Relations

Exercise 2A

Q. 1. Find the values of a and b, when:

(i) (a + 3, b –2) = (5, 1)
(ii) (a + b, 2b – 3) = (4, –5)
(iii) $\left(\frac{a}{3}+1, b-\frac{1}{3}\right)=\left(\frac{5}{3}, \frac{2}{3}\right)$
(iv) (a – 2, 2b + 1 = (b – 1, a + 2)

Answer : Since, the ordered pairs are equal, the corresponding elements are equal.

∴, a + 3 = 5 …(i) and b – 2 = 1 …(ii)

Solving eq. (i), we get

a + 3 = 5

⇒ a = 5 – 3

⇒ a = 2

Solving eq. (ii), we get

b – 2 = 1

⇒ b = 1 + 2

⇒ b = 3

Hence, the value of a = 2 and b = 3.

(ii) Since, the ordered pairs are equal, the corresponding elements are equal.

∴, a + b = 4 …(i) and 2b – 3 = -5 …(ii)

Solving eq. (ii), we get

2b – 3 = -5

⇒ 2b = -5 + 3

⇒ 2b = -2

⇒ b = -1

Putting the value of b = - 1 in eq. (i), we get

a + (-1) = 4

⇒ a – 1 = 4

⇒ a = 4 + 1

⇒ a = 5

Hence, the value of a = 5 and b = -1.

(iii) Since the ordered pairs are equal, the corresponding elements are equal.

$\therefore \frac{a}{3}+1=\frac{5}{3}$ …(i)

$\& \ b-\frac{1}{3}=\frac{2}{3}$ …(ii)

Solving Eq. (i), we get

$$\frac{a}{3}+1=\frac{5}{3}$$

$$\Rightarrow \frac{a}{3}=\frac{5}{3}-1$$

$$\Rightarrow a=3\left(\frac{5}{3}-1\right)$$

⇒ a = 5 – 3

⇒ a = 2

Solving eq. (ii), we get

$$b-\frac{1}{3}=\frac{2}{3}$$

$$\Rightarrow b=\frac{2}{3}+\frac{1}{3}$$

$$\Rightarrow b=\frac{3}{3}$$

⇒ b = 1

Hence, the value of a = 2 and b = 1.

(iv) Since, the ordered pairs are equal, the corresponding elements are equal.

$\therefore$, $a - 2 = b - 1$ …(i)

& $2b + 1 = a + 2$ …(ii)

Solving eq. (i), we get

$a - 2 = b - 1$

$\Rightarrow a - b = -1 + 2$

$\Rightarrow a - b = 1$ … (iii)

Solving eq. (ii), we get

$2b + 1 = a + 2$

$\Rightarrow 2b - a = 2 - 1$

$\Rightarrow -a + 2b = 1$ …(iv)

Adding eq. (iii) and (iv), we get

$a - b + (-a) + 2b = 1 + 1$

$\Rightarrow a - b - a + 2b = 2$

$\Rightarrow b = 2$

Putting the value of $b = 2$ in eq. (iii), we get

$a - 2 = 1$

$\Rightarrow a = 1 + 2$

$\Rightarrow a = 3$

Hence, the value of $a = 3$ and $b = 2$.

Q. 2. If A = {9, 1} and B = {1, 2, 3}, show that A × B ≠ B × A.

Answer : Given: $A = \{9, 1\}$ and $B = \{1, 2, 3\}$

To show: $A \times B \neq B \times A$

Now, firstly we find the $A \times B$ and $B \times A$

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, .i.e.

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

Here, $A = (9, 1)$ and $B = (1, 2, 3)$. So,

$A \times B = (9, 1) \times (1, 2, 3)$

$= \{(9, 1), (9, 2), (9, 3), (1, 1), (1, 2), (1, 3)\}$

$B \times A = (1, 2, 3) \times (9, 1)$

$= \{(1, 9), (2, 9), (3, 9), (1, 1), (2, 1), (3, 1)\}$

Since by the definition of equality of ordered pairs .i.e. the corresponding first elements are equal and the second elements are also equal, but here, the pair (9, 1) is not equal to the pair (1, 9)

$\therefore A \times B \neq B \times A$

Hence proved

Q. 3. If P = {a, b} and Q = {x, y, z}, show that P × Q ≠ Q × P.

Answer : Given: $P = \{a, b\}$ and $Q = \{x, y, z\}$

To show: $P \times Q \neq Q \times P$

Now, firstly we find the $P \times Q$ and $Q \times P$

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, .i.e.

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

Here, $P = (a, b)$ and $Q = (x, y, z)$. So,

$P \times Q = (a, b) \times (x, y, z)$

$= \{(a, x), (a, y), (a, z), (b, x), (b, y), (b, z)\}$

$Q \times P = (x, y, z) \times (a, b)$

$= \{(x, a), (y, a), (z, a), (x, b), (y, b), (z, b)\}$

Since by the definition of equality of ordered pairs .i.e. the corresponding first elements are equal and the second elements are also equal, but here the pair (a, x) is not equal to the pair (x, a)

$\therefore P\times Q \neq Q \times P$

Hence proved

Q. 4. If A = {2, 3, 5} and B = {5, 7}, find:

(i) A × B

(ii) B × A

(iii) A × A

(iv) B × B

Answer : (i) Given: A = {2, 3, 5} and B = {5, 7}

To find: A × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {2, 3, 5} and B = {5, 7}. So,

A × B = (2, 3, 5) × (5, 7)

= {(2, 5), (3, 5), (5, 5), (2, 7), (3, 7), (5, 7)}

(ii) Given: A = {2, 3, 5} and B = {5, 7}

To find: B × A

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {2, 3, 5} and B = {5, 7}. So,

B × A = (5, 7) × (2, 3, 5)

= {(5, 2), (5, 3), (5, 5), (7, 2), (7, 3), (7, 5)}

(iii) Given: A = {2, 3, 5} and B = {2, 3, 5}

To find: A × A

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {2, 3, 5} and A = {2, 3, 5}. So,

A × A = (2, 3, 5) × (2, 3, 5)

= {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

(iv) Given: B = {5, 7}

To find: B × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, B = {5, 7} and B = {5, 7}. So,

B × B = (5, 7) × (5, 7)

= {(5, 5), (5, 7), (7, 5), (7, 7)}

Q. 5. If A = {x ϵ N : x ≤ 3} and {x ϵ W : x < 2}, find (A × B) and (B × A). Is (A × B) = (B × A)?

Answer : Given:

A = {x ϵ N: x ≤ 3}

Here, N denotes the set of natural numbers.

∴ A = {1, 2, 3}

[∵ It is given that the value of x is less than 3 and natural numbers which are less than 3 are 1 and 2]

and B = {x ϵ W: x < 2}

Here, W denotes the set of whole numbers (non – negative integers).

