ - does not belongs to

(c) Here, A = {0, 2, 4, 6, 8}

As 3 is not in a set A

Hence, 3 ∉ A

(d) Here, A = {0, 2, 4, 6, 8}

∈ - belongs to
∉ - does not belongs to

As 6 is in set A

Hence, 6 ∈ A

Q. 3 A. Write the following sets in roster from:

A = {x : x is a natural number, 30 ≤ x < 36}.

Answer : Natural numbers = 1, 2, …, 30, 31, 32, 33, 34, 35, 36, …

The elements of this set are 30, 31, 32, 33, 34 and 35 only

So, A = {30, 31, 32, 33, 34, 35}

Q. 3 B. Write the following sets in roster from:

B = {x : x is an integer and –4 < x < 6}.

Answer : Integers = …-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7,…

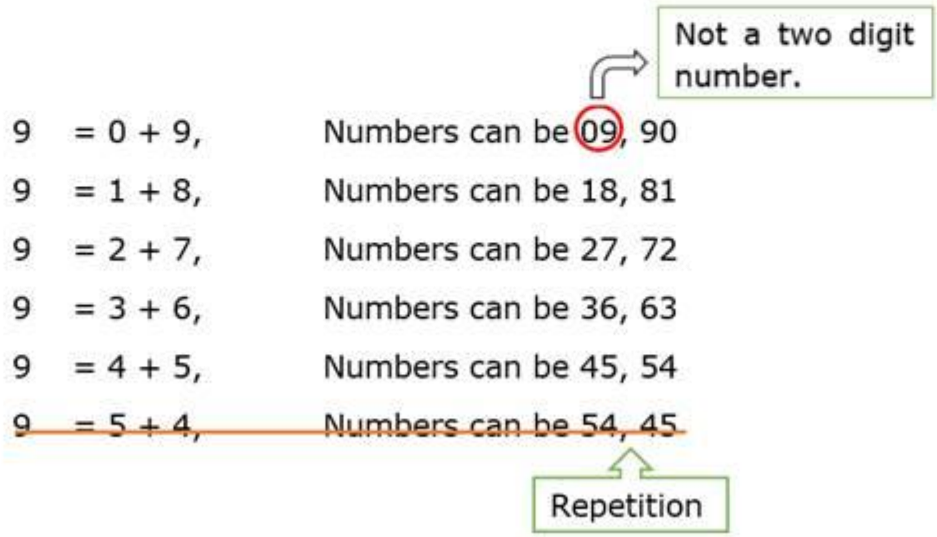
The elements of this set are -3, -2, -1, 0, 1, 2, 3, 4 and 5 only.

So, B = {-3, -2, -1, 0, 1, 2, 3, 4, 5}

Q. 3 C. Write the following sets in roster from:

C = {x : x is a two-digit number such that the sum of its digits is 9}.

Answer :



The elements of this set are 18, 27, 36, 45, 54, 63, 72, 81 and 90

So, C = {18, 27, 36, 45, 54, 63, 72, 81, 90}

Q. 3 D. Write the following sets in roster from:

D = {x : x is an integer, $x^2 \leq 9$}.

Answer : Integers = …, -4, -3, -2, -1, 0, 1, 2, 3, 4, …

$x = -4, x^2 = (-4)^2 = 16 > 9$

$x = -3, x^2 = (-3)^2 = 9$

$x = -2, x^2 = (-2)^2 = 4$

$x = -1, x^2 = (-1)^2 = 1$

$x = 0, x^2 = (0)^2 = 0$

$x = 1, x^2 = (1)^2 = 1$

$x = 2, x^2 = (2)^2 = 4$

$x = 3, x^2 = (3)^2 = 9$

$x = 4, x^2 = (4)^2 = 16$

The elements of this set are -3, -2, -1, 0, 1, 2, 3

So, D = {-3, -2, -1, 0, 1, 2, 3}

Q. 3 E. Write the following sets in roster from:

E = {x : x is a prime number, which is a divisor of 42}.

Answer : Prime number = Those number which is divisible by 1 and the number itself.

Prime numbers are 2, 3, 5, 7, 11, 13, …

Divisor of 42:

$42 = 1 \times 42$

$42 = 2 \times 21$

$42 = 3 \times 14$

$42 = 6 \times 7$

So, divisors of 42 are 1, 2, 3, 6, 7, 14, 21, 42

The elements which are prime and divisor of 42 are 2, 3, 7

So, E = {2, 3, 7}

Q. 3 F. Write the following sets in roster from:

F = {x : x is a letter in the word' MATHEMATICS'}.

Answer : There are 11 letters in the word MATHEMATICS, out of which M, A and T are repeated.

So, F = {M, A, T, H, E, I, C, S}

Q. 3 G. Write the following sets in roster from:

G = {x : x is a prime number and 80 < x < 100}.

Answer : Prime number = Those number which is divisible by 1 and the number itself.

Prime numbers are 2, 3, 5, 7, 11, 13,…

The numbers 80 < x < 100 are

81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

The elements which are prime and lies between 80 and 100 are 83, 89, 97

So, G = {83, 89, 97}

Q. 3 H. Write the following sets in roster from:

H = {x : x is a perfect square and x < 50}.

Answer : Perfect squares are:

$0^2 = 0$

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

$8^2 = 64 > 50$

The elements which are perfect square and x < 50 are 0, 1, 2, 3, 4, 5, 6, 7

Q. 3 I. Write the following sets in roster from:

J = {x : ϵ R and $x^2 + x - 12 = 0$}.

Answer : The given equation is:

$x^2 + x - 12 = 0$

$\Rightarrow x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x + 4) - 3(x + 4) = 0$

$\Rightarrow (x - 3)(x + 4) = 0$

$\Rightarrow x - 3 = 0$ or $x + 4 = 0$

$\Rightarrow x = 3$ or $x = -4$

therefore, the solution set of the given equation can be written in roaster form as {3, -4}

So, J = {3, -4}

Q. 3 J. Write the following sets iroster from:

K = {x : ϵ N, x is a multiple of 5 and x^2 < 400}.

Answer : Multiple of 5 are 5, 10, 15, 20, 25, 30, …

So, $5^2 = 25$

$10^2 = 100$

$15^2 = 225$

$20^2 = 400$

$25^2 = 625 > 400$

The elements which are multiple of 5 and $x^2 < 400$ are 5, 10, 15

So, K = {5, 10, 15}

Q. 4 A. List all the elements of each of the sets given below.

A = {x : x = 2n, n ϵ N and n ≤ 5}.

Answer : Given: $x = 2n$ and $n \leq 5$

$\Rightarrow n = 1, 2, 3, 4$ and 5 [$\because n \in N$]

Given $x = 2n$

$n = 1, x = 2 \times 1 = 2$

$n = 2, x = 2 \times 2 = 4$

$n = 3, x = 2 \times 3 = 6$

$n = 4, x = 2 \times 4 = 8$

$n = 5, x = 2 \times 5 = 10$

So, the elements of A are 2, 4, 6, 8 and 10

$\therefore$, A = {2, 4, 6, 8, 10}

Q. 4 B. List all the elements of each of the sets given below.

$B = \{x : x = 2n + 1, n \in W \text{ and } n \le 5\}$.

Answer : Given: $x = 2n + 1$ and $n \le 5$

$\Rightarrow n = 0, 1, 2, 3, 4$ and 5 [$\because n \in W$]

Given $x = 2n + 1$

$n = 0, x = 2 \times 0 + 1 = 1$

$n = 1, x = 2 \times 1 + 1 = 3$

$n = 2, x = 2 \times 2 + 1 = 5$

$n = 3, x = 2 \times 3 + 1 = 7$

$n = 4, x = 2 \times 4 + 1 = 9$

$n = 5, x = 2 \times 5 + 1 = 11$

So, the elements of B are 1, 3, 5, 7, 9 and 11

$\therefore$, B = {1, 3, 5, 7, 9, 11}

Q. 4 C. List all the elements of each of the sets given below.

$$C = \{x : x = \frac{1}{n}, n \in N \text{ and } n \le 6\}.$$

Answer : Here, $x = \frac{1}{n}$ and n ≤ 6

So, n = 1, 2, 3, 4, 5 and 6 [∵n ∈ N]

Given: $x = \frac{1}{n}$

$n = 1, x = \frac{1}{1} = 1$

$n = 2, x = \frac{1}{2}$

$n = 3, x = \frac{1}{3}$

$n = 4, x = \frac{1}{4}$

$n = 5, x = \frac{1}{5}$

$n = 6, x = \frac{1}{6}$

$So, C = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\right\}$

Q. 4 D. List all the elements of each of the sets given below.

D = {x : x = n^2, n ϵ N and 2 ≤ n ≤ 5}.

Answer : Here, x = n^2 and 2 ≤ n ≤ 5

∴ n = 2, 3, 4, 5

[it is given that n is less than equal to 2 and greater than equal to 5]

If

n = 2, x = $(2)^2$ = 4

$n = 3, x = (3)^2 = 9$

$n = 4, x = (4)^2 = 16$

$n = 5, x = (5)^2 = 25$

So, D = {4, 9, 16, 25}

Q. 4 E. List all the elements of each of the sets given below.
E = {x : x ϵ Z and $x^2 = x$}.

Answer : Given: $x \in Z$ and $x^2 = x$

Z is a set of integers

Integers are …-2 , -1, 0, 1, 2, …

Now, if we take x = -2 then we have to check that it satisfies the given condition $x^2 = x$

$(-2)^2 = 4 \neq 2$

So, $-2 \notin E$

If x = -1 then $(-1)^2 = 1 \neq -1$ [not satisfying $x^2 = x$]

So, $-1 \notin E$

If x = 0 then $(0)^2 = 0$ [satisfying $x^2 = x$]

$\therefore 0 \in E$

If x = 1 then $(1)^2 = 1$ [satisfying $x^2 = x$]

$\therefore 1 \in E$

If x = 2 then $(2)^2 = 4 \neq 2$ [not satisfying $x^2 = x$]

$\Rightarrow 2 \notin E$

So, E = {0, 1}

Q. 4 F. List all the elements of each of the sets given below.

$$F = \{x : x = \in Z \text{ and } -\frac{1}{2} < X\frac{13}{2}\}.$$

Answer : Given x ∈ Z and

$$-\frac{1}{2} < x < \frac{13}{2}$$

It can be seen that

$$-\frac{1}{2} = -0.5 \ \& \ \frac{13}{2} = 6.5$$

We know that, Z means Set of integers

∴ F = {0, 1, 2, 3, 4, 5, 6}

Q. 4 G. List all the elements of each of the sets given below.

$$G = \{x : x = \frac{1}{(2n-1)}, n \in N \text{ and } 1 \le n \le 5$$

Answer : Given:

$$x = \frac{1}{2n-1} \, and \ 1 \le n \le 5$$

So, n = 1, 2, 3, 4, 5

$$If\ n = 1, then\ x = \frac{1}{2n-1} = \frac{1}{2(1)-1} = \frac{1}{1} = 1$$

$$If\ n = 2, then\ x = \frac{1}{2n-1} = \frac{1}{2(2)-1} = \frac{1}{4-1} = \frac{1}{3}$$

$$If\ n = 3, then\ x = \frac{1}{2n-1} = \frac{1}{2(3)-1} = \frac{1}{6-1} = \frac{1}{5}$$

$$If\ n = 4, then\ x = \frac{1}{2n-1} = \frac{1}{2(4)-1} = \frac{1}{8-1} = \frac{1}{7}$$

$$If\ n = 5, then\ x = \frac{1}{2n-1} = \frac{1}{2(5)-1} = \frac{1}{10-1} = \frac{1}{9}$$

So, $G = \left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}\right\}$

Q. 4 H. List all the elements of each of the sets given below.

H = {x : x ϵ Z, |x| ≤ 2}.

Answer : Given x ∈ Z and |x| ≤ 2

Z is a set of integers

Integers are …-3, -2 , -1, 0, 1, 2, 3, …

Now, if we take x = -3 then we have to check that it satisfies the given condition |x| ≤ 2

|-3| = 3 > 2

So, -3 ∉ H

If x = -2 then |-2| = 2 [satisfying |x| ≤ 2]

So, -2 ∈ H

If x = -1 then |-1| = 1 [satisfying |x| ≤ 2]

∴ -1 ∈ H

If x = 0 then |0| = 0 [satisfying |x| ≤ 2]

∴ 0 ∈ H

If x = 1 then |1| = 1 [satisfying |x| ≤ 2]

⇒ 1 ∈ H

If x = 2 then |2| = 2 [satisfying |x| ≤ 2]

So, 2 ∈ H

If x = 3 then |3| = 3 > 2 [satisfying |x| ≤ 2]

So, 3 ∉ H

So, H = {-2, -1, 0, 1, 2}

So, E = {0, 1}

Q. 5. Write each of the sets given below in set7builder from:

(i)

$$A = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\right\}.$$

(ii)

$$B = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}.$$

(iii) C = {53, 59, 61, 67, 71, 73, 79}.
(iv) D = {–1, 1}.
(v) E = {14, 21, 28, 35, 42, …., 98}.

Answer :

$$A = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\right\}$$

$$= \left\{\left(\frac{1}{1}\right)^2, \left(\frac{1}{2}\right)^2, \left(\frac{1}{3}\right)^2, \left(\frac{1}{4}\right)^2, \left(\frac{1}{5}\right)^2, \left(\frac{1}{6}\right)^2, \left(\frac{1}{7}\right)^2\right\}$$

Hence, we may write the set as

$$A = \left\{x : x = \frac{1}{n^2}, n \in N \; and \; 1 \leq n \leq 7\right\}$$

(ii) $B = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$

$$= \left\{\frac{1}{1+1}, \frac{2}{4+1}, \frac{3}{9+1}, \frac{4}{16+1}, \frac{5}{25+1}, \frac{6}{36+1}, \frac{7}{49+1}\right\}$$

$$= \left\{\frac{1}{1^2+1}, \frac{2}{2^2+1}, \frac{3}{3^2+1}, \frac{4}{4^2+1}, \frac{5}{5^2+1}, \frac{6}{6^2+1}, \frac{7}{7^2+1}\right\}$$

Hence, we may write the set as

$$B = \left\{x : x = \frac{n}{n^2+1}, n \in N \; and \; 1 \leq n \leq 7\right\}$$

(iii) C = {53, 59, 61, 67, 71, 73, 79}

We know that prime numbers are those numbers which are divisible by 1 and the number itself.

e.g. $\frac{3}{1} = 3 \; and \; \frac{3}{3} = 1$

Here, all the given numbers are consecutive prime numbers greater than 50.

So, C = {x: x is a prime number and 50 < x < 80}

(iv) Here, in set D there are two elements -1 and 1

-1 and 1 are integers

So, the given set can be write as

D = {x: x is an integer and -2 < x < 2}

(v) 14 = 7 × 2

21 = 7 × 3

28 = 7 × 4

35 = 7 × 5

42 = 7 × 6

.

.

.

.