∴ B = {0, 1}

[∵ It is given that x < 2 and the whole numbers which are less than 2 are 0 and 1]

So, A × B = {1, 2, 3} × {0, 1}

-

[By the definition of equality of ordered pairs .i.e. the corresponding first elements are equal and the second elements are also equal, but here the pair (1, 0) is not equal to the pair (0, 1)]

Q. 6. If A = {1, 3, 5) B = {3, 4} and C = {2, 3}, verify that:

(i) A × (B ∪ C) = (A × B) ∪ (A × C)

(ii) A × (B ∩ C) = (A × B) ∩ (A × C)

Answer : (i) Given: A = {1, 3, 5}, B = {3, 4} and C = {2, 3}

L. H. S = A × (B ∪ C)

By the definition of the union of two sets, (B ∪ C) = {2, 3, 4}

= {1, 3, 5} × {2, 3, 4}

Now, by the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

= {(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4), (5, 2), (5, 3), (5, 4)}

R. H. S = (A × B) ∪ (A × C)

Now, A × B = {1, 3, 5} × {3, 4}

= {(1, 3), (1, 4), (3, 3), (3, 4), (5, 3), (5, 4)}

and A × C = {1, 3, 5} × {2, 3}

= {(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)}

Now, we have to find (A × B) ∪ (A × C)

So, by the definition of the union of two sets,

(A × B) ∪ (A × C) = {(1, 2), (1, 3), (1, 4), (3, 2), (3, 3), (3, 4), (5, 2), (5, 3), (5, 4)}

= L. H. S

∴ L. H. S = R. H. S is verified

(ii) Given: A = {1, 3, 5}, B = {3, 4} and C = {2, 3}

L. H. S = A × (B ∩ C)

By the definition of the intersection of two sets, (B ∩ C) = {3}

= {1, 3, 5} × {3}

Now, by the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

= {(1, 3), (3, 3), (5, 3)}

R. H. S = (A × B) ∩ (A × C)

Now, A × B = {1, 3, 5} × {3, 4}

$= \{(1, 3), (1, 4), (3, 3), (3, 4), (5, 3), (5, 4)\}$

and $A \times C = \{1, 3, 5\} \times \{2, 3\}$

$= \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$

Now, we have to find $(A \times B) \cap (A \times C)$

So, by the definition of the intersection of two sets,

$(A \times B) \cap (A \times C) = \{(1, 3), (3, 3), (5, 3)\}$

= L. H. S

∴ L. H. S = R. H. S is verified

Q. 7. Let $A = \{x \in W : x < 2\}$, $B = \{x \in N : 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Answer : (i) Given:

$A = \{x \in W : x < 2\}$

Here, W denotes the set of whole numbers (non – negative integers).

$\therefore A = \{0, 1\}$

[∵ It is given that $x < 2$ and the whole numbers which are less than 2 are 0 & 1]

$B = \{x \in N : 1 < x \leq 4\}$

Here, N denotes the set of natural numbers.

$\therefore B = \{2, 3, 4\}$

[∵ It is given that the value of x is greater than 1 and less than or equal to 4]

and $C = \{3, 5\}$

L. H. S $= A \times (B \cup C)$

By the definition of the union of two sets, $(B \cup C) = \{2, 3, 4, 5\}$

$= \{0, 1\} \times \{2, 3, 4, 5\}$

Now, by the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, .i.e.

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

$= \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

R. H. S = $(A \times B) \cup (A \times C)$

Now, $A \times B = \{0, 1\} \times \{2, 3, 4\}$

$= \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

and $A \times C = \{0, 1\} \times \{3, 5\}$

$= \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

Now, we have to find $(A \times B) \cup (A \times C)$

So, by the definition of the union of two sets,

$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

= L. H. S

∴ L. H. S = R. H. S is verified

(ii) Given:

$A = \{x \in W : x < 2\}$

Here, W denotes the set of whole numbers (non – negative integers).

∴ $A = \{0, 1\}$

[∵ It is given that $x < 2$ and the whole numbers which are less than 2 are 0, 1]

$B = \{x \in N : 1 < x \leq 4\}$

Here, N denotes the set of natural numbers.

∴ $B = \{2, 3, 4\}$

[∵ It is given that the value of x is greater than 1 and less than or equal to 4]

and $C = \{3, 5\}$

L. H. S = $A \times (B \cap C)$

By the definition of the intersection of two sets, $(B \cap C) = \{3\}$

$= \{0, 1\} \times \{3\}$

Now, by the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product $P \times Q$ is the set of all ordered pairs of elements from P and Q, .i.e.

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

= {(0, 3), (1, 3)}

R. H. S = (A × B) ∩ (A × C)

Now, A × B = {0, 1} × {2, 3, 4}

= {(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)}

and A × C = {0, 1} × {3, 5}

= {(0, 3), (0, 5), (1, 3), (1, 5)}

Now, we have to find (A × B) ∩ (A × C)

So, by the definition of the intersection of two sets,

(A × B) ∩ (A × C) = {(0, 3), (1, 3)}

= L. H. S

∴ L. H. S = R. H. S is verified

Q. 8. If A × B = {(–2, 3), (–2, 4), (0, 4), (3, 3), (3, 4), find A and B.

Answer : Here, A × B = {(–2, 3), (–2, 4), (0, 4), (3, 3), (3, 4)}

To find: A and B

Clearly, A is the set of all first entries in ordered pairs in A × B

{(–2, 3), (–2, 4), (0, 4), (3, 3), (3, 4)}

∴ A = {-2, 0, 3}

and B is the set of all second entries in ordered pairs in A × B

{(–2, 3), (–2, 4), (0, 4), (3, 3), (3, 4)}

∴ B = {3, 4}

Q. 9. Let A = {2, 3} and B = {4, 5}. Find (A × B). How many subsets will (A × B) have?

Answer : Given: A = {2, 3} and B = {4, 5}

To find: A × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {2, 3} and B = {4, 5}. So,

A × B = (2, 3) × (4, 5)

= {(2, 4), (2, 5), (3, 4), (3, 5)}

∴Number of elements of A× B = n = 4

Number of subsets of A × B = 2^n

= 2^4

= 2 × 2 × 2 × 2

= 16

∴, the set A × B has 16 subsets.

Q. 10. Let A × B = {(a, b): b = 3a – 2}. if (x, –5) and (2, y) belong to A × B, find the values of x and y.

Answer : Given: A × B = {(a, b): b = 3a – 2}

and {(x, -5), (2, y)} Є A × B

For (x, -5) Є A × B

b = 3a – 2

⇒ -5 = 3(x) – 2

⇒ -5 + 2 = 3x

⇒ -3 = 3x

⇒ x = -1

For (2, y) Є A × B

b = 3a – 2

⇒ y = 3(2) – 2

⇒ y = 6 – 2

⇒ y = 4

Hence, the value of x = -1 and y = 4

Q. 11. Let A and B be two sets such that n(A) = 3 and n(B) = 2.

If a ≠ b ≠ c and (a, 0), (b, 1), (c, 0) is in A × B, find A and B.

Answer : Since, (a, 0), (b, 1), (c, 0) are the elements of A × B.

∴ a, b, c Є A and 0, 1 Є B

It is given that n(A) = 3 and n(B) = 2

∴ a, b, c Є A and n(A) = 3

⇒ A = {a, b, c}

and 0, 1 Є B and n(B) = 2

⇒ B = {0, 1}

Q. 12. Let A = {–2, 2} and B = (0, 3, 5). Find:

(i) A × B

(ii) B × A

(iii) A × A

(iv) B × B

Answer : (i) Given: A = {-2, 2} and B = {0, 3, 5}

To find: A × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {-2, 2} and B = {0, 3, 5}. So,

A × B = {(-2, 0), (-2, 3), (-2, 5), (2, 0), (2, 3), (2, 5)}

(ii) Given: A = {-2, 2} and B = {0, 3, 5}

To find: B × A

By the definition of the Cartesian product, Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {-2, 2} and B = {0, 3, 5}. So,

B × A = {(0, -2), (0, 2), (3, -2), (3, 2), (5, -2), (5, 2)}

(iii) Given: A = {-2, 2}

To find: A × A

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {-2, 2} and A = {-2, 2}.So,

A × A = {(-2, -2), (-2, 2), (2, -2), (2, 2)}

(iv) Given: B = {0, 3, 5}

To find: B × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, B = {0, 3, 5} and B = {0, 3, 5}. So,

B × B = {(0, 0), (0, 3), (0, 5), (3, 0), (3, 3), (3, 5), (5, 0), (5, 3), (5, 5)}

Q. 13. If A = {5, 7), find (i) A × A × A.

Answer : We have, A = {5, 7}

So, By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {5, 7} and A = {5, 7}.So,

A × A = {(5, 5), (5, 7), (7, 5), (7, 7)}

Now again, we apply the definition of Cartesian product to find A × A × A

Here, A = {5, 7} and A × A = {(5, 5), (5, 7), (7, 5), (7, 7)}

∴ A × A × A = {(5, 5, 5), (5, 5, 7), (5, 7, 5), (5, 7, 7), (7, 5, 5), (7, 5, 7), (7, 7, 5), (7, 7, 7)}

Q. 14. Let A = {–3, –1}, B = {1, 3) and C = {3, 5). Find:

(i) A × B

(ii) (A × B) × C

(iii) B × C

(iv) A × (B × C)

Answer : (i) Given: A = {-3, -1} and B = {1, 3}

To find: A × B

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, A = {-3, -1} and B = {1, 3}. So,

A × B = {-3, -1} × {1, 3}

= {(-3, 1), (-3, 3), (-1, 1), (-1, 3)}

(ii) Given: C = {3, 5}

From part (i), we get A × B = {(-3, 1), (-3, 3), (-1, 1), (-1, 3)}

So,

(A × B) × C = {(-3, 1), (-3, 3), (-1, 1), (-1, 3)} × (3, 5)

= (-3, 1, 3), (-3, 1 , 5), (-3, 3, 3), (-3, 3, 5), (-1, 1, 3), (-1, 1, 5), (-1, 3, 3), (-1, 3, 5)}

(iii) Given: B = {1, 3} and C = {3, 5}

To find: B × C

By the definition of the Cartesian product,

Given two non – empty sets P and Q. The Cartesian product P × Q is the set of all ordered pairs of elements from P and Q, .i.e.

P × Q = {(p, q) : p Є P, q Є Q}

Here, B = {1, 3} and C = {3, 5}. So,

B × C = (1, 3) × (3, 5)

= {(1, 3), (1, 5), (3, 3), (3, 5)}

(iv) Given: A = {-3, -1}

From part (iii), we get B × C = {(1, 3), (1, 5), (3, 3), (3, 5)}

So,

A × (B × C) = {-3, -1} × {(1, 3), (1, 5), (3, 3), (3, 5)}

= (-3, 1, 3), (-3, 1, 5), (-3, 3, 3), (-3, 3, 5), (-1, 1, 3), (-1, 1, 5), (-1, 3, 3), (-1, 3, 5)}

Exercise 2B

Q. 1 A. For any sets A, B and C prove that:

A × (B ∪ C) = (A × B) ∪ (A × C)

Answer : Given: A, B and C three sets are given.

Need to prove: A × (B ∪ C) = (A × B) ∪ (A × C)

Let us consider, (x, y) ∈ A × (B ∪ C)

⇒ x ∈ A and y ∈ (B ∪ C)

$\Rightarrow x \in A$ and ($y \in B$ or $y \in C$)

$\Rightarrow$ ($x \in A$ and $y \in B$) or ($x \in A$ and $y \in C$)

$\Rightarrow (x, y) \in (A \times B)$ or $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$From this we can conclude that,

$\Rightarrow A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ---- (1)

Let us consider again, $(a, b) \in (A \times B) \cup (A \times C)$

$\Rightarrow (a, b) \in (A \times B)$ or $(a, b) \in (A \times C)$

$\Rightarrow$ ($a \in A$ and $b \in B$) or ($a \in A$ and $b \in C$)

$\Rightarrow a \in A$ and ($b \in B$ or $b \in C$)

$\Rightarrow a \in A$ and $b \in (B \cup C)$

$\Rightarrow (a, b) \in A \times (B \cup C)$

From this, we can conclude that,

$\Rightarrow (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ---- (2)

Now by the definition of the set we can say that, from (1) and (2),

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ [Proved]

Q. 1 B. For any sets A, B and C prove that:

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Answer : Given: A, B and C three sets are given.

Need to prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let us consider, $(x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and ($y \in B$ and $y \in C$)

$\Rightarrow$ ($x \in A$ and $y \in B$) and ($x \in A$ and $y \in C$)

$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

From this we can conclude that,

$\Rightarrow A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ---- (1)

Let us consider again, $(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow (a, b) \in (A \times B)$ and $(a, b) \in (A \times C)$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow a \in A$ and $b \in (B \cap C)$

$\Rightarrow (a, b) \in A \times (B \cap C)$

From this, we can conclude that,

$\Rightarrow (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ---- (2)

Now by the definition of the set we can say that, from (1) and (2),

$A \times (B \cap C) = (A \times B) \cap (A \times C)$ [Proved]

Q. 1 C. For any sets A, B and C prove that:

$A \times (B - C) = (A \times B) - (A \times C)$

Answer : Given: A, B and C three sets are given.

Need to prove: $A \times (B - C) = (A \times B) - (A \times C)$

Let us consider, $(x, y) \in A \times (B - C)$

$\Rightarrow x \in A$ and $y \in (B - C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C)$

⇒ $(x, y) \in (A \times B)$ and $(x, y) \notin (A \times C)$

⇒ $(x, y) \in (A \times B) – (A \times C)$

From this we can conclude that,

⇒ $A \times (B – C) \subseteq (A \times B) – (A \times C)$ ---- (1)

Let us consider again, $(a, b) \in (A \times B) – (A \times C)$

⇒ $(a, b) \in (A \times B)$ and $(a, b) \notin (A \times C)$

⇒ ($a \in A$ and $b \in B$) and ($a \in A$ and $b \notin C$)

⇒ $a \in A$ and ($b \in B$ and $b \notin C$)

⇒ $a \in A$ and $b \in (B – C)$

⇒ $(a, b) \in A \times (B \cup C)$

From this, we can conclude that,

⇒ $(A \times B) – (A \times C) \subseteq A \times (B – C)$ ---- (2)

Now by the definition of set we can say that, from (1) and (2),

$A \times (B – C) = (A \times B) – (A \times C)$ [Proved]

Q. 2. For any sets A and B, prove that

$(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$.