98 = 7 × 14

So, the given set can be write as

E = {x: x = 7n, n ∈ N and 1 ≤ n ≤ 14}

Q. 6. Match each of the sets on the left described in the roster from with the same set on the right described in the set-builder from:

	Column I		Column II
(i)	{–5, 5}	**(a)**	{x : x ϵ Z and x^2 < 16}
(ii)	{1, 2, 3, 6, 9, 18}	**(b)**	{x : x ϵ N and x^2 = x}
(iii)	{–3, –2, –1, 0, 1, 2, 3}	**(c)**	{x : x = ϵ Z and x^2 =25}
(iv)	{P, R, I, N, C, A, L}	**(d)**	{x : x ϵ N and x is a factor of 18}
(v)	{1}	**(e)**	{x : x is a letter in the word 'PRINCIPAL'}

Answer : (i) {-5, 5}

It can be seen that if we take the square of -5 and 5, the result will be 25

If x = -5, then $(-5)^2 = 25$

If x = 5, then $(5)^2 = 25$

and -5, 5 both are integers

So, {x : x ∈Z and x^2 = 25}

∴ **(i)** matches (c)

(ii) {1, 2, 3, 6, 9, 18}

Divisor of 18 are
18 = 18 × 1

18 = 9 × 2

18 = 6 × 3

1, 2, 3, 6, 9, 18 are divisors of 18

So, {x : x ∈ N and x is a factor of 18}

∴ **(ii)** matches (d)

(iii) {–3, –2, –1, 0, 1, 2, 3}

$(-3)^2 = 9 < 16$

$(-2)^2 = 4 < 16$

$(-1)^2 = 1 < 16$

$(0)^2 = 0 < 16$

$(1)^2 = 1 < 16$

$(2)^2 = 4 < 16$

$(3)^2 = 9 < 16$

All are the given elements are integers and satisfying $x^2 < 16$

So, **(iii)** matches (a)

(iv) {P, R, I, N, C, A, L}

There are 9 letters in the word PRINCIPAL out of which P and I are repeated.

So, {x : x is a letter in the word 'PRINCIPAL'}

∴ (iv) matches (e)

(v) {1}

Since, $1 \in N$ and $(1)^2 = 1$

So, $\{x : x \in N \text{ and } x^2 = x\}$

∴ (v) matches (b)

	Column I		Column II
(i)	{–5, 5}	(a)	{x : x ϵ Z and x^2 < 16}
(ii)	{1, 2, 3, 6, 9, 18}	(b)	{x : x ϵ N and x^2 = x}
(iii)	{–3, –2, –1, 0, 1, 2, 3}	(c)	{x : x = ϵ Z and x^2 = 25}
(iv)	{P, R, I, N, C, A, L}	(d)	{x : x ϵ N and x is a factor of 18}
(v)	{1}	(e)	{x : x is a letter in the word 'PRINCIPAL'}

Exercise 1B

Q. 1 A. Which of the following are examples of the null set?

Set of odd natural numbers divisible by 2.

Answer : Natural numbers = 1, 2, 3, 4, 5,…

Odd Natural numbers = 1, 3, 5, 7, 9, 11, …

No odd natural number is divisible by 2.

∴ no elements in this set

∴ It is a null set.

Q. 1 B. Which of the following are examples of the null set?

Set of even prime numbers.

Answer : Prime numbers = Those numbers which are divisible by 1 and number itself.

Prime numbers = 2, 3, 5, 7, 11, 13,…

Even Prime number = 2

∴ set is not empty.

∴ It is not a null set

Q. 1 C. Which of the following are examples of the null set?

A = {x : x ϵ N, 1 < x ≤ 2}.

Answer : Natural numbers = 1, 2, 3, 4, 5, 6, 7,…

Natural number greater than 1 $(1 < x) = 2, 3, 4, 5, ..$

Natural number less than or equal to 2 $(x \geq 2) = 1$

A number cannot be simultaneously greater than 1 and less than equal to 2

∴ no elements in this set

∴ It is a null set.

Q. 1 D. Which of the following are examples of the null set?

B = $\{x : x \in N, 2x + 3 = 4\}$.

Answer : Natural numbers = 1, 2, 3, 4, 5, 6,…

If $x = 1$, then $2x + 3 = 2(1) + 3 = 2 + 3 = 5 \neq 4$

∴ no elements in the set B because the equation $2x + 3 = 4$ is not satisfied by any natural number of x.

∴ It is a null set.

Q. 1 E. Which of the following are examples of the null set?

C = $\{x : x \text{ is prime}, 90 < x < 96\}$.

Answer : Prime numbers = Those numbers which are divisible by 1 and number itself.

Prime numbers = 2, 3, 5, 7, 11, 13,…, 83, 89, 97, …

Prime number greater than 90 = 97

Prime number less than 96 = 89

Prime number less than 96 but greater than 90 = ϕ

∴ The set is empty

∴ It is a null set

Q. 1 F. Which of the following are examples of the null set?

D = $\{x : x \in N, x^2 + 1 = 0\}$.

Answer : Natural numbers = 1, 2, 3, 4, 5, 6,…

If $x = 1$, then $x^2 + 1 = (1)^2 + 1 = 1 + 1 = 2 \neq 0$

∴ no elements in the set B because the equation $2x + 3 = 4$ is not satisfied by any natural number of x.

∴ It is a null set.

Q. 1 G. Which of the following are examples of the null set?

E = {x : x ϵ W, x + 3 ≤ 3}.

Answer : Whole numbers = 0, 1, 2, 3, …

If we take x = 0 then x + 3 = 0 + 3 = 3

If we take x = 1 then x + 3 = 1 + 3 = 4 > 3

So, 0 is the element of set E because it satisfies the given equation.

∴ It is not a null set.

Q. 1 H. Which of the following are examples of the null set?

F = {x : x ϵ Q, 1 < x < 2}.

Answer : Here, x ∈ Q

i.e. x is a rational number

We know that,

If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$

So, the rational number 1 and 2 is

$$\frac{1+2}{2} = \frac{3}{2}$$

∴ the set is not empty

∴ It is not a null set.

Q. 1 I. Which of the following are examples of the null set?

G = {0}

Answer : Since, 0 ∈ G

∴ the set is not empty

∴ It is not a null set

Q. 2. Which of the following are examples of the singleton set?

(i) {*x* : *x* ϵ Z, $x^2 = 4$}.
(ii) {x : x ϵ Z, x + 5 = 0}.
(iii) {x : x ϵ Z, |x| = 1}.
(iv) {x : x ϵ N, $x^2 = 16$}.
(v) {x : x is an even prime number}

Answer : (i) Integers = …-3, -2, -1, 0, 1, 2, 3, …

Given equation:

$x^2 = 4$

$\Rightarrow x = \sqrt{4}$

$\Rightarrow x = \pm 2$

If x = -2, then $x^2 = (-2)^2 = 4$

If x = 2, then $x^2 = (2)^2 = 4$

So, there are two elements in a set.

∴ It is not a singleton set.

(ii) Integers = -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, …

Given equations:

$x + 5 = 0$

$\Rightarrow x + 5 – 5 = 0 – 5$

$\Rightarrow x = -5$

So, there is only 1 element in a given set.

∴ It is a singleton set.

(iii) Integers = …, -2, -1, 0, 1, 2, …

Given equation: |x| = 1

If x = -1, then |x| = |-1| = 1

If x = 1, then |x| = |1| = 1

So, there are 2 elements in a given set

∴ It is not a singleton set.

(iv) Natural Numbers = 1, 2, 3, …

Given equation:

$x^2 = 16$

⇒ x = √16

⇒ x = ± 4

⇒ x = -4, 4

but x = -4 not possible because x ∈ N

So, there is only 1 element in a set.

∴ It is a singleton set.

(v) Prime number = 2, 3, 5, 7, 11, …

Even Prime number = 2

∴ It is a singleton set.

Q. 3 A. Which of the following are pairs of equal sets?

A = set of letters in the word, 'ALLOY.'

B = set of letters in the word, 'LOYAL.'

Answer : Equal Sets = Two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have,

A = set of letters in the word, ALLOY

A = {A, L, O, Y}

and B = set of letters in the word, LOYAL

B = {L, O, Y, A}

Here, A = B because the elements in both the sets are equal. The repetition of elements in a set does not change a set.

Thus, A and B are equal sets.

Q. 3 B. Which of the following are pairs of equal sets?

C = set of letters in the word, 'CATARACT.'

D = set of letters in the word, 'TRACT.'

Answer : Equal Sets = Two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have,

C = set of letters in the word, 'CATARACT.'

C = {C, A, T, R}

and D = set of letters in the word, 'TRACT.'

D = {T, R, A, C}

Here, C = D because the elements in both the sets are equal. The repetition of elements in a set does not change a set.

Thus, C and D are equal sets.

Q. 3 C. Which of the following are pairs of equal sets?

$E = \{x : x \in Z, x^2 \leq 4\}$ and $F = \{x : x \in Z, x^2 = 4\}$.

Answer : Equal Sets = Two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have,

$E = \{x : x \in Z, x^2 \leq 4\}$

Here, $x \in Z$ and $x^2 \leq 4$

If $x = -2$, then $x^2 = (-2)^2 = 4 = 4$

If $x = -1$, then $x^2 = (-1)^2 = 1 < 4$

If $x = 0$, then $x^2 = (0)^2 = 0 < 4$

If $x = 1$, then $x^2 = (1)^2 = 1 < 4$

If $x = 2$, then $x^2 = (2)^2 = 4 = 4$

So,

$E = \{-2, -1, 0, 1, 2\}$

and $F = \{x : x \in Z, x^2 = 4\}$

Here, $x \in Z$ and $x^2 = 4$

If $x = -2$, then $x^2 = (-2)^2 = 4 = 4$

If $x = 2$, then $x^2 = (2)^2 = 4 = 4$

So, $F = \{-2, 2\}$

$\therefore E \neq F$ because the elements in the both the sets are not equal.

Q. 3 D. Which of the following are pairs of equal sets?

$G = \{-1, 1\}$ and $H = \{x : x \in Z, x^2 - 1 = 0\}$.

Answer : Equal Sets = Two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have,

$G = \{-1, 1\}$

and $H = \{x : x \in Z, x^2 - 1 = 0\}$

Here, $x \in Z$ and $x^2 - 1 = 0$

The given equation can be solved as:

$x^2 - 1 = 0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \sqrt{1}$

$\Rightarrow x = \pm 1$

$\therefore x = -1$ and 1

$\therefore H = \{-1, 1\}$

$\Rightarrow G = H$ because elements of both the sets are equal.

Q. 3 E. Which of the following are pairs of equal sets?

J = {2, 3} and K = {x : x ϵ Z, (x^2 + 5x + 6) = *0*}

Answer : Equal Sets = Two sets A and B are said to be equal if they have exactly the same elements & we write A = B

We have,

J = {2, 3}

and K = {x : x ϵ Z, (x^2 + 5x + 6) = *0*}

Here, $x \in Z$ and $x^2 + 5x + 6 = 0$

The given equation can be solved as:

$x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 2x + 3x + 6 = 0$

$\Rightarrow x(x + 2) + 3(x + 2) = 0$

$\Rightarrow (x + 2)(x + 3) = 0$

$\Rightarrow x = -2$ and -3

$\therefore K = \{-2, -3\}$

$\therefore J \neq K$ because elements of both the sets are not equal.

Q. 4. Which of the following are pairs of equivalent sets?

(i) A = {–1, –2, 0} and B = { 1, 2, 3,}

(ii) C = {x : x ϵ N, x < 3} and D ={x : x ϵ W, x < 3}

(iii) E = {a, e, i, o, u} and F = {p, q, r, s, t}

Answer : (i) Equivalent Sets can have different or same elements but have the same amount of elements.

We have,

A = {–1, –2, 0} and B = {1, 2, 3,}

∴ A and B are equivalent sets because both have 3 elements in their set.

(ii) Equivalent Sets can have different or same elements but have the same amount of elements.

We have,

C = {x : x ϵ N, x < 3}

Natural numbers = 1, 2, 3, 4, …

Natural numbers less than 3 (x < 3) = 1, 2

So, C = {1, 2}

and D ={x : x ϵ W, x < 3}

Whole numbers = 0, 1, 2, 3, 4, …

Whole numbers less than 3 (x < 3) = 0, 1, 2

So, D = {0, 1, 2}

∴ C and D are not equivalent sets because their cardinality is not same.

(iii) Equivalent Sets can have different or same elements but have the same amount of elements.

We have,

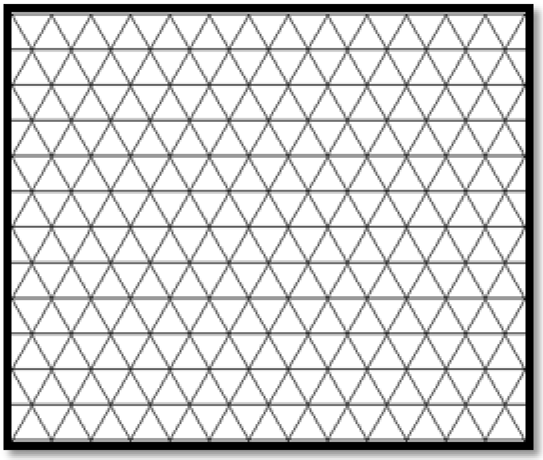
E = {a, e, i, o, u} and F = {p, q, r, s, t}

∴ E and F are equivalent sets because both have 5 elements in their set.

Q. 5 A. State whether any given set is finite or infinite:

A = Set of all triangles in a plane.

Answer :

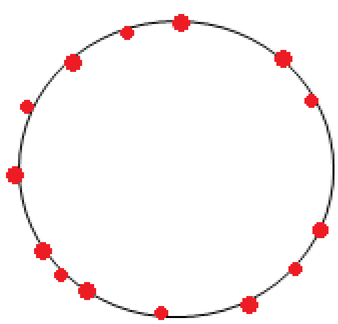


The set of all triangles in a plane is an infinite set because in a plane there is an infinite number of triangles.

Q. 5 B. State whether any given set is finite or infinite:

B = Set of all points on the circumference of a circle.

Answer :



There are infinite numbers of points on the circumference of the circle.

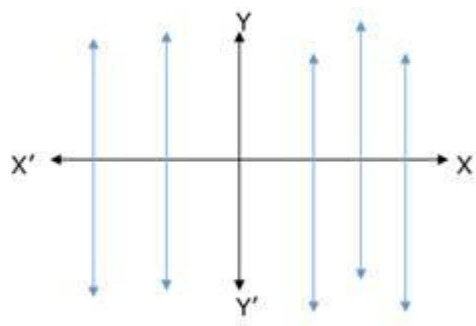
so the set will have infinite elements.

So, the given set is infinite.

Q. 5 C. State whether any given set is finite or infinite:

C = set of all lines parallel to the y-axis

Answer :



There are infinite lines parallel to y-axis,

so the set will have infinite elements.

So, the given set is infinite.

Q. 5 D. State whether any given set is finite or infinite:

D = set of all leaves on a tree

Answer : Here, the set is finite because infinite means never-ending.

Definitely, number will be huge but by definition, it has to be finite.

Q. 5 E. State whether any given set is finite or infinite:

E = set of all positive integers greater than 500

Answer : Positive Integers = 0, 1, 2, 3, …500

Positive Integers greater than 500 = 501, 502, 503, …

There are infinite positive integers which are greater than 500.

So, the given set is infinite.

Q. 5 F. State whether any given set is finite or infinite:

F = {x ϵ R: 0 < x < 1].

Answer : R means set of Real numbers

Real numbers include both rational and irrational numbers

Real numbers between 0 and 1 are infinite.

So, the given set is infinite.

Q. 5 G. State whether any given set is finite or infinite:

G = {x ϵ Z: x < 1].

Answer : Integers = -3, -2, -1, 0, 1, 2, 3, …

Integers less than 1 (x < 1) = …-4, -3, -2, -1, 0

There are infinite integers which are less than 1.

∴ the given set is infinite.

Q. 5 H. State whether any given set is finite or infinite:

H = {x ϵ Z: –15 < x < 15].

Answer : Integers = -3, -2, -1, 0, 1, 2, 3, …

The integers lies between -15 and 15 are finite.

∴ the given set is finite.