Answer : Given: A and B two sets are given.

Need to prove: $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$

Let us consider, $(x, y) \in (A \times B) \cap (B \times A)$

⇒ $(x, y) \in (A \times B)$ and $(x, y) \in (B \times A)$

⇒ ($x \in A$ and $y \in B$) and ($x \in B$ and $y \in A$)

⇒ (x $\in$ A and x $\in$ B) and (y $\in$ B and y $\in$ A)

⇒ x $\in$ (A × B) and y $\in$ (B × A)

⇒ (x, y) $\in$ (A × B) ∩ (B × A)

From this, we can conclude that,

⇒ (A × B) ∩ (B × A) ⊆ (A ∩ B) × (B ∩ A) ---- (1)

Let us consider again, (a, b) $\in$ (A ∩ B) × (B ∩ A)

⇒ a $\in$ (A ∩ B) and b $\in$ (B ∩ A)

⇒ (a $\in$ A and a $\in$ B) and (b $\in$ B and b $\in$ A)

⇒ (a $\in$ A and b $\in$ B) and (a $\in$ B and b $\in$ A)

⇒ (a, b) $\in$ (A × B) and (a, b) $\in$ (B × A)

⇒ (a, b) $\in$ (A × B) ∩ (B × A)

From this, we can conclude that,

⇒ (A ∩ B) × (B ∩ A) ⊆ (A × B) ∩ (B × A) ---- (2)

Now by the definition of set we can say that, from (1) and (2),

(A × B) ∩ (B × A) = (A ∩ B) × (B ∩ A) [Proved]

Q. 3. If A and B are nonempty sets, prove that

A × B = B × A ⇔ A = B

Answer : Given: A = B, where A and B are nonempty sets.

Need to prove: A × B = B × A

Let us consider, (x, y) $\in$ (A × B)

That means, x $\in$ A and y $\in$ B

As given in the problem A = B, we can write,

⇒ x $\in$ B and y $\in$ A

⇒ (x, y) $\in$ (B × A)

That means, $(A \times B) \subseteq (B \times A)$ ---- (1)

Similarly we can prove,

$\Rightarrow (B \times A) \subseteq (A \times B)$ ---- (2)

So, by the definition of set we can say from (1) and (2),

$A \times B = B \times A$ [Proved]

Q. 4. (i) If $A \subseteq B$, prove that $A \times C \subseteq B \times C$ for any set C.

(ii) If $A \subseteq B$ and $C \subseteq D$ then prove that $A \times C \subseteq B \times D$.

Answer : (i) Given: $A \subseteq B$

Need to prove: $A \times C \subseteq B \times C$

Let us consider, $(x, y) \in (A \times C)$

That means, $x \in A$ and $y \in C$

Here given, $A \subseteq B$

That means, x will surely be in the set B as A is the subset of B and $x \in A$.

So, we can write $x \in B$

Therefore, $x \in B$ and $y \in C \Rightarrow (x, y) \in (B \times C)$

Hence, we can surely conclude that,

$A \times C \subseteq B \times C$ [Proved]

(ii) Given: $A \subseteq B$ and $C \subseteq D$

Need to prove: $A \times C \subseteq B \times D$

Let us consider, $(x, y) \in (A \times C)$

That means, $x \in A$ and $y \in C$

Here given, $A \subseteq B$ and $C \subseteq D$

So, we can say, $x \in B$ and $y \in D$

$(x, y) \in (B \times D)$

Therefore, we can say that, $A \times C \subseteq B \times D$ [Proved]

Q. 5. If A × B ⊆ C × D and A × B ≠ ϕ, prove that A ⊆ C and B ⊆ D.

Answer : Given: A × B ⊆ C × D and A × B ≠ ϕ

Need to prove: A ⊆ C and B ⊆ D

Let us consider, (x, y) ∈ (A × B) ---- (1)

⇒ (x, y) ∈ (C × D) [as A × B ⊆ C × D] ---- (2)

From (1) we can say that,

x ∈ A and y ∈ B ---- (a)

From (2) we can say that,

x ∈ C and y ∈ D ---- (b)

Comparing (a) and (b) we can say that,

⇒ x ∈ A and x ∈ C

⇒ A ⊆ C

Again,

⇒ y ∈ B and y ∈ D

⇒ B ⊆ D [Proved]

Q. 6. If A and B be two sets such that n(A) = 3, n(B) = 4 and n(A ∩ B) = 2 then find.

(i) n(A × B)

(ii) n(B × A)

(iii) n(A × B) ∩ (B × A)

Answer : Given: n(A) = 3, n(B) = 4 and n(A ∩ B) = 2

(i) n(A × B) = n(A) × n(B)

⇒ n(A × B) = 3 × 4

⇒ n(A × B) = 12

(ii) n(B × A) = n(B) × n(A)

⇒ n(B × A) = 4 × 3

⇒ n(B × A) = 12

(iii) n((A × B) ∩ (B × A)) = n(A × B) + n(B × A) – n((A × B) ∪ (B × A))

n((A × B) ∩ (B × A)) = n(A × B) + n(B × A) – n(A × B) + n(B × A)

n((A × B) ∩ (B × A)) = 0

Q. 7. For any two sets A and B, show that A × B and B × A have an element in common if and only if A and B have an element in common.

Answer : We know,

(A × B) ∩ (B × A) = (A ∩ B) × (B ∩ A)

Here A and B have an element in common i.e., n(A ∩ B) = 1 = (B ∩ A)

So, n((A × B) ∩ (B × A)) = n((A ∩ B) × (B ∩ A)) = n(A ∩ B) × n(B ∩ A) = 1 × 1 = 1

That means, A × B and B × A have an element in common if and only if A and B have an element in common. [Proved]

Q. 8. Let A = {1, 2} and B = {2, 3}. Then, write down all possible subsets of A × B.

Answer : Given: A = {1, 2} and B = {2, 3}

Need to write: All possible subsets of A × B

A = {1, 2} and B = {2, 3}

So, all the possible subsets of A × B are:

(A × B) = {(x, y): x ∈ A and y ∈ B}

= {(1, 2), (1, 3), (2,2), (2,3)}

Q. 9. Let A = {a, b, c, d}, B = {c, d, e} and C = {d, e, f, g}. Then verify each of the following identities:

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × (B – C) = (A × B) – (A × C)

(iii) (A × B) ∩ (B × A) = (A ∩ B) × (A ∩ B)

Answer : Given: A = {a, b, c, d,}, B = {c, d, e} and C = {d, e, f, g}

(i) Need to prove: A × (B ∩ C) = (A × B) ∩ (A × C)

Left hand side,

(B ∩ C) = {d, e}

⇒ A × (B ∩ C) = {(a, d), (a, e), (b, d), (b, e), (c, d), (c, e), (d, d), (d, e)}

Right hand side,

$(A \times B) = \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, c), (d, d), (d, e)\}$

$(A \times C) = \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g), (d, d), (d, e), (d, f), (d, g)\}$

Now,

$(A \times B) \cap (A \times C) = \{(a, d), (a, e), (b, d), (b, e), (c, d), (c, e), (d, d), (d, e)\}$

Here, right hand side and left hand side are equal.