Q. 5 I. State whether any given set is finite or infinite:

J = {x : x ϵ N and x is prime}.

Answer : The given set is the set of all prime numbers and since the set of prime numbers is infinite. Hence, the given set is infinite.

Q. 5 J. State whether any given set is finite or infinite:

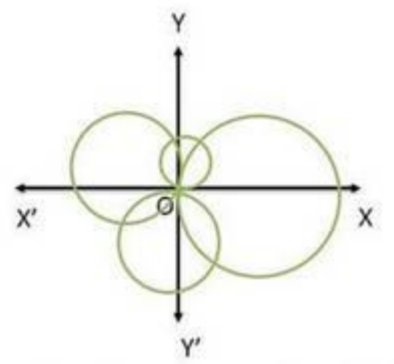
K = {x : x ϵ N and x is prime}.

Answer : The given set is the set of all prime numbers and since the set of prime numbers is infinite. Hence, the given set is infinite.

Q. 5 K. State whether any given set is finite or infinite:

L = set of all circles passing through the origin (0, 0)

Answer :



Infinitember of circles can pass through the origin, so the set will have infinite elements.

So, the given set is infinite

Q. 6. Rewrite the following statements using the set notation:

(i) a is an element of set A.

(ii) b is not an element of A.

(iii) A is an empty set and B is a nonempty set.

(iv) A number of elements in A is 6.

(v) 0 is a whole number but not a natural number.

Answer : (i) Given: a is an element of set A

this means $a \in A$

(ii) Given: b is not an element of A

this means $b \notin A$

(iii) Given: A is an empty set

and B is a non – empty set

this means $A = \phi$

and $B \neq \phi$

(iv) Given: In set A, total number of elements is 6

This means, $|A| = 6$

(v) Given: 0 is a whole number but not a natural number

This means $0 \in W$ but $0 \notin N$

Exercise 1C

Q. 1 A. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {0, 1, 2, 3,}, B = {1, 2, 3, 4, 5}

Answer : $A \not\subset B$

Explanation: $A \not\subset B$ since $0 \in A$ and $0 \notin B$.

Q. 1 B. State in each case whether $A \subset B$ or $A \not\subset B$.

A =ϕ, B = {0}

Answer : $A \subset B$

Explanation: A is a null set. Since, ϕ is a subset of every set therefore $A \subset B$.

Q. 1 C. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {1, 2, 3,}, B = {1, 2, 4}

Answer : $A \not\subset B$

Explanation: $A \not\subset B$ since $3 \in A$ and $3 \notin B$.

Q. 1 D. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : $x \in Z$, $x^2 = 1$}, B = {x : $x \in N$, $x^2 = 1$}

Answer : $A \not\subset B$

Explanation: we have, A = { -1,1} and B={1}

Since, $-1 \in A$ and $-1 \notin B$ thus $A \not\subset B$.

Q. 1 E. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is an even natural number,}, B = {x : x is an integer}

Answer : $A \subset B$

Explanation: we have, A = {2,4,6,8,...} and B = {. . ., –3, –2, –1, 0, 1, 2, 3, . . .}. since, even natural numbers are also integers, we observe that elements of A belongs to B.

Thus , $A \subset B$.

Q. 1 F. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is an integer}, B = {x: x is a rational number}

Answer : A = {….-2, -1, 0, 1, 2, 3….}

B = {-∞, ……..0, ……∞ }

$A \subset B$ as integers are contained in rational numbers.

Q. 1 G State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is a real number,}, B = {x : x is a complex number}

Answer : $A \subset B$

Explanation: we have, A = set of real numbers and B = set of complex numbers, a combination of the real and imaginary number in the form of a+ib, where a and b are real, and i is imaginary.

Since, any real number can be expressed as complex number, $A \subset B$.

Q. 1 H. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is an isosceles triangle in the plane}, B = {x : x is an equilateral triangle in the same plane}

Answer : $A \not\subset B$

Explanation: since all isosceles triangles are not equilateral triangles. Therefore set of isosceles triangle is not contained in the set of equilateral triangle.

Q. 1 I. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is a square in a plane}, B = {x : x is a rectangle in the same plane}

Answer : $A \subset B$

Explanation: Set of squares is a subset of set of rectangles since all squares are rectangles.

Q. 1 J. State in each case whether $A \subset B$ or $A \not\subset B$.

A = {x : x is a triangle in a plane}, B = {x : x is a rectangle in the same plane}

Answer : $A \not\subset B$

We have, A = set of triangles and B = set of rectangles.

Now, we can see elements of A does not belong to B since the set of rectangles does not include a set of triangles.

Q. 1 K. State in each case whether A ⊂ B or A ⊄ B.

A = {x : x is an even natural number less than 8}, B = {x : x is a natural number which divides 32}

Answer : A ⊄ B

Explanation: we have, A = {2,4,6} and B = {1,2,4,8,16,32}.

Thus , A ⊄ B , since 6 ∉ B.

Q. 2. Examine whether the following statements are true of false:

(i) {a, b} ⊄{b, c, a}

(ii) {a}ϵ {a, b, c}

(iii) ϕ ⊂{a, b, c}

(iv) {a, e} ⊂{x : x is a vowel in the English alphabet}

(v) {x : x ϵ W, x + 5 = 5} =ϕ

(vi) a ϵ {{a}, b}

(vii) {a} ⊂ {{a}, b}

(viii) {b, c} ⊂{a, {b, c}}

(ix) {a, a, b, b} = {a, b}

(x) {a, b, a, b, a, b, ….} is an infinite set.

(xi) If A = set of all circles of unit radius in a plane and B = set of all circles in the same plane then A⊂B.

Answer : (i) False

Explanation: Since elements of {a,b} are also elements of {b,c,a} hence {a, b}⊂{b, c, a}.

(ii) False

{a} is not in {a,b,c}. Hence, {a} ∉ {a, b, c}.

(iii) True

Explanation: ϕ is a subset of every set.

(iv) True

Explanation: a, e are vowels of English alphabet.

(v) False

Explanation: 0+5 = 5 , 0 ϵ W

Hence, {0} ≠ ϕ

(vi) False

Explanation: a is not an element of {{a}, b}

(vii) False

As a is not an element of set {{a}, b}

(viii) False

Explanation: {b,c} is an element of {a, {b, c}} and element cannot be subset of set.

(ix) True

Explanation: In a set all the elements are taken as distinct. Repetition of elements in a set do not change a set.

(x) False

Explanation: Given set is {a,b}, which is finite set. In a set all the elements are taken as distinct.Repetition of elements in a set do not change a set.

(xi) True

Explanation: Circle in a plane with unit radius is subset of circle in a plane of any radius.

Q. 3. If A = {1} and B = {{1}, 2} then show that A ⊄ B.

Hint 1 ϵ A but 1⊄ B.

Answer : There is only one element in set A.

Now, 1 is not an element of B.

Therefore, A ⊄ B.

Q. 4. Write down all subsets of each of the following sets:

(i) A = {A}

(ii) B = {a, b}

(iii) C = {–2, 3}

(iv) D = {–1, 0, 1}

(v) E =ϕ

(vi) F = {2, {3}}

(vii) G = {3, 4, {5, 6}}

Answer : (i) The subsets of {A} are ϕ and {A}

(ii) The subsets of {a, b} are ϕ, {a}, {b}, and {a, b}.

(iii) The subsets of {–2, 3} are ϕ, {-2}, {3}, and {-2, 3}.

(iv) The subsets of {–1,0,1} are ϕ ,{-1},{0},{1},{-1,0},{0,1},{-1,1},{–1, 0, 1}

(v)ϕ has only one subset ϕ.

(vi) let x = {3}

Then, F = {2,x}

The subsets of {2,x} are ϕ , {2},{x},{2,x}

i.e ϕ , {2},{{3}},{2,{3}}

(vii) let x = {5,6}

Then, G = {3,4,x}

The subsets of {3,4,x } are ϕ ,{3},{4},{x},{3,x},{4,x},{3,4}, {3,4,x} i.e. ϕ ,{3},{4},{{5,6}},{3,{5,6}},{4,{5,6}},{3,4},{3,4,{5,6}}

Q. 5. Express each of the following sets as an interval:

(i) A = {x : x ϵR, –4 < x < 0}

(ii) B = {x : x ϵ R, 0 ≤ x < 3}

(iii) C = {x : x ϵ R, 2 < x ≤ 6}

(iv) D = {x : x ϵ R, –5 ≤ x ≤ 2}

(v) E = {x : x ϵ R, –3 ≤ x < 2}

(vi) F = {x : x ϵ R, –2 ≤ x < 0}

Answer : (i) A = (-4,0)

Explanation: All the points between -4 and 0 belong to the open interval (-4,0) but -4 ,0 themselves do not belong to this interval.

(ii) B = [0,3)

Explanation: B = {x : x ϵ R, 0 ≤ x < 3} is an open interval from 0 to 3, including 0 but excluding 3.

(iii) C = (2,6]

Explanation: C = {x : x ϵ R, 2 < x ≤ 6} is an open interval from 2 to 6, including 6 but excluding 2.

(iv) D = [-5,2]

Explanation: D = {x : x ϵ R, –5 ≤ x ≤ 2} is a closed interval from -5 to 2 and contains the end points.

(v) E = [-3,2)

Explanation: E = {x : x ϵ R, –3 ≤ x < 2} is an open interval from -3 to 2, including -3 but excluding 2.

(vi) F = [-2,0)

Explanation: F = {x : x ϵ R, –2 ≤ x < 0} is an open interval from -2 to 0, including -2 but excluding 0.

Q. 6. Write each of the following intervals in the set-builder from:

(i) A = (–2, 3)

(ii) B = [4, 10]

(iii) C = [–1, 8)

(iv) D = (4, 9]

(v) E = [–10, 0)

(vi) F = (0, 5]

Answer : (i) $A = \{x : x \in R, -2 < x < 3\}$

(ii) $B = \{x : x \in R, 4 \leq x \leq 10\}$

(iii) $C = \{x : x \in R, -1 \leq x < 8\}$

(iv) $D = \{x : x \in R, 4 < x \leq 9\}$

(v) $E = \{x : x \in R, -10 \leq x < 0\}$

(vi) $F = \{x : x \in R, 0 < x \leq 5\}$

Q. 7. if A = {3, {4, 5}, 6} find which of the following statements are true.

(i) $\{4, 5\} \not\subset A$

(ii) $\{4, 5\} \in A$

(iii) $\{\{4, 5\}\} \subseteq A$

(iv) $4 \in A$

(v) $\{3\} \subseteq A$

(vi) $\{\phi\} \subseteq A$

(vii) $\phi \subseteq A$

(viii) $\{3, 4, 5\} \subseteq A$

(ix) $\{3, 6\} \subseteq A$

Answer : (i) True

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

4,5 is not in A, {4,5} is an element of A and element cannot be subset of set,thus {4, 5} $\not\subset$A.

(ii) True

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

Now, x is in A.

So, x $\in$ A.

Thus, {4, 5} $\in$ A

(iii) True

Explanation: {4,5} is an element of set {{4,5}}.

Let {4,5} = x

{{4,5}} = {x}

we have, A = {3, {4, 5}, 6}

Now, A = {3, x, 6}

So, x is in {x} and x is also in A.

So , {x} is a subset of A.

Hence, {{4, 5}} $\subseteq$A

(iv) False

Explanation: 4 is not an element of A.

(v) True

Explanation: 3 is in {3} and also 3 is in A.

(vi) False

Explanation: ϕ is an element in { ϕ} but not in A.

Thus, {ϕ} ⊄ A

(vii) True

Explanation: ϕ is a subset of every set.

(viii) False

Explanation: we have, A = {3, {4, 5}, 6}

Let {4,5} = x

Now, A = {3, x, 6}

4,5 is in {3,4,5} but not in A, thus {3,4, 5} ⊄A.

(ix) True

Explanation: 3,6 is in {3,6} and also in A, thus {3, 6} ⊆A.

Q. 8. If A = {a, b, c}, find P(A) and n{P(A)}.

Answer : The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).

Now, We know that ϕ is a subset of every set. So, ϕ is a subset of {a, b, c}.

Also, {a},{b},{c},{a,b},{b,c},{a,c} are also subsets of {a, b, c}

We know that every set is a subset of itself. So, {a, b, c} is a subset of {a, b, c}.

Thus, the set {a, b, c} has, in all eight subsets, viz. ϕ , {a},{b},{c},{a,b},{b,c},{a,c},{a, b, c}.

∴ P(A) = { ϕ , {a},{b},{c},{a,b},{b,c},{a,c},{a, b, c}}

Now, n{P(A)} = 2^m , where m = n(A) = 3

⇒ $n\{P(A)\} = 2^3 = 8$

Q. 9. If A = {1, {2, 3}}, find P(A) and n {P(A)}.

Answer : Let {2,3} = x

Now, A = {1,x}

Subsets of A are ϕ , {1} , {x} , {1,x}

⇒ Subsets of A are ϕ , {1} , {2,3} , {1, {2,3}}

Now, $n\{P(A)\} = 2^m$, where m = n(A) = 2

⇒ $n\{P(A)\} = 2^2 = 4$

Q. 10. If A = ϕ then find n{P(A)}.

Answer : We have, A = ϕ , i.e. A is a null set.

Then, n(A)= 0

∴ $n\{P(A)\} = 2^m$, where m = n(A)

⇒ $n\{P(A)\} = 2^0 = 1$.

Thus, P(A) has one element.

Q. 11. If A = {1, 3, 5}, B = {2, 4, 6} and C = {0, 2, 4, 8} then find the universal set.

Answer : Elements of A+B+C = {1,3,5,2,4,6,0,8}

Thus, the universal set for A,B and C = {0,1,2,3,4,5,6,8}

Q. 12. Prove that A ⊆ B, B⊆ C and C⊆ A ⇒A = C.

Answer : We have A ⊆B , B⊆ C and C⊆ A

Now , A is a subset of B and B is a subset of C, So A is a subset of C.

Given that C⊆ A.

Hence, A = C.

Q. 13. For any set A, prove that A⊆ϕ ⇔ A =ϕ

Answer : Let $A \subseteq \phi$

A is a subset of the null set , then A is also an empty set.

$\Rightarrow A = \phi$

Now, let $A = \phi$

$\Rightarrow$ A is an empty set.

Since, every set is a subset of itself.

$\Rightarrow A \subseteq \phi$

Hence, for any set A, $A \subseteq \phi \Leftrightarrow A = \phi$

Q. 14. State whether the given statement is true false:

(i) If $A \subset B$ and $x \notin B$ than $x \notin A$.

(ii) If $A \subseteq \phi$ then $A = \phi$

(iii) If A, B and C are three sets such than $A \in B$ and $B \subset C$ then $A \subset C$.

(iv) If A, B and C are three sets such than $A \subset B$ and $B \in C$ then $A \in C$.

(v) If A, B and C are three sets such that $A \not\subset B$ and $B \not\subset C$ then $A \not\subset C$.

(vi) If A and B are sets such that x A and $A \in B$ then $x \in B$.

Answer : (i) True

Explanation: We have $A \subset B$ since A is a subset of B then all elements of A should be in B.

Let A = {1,2} and B = {1,2,3}

Let $x = 4 \notin B$

Also we observe that $4 \notin A$.

Hence, If $A \subset B$ and $x \notin B$ than $x \notin A$.

(ii) True

Explanation: We have, $A \subseteq \phi$

Now, A is a subset of null set , this implies A is also an empty set.

⇒ A =ϕ

(iii) False

Explanation: Let A = {a}, B = {{a}, b}

here , A ϵ B

Now, let C = {{a}, b, c}.

Since, {a},b is in B and also in C thus, B ⊂ C.

But, A ={a} and {a} is an element of C, since the element of a set cannot be a subset of a set.

Hence,A ⊄ C.

(iv) False

Explanation: Let A = {a},B = {a, b} and C = {{a, b}, c}.

Then,A⊂ B and B ϵC. But, A ∉ C since {a} is not an element of C.

(v) False.

Explanation: Let A = {a}, B = {b, c} and C = {a, c}.

Since a ∈ A and a ∉ B.Then, A ⊄ B

Now, b ∈ B and b ∉ C ⇒ B ⊄C.

But, A ⊂ C since, a ∈ A and a ∈ C.

(vi) False.

Explanation: Let A = {x}, B = {{x}, y}

Now, x ϵ A and {x} is an element of B ⇒ A ϵ B

But, x is not an element of B. Thus, x∉B.

Exercise 1D

Q. 1. If A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}, find:

(i) A ∪ B

(ii) B ∪ C

(iii) B ∪ C

(iv) C ∩ A

(vi) A ∩ B

Answer : Given; A = {a, b, c, d, e, f}, B = {c, e, g, h} and C = {a, e, m, n}

(i) A ∪ B = {a, b, c, d, e, f, g, h}

(ii) B ∪ C = {a, c, e, g, h, m, n}

(iii) B ∪ C = {a, c, e, g, h, m, n}

(iv) C ∩ A = {a, e}

(vi) A ∩ B = {c, e}

Q. 2. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8} and C = {10, 11, 12, 13, 14}, find:

(i) A ∪ B

(ii) B ∪ C

(iii) A ∪ C

(iv) B ∪ D

(v) (A ∪ B) ∪ C

(vi) (A ∪B) ∩C

(vii) (A ∩ B) ∪D

(viii) (A ∩ B) ∪ (B ∩ C)

(ix) (A ∩ C) ∩ (C ∪ D

Answer : Given; A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8} and C = {10, 11, 12, 13, 14}

(i) A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8}

(ii) B ∪ C = {4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(iii) A ∪ C = {1, 2, 3, 4, 5, 10, 11, 12, 13, 14}

(iv) B ∪ D

(v) (A ∪ B) ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(vi) (A ∪B) ∩C = Φ or {}

(vii) (A ∩ B) ∪D =

(viii) (A ∩ B) ∪ (B ∩ C) = Φ or {}

(ix) (A ∩ C) ∩ (C ∪ D =

Q. 3. If A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13} and C = {11, 13, 15}, and D = {15, 17}, find:

(i) A ∩ B

(ii) A ∩ C

(iii) B ∩ C

(iv) B ∩ D

(v) B ∩ (C ∪ D)

(vi) A ∩ (B ∪ C)

Answer : Given; A = {3, 5, 7, 9, 11}, B = {7, 9, 11, 13} and C = {11, 13, 15}, and D = {15, 17}

(i) A ∩ B = {7, 9, 11}

(ii) A ∩ C = {11}

(iii) B ∩ C = {11, 13}

(iv) B ∩ D = Φ or {}

(v) B ∩ (C ∪ D) = {11, 13}

(vi) A ∩ (B ∪ C) = {7, 9, 11}

Q. 4. If A = {x : x ϵ N}, B = {x : x ϵ N and x is even), C = {x : x ϵ N and x is odd} and D = {x : x ϵ N and x is prime} then find:

(i) A ∩ B

(ii) A ∩ C

(iii) A ∩ D

(iv) B ∩ C

(v) B ∩ D

(vi) C ∩ D

Answer : Given; A = {x : x ϵ N}, B = {x : x ϵ N and x is even), C = {x : x ϵ N and x is odd} and D = {x : x ϵ N and x is prime}

(i) A ∩ B = {x : x ϵ N and x is even}

(ii) A ∩ C = {x : x ϵ N and x is odd}

(iii) A ∩ D = {x : x ϵ N and x is prime}

(iv) B ∩ C = Φ or {}

(v) B ∩ D = {2} [∵ 2 is the only even prime number]

(vi) C ∩ D = {x : x ϵ N and x is prime and x ≠ 2}

Q. 5. If A = {2x : x ϵ N}, 1 ≤ x < 4}, B = {x + 2) : x ϵ N and 2 ≤ x < 5} and C = {x : x ϵ N and 4 < x < 8}, find:

(i) A ∩ B

(ii) A ∪ B

(iii) (A ∪ B) ∩ C

Answer : Given; A = {2x : x ϵ N}, 1 ≤ x < 4}, B = {x + 2) : x ϵ N and 2 ≤ x < 5} and C = {x : x ϵ N and 4 < x < 8}

According to the given conditions; A = {2, 4, 6}, B = {4, 5, 6} and C = {5, 6, 7}

(i) A ∩ B = {4, 6}

(ii) A ∪ B = {2, 4, 5, 6}

(iii) (A ∪ B) ∩ C = {5, 6}

Q. 6. If A = {2, 4, 6, 8, 10, 12}, B = {3, 4, 5, 6, 7, 8, 10}, find:

(i) (A – B)

(ii) (B – A)

(iii) (A – B) ∪ (B – A)

Answer : Given; A = {2, 4, 6, 8, 10, 12}, B = {3, 4, 5, 6, 7, 8, 10}

(i) (A – B) = {2, 12}

(ii) (B – A) = {5, 7}

(iii) (A – B) ∪ (B – A) = Φ or {}

Q. 7. If A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}, find:

(i) A ∩ (B – C)

(ii) A – (B ∪ C)

(iii) A – (B ∩ C)

Answer : Given; A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}

(i) A ∩ (B – C) = {a, c}

(ii) A – (B ∪ C) = {d}

(iii) A – (B ∩ C) = {a, b, c, d}

Q. 8. If $A = \left\{\frac{1}{x} : x \in N\right\}$ **and x < 8}, and B =** $\left\{\frac{1}{2x} : x \in N \text{ and } x \le 4\right\}$**, find :**

(i) A ∪ B
(ii) A ∩ B
(iii) A – B
(vi) B – A

Answer : Given; $A = \left\{\frac{1}{x} : x \epsilon N\right\}$ and x < 8 and $B = \left\{\frac{1}{2x} : x \epsilon N\right\}$ and x ≤ 4

According to the given conditions;

$$A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}\right\} \ and \ B = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}\right\}$$

(i) A ∪ B =

$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}\right\}$

(ii) A ∩ B =

$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}\right\}$

(iii) A – B =

$\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}\right\}$

(vi) B – A =

$\left\{\frac{1}{8}\right\}$

Q. 9. If R is the set of all real numbers and Q is the set of all rational numbers then what is the set (R – Q)?

Answer : Given; R is the set of all real numbers and Q is the set of all rational numbers.

Then (R – Q) is the set of all irrational numbers.

Q. 10. If A = {2, 3, 5, 7, 11} and B = ϕ, find:

(i) A ∪ B

(ii) A ∩ B

Answer : Given; A = {2, 3, 5, 7, 11} and B = ϕ

(i) A ∪ B = {2, 3, 5, 7, 11}

(ii) A ∩ B = ϕ

Q. 11. If A and B are two sets such that A ⊆ B then find:

(i) A ∪ B

(ii) A ∩ B

Answer : Given; A and B are two sets such that A ⊆ B.

(i) A ∪ B = A

(ii) A ∩ B = B

Q. 12. Which of the following sets are pairs of disjoint sets? Justify your answer.

(i) A = {3, 4, 5, 6} and B = {2, 5, 7, 9}

(ii) C = {1, 2, 3, 4, 5} and D = {6, 7, 9, 11}

(iii) E = {x : x ϵ N, x is even and x < 8}

F = {x : x = 3n, n ϵ N, and x < 4}

(vi) G = {x : x ϵ N, x is even} and H {x : x ϵ N, x is prime}

(v) J = {x : x ϵ N, x is even} and K = {x : x ϵ N, x is odd}

Answer : Disjoint sets have their intersections as Φ.

(i) A = {3, 4, 5, 6} and B = {2, 5, 7, 9} Are pairs of disjoint sets.

(ii) C = {1, 2, 3, 4, 5} and D = {6, 7, 9, 11} Are pairs of disjoint sets.

(iii) E = {x : x ϵ N, x is even and x < 8} = {2, 4, 6} and

F = {x : x = 3n, n ϵ N, and x < 4} = {3, 6, 9} Are not pairs of disjoint sets.

(iv) G = {x : x ϵ N, x is even} and H {x : x ϵ N, x is prime}

∵ 2 is an even prime number; their intersection is not Φ

Are not pairs of disjoint sets.

(v) J = {x : x ϵ N, x is even} and K = {x : x ϵ N, x is odd}

∵ there is no number which is both odd and even.

∴ J and K are pairs of disjoint sets.

Q. 13. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A A = {1, 2, 3, 4,}, B = {2, 4, 6, 8} and = {1, 4, 5, 6}, find:

(i) A’

(ii) B’

(iii) C’

(iv) (B’)’

(v) (A ∪ B)’

(vi) (A ∩ C)’

(vii) (B – C)’

Answer : Given; U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2, 3, 4,}, B = {2, 4, 6, 8} and C = {1, 4, 5, 6}

(i) A’ = {5, 6, 7, 8, 9}

(ii) B’ = {1, 3, 5, 7, 9}

(iii) C’ = {2, 3, 7, 8, 9}

(iv) (B’)’ = {2, 4, 6, 8}

(v) (A ∪ B)’ = {5, 7, 9}

(vi) (A ∩ C)’ = {2, 3, 5, 6, 7, 8, 9}

(vi) (B – C)’ = {1, 3, 4, 5, 6, 7, 9}

Q. 14. if U = {a, b, c} and A = {a, c, d, e} then verify that:

(i) (A ∪ B)’ = (A’ ∩ B’)

(ii) (A ∩ B)’ = (A’ ∪ B’)

Answer : Given; U = {a, b, c} and B = {a, c, d, e}

(i) (A ∪ B)’ = (A’ ∩ B’)

(ii) (A ∩ B)’ = (A’ ∪ B’)

Q.15. if U is the universal set and A U then fill in the blanks.

(i) A ∪ A’ = ….

(ii) A ∩ A’ = ….

(iii) ϕ ‘∩ A = ….

(iv) U’ ∩ A = ….

Answer : Given; U is the universal set and A ⊂ U

(i) A ∪ A’ = U

(ii) A ∩ A’ = Φ or {}

(iii) ϕ ‘∩ A = Φ

(iv) U’ ∩ A = Φ or {}

Exercise 1E

Q. 1. If A = {a, b, c, d, e}, B = {a, c, e, g}, verify that:

(i) A ∪ B = B ∪ A
(ii) A ∪ C = C ∪ A
(iii) B ∪ C = C ∪ B
(iv) A ∩ B = B ∩ A
(v) B ∩ C = C ∩ B
(vi) A ∩ C = C ∩ A
(vii) (A ∪ B ∪ C = A ∪ (B ∪ C)
(viii) (A ∩ B) ∩ C = A ∩ (B ∩ C)

Answer : (i) LHS = A ∪ B

= {a, b, c, d, e}∪ {a, c, e, g}

= { a, b, c, d, e, g}

= {a, c, e, g}∪{a, b, c, d, e}

= B ∪ A

= RHS

Hence proved.

(ii) To prove: A ∪ C = C ∪ A

Since the element of set C is not provided,

let x be any element of C.

LHS = A ∪ C

= {a, b, c, d, e} ∪ { x |x$^{\in}$C}

= { a, b, c, d, e, x}

= {x, a, b, c, d, e }

= { x |x$^{\in}$C} ∪ { a, b, c, d, e}

= C ∪ A

= RHS

Hence proved.

(iii) To prove: $B \cup C = C \cup B$

Since the element of set C is not provided,

let x be any element of C.

LHS = $B \cup C$

$= \{ a, c, e, g \} \cup \{ x \mid x \in C\}$

$= \{ a, c, e, g, x\}$

$= \{x, a, c, e, g \}$

$= \{ x \mid x \in C\} \cup \{ a, c, e, g \}$

$= C \cup B$

= RHS

Hence proved.

(iv) LHS = $A \cap B$

$= \{a, b, c, d, e\} \cup \{a, c, e, g\}$

$= \{a, c, e\}$

RHS = $B \cap A$

$= \{a, c, e, g\} \cap \{a, b, c, d, e\}$

$= \{a, c, e\}$

$\therefore A \cap B = B \cap A$

(v) Let x be an element of $B \cap C$

$\Rightarrow x \in B \cap C$

$\Rightarrow x \in B$ and $x \in C$

$\Rightarrow x \in C$ and $x \in B$ [by definition of intersection]

$\Rightarrow x \in C \cap B$

$\Rightarrow B \cap C \subset C \cap B$ ….(i)

Now let x be an element of $C \cap B$

Then, $x \in C \cap B$

$\Rightarrow x \in C$ and $x \in B$

$\Rightarrow x \in B$ and $x \in C$ [by definition of intersection]

$\Rightarrow x \in B \cap C$

$\Rightarrow C \cap B \subset B \cap C$ ….(ii)

From (i) and (ii) we have,

$B \cap C = C \cap B$ [every set is a subset of itself]

Hence proved.

(vi) Let x be an element of $A \cap C$

$\Rightarrow x \in A \cap C$

$\Rightarrow x \in A$ and $x \in C$

$\Rightarrow x \in C$ and $x \in A$ [by definition of intersection]

$\Rightarrow x \in C \cap A$

$\Rightarrow A \cap C \subset C \cap A$ ….(i)

Now let x be an element of $C \cap A$

Then, $x \in C \cap A$

$\Rightarrow x \in C$ and $x \in A$

$\Rightarrow x \in A$ and $x \in C$ [by definition of intersection]

$\Rightarrow x \in A \cap C$

$\Rightarrow C \cap A \subset A \cap C$ ….(ii)

From (i) and (ii) we have,

$A \cap C = C \cap A$ [every set is a subset of itself]

Hence proved.

(vii) Let x be any element of $(A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$

$\Rightarrow (A \cup B) \cup C \subset A \cup (B \cup C)$ …..(i)

Now, let x be an element of $A \cup (B \cup C)$

Then, $x \in A$ or $(B \cup C)$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in (A \cup B) \cup C$

$\Rightarrow A \cup (B \cup C) \subset (A \cup B) \cup C$ …..(ii)

From i and ii, $(A \cup B) \cup C = A \cup (B \cup C)$

[every set is a subset of itself]

Hence , proved.

(viii) Let x be any element of $(A \cap B) \cap C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow x \in A$ and $x \in (B \cap C)$

$\Rightarrow x \in A \cap (B \cap C)$

$\Rightarrow (A \cap B) \cap C \subset A \cap (B \cap C)$ …..(i)

Now, let x be an element of $A \cap (B \cap C)$

Then, $x \in A$ and $(B \cap C)$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow x \in (A \cap B)$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

$\Rightarrow A \cap (B \cap C) \subset (A \cap B) \cap C$ …..(ii)

From i and ii, $(A \cap B) \cap C = A \cap (B \cap C)$

[every set is a subset of itself]

Hence, proved.