That means, $A \times (B \cap C) = (A \times B) \cap (A \times C)$ [Proved]

(ii) Need to prove: $A \times (B - C) = (A \times B) - (A \times C)$

Left hand side,

$(B - C) = \{c\}$

$\Rightarrow A \times (B - C) = \{(a, c), (b, c), (c, c), (d, c)\}$

Right hand side,

$(A \times B) = \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, c), (d, d), (d, e)\}$

$(A \times C) = \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g), (d, d), (d, e), (d, f), (d, g)\}$

Therefore, $(A \times B) - (A \times C) = \{(a, c), (b, c), (c, c), (d, c)\}$

Here, right hand side and left hand side are equal.

That means, $A \times (B - C) = (A \times B) - (A \times C)$ [Proved]

(iii) Need to prove:

$(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$

Left hand side,

$(A \times B) = \{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e), (c, c), (c, d), (c, e), (d, c), (d, d), (d, e)\}$

$(B \times A) = \{(c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d), (e, a), (e, b), (e, c), (e, d)\}$

Now, $(A \times B) \cap (B \times A) = \{(c, c), (c, d), (d, c), (d, d)\}$

Right hand side,

$(A \cap B) = \{c, d\}$

So, $(A \cap B) \times (A \cap B) = \{(c, c), (c, d), (d, c), (d, d)\}$

Here, right hand side and left hand side are equal.

That means, $(A \times B) \cap (B \times A) = (A \cap B) \times (A \cap B)$ [Proved]

Exercise 2C

Q. 1. Let A and B be two nonempty sets.

(i) What do you mean by a relation from A to B?
(ii) What do you mean by the domain and range of a relation?

Answer : (i) If A and B are two nonempty sets, then any subset of the set (A × B) is said to a relation R from set A to set B.

That means, if R be a relation from A to B then $R \subseteq (A \times B)$.

Therefore, $(x, y) \in R \Rightarrow (x, y) \in (A \times B)$

That means x is in relation to y. Or we can write xRy.

(ii) Let R be a relation from A to B. Then, the set containing all the first elements of the ordered pairs belonging to R is called Domain.

For the relation R, $Dom(R) = \{x: (x, y) \in R\}$

And the set containing all the second elements of the ordered pair belonging to R is called Range.

For the relation R, $Range(R) = \{y: (x, y) \in R\}$

Q. 2. Find the domain and range of each of the relations given below:

(i) R = {(–1, 1), (1, 1), (–2, 4), (2, 4), (2, 4), (3, 9)}

(ii) $R = \left\{ \left(x, \frac{1}{x} \right) : x \text{ is an interger}, 0 < x < 5 \right\}$

(iii) R ={(x, y) : x + 2y = 8 and x, y ϵ N}

(iv) R = {(x, y), : y = |x – 1|, x ϵ Z and |x| ≤ 3}

Answer : (i) Given: R = {(–1, 1), (1, 1), (–2, 4), (2, 4), (2, 4), (3, 9)}

Dom(R) = {x: (x, y) ∈ R} = {-2, -1, 1, 2, 3}

Range(R) = {y: (x, y) ∈ R} = {1, 4, 9}

(ii) Given:

$$R = \left\{ \left(x, \frac{1}{x} \right) : x \text{ is an interger}, 0 < x < 5 \right\}$$

That means,

$$R = \{(1,1), (2, \frac{1}{2}), (3, \frac{1}{3}), (4, \frac{1}{4})\}$$

Dom(R) = {x: (x, y) ∈ R} = {1, 2, 3, 4}

$\text{Range}(R) = \left\{ y: (x, y) \in R \right\} = \left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \right\}$

(iii) Given: R = {(x, y): x + 2y = 8 and x, y ϵ N}

That means, R = {(2, 3), (4, 2), (6, 1)}

Dom(R) = {x: (x, y) ∈ R} = {2, 4, 6}

Range(R) = {y: (x, y) ∈ R} = {1, 2, 3}

(iv) Given: R = {(x, y): y = |x – 1|, x ϵ Z and |x| ≤ 3}

Dom(R) = {x: (x, y) ∈ R} = {-3, -2, -1, 0, 1, 2, 3}

Range(R) = {y: (x, y) ∈ R} = {0, 1, 2, 3, 4}

Q. 3. Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8}.

Let R = {(x, y), : x ϵ A, y ϵ B and x > y}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

(iii) Depict R by an arrow diagram.

Answer : Given: A = {1, 3, 5, 7} and B = {2, 4, 6, 8}

(i) R = {(x, y), : x ϵ A, y ϵ B and x > y}

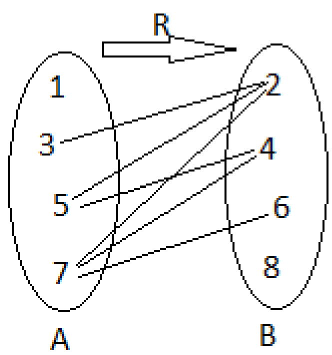
So, R in Roster Form,

R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)}

(ii) Dom(R) = {3, 5, 7}

Range(R) = {2, 4, 6}

(iii)



Q. 4. Let A = {2, 4, 5, 7} and b = {1, 2, 3, 4, 5, 6, 7, 8}.

Let R = {(x, y) x ϵ A, y ϵ B and x divides y}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

Answer : Given: A = {2, 4, 5, 7} and b = {1, 2, 3, 4, 5, 6, 7, 8}

(i) R = {(x, y) x ϵ A, y ϵ B and x divides y}

So, R in Roster Form,

R = {(2, 2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8), (5, 5), (7, 7)}

(ii) Dom(R) = {2, 4, 5, 7}

Range(R) = {2, 4, 5, 7, 6, 7, 8}

Q. 5. Let A = {2, 3, 4, 5} and B = {3, 6, 7, 10}.

Let R = {(x, y): x ϵ A, y ϵ B and x is relatively prime to y}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

Answer : Given: A = {2, 3, 4, 5} and B = {3, 6, 7, 10}

(i) R = {(x, y), : x ϵ A, y ϵ B and x is relatively prime to y}

So, R in Roster Form,

R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}

(ii) Dom(R) = {2, 3, 4, 5}

Range(R) = {3, 6, 7, 10}

Q. 6. Let A = {1, 2, 3, 5} AND B = {4, 6, 9}.

Let R = {(x, y): x ϵ A, y ϵ B and (x – y) is odd}.

Write R in roster form.