Q. 2. If A = {a, b, c, d, e}, B = {a, c, e, g}, and C = {b, e, f, g} verify that:

(i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

Answer : (i) B - C represents all elements in B that are not in C

B - C = {a, c}

$A \cap (B - C) = \{a, c\}$

$A \cap B = \{a, c, e\}$

$A \cap C = \{b, e\}$

$(A \cap B) - (A \cap C) = \{a, c\}$

$\Rightarrow A \cap (B - C) = (A \cap B) - (A \cap C)$

Hence proved

(ii) $B \cap C = \{e, g\}$

$A - (B \cap C) = \{a, b, c, d\}$

$(A - B) = \{b, d\}$

$(A - C) = \{a, c, d\}$

$(A - B) \cup (A - C) = \{a, b, c, d\}$

$\Rightarrow A - (B \cap C) = (A - B) \cup (A - C)$

Hence proved

Q. 3. If $A = \{x : x \in N, x \leq 7\}$, $B = \{x : x \text{ is prime}, x < 8\}$ and $C = \{x : x \in N, x \text{ is odd and } x < 10\}$, verify that:

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Answer : Natural numbers start from 1

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{1, 3, 5, 7, 9\}$

(i) $B \cap C = \{3, 5, 7\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup C = \{1, 2, 3, 4, 5, 6, 7, 9\}$

$(A \cup B) \cap (A \cup C) = \{1, 23, 4, 5, 6, 7\}$

$\Rightarrow A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved

(ii) $B \cup C = \{1, 2, 3, 5, 7, 9\}$

$A \cap (B \cup C) = \{1, 2, 3, 5, 7\}$

$A \cap B = \{2, 3, 5, 7\}$

$A \cap C = \{1, 3, 5, 7\}$

$(A \cap B) \cup (A \cap C) = \{1, 2, 3, 5, 7\}$

$\Rightarrow A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Q. 4. If U = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {2, 4, 6, 8}, and = {2, 3, 5, 7} verify that:

(i) $(A \cup B)' = (A' \cap B')$

(ii) $(A \cap C)' = (A' \cup B')$

Answer : (i) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{1, 9\}$

$A' = \{1, 3, 5, 7, 9\}$

$B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$

$\Rightarrow (A \cup B)' = A' \cap B'$

Hence proved

(ii) $A \cap B = \{2\}$

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' \cup B'$ = {1, 3, 4, 5, 6, 7, 8, 9}

$\Rightarrow (A \cap B)' = A' \cup B'$

Hence proved

These are also known as De Morgan's theorem

Q. 5. Let A = {a, b, c}, B = {b, c, d, e} and = {c, d, e, f} be subsets of U = {a, b, c, d, e, f}. Then verify that:

(i) (A')' = A
(ii) (A ∪ B)' = (A' ∩ B')
(iii) (A ∩ B)' = (A' ∪ B')

Answer : (i) A' = {d, e, f}

(A')' = {a, b, c} = A

Hence proved

(ii) $A \cup B$ = {a, b, c, d, e}

$(A \cup B)'$ = {f}

A' = {d, e, f}

B' = {a, f}

$A' \cap B'$ = {f}

$\Rightarrow (A \cup B)' = (A' \cap B')$

Hence proved

(iii) $A' \cup B'$ = {a, d, e, f}

$A \cap B$ = (b, c}

$(A \cap B)'$ = {a, d, e, f}

$\Rightarrow (A \cap B)' = A' \cup B'$

Hence proved

Q. 6. Given an example of three sets A, B, C such that A ∩ C ≠ ϕ, B ∩ C ≠ ϕ, A ∩ C ≠ ϕ, and A ∩ B ∩ C = ϕ

Answer : Let A = {1, 2}

B = {2, 3}

C = {1, 3, 4}

$A \cap B = \{2\}$

$A \cap C = \{1\}$

$B \cap C = \{3\}$

$A \cap B \cap C = \{2\} \cap \{1, 3, 4\} = ø$

So the three sets are valid and satisfy the given conditions

Q. 7. For any sets A and B, prove that:

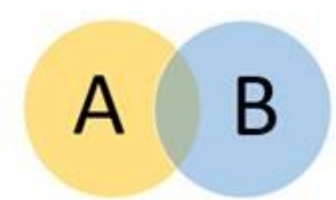
(i) (A – B) ∩ B = ϕ
(ii) A ∪ (B – A) = A ∪ B
(iii) (A – B) ∪ (A ∩ B) = A
(iv) (A ∪ B) – B = A – B
(iv) A – (A ∪ B) = A – B

Answer : Two sets are shown with the following Venn Diagram

The yellow region is denoted by 1.

Blue region is denoted by 2.

The common region is denoted by 3.



(i) A - B denotes region 1

B denotes region (2+3)

So their intersection is a null set

$\Rightarrow (A - B) \cap B = ø$

(ii) B - A denotes region 2

A denotes region (1+3)

So their union denotes region (1+2+3) which is the union of A and B

$\Rightarrow A \cup (B - A) = A \cup B$

(iii) A - B denotes region 1

$A \cup B$ denotes region 3

Their union denotes region (1+3) which is set A$\Rightarrow (A - B) \cup (A \cap B) = A$

(iv) $A \cup B$ denotes region (1+2+3)

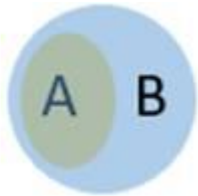
(AUB) - B denotes region (1+2+3) - (2+3) = 1A - B denotes region 1$\Rightarrow (A \cup B) - B = A - B$

(v) Wrong question

Q. 8. For any sets A and B, prove that:

(i) $A \cap B' = \phi \Rightarrow A \subset B$
(ii) $A' \cup B' = U \Rightarrow A \subset B$

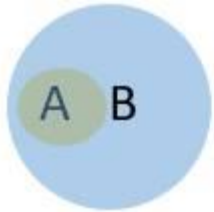
Answer : (i) The Venn Diagram for the given condition is given below



As can be seen from the Venn Diagram, A is a proper subset of B

$\Rightarrow A \subset B$

(ii) Wrong question. If A is a proper subset of B then $A' \cup B' \neq U$



Exercise 1F

Q. 1. Let A = {a, b, c, e, f} B = {c, d, e, g} and C = {b, c, f, g} be subsets of the set U = {a, b, c, d, e, f, g, h}.

(i) $A \cap B$
(ii) $A \cup (B \cap C)$
(iii) A – B
(iv) B – A
(v) $A - (B \cap C)$
(vi) $(B - C) \cup (C - B)$

Answer : (i) $A \cap B$ will contain the common elements of A and B

$A \cap B = \{c, e\}$

(ii) $AU(B \cap C)$

$B \cap C = \{c, d, g\}$

$AU(B \cap C) = \{a, b, c, d, e, f, g\}$

(iii) A - B implies the set of all elements in A that are not in B

A - B = {a, b, f}

(iv) B - A implies the set of all elements in B that are not in A

B - A = {d, g}

(v) $A - (B \cap C)$ denotes elements of A that are not in $B \cap C$

$A - (B \cap C) = \{a, b, e, f\}$

(vi) (B - C)U(C - B) implies the union of sets B - C and C – B

B - C = {d, e}

C - B = {b, f}

(B - C)U(C - B) = {b, d, e, f}

Q. 2. Let A = {2, 4, 6, 8, 10}, B = {4, 8, 12, 16} and C = {6, 12, 18, 24}.

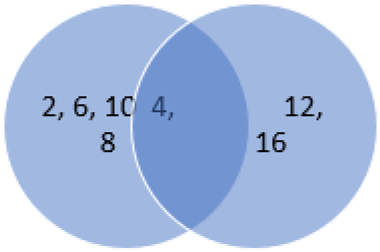
Using Venn diagrams, verify that:

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

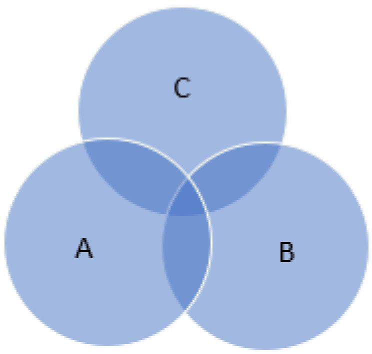
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$.

Answer : (i) LHS:

$A \cup B = \{2, 4, 6, 8, 10, 12, 16\}$ $(A \cup B) \cup C = \{2, 4, 6, 8, 10, 12, 16, 18, 24\}$



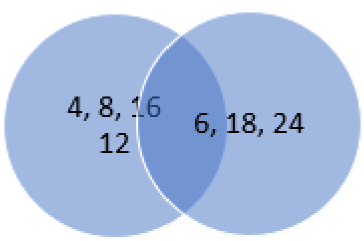
A



B

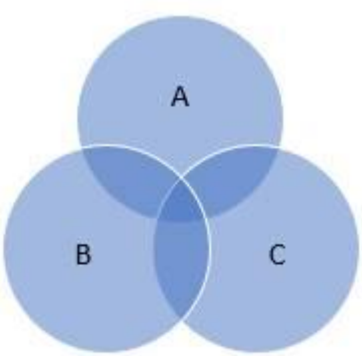
RHS:

$B \cup C$ = {4, 6, 8, 10, 12, 16, 18, 24}



B C

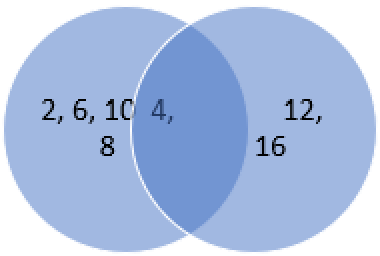
$A \cup (B \cup C)$ = {2, 4, 6, 8, 10, 12, 16, 18, 24}



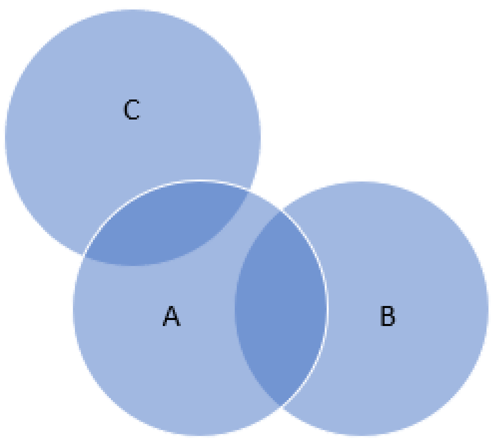
LHS = RHS. [Verified]

(ii) LHS:

$A \cap B = \{4, 8\}$ $(A \cap B) \cap C = \{\}$ or ø



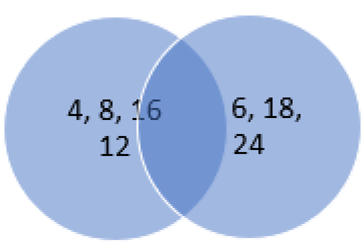
A



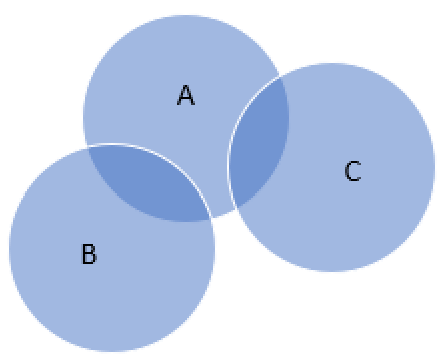
B

RHS:

$B \cap C = \{12\}$ $A \cap (B \cap C) = \{\}$



B



C

Q. 3. Let A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

Using Venn diagrams, verify the following

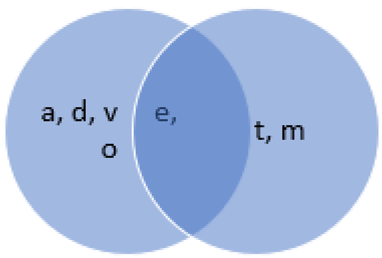
(i) A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

(ii) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C).

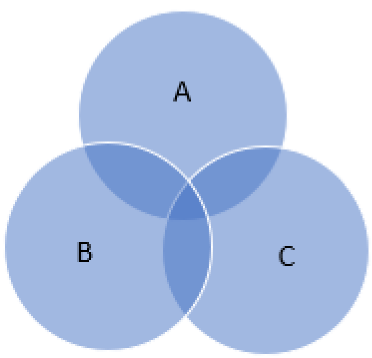
Answer : (i) Given:

A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

B∩C = {e, o} and A∪(B∩C) = {a, e, I, o, u}



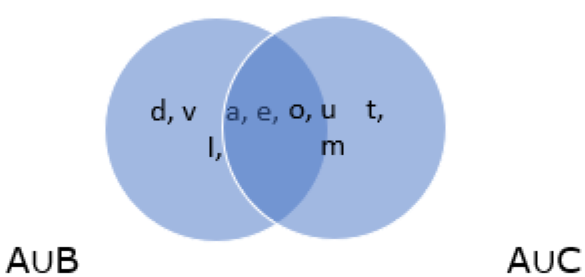
B



C

RHS:

A∪B = {a, d, e, I, o, u, v} and A∪C = {a, e, I, o, u, t, m}



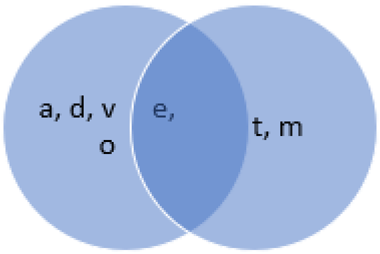
(A ∪ B) ∩ (A ∪ C) = {a, e, I, o, u}

LHS = RHS. [Verified].

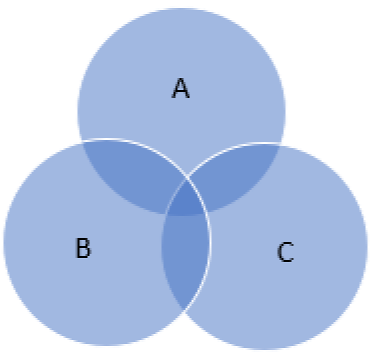
(ii) Given:

A = {a, e, I, o, u}, B = {a, d, e, o, v} and C = {e, o, t, m}.

$B \cup C$ = {a, d, v, e, o, t, m} and $A \cap (B \cup C)$ = {a, e, o}



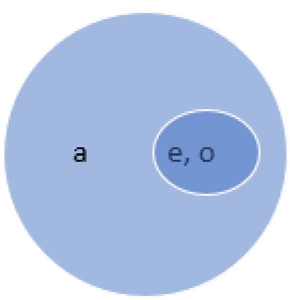
B



C

RHS:

$A \cap B$ = {a, e, o} and $A \cap C$ = {e, o}



A∩B A∩C

(A ∩ B) ∪ (A ∩ C) = {a, e, o}

LHS = RHS. [Verified]

Q. 4. Let A ⊂ B ⊂ U. Exhibit it in a Venn diagram.

Answer : Given: A⊂B⊂U.

Corresponding Venn diagram –

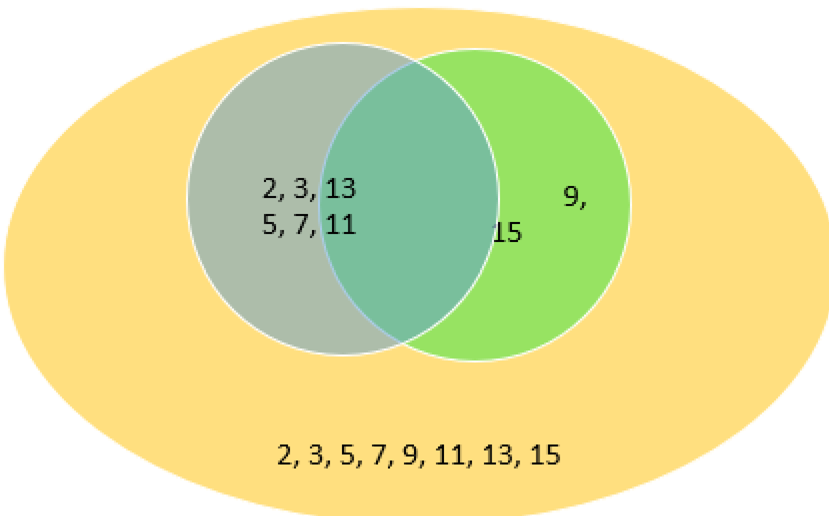
Q. 5. Let A = {2, 3, 5, 7, 11, 13}, B = {5, 7, 9, 11, 15} be subsets of U = {2, 3, 5, 7, 9, 11, 13, 15}.