Answer : Given: A = {1, 2, 3, 5} AND B = {4, 6, 9}

R = {(x, y): x ϵ A, y ϵ B and (x – y) is odd}

Therefore, R in Roster Form is,

R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

Q. 7. Let A = {(x, y): x + 3y = 12, x ϵ N and y ϵ N}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

Answer : Given: A = {(x, y): x + 3y = 12, x ϵ N and y ϵ N}

(i) So, R in Roster Form is,

R = {(3, 3), (6, 2), (9, 1)}

(ii) Dom(R) = {3, 6, 9}

Range(R) = {1, 2, 3}

Q. 8. Let A = {1, 2, 3, 4, 5, 6}.

Define a relation R from A to A by R = {(x, y): y = x + 1}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

(iii) What is its co-domain?

(iv) Depict R by using arrow diagram.

Answer : Given: A = {1, 2, 3, 4, 5, 6}

(i) R = {(x, y): y = x + 1}

So, R is Roster Form is,

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

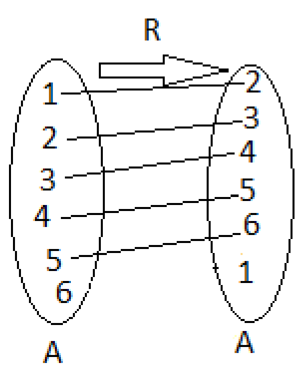
(ii) Dom(R) = {1, 2, 3, 4, 5}

Range(R) = {2, 3, 4, 5, 6}

(iii) Here, y = x + 1

So, the CoD(R) = {1, 2, 3, 4, 5, 6,………… }

(iv)



Q. 9. Let R = {(x, x + 5): x ϵ {9, 1, 2, 3, 4, 5}}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

Answer : Given: R = {(x, x + 5): x ϵ {9, 1, 2, 3, 4, 5}}

(i) R is Foster Form is,

R = {(9, 14), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

(ii) Dom(R) = {1, 2, 3, 4, 5, 9}

Range(R) = {6, 7, 8, 9, 10, 14}

Q. 10. Let A = {1, 2, 3, 4, 6} and R = {(a, b) : a, b ϵ A, and a divides b}.

(i) Write R in roster form.

(ii) Find dom (R) and range (R).

Answer : Given: A = {1, 2, 3, 4, 6}

(i) R = {(a, b) : a, b ϵ A, and a divides b}

R is Foster Form is,

R = {(1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Dom(R) = {1, 2, 3, 4, 6}

Range(R) = {2, 3, 4, 6}

Q. 11. Define a relation R from Z to Z, given by

R = {(a, b): a, b ϵ Z and (a – b) is an integer.

Find dom (R) and range (R).

Answer : Given: R = {(a, b): a, b ϵ Z and (a – b) is an integer

The condition satisfies for all the values of a and b to be any integer.

So, R = {(a, b): for all a, b ϵ (-∞, ∞)}
Dom(R) = {-∞, ∞}

Range(R) = {-∞, ∞}

Q. 12. Let R = {(x, y): x, y ϵ Z and $x^2 + y^2 \leq 4$}.

(i) Write R in roster form.
(ii) Find dom (R) and range (R).

Answer : Given: R = {(x, y): x, y ϵ Z and $x^2 + y^2 \leq 4$}

(i) R is Foster Form is,

R = {(-2, 0), (-1, -1), (-1, 0), (-1, 1), (0, -2), (0, -1), (0, 0), (0, 1), (0, 2), (1, -1), (1, 0), (1, 1), (2, 0)}

(ii) Dom(R) = {-2, -1, 0, 1, 2}

Range(R) = {-2, -1, 0, 1, 2}

Q. 13. Let A = {2, 3} and B= {3, 5}

(i) Find (A × B) and n(A × B).
(ii) How many relations can be defined from A to B?

Answer : Given: A = {2, 3} and B= {3, 5}

(i) (A × B) = {(2, 3), (2, 5), (3, 3), (3, 5)}

Therefore, n(A × B) = 4

(ii) No. of relation from A to B is a subset of Cartesian product of (A × B).

Here no. of elements in A = 2 and no. of elements in B = 2.

So, (A × B) = 2 × 2 = 4

So, the total number of relations can be defined from A to B is

$= 2^4 = 16$

Q. 14. Let A = {3, 4} and B = {7, 9}. Let R = {(a, b): a ϵ A, b ϵ B and (a – b) is odd}.

Show that R is an empty relation from A to B.

Answer : Given: A = {3, 4} and B = {7, 9}

R = {(a, b): a ϵ A, b ϵ B and (a – b) is odd}

So, R = {(4, 7), (4, 9)}

An empty relation means there is no elements in the relation set.

Here we get two relations which satisfy the given conditions.

Therefore, the given relation is not an Empty Relation.

The given relation would be an Empty Relation if,

1) A = {3} or,

2) A = {3, any odd number} or,

Exercise 2D

Q. 1. What do you mean by a binary relation on a set A? Define the domain and range of relation on A.

Answer : Any subset of (A × A) is called a binary relation to A. Here, (A × A) is the cartesian product of A with A.

Let A = {4, 5, 6) and R = {(4, 5), (6, 4), (5, 6)}

Here, R is a binary relation to A.

The domain of R is the set of first co-ordinates of R

Dom(R) = {4, 6, 5}

The range of R is the set of second co-ordinates of R

Range(R) = {5, 4, 6}

Q. 2. Let A = {2, 3, 5} and R = {(2, 3), (2, 5), (3, 3), (3, 5)}. Show that R is a binary relation on A. Find its domain and range.

Answer : First, calculate A×A.

A×A = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

Since, R is a subset of A × A, it's a binary relation on A.

The domain of R is the set of first co-ordinates of R

Dom(R) = {2, 3}

The range of R is the set of second co-ordinates of R

Range(R) = {3, 5}

Q. 3. Let A = {0, 1, 2, 3, 4, 5, 6, 7, 8} and let R = {(a, b) : a, b ϵ A and 2a + 3b = 12}.

Express R as a set of ordered pairs. Show that R is a binary relation on A. Find its domain and range.

Answer : A = {0, 1, 2, 3, 4, 5, 6, 7, 8}

2a + 3b = 12

$$b = \frac{12 - 2a}{3}$$

a=0 è b=4

a=3 è b=2

a=6 è b=0

R = {(0, 4), (3, 2), (6, 0)}

Since, R is a subset of A × A, it a relation to A.

The domain of R is the set of first co-ordinates of R

Dom(R) = {0, 3, 6}

The range of R is the set of second co-ordinates of R

Range(R) = {4, 2, 0}

Q. 4. If R is a binary relation on a set A define R^{-1} on A.

Let R = {(a, b) : a, b ϵ W and 3a + 2b = 15} and 3a + 2b = 15}, where W is the set of whole numbers.
Express R and R^{-1} as sets of ordered pairs.
Show that (i) dom (R) = range (R^{-1}) (ii) range (R) = dom (R^{-1})

Answer : 3a + 2b = 15

$$b = \frac{15 - 3a}{2}$$

a=1 è b=6

a=3 è b=3

a=5 è b=0

R = {(1, 6), (3, 3), (5, 0)}

R^{-1} = {(6, 1), (3, 3), (0, 5)}

The domain of R is the set of first co-ordinates of R

Dom(R) = {1, 3, 5}

The range of R is the set of second co-ordinates of R

Range(R) = {6, 3, 0}

The domain of R^{-1} is the set of first co-ordinates of R^{-1}

Dom(R^{-1}) = {6, 3, 0}

The range of R^{-1} is the set of second co-ordinates of R^{-1}

Range(R^{-1}) = {1, 3, 5}

Thus,

dom (R) = range (R^{-1})

range (R) = dom (R^{-1})

Q. 5. What is an equivalence relation?

Show that the relation of 'similarity' on the set S of all triangles in a plane is an equivalence relation.