Using Venn diagrams, verify that:

(i) (A ∪ B’) = (A’ ∩ B’)

(ii) (A ∩ B)’ = (A’ ∪ B’)

Answer :



(i) Here blue region denotes set A – B

The green region denotes set B – A

The overlapping region denotes A∩B, and the orange region denotes the universal set U.

From the Venn diagram we get (A∪B’) = {2, 3, 5, 7, 11, 13} (B’ is the set excluding those elements present in set B i.e. A - B region)

A’ = {9, 15} and B’ = {2, 3, 13}

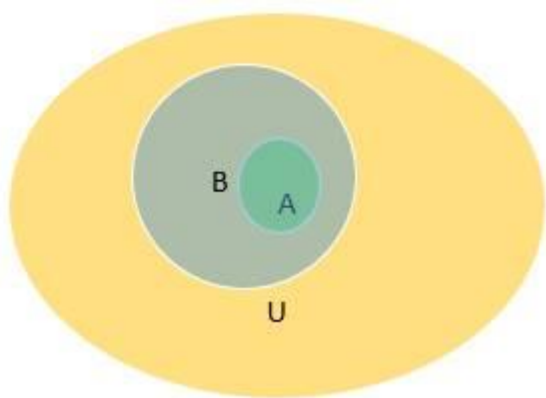
Therefore A’∩B’ = {}

Therefore (A ∪ B’) ≠ (A’ ∩ B’) [Verified]

(ii) From the Venn diagram we get (A ∩ B)' = {2, 3, 9, 13, 15} (elements except those present in A ∩ B)

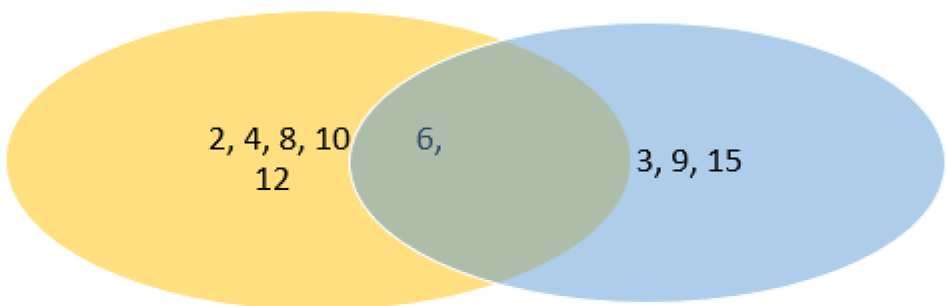
(A' ∪ B') = {2, 3, 9, 13, 15}

Therefore, (A ∩ B)' = (A' ∪ B') [Verified]



Q. 6. Using Venn diagrams, show that (A – B), A ∩ B) and (B – A) are disjoint sets, taking A = {2, 4, 6, 8, 10, 12} and B = {3, 6, 9, 12, 15, }.

Answer :



A - B is denoted by the yellow region only

B - A is denoted by the blue region only

AB is denoted by the common region (blue +yellow)

There is no intersection between these three regions

Hence the three sets are disjoint sets.

Exercise 1G

Q. 1. If A and B are two sets such that n(A) = 37, n(B) = 26 and n (A ∪ B) = 51, find n (A ∩ B).

Answer : Given:

$n(A) = 37$

$n(B) = 26$

$n(A \cup B) = 51$

To Find: $n(A \cap B)$

We know that,

$|A \cup B| = |A| + |B| - |A \cap B|$ (where A and B are two finite sets)

Therefore,

$n(A \cup B) = n(A) + n(B) – n(A \cap B)$

$51 = 37 + 26 – n(A \cap B)$

$n(A \cap B) = 63 – 51 = 12$

Therefore,

$n(A \cap B) = 12$

Q. 3. If A and B are two sets such that n(A) = 24, n(B) = 22 and n(A ∩ B) = 8, find:

(i) n(A ∪ B)

(ii) n(A – B)

(iii) n(B – A)

Answer : Given:

$n(A) = 24$, $n(B) =22$ and $n(A \cap B) = 8$

To Find:

(i) $n(A \cup B)$

$n(A \cup B) = n(A) + n(B) – n(A \cap B)$

$= 24 + 22 – 8$

$= 38$

Therefore,

$n(A \cup B) = 38$

(ii) $n(A - B)$

We know that,

$n(A - B) = n(A) – n(A \cap B)$

$= 24 – 8$

$= 16$

Therefore,

$n(A - B) = 16$

(iii) $n(B - A)$

We know that,

$n(B - A) = n(B) – n(A \cap B)$

$= 22 – 8$

$= 14$

Therefore,

$n(B - A) = 14$

Q. 4. If A and B are two sets such that $n(A –B) = 24$, $n(B – A) = 19$ and $n(A \cap B) = 11$, find:

(i) $n(A)$

(ii) $n(B)$

(iii) $n(A \cup B)$

Answer : Given:

$n(A - B) = 24$, $n(B - A) =19$ and $n(A \cap B) = 11$

To Find:

(i) $n(A)$

We know that,

$n(A) = n(A - B) + n(A \cap B)$

$= 24 + 11$

$= 35$

Therefore, $n(A) = 35$ …(1)

(ii) $n(B)$

We know that,

$n(B) = n(B - A) + n(A \cap B)$

= 19 + 11

= 30

Therefore,

n(B) = 30 …(2)

(iii) n(A ∪ B)

We know that,

n(A ∪ B) = n(A) + n(B) – n(A ∩ B) {From (1) & (2) n(A) =35

and n(B) =30}

= 35 + 30 - 11

= 54

Therefore,

n(A ∪ B) = 54

Q. 5. In a committee, 50 people speak Hindi, 20 speak English and 10 speak both Hindi and English. How many speak at least one of these two languages?

Answer : Given:

People who speak Hindi = 50

People who speak English = 20

People who speak both English and Hindi = 10

To Find: People who speak at least one of these two languages

Let us consider,

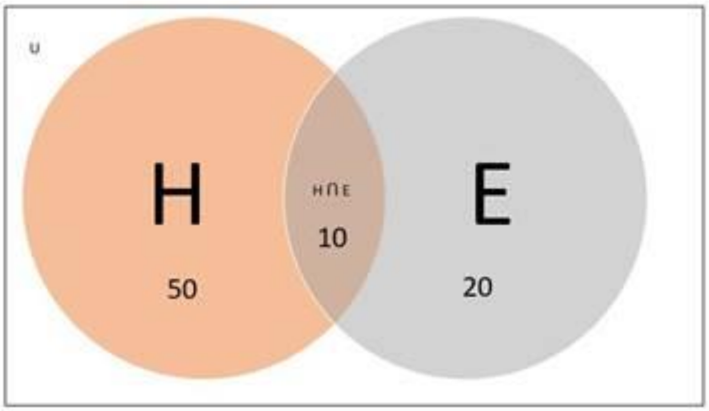
People who speak Hindi = n(H) = 50

People who speak English = n(E) = 20

People who speak both Hindi and English = n(H ∩ E) = 10

People who speak at least one of the two languages = n(H ∪ E)

Venn diagram:



Now, we know that,

n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

Therefore,

n(H ∪ E) = n(H) + n(E) – n(H ∩ E)

= 50 + 20 – 10

= 60

Thus, People who speak at least one of the two languages are 60.

Q. 6. In a group of 50 persons, 30 like tea, 25 like coffee and 16 like both. How many like

(i) either tea or coffee?

(ii) neither tea nor coffee?

Answer : Given:

In a group of 50 persons,

-30 like tea

-25 like coffee

-16 like both tea and coffee

To find:

(i) People who like either tea or coffee.

Let us consider,

Total number of people = n(X) = 50

People who like tea = n(T) = 30

People who like coffee = n(C) = 25

People who like both tea and coffee = n(T ∩ C) = 16

People who like either tea or coffee = n(T ∪ C)

Venn diagram:

Therefore,

n(T ∪ C) = n(T) + n(C) - n(T ∩ C)

= 30 + 25 – 16

= 39

Thus, People who like either tea or coffee = 39

(ii) People who like neither tea nor coffee.

People who like neither tea nor coffee = n(X) – n(T ∪ C)

= 50 – 39

= 11

Therefore, People who like neither tea nor coffee = 11

Q.7. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2, and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to

(i) Chemical C_1 but not chemical C_2
(ii) Chemical C_2 but not chemical C_1
(iii) Chemical C_1 or chemical C_2

Answer : Given:

Total number of individuals with skin disorder = 200

Individuals exposed to chemical C_1 = 120

Individuals exposed to chemical C_2 = 50

Individuals exposed to chemicals C_1 and C_2 both = 30

To Find:

(i) Individuals exposed to Chemical C_1 but not C_2

Let us consider,

Total number of individuals with skin disorder = n(C) = 200

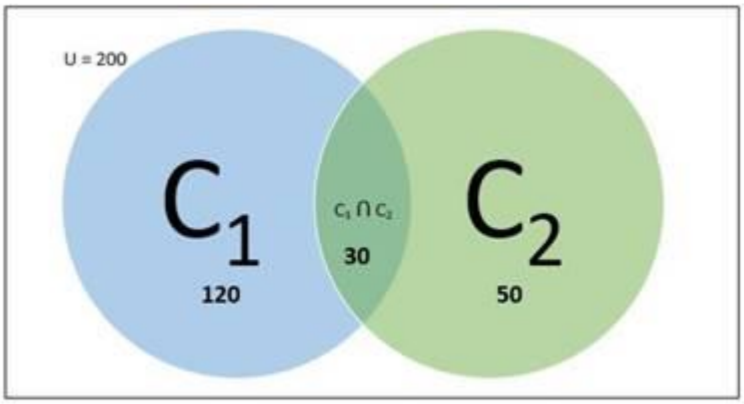
Individuals exposed to chemical C_1 = $n(C_1)$ = 120

Individuals exposed to chemical C_2 = $n(C_2)$ = 50

Individuals exposed to chemicals C_1 and C_2 both = $n(C_1 \cap C_2)$ = 30

Individuals exposed to Chemical C_1 but not C_2 = $n(C_1 - C_2)$

Venn diagram:



Now,

$n(C_1 - C_2) = n(C_1) – n(C_1 \cap C_2)$

= 120 – 30

= 90

Therefore, number of individuals exposed to chemical C_1 but not C_2 = 90

(ii) Individuals exposed to Chemical C_2 but not C_1

Let us consider number of Individuals exposed to Chemical C_2 but not C_1 = $n(C_2 – C_1)$

Now,

$n(C_2 – C_1) = n(C_2) – n(C_1 \cap C_2)$

= 50 – 30

= 20

Therefore, number of individuals exposed to chemical C_2 but not C_1 = 20

(iii) Individuals exposed to Chemical C_1 or chemical C_2

Let us consider number of Individuals exposed to Chemical C_1 or chemical C_2 = $n(C_1 \cup C_2)$

Now,

$n(C_1 \cup C_2) = n(C_1) + n(C_2) - n(C_1 \cap C_2)$

= 120 + 50 – 30

= 140

Therefore, number of individuals exposed to chemical C_1 or C_2 = 140

Q. 8. In a class of a certain school, 50 students, offered mathematics, 42 offered biology and 24 offered both the subjects. Find the number of students offering

(i) mathematics only,

(ii) biology only,

(iii) any of the two subjects.

Answer : Given:

Number of students offered Mathematics = 50

Number of students offered Biology = 42

Number of students offered both Mathematics and Biology = 24

To Find:

(i) Number of students offered Mathematics only

Let us consider,

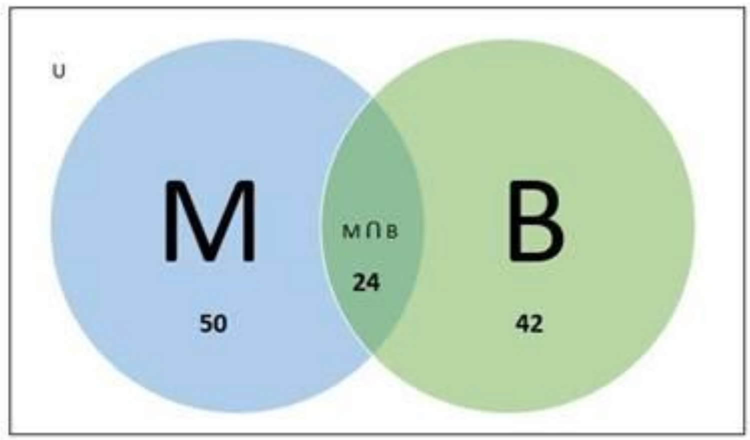
Number of students offered Mathematics = n(M) = 50

Number of students offered Biology = n(B) = 42

Number of students offered Mathematics & Biology both = $n(M \cap B)$ = 24

Number of students offered Mathematics only = n(M - B)

Venn diagram:



Now,

n(M - B) = n(M) – n(M ∩ B)

= 50 – 24

= 26

Therefore, Number of students offered Mathematics only= 26

(ii) Number of students offered Biology only

Number of students offered Biology only= n(B – M)

Now,

n(B – M) = n(B) – n(M ∩ B)

= 42 – 24

= 18

Therefore, Number of students offered Biology only = 18

(iii) Number of students offered any of two subjects

Number of students offered any of two subjects = n(M ∪ B)

Now,

n(M ∪ B) = n(M) + n(B) - n(M ∩ B)

= 50 + 42 – 24

= 140

Therefore, Number of students offered any of two subjects = 68

Q. 9. In an examination, 56% of the candidates failed in English and 48% failed in science. If 18% failed in both English and science, find the percentage of those who passed in both the subjects.

Answer : Given:

In an examination:

- 56% of candidates failed in English

- 48% of candidates failed in science

- 18% of candidates failed in both English and Science

To Find;

Percentage of students who passed in both subjects.

Let us consider,

Percentage of candidates who failed in English = n(E) = 56

Percentage of candidates who failed in Science = n(S) = 48

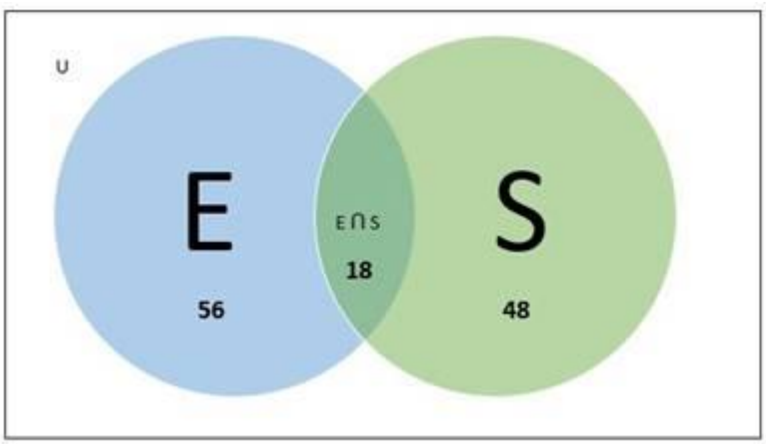
Percentage of candidates who failed in English and Science both

= n(E ∩ S) = 18

Percentage of candidates who failed in English only = n(E - S)

Percentage of candidates who failed in Science only = n(S - E)

Venn diagram:



Now,

n(E - S) = n(E) - n(E ∩ S)

= 56 – 18

= 38

n(S - E) = n(S) - n(E ∩ S)

= 48 – 18

= 30

Therefore,

Percentage of total candidates who failed =

n(E - S) + n(S - E) + n(E ∩ S)

= 38 + 30 + 18 = 86%

Now,

The percentage of candidates who passed in both English and

Science = 100 - 86 = 14%

Hence,

The percentage of candidates who passed in both English and

Science = 14%

Q. 10. In a group of 65 people, 40 like cricket and 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Answer : Given:

In a group of 65 people:

- 40 people like cricket

- 10 like both cricket and tennis

To Find:

- Number of people like tennis only

- Number of people like tennis

Let us consider,

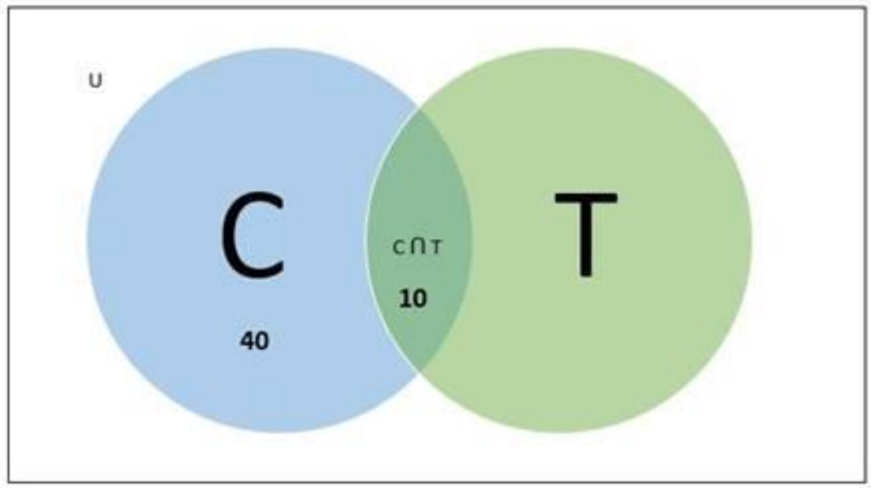
Number of people who like cricket = n(C) = 40

Number of people who like tennis = n(T)

Number of people who like cricket or tennis = n(C ∪ T) = 65

Number of people who like cricket and tennis both = n(C ∩ T) = 10

Venn diagram:



Now,

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$65 = 40 + n(T) - 10$

$n(T) = 65 - 40 + 10$

$= 35$

Therefore, number of people who like tennis = 35

Now,

Number of people who like tennis only = $n(T - C)$

$n(T - C) = n(T) - n(C \cap T)$

$= 35 - 10$

$= 25$

Therefore, the number of people who like tennis only = 25

Q. 11. A school awarded 42 medals in hockey, 18 in basketball and 23 in cricket. if these medals were bagged by a total of 65 students and only 4 students got medals in all the three sports, how many students received medals in exactly two of the three sports?

Answer : Given:

- Total number of students = 65

- Medals awarded in Hockey = 42

- Medals awarded n Basketball = 18

- Medals awarded in Cricket = 23

- 4 students got medals in all the three sports.

To Find:

Number of students who received medals in exactly two of the three sports.

Total number of medals = Medals awarded in Hockey + Medals awarded in Basketball + Medals awarded in Cricket

Total number of medals = 42 + 28 + 23

= 83

It is given that 4 students got medals in all the three sports.

Therefore, the number of medals received by those 4 students = 4 × 3 = 12

Now, the number of medals received by the rest of 61 students = 83 – 12 = 71

Among these 61 students, everyone at least received 1 medal.

Therefore, the number of extra medals = 71 – 1 × 61 = 10

Therefore, we can say that 10 students received more than one and less than three medals, or we can say that 10 students received medals in exactly two of three sports.

Q. 12. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read the newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspapers. Find

(i) The number of people who read at least one of the newspapers,
(ii) The number of people who read exactly one newspaper.

Answer : Given:

- Total number of people = 60

- Number of people who read newspaper H = 25

- Number of people who read newspaper T = 26

- Number of people who read newspaper I = 26

- Number of people who read newspaper H and I both = 9

- Number of people who read newspaper H and T both = 11

- Number of people who read newspaper T and I both = 8

- Number of people who read all three newspapers = 3

To Find:

(i) The number of people who read at least one of the newspapers

Let us consider,

Number of people who read newspaper H = n(H)= 25

Number of people who read newspaper T = n(T) = 26

Number of people who read newspaper I = n(I) =26

Number of people who read newspaper H and I both = n(H ∩ I) = 9

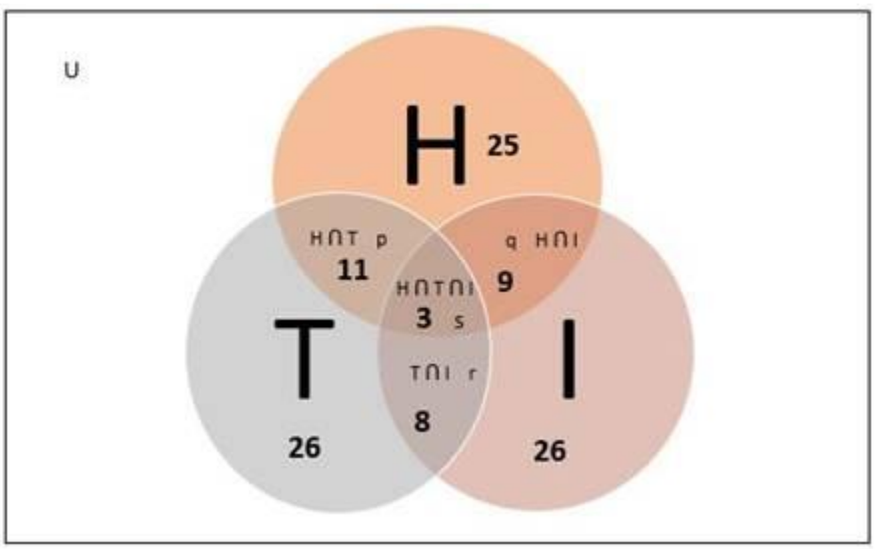
Number of people who read newspaper H and T both = n(H ∩ T) = 11

Number of people who read newspaper T and I both = n(T ∩ I) = 8

Number of people who read all three newspapers = n(H ∩ T ∩ I) = 3

Number of people who read at least one of the three newspapers= n(H ∪ T ∪ I)

Venn diagram:



We know that,

n(H ∪ T ∪ I) = n(H) + n(T) + n(I) – n(H ∩ I) – n(H ∩ T) – n(T ∩ I) + n(H ∩ T ∩ I)

= 25 + 26 + 26 – 9 – 11 – 8 + 3

= 52

Therefore,

Number of people who read at least one of the three newspapers = 52

(ii) The number of people who read exactly one newspaper

Number of people who read exactly one newspaper =

$n(H \cup T \cup I) - p - q - r - s$

Where,

p = Number of people who read newspaper H and T but not I

q = Number of people who read newspaper H and I but not T

r = Number of people who read newspaper T and I but not H

s = Number of people who read all three newspapers = 3

$p + s = n(H \cap T)$ …(1)

$q + s = n(H \cap I)$ …(2)

$r + s = n(T \cap I)$ …(3)

Adding (1), (2) and (3)

$p + s + q + s + r + s = n(H \cap T) + n(H \cap I) + n(T \cap I)$

$p + q + r + 3s = 9 + 11 + 8$

$p + q + r + s + 2s = 28$

$p + q + r + s = 28 - 2\times3$

$p + q + r + s = 22$

Now,

$n(H \cup T \cup I) - p - q - r - s = n(H \cup T \cup I) - (p + q + r + s)$

$= 52 - 22$

$= 30$

Hence, 30 people read exactly one newspaper.

Q. 13. In a survey of 100 students, the number of students studying the various languages is found as English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find

(i) how many students are studying Hindi?
(ii) how many students are studying English and Hindi both?

Answer : Given:

- Total number of students = 100

- Number of students studying English(E) only = 18

- Number of students learning English but not Hindi(H) = 23

- Number of students learning English and Sanskrit(S) = 8

- Number of students learning Sanskrit and Hindi = 8

- Number of students learning English = 26

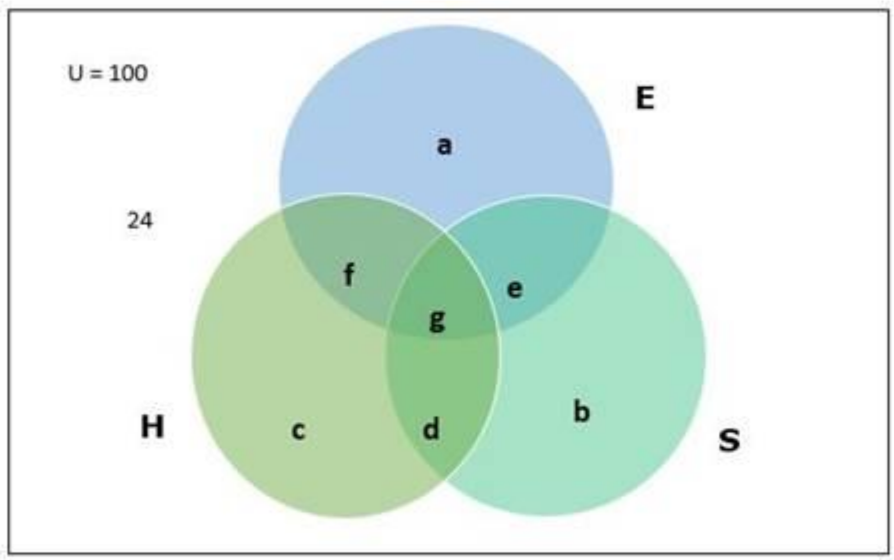
- Number of students learning Sanskrit = 48

- Number of students learning no language = 24

To Find:

(i) Number of students studying Hindi

Venn diagram:



From the above Venn diagram,

a = Number of students who study only English = 18

b = Number of students who study only Sanskrit

c = Number of students who study only Hindi

d = Number of students learning Hindi and Sanskrit but not English

e = Number of students learning English and Sanskrit but not Hindi

f = Number of students learning Hindi and English but not Sanskrit

g = Number of students learning all the three languages

e + g = Number of students learning English and Sanskrit = 8

$= n(E \cap S)$

$g + d$ = Number of students learning Hindi and Sanskrit = 8

$= n(H \cap S)$

$E = a + e + f + g$ = Number of students learning English

$26 = 18 + 8 + f$

$f = 26 - 26 = 0$

Therefore, $f = 0$

Now,

Number of students learning English but not Hindi = $a + e = 23$

$23 = 18 + e$

Therefore, $e = 5$

Now, $e + g = 8$

$5 + g = 8$

Therefore, $g = 3$

$S = b + e + d + g$ = Number of students studying Sanskrit

$48 = b + 5 + 8$ (Because, $d + g = 8$)

$b = 48 – 13$

Therefore, $b = 35$ (Number of students studying Sanskrit only)

Also, $d + g = 8$

$d + 3 = 8$

Therefore, $d = 5$

Now,

Number of students studying Hindi only = c

$c = 100 – (a + e + b + d + f + g) – 24$

$= 100 – (18 + 5 + 35 + 5 + 0 + 3) - 24$

$= 100 – 66 – 24$

$= 100 – 90 = 10$

Number of students studying Hindi = $c + f + g + d$

$= 10 + 0 + 3 + 5$

= 18

Therefore, number of students studying Hindi = 18

(ii) Number of students studying English and Hindi both

Number of students studying English and Hindi both = f + g

= 0 + 3 = 3

Therefore, Number of students studying English and Hindi both = 3

Q. 14. In a town of 10,000 families, it was found that 40% of the families buy newspaper A, 20% buy newspaper B, 10% buy newspaper C, 5% buy A and B; 3% buy B and C, and 4% buy A and C. IF 2% buy all the three newspapers, find the number of families which buy

(i) A only,
(ii) B only,
(iii) none of A, B, and C.

Answer : Given:

Total number of families = 10000

Percentage of families that buy newspaper A = 40

Percentage of families that buy newspaper B = 20

Percentage of families that buy newspaper C = 10

Percentage of families that buy newspaper A and B = 5

Percentage of families that buy newspaper B and C = 3

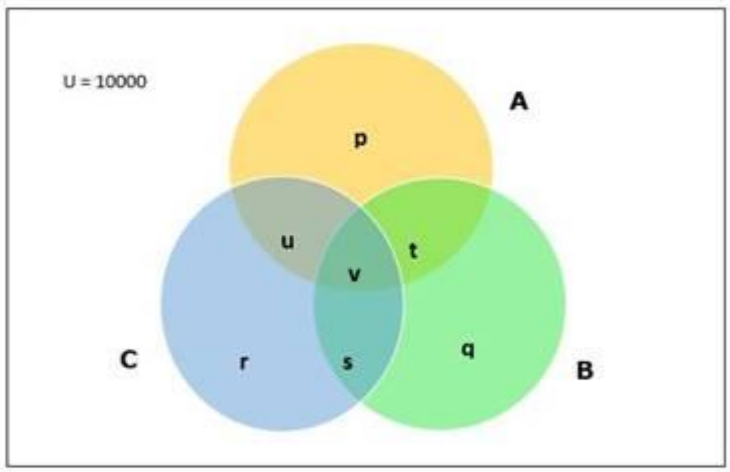
Percentage of families that buy newspaper A and C = 4

Percentage of families that buy all three newspapers = 2

To find:

(i) Number of families that buy newspaper A only

Consider the Venn Diagram below:



Number of families that buy newspaper A = n(A) = 40% of 10000

= 4000

Number of families that buy newspaper B = n(B) = 20% of 10000 = 2000

Number of families that buy newspaper C = n(C) = 10% of 10000

= 1000

Number of families that buy newspaper A and B = n(A ∩ B)

= 5% of 10000

= 500

Number of families that buy newspaper B and C = n(B ∩ C)

= 3% of 10000

= 300

Number of families that buy newspaper A and C = n(A ∩ C)

= 4% of 10000

= 400

Number of families that buys all three newspapers = n(A ∩ B ∩ C)=v

= 2% of 10000

= 200

We have,

n(A ∩ B) = v + t

500 = 200 + t

$t = 500 – 200 = 300$

$n(B \cap C) = v + s$

$300 = 200 + s$

$s = 300 – 200 = 100$

$n(A \cap C) = v + u$

$400 = 200 + u$

$u = 400 – 200 = 200$

p = Number of families that buy newspaper A only

We have,

$A = p + t + v + u$

$4000 = p + 300 + 200 + 200$

$p = 4000 – 700$

$p = 3300$

Therefore,

Number of families that buy newspaper A only = 3300

(ii) Number of families that buy newspaper B only

q = Number of families that buy newspaper B only

$B = q + s + v + t$

$2000 = q + 100 + 200 + 300$

$q = 2000 – 600 =1400$

Therefore,umber of families that buy newspaper B only = 1400

(iii) Number of families that buys none of the newspaper

Number of families that buy none of the newspaper =

$10000 – \{n(A) + n(B) + n(C) – n(A \cap B) – n(B \cap C) – n(A \cap C) + n(A \cap B \cap C)\}$

$= 10000 – (4000 + 2000 + 1000 – 500 – 300 – 400 + 200)$

$= 10000 – 6000$

$= 4000$

Therefore,

Number of families that buy none of the newspaper = 4000

Q. 15. A class has 175 students. The following description gives the number of students one or more of the subjects in this class: mathematics 100, physics 70, chemistry 46, mathematics and physics 30; mathematics and chemistry 28; physics and chemistry 23; mathematics, physics and chemistry 18. Find

(i) how many students are enrolled in mathematics alone, physics alone and chemistry alone,
(ii) The number of students who have not offered any of these subjects.