Answer : An equivalence relation is one which possesses the properties of reflexivity, symmetry and transitivity.

(i) Reflexivity: A relation R on A is said to be reflexive if (a, a) є R for all a є A.

(ii) Symmetry: A relation R on A is said to be symmetrical if (a,b) є R è(b, a) є R

for all (a, b) є A.

(iii) Transitivity: A relation R on A is said to be transitive if (a, b) є R and (b, c) є R è (a, c) є R for all (a, b, c) є A.

Let S be a set of all triangles in a plane.

(i) Since every triangle is similar to itself, it is reflexive.

(ii) If one triangle is similar to another triangle, it implies that the other triangle is also similar to the first triangle. Hence, it is symmetric.

(iii) If one triangle is similar to a triangle and another triangle is also similar to that triangle, all the three triangles are similar. Hence, it is transitive.

Q. 6. Let R = {(a, b) : a, b ϵ Z and (a – b) is even}.

Then, show that R is an equivalence relation on Z.

Answer : (i) Reflexivity: Let a ϵ Z, a - a = 0 ϵ Z which is also even.

Thus, (a, a) ϵ R for all a ϵ Z. Hence, it is reflexive

(ii) Symmetry: Let (a, b) ϵ R

(a, b) ϵ R è a - b is even

-(b - a) is even

(b - a) is even

(b, a) ϵ R

Thus, it is symmetric

(iii) Transitivity: Let (a, b) ϵ R and (b, c) ϵ R

Then, (a – b) is even and (b – c) is even.

[(a - b) + (b - c)] is even

(a - c) is even.

Thus (a, c) ϵ R.

Hence, it is transitive.

Since, the given relation possesses the properties of reflexivity, symmetry and transitivity, it is an equivalence relation.

Q. 7. Let A = {1, 2, 3} and R = {(a, b) : a, b ϵ A and $|a^2 – b^2| \leq 5$.

Write R as a set of ordered pairs.

Mention whether R is (i) reflexive (ii) symmetric (iii) transitive. Give reason in each case.

Answer : Put a = 1 , b = 1 $|1^2 – 1^2| \le 5$, (1, 1) is an ordered pair.

Put a = 1 , b = 2 $|1^2 – 2^2| \le 5$, (1, 2) is an ordered pair.

Put a = 1 , b = 3 $|1^2 – 3^2| > 5$, (1, 3) is not an ordered pair.

Put a = 2 , b = 1 $|2^2 – 1^2| \le 5$, (2, 1) is an ordered pair.

Put a = 2 , b = 2 $|2^2 – 2^2| \le 5$, (2, 2) is an ordered pair.

Put a = 2 , b = 3 $|2^2 – 3^2| \le 5$, (2, 3) is an ordered pair.

Put a = 3 , b = 1 $|3^2 – 1^2| > 5$, (3, 1) is not an ordered pair.

Put a = 3 , b = 2 $|3^2 – 2^2| \le 5$, (3, 2) is an ordered pair.

Put a = 3 , b = 3 $|3^2 – 3^2| \le 5$, (3, 3) is an ordered pair.

R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

(i) For (a, a) є R

$|a^2 – a^2| = 0 \le 5$. Thus, it is reflexive.

(ii) Let (a, b) є R

(a, b) є R è $|a^2 – b^2| \le 5$

$|b^2 – a^2| \le 5$

(b, a) є R

Hence, it is symmetric

(iii) Put a = 1 , b = 2 , c = 3.

$|1^2 – 2^2| \le 5$

$|2^2 – 3^2| \le 5$

But $|1^2 – 3^2| > 5$

Thus, it is not transitive.

Q. 8. Let R = {(a, b) : a, b ϵ Z and b = 2a – 4}. If (a, –2} ϵ R and (4, b^2) ϵ R. Then, write the values of a and b.

Answer : b = 2a – 4

$$a = \frac{b+4}{2}$$

Put b = -2 , a = 1

Put a = 4 , b = 4

a = 1 , b = 4

Q. 8. Let R = {(a, b) : a, b ϵ Z and b = 2a – 4}. If (a, –2} ϵ R and (4, b^2) ϵ R. Then, write the values of a and b.

Answer : b = 2a – 4

$$a = \frac{b+4}{2}$$

Put b = -2 , a = 1

Put a = 4 , b = 4

a = 1 , b = 4

Q. 9. Let R be a relation on Z, defined by (x, y) ϵ R ↔ $x^2 + y^2 = 9$. Then, write R as a set of ordered pairs. What is its domain?

Answer : $x^2 + y^2 = 9$

We can have only integral values of x and y.

Put x = 0 , y = 3 , $0^2 + 3^2 = 9$

Put x = 3 , y = 0 , $3^2 + 0^2 = 9$

R = {(0, 3) , (3, 0) , (0 , -3) , (-3 , 0)}

The domain of R is the set of first co-ordinates of R

Dom(R) = {-3 , 0 , 3}

The range of R is the set of second co-ordinates of R

Range(R) = {-3 , 0 , 3}

Q. 10. Let A be the set of first five natural numbers and let R be a relation on A, defined by (x, y) ϵ R ↔ x ≤ y.

Express R and R^{-1} as sets of ordered pairs.
Find: dom (R^{-1}) and range (R).

Answer : A = {1, 2, 3, 4, 5}

Since, x ≤ y

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3) ,(2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5) }

The domain of R is the set of first co-ordinates of R

Dom(R) = {1, 2, 3, 4, 5}

The range of R is the set of second co-ordinates of R

Range(R) = {1, 2, 3, 4, 5}

Q. 11. Let R = (x, y) : x, y ϵ Z and $x^2 + y^2 = 25$}.

Express R and R^{-1} as sets of ordered pairs. Show that R = R^{-1}.

Answer : $x^2 + y^2 = 25$

Put x = 0, y = 5, $0^2 + 5^2 = 25$

Put x = 3, y = 4, $3^2 + 4^2 = 25$

R = {(0, 5), (0, -5), (5, 0), (-5, 0), (3, 4), (-3, 4), (-3, -4), (3, -4)}

Since, x and y get interchanged in the ordered pairs, R and R^{-1}

are same.

Q. 12. Find R^{-1}, when

(i) R = {(1, 2), (1, 3), (2, 3), (3, 2), (4, 5)}
(ii) R = {(x, y) : x, y ϵ N, x + 2y = 8}.

Answer : (i) R = {(1, 2), (1, 3), (2, 3), (3, 2), (4, 5)}

R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (5, 4)}

(ii) R = {(x, y) : x, y ∈ N, x + 2y = 8}.

$$y = \frac{8 - x}{2}$$

Put x = 2, y = 3

Put x = 4, y = 2

Put x = 6, y = 1

R = {(2, 3), (4, 2), (6, 1)}

R^{-1} = {(3, 2), (2, 4), (1, 6)}

Q. 13. Let A = {a, b}. List all relation on A and find their number.

Answer : Any relation on A is a subset of A×A.