Answer : Given:

- Number of students in class = 175

- Number of students enrolled in Mathematics = 100

- Number of students enrolled in Physics = 70

- Number of students enrolled in Chemistry = 46

- Number of students enrolled in Mathematics and Physics = 30

- Number of students enrolled in Physics and Chemistry = 23

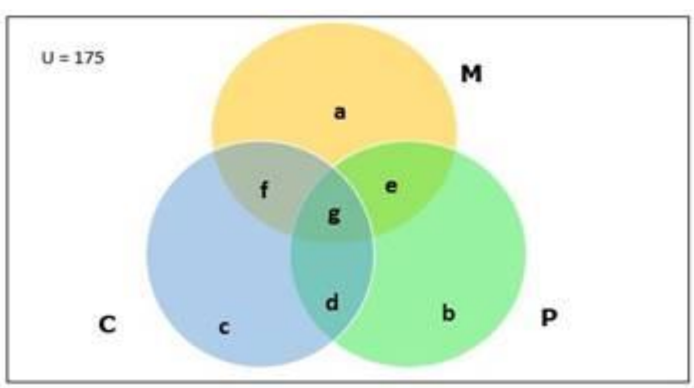
- Number of students enrolled in Mathematics and Physics = 28

- Number of students enrolled in all three subjects = 18

To find:

(i) Number of students enrolled in Mathematics alone, Physics alone and Chemistry alone

Venn diagram:



Number of students enrolled in Mathematics = 100 = n(M)

Number of students enrolled in Physics = 70 = n(P)

Number of students enrolled in Chemistry = 46 = n(C)

Number of students enrolled in Mathematics and Physics

= 30 = $n(M \cap P)$

Number of students enrolled in Mathematics and Chemistry

= 28 = $n(M \cap C)$

Number of students enrolled in Physics and Chemistry

= 23 = $n(P \cap C)$

Number of students enrolled in all the three subjects

= 18 = $n(M \cap P \cap C)$ = g

We have,

$n(M \cap P) = e + g$

30 = e + 18

e = 30 – 18 = 12

$n(M \cap C) = f + g$

28 = f + 18

f = 28 – 18 = 10

$n(P \cap C) = d + g$

23 = d + 18

d = 23 – 18 = 5

a = Number of students enrolled only in Mathematics

b = Number of students enrolled only in Physicsc = Number of students enrolled only in Chemistry

We have,

M = a + e + f + g

100 = a + 12 + 10 + 18

a = 100 – 40

a = 60

Therefore,

Number of students enrolled only in Mathematics = 60

P = b + e + d + g

70 = b + 12 + 5 + 18

b = 70 – 35

b = 35

Therefore,

Number of students enrolled only in Physics = 35

C = c + f + d + g

46 = c + 10 + 5 + 18

c = 46 – 33

c = 13

Therefore,

Number of students enrolled only in Chemistry = 13

(ii) Number of students who have not offered any of these subjects

Number of students who have not offered any of these subjects

$= 175 - \{n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)\}$

$= 175 - (100 + 70 + 46 - 30 - 28 - 23 + 18)$

$= 175 - 153$

$= 22$

Therefore,

Number of students who have not offered any of these subjects = 22

Exercise 1H

Q. 1. If a set A and n elements then find the number of elements in its power set P(A).

Answer : The power set of set A is a collection of all subsets of A.

For example: if the set A is {1, 2} then all possible subsets of A would be {} (empty set), {1}, {2}, {1, 2}

Hence powerset of A that is P(A) will be {ϕ, {1}, {2}, {1,2}}

Now if the number of elements in set A is n then the number of elements in power set of A P(A) is 2^n

Q. 2. If A = ϕ then write P(A).

Answer : The power set of set A is a collection of all subsets of A.

Here A = {ϕ}

Hence the subset of A will only be a null set ϕ

Hence P(A) = {ϕ}

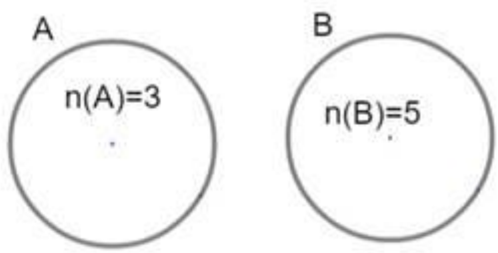
Q.3. If n(A) = 3 and n(B) = 5, find:

(i) The maximum number of elements in A ∪ B,
(ii) The minimum number of elements in A ∪ B.

Answer : Number of elements in set A n(A) = 3 and number of elements in set B n(B) = 5

The number of elements in A ∪ B is n(A ∪ B).

i) Now for elements in A ∪ B to be maximum, there should not be any intersection between both sets that is A and B both sets must be disjoint sets as shown.



Hence the number of elements in A ∪ B is n(A ∪ B) = n(A) + n(B)

⇒ n(A ∪ B) = 3 + 5

⇒ n(A ∪ B) = 8

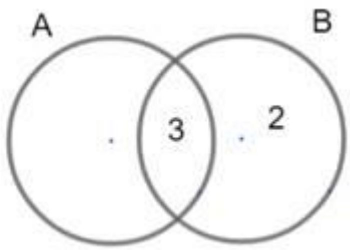
Hence maximum number of elements in A ∪ B is 8

ii) Now for a number of elements in A ∪ B to be minimum, there should be an ntersection between sets A and B so that some elements are common

The count will be minimum when all the elements from set A are also in set B the reverse are not possible because n(A) < n(B)

Hence if the 3 elements of A are in the intersection of A and B, then the number of elements only in B will be 2 because n(B) = 5

Visually it is represented as,



As seen from the figure the number of elements in A ∪ B is 5 hence minimum number of elements in A ∪ B = 5

Q. 4. If A and B are two sets such than n(A) = 8, n(B) = 11 and n(A ∪ B) = 14 then find n(A ∩ B).

Answer : Given: n(A) = 8, n(B) = 11, n(A ∪ B) = 14

We know that n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

⇒ 14 = 8 + 11 – n(A ∩ B)

⇒ 14 = 19 – n(A ∩ B)

⇒ n(A ∩ B) = 19 – 14

⇒ n(A ∩ B) = 5

Hence n(A ∩ B) = 5

Q. 5. If A and B are two sets such that n(A) = 23, n(b) = 37 and n(A – B) = 8 then find n(A ∪ B).

Hint n(A) = n(A – B) + n(A ∩ B) n(A ∩ B) = (23 – 8) = 15.

Answer : Given: n(A) = 23, n(B) = 37, n(A – B) = 8

Using the hint

n(A) = n(A – B) + n(A ∩ B)

⇒ 23 = 8 + n(A ∩ B)

⇒ n(A ∩ B) = 23 – 8

⇒ n(A ∩ B) = 15

Visualizing the hint given,

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cup B) = 23 + 37 - 15$

$\Rightarrow n(A \cup B) = 45$

Hence $n(A \cup B) = 45$

Q. 6. If A and B are two sets such than $n(A) = 54$, $n(B) = 39$ and $n(B - A) = 13$ then find $n(A \cup B)$.

Hint $n(B) = n(B - A) + n(A \cap B) \Rightarrow n(A \cap B) = (39 - 13) = 26$.

Answer : Given: $n(A) = 54$, $n(B) = 39$, $n(B - A) = 13$

Using the hint

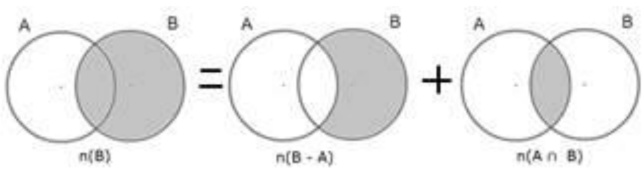
$n(B) = n(B - A) + n(A \cap B)$

$\Rightarrow 39 = 13 + n(A \cap B)$

$\Rightarrow n(A \cap B) = 39 - 13$

$\Rightarrow n(A \cap B) = 26$

Visualizing the hint given,



We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

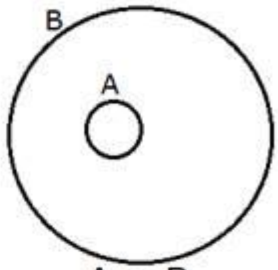
$\Rightarrow n(A \cup B) = 54 + 39 - 26$

$\Rightarrow n(A \cup B) = 67$

Hence $n(A \cup B) = 67$

Q. 7. If $A \subset B$, prove that $B' \subset A'$.

Answer : As $A \subset B$ the set A is inside set B



$A \subset B$

Hence A ∪ B = B

Taking compliment

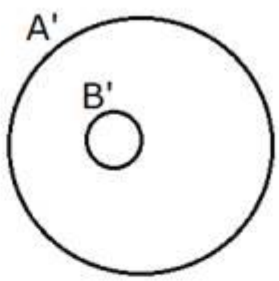
⇒ (A ∪ B)’ = B.’

Using de-morgans law (A ∪ B)’ = A’ ∩ B.’

⇒ A’ ∩ B’ = B.’

A’ ∩ B’ = B’ means that the set B’ is inside the set A.’

Representing in Venn diagram,

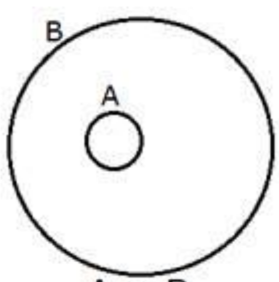


As seen from Venn diagram B’ ⊂ A.’

Hence proved

Q. 8. If A ⊂ B, show that (B’ – A’) = ϕ.

Answer : As A ⊂ B the set A is inside set B



$A \subset B$

Hence A ∪ B = B

Taking compliment

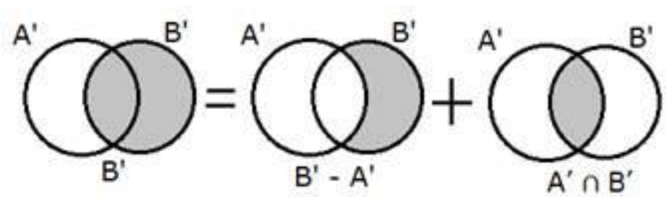
$\Rightarrow (A \cup B)' = B'$

Using De-Morgan's law $(A \cup B)' = A' \cap B'$

$\Rightarrow A' \cap B' = B'$ …(i)

Now we know that

$B' = (B' - A') + (A' \cap B')$



Using (i)

$\Rightarrow B' = (B' - A') + B'$

$\Rightarrow (B' - A') = B' - B'$

$\Rightarrow (B' - A') = 0$

$\Rightarrow (B' - A') = \{\phi\}$

Hence proved

Q. 9. Let A = {x : x = 6n $\in$ N) and B = {x : x = 9n, n $\in$ N}, find A ∩ B.

Answer : $A = \{x : x = 6n \ \forall \ n \in N)$

As x = 6n hence for n = 1, 2, 3, 4, 5, 6… x = 6, 12, 18, 24, 30, 36…

Hence A = {6, 12, 18, 24, 30, 36…}

$B = \{x : x = 9n \ \forall \ n \in N)$

As x = 9n hence for n = 1, 2, 3, 4… x = 9, 18, 27, 36…

Hence B = {9, 18, 27, 36…}

$A \cap B$ means common elements to both sets

The common elements are 18, 36, 54, …

Hence $A \cap B = \{18, 36, 54, \ldots\}$

All the elements are multiple of 18

Hence $A \cap B = \{x: x = 18n \ \forall \ n \in N\}$

Q. 10. If A = {5, 6, 7}, find P(A).

Answer : A = {5, 6, 7}

We have to find P(A) which is power set of A

The power set of set A is collection of all possible subsets of A

The possible subsets of A are {ϕ}, {5}, {6}, {7}, {5,6}, {5,7}, {6, 7}, {5, 6, 7}

Hence the power set P(A) will be

P(A) = {{ϕ}, {5}, {6}, {7}, {5,6}, {5,7}, {6, 7}, {5, 6, 7}}

Q. 11. If A = {3, {2}}, find P(A).

Answer : A = {3, {2}}

We have to find P(A) which is power set of A

The power set of set A is collection of all possible subsets of A

The possible subsets of A are {ϕ}, {3}, {{2}}, {3, {2}}

Hence the power set P(A) will be

P(A) = {{ϕ}, {3}, {{2}}, {3, {2}}}

Q. 12. Prove that A ∩ (A ∪ B)' = ϕ

Answer : LHS = A ∩ (A ∪ B)'

Using De-Morgan's law (A ∪ B)' = (A' ∩ B')

⇒ LHS = A ∩ (A' ∩ B')

⇒ LHS = (A ∩ A') ∩ (A ∩ B')

We know that A ∩ A' = ϕ

⇒ LHS = ϕ ∩ (A ∩ B')

We know that intersection of null set with any set is null set only

Let (A ∩ B') be any set X hence

⇒ LHS = ϕ ∩ X

⇒ LHS = ϕ

⇒ LHS = RHS

Hence proved

Q. 13. Find the symmetric difference A Δ B, when A = {1, 2, 3} and B = {3, 4, 5}.

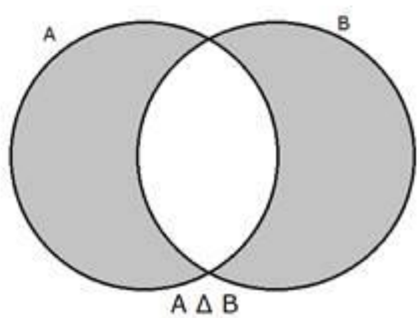
Answer : A = {1, 2, 3}

B = {3, 4, 5}

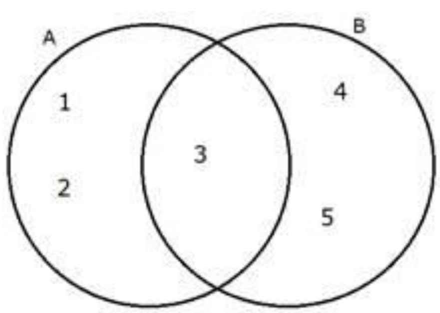
The symmetric difference A Δ B is given by

A Δ B = (A – B) ∪ (B – A)

Venn diagram representation:



Representing the given sets A and B through venn diagram



Hence as seen the elements in A Δ B are 1, 2, 4 and 5

Hence the symmetric difference A Δ B = {1, 2, 4, 5}

Q. 14. Prove that A – B = A ∩ B.'

Answer : Let x be some element in set A – B that is x ∈ (A – B)

Now if we prove that x ∈ (A ∩ B') then (A – B) = (A ∩ B')

x ∈ (A – B) means x ∈ A and x ∉ B

Now x ∉ B means x ∈ B.'

Hence we can say that x ∈ A and x ∈ B.'

Hence x ∈ A ∩ B.'

And as x ∈ A ∩ B' and also x ∈ A – B we can conclude that

A – B = A ∩ B.'

Q. 15. If A = {x : x ϵ R, x < 5} and B = {x : x ϵ R, x > 4}, find A ∩ B.

Answer : A = {x: x ∈ R, x < 5}

As x takes all real values upto 5 hence the set A will contain all numbers from -∞ to 5

A = (-∞, 5)

B = {x: x ∈ R, x > 4}

As x takes all real values greater than 4 hence the set B will contain values from 4 to ∞

B = (4, -∞)

Hence their intersection or the common part between sets A and B would be values from 4 to 5

Hence A ∩ B = (4, 5)

Representing the sets on number line