A×A = {(a, a), (a, b), (b, a), (b, b)}

The subsets are.

{} empty set

{(a, a)}

{(a, b)}

{(a, a), (a, b)}

{(b, a)}

{(b, b)}

{(b, a), (b, b)}

{(a, a), (b, a)}

{(a, b), (b, a)}

{(a, a), (b, a), (b, b)}

{(a, a), (b, b)}

{(a, a), (a, b), (b, a)}

{(a, a), (a, b), (b, b)}

{(a, b), (b, a), (b, b)}

{(a, a), (a, b), (b, a), (b, b)}

Thus, there are 16 total relations.

Q. 14. Let R = {(a, b) : a, b, ϵ N and a < b}.

Show that R is a binary relation on N, which is neither reflexive nor symmetric. Show that R is transitive.

Answer : N is the set of all the natural numbers.

N = {1, 2, 3, 4, 5, 6, 7…..}

R = {(a, b) : a, b, ϵ N and a < b}

R = {(1, 2), (1, 3), (1, 4) …. (2, 3), (2, 4), (2, 5) ……}

For reflexivity,

A relation R on N is said to be reflexive if (a, a) ϵ R for all a ϵ N.

But, here we see that a < b, so the two co-ordinates are never equal. Thus, the relation is not reflexive.

For symmetry,

A relation R on N is said to be symmetrical if (a, b) ϵ R è(b, a) ϵ R

Here, (a, b) ϵ R does not imply (b, a) ϵ R . Thus, it is not symmetric.

For transitivity,

A relation R on A is said to be transitive if (a, b) ϵ R and (b, c) ϵ R è (a, c) ϵ R for all (a, b, c) ϵ N.

Let's take three values a, b and c such that a < b < c. So, (a, b) ϵ R and (b, c) ϵ R è (a, c) ϵ R. Thus, it is transitive.

Exercise 2E

Q. 1. Let A and B be two sets such that $n(A) = 5$, $n(B) = 3$ and $n(A \cap B) = 2$.

(i) $n(A \cup B)$

(ii) $n(A \times B)$

(iii) $n(A \times B) \cap (B \times A)$

Answer : (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 5 + 3 – 2$

$=6$

(ii) $n(A \times B) = n(A) \times n(B)$

$= 5 \times 3$

$= 15$

(iii) $n(A \times B) \cap (B \times A) = n(A \times B) + n(B \times A) –$

Q. 3. If A = {1, 2}, find A × A × A.

Answer : $A = \{1, 2\}$

$A \times A = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

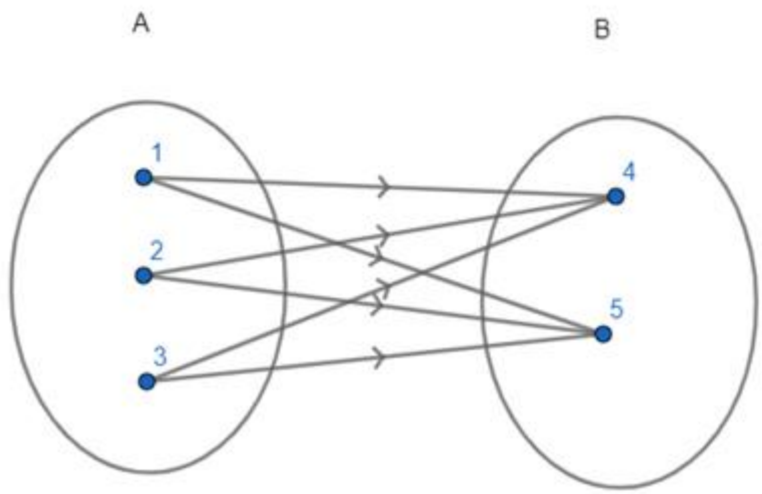
$A \times A \times A = \{1, 2\} \times \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

Therefore

$A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)\}$

Q. 4. If A = {2, 3, 4} and B = {4, 5}, draw an arrow diagram represent (A × B).

Answer :



Q. 5. If A = {3, 4}, B = {4, 5} and C = {5, 6}, find A × (B × C).

Answer : A = {3, 4}, B = {4, 5} and C = {5, 6}

B × C = {(4, 5), (4, 6), (5, 5), (5, 6)}

A × (B × C) = {(3, 4, 5), (3, 4, 6), (3, 5, 5), (3, 5, 6), (4, 4, 5), (4, 4, 6), (4, 5, 5), (4, 5, 6)}

Q. 6. If A ⊆ B, prove that A × C = B × C

Answer : Given: A ⊆ B

Then, A = B at some value

Multiplying by C both sides, we get,

A × C = B × C

Hence, Proved.

Q. 7. Prove that A × B = B × A ⇒ A = B.

Answer : Let A and B be any two sets such that

A × B = {(a, b): a ϵ A, b ϵ B}

Now,

B × A = {(b, a): a ϵ A, b ϵ B}

$A \times B = B \times A$

(a, b) = (b, a)

We can see that this is possible only when the ordered pairs are equal.

Therefore,

a = b and b = a

Hence, Proved.

Q. 8. If A = {5} and B = {5, 6}, write down all possible subsets of A × B.

Answer : A = {5}

B = {5, 6}

A × B = {(5, 5), (5, 6)}

All the possible subsets of A × B are,

{}

{(5, 5)}

{(5, 6)}

{(5, 6), (5, 6)}

Q. 9. Let R = {(x, x^2) : x is a prime number less than 10}.

(i) Write R in roster form.
(ii) Find dom (R) and range (R).

Answer : i) {(x, x^2) : x is a prime number less than 10}.

Roster form: R = {(1, 1), (2, 4), (3, 9), (5, 25), (7, 49)}

ii) The domain of R is the set of first co-ordinates of R

Dom(R) = {1, 2, 3, 5, 7}

The range of R is the set of second co-ordinates of R

Range(R) = {1, 4, 9, 25, 49}

Q. 10. Let A = (1, 2, 3} and B = {4}

How many relations can be defined from A to B.

Answer : The number of relations from set A to set B =

$2^{n(A) \times n(B)}$

n(A) = Number of elements in set A

n(B) = Number of elements in set B

Here,

n(A) = 3

n(B) = 1

Total number of relations =

$2^{3 \times 1}$

= 8

Q. 11. Let A = {3, 4, 5, 6} and R = {(a, b) : a, b ϵ A and a <b

(i) Write R in roster form.
(ii) Find: dom (R) and range (R)
(iii) Write R^{-1} in roster form

Answer : (i) R = {(3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)}

(ii) The domain of R is the set of first co-ordinates of R

Dom(R) = {3, 4, 5}

The range of R is the set of second co-ordinates of R

Range(R) = {4, 5, 6}

(iii) R^{-1} = {(4, 3), (5, 3), (6, 3), (5, 4), (6, 4), (6, 5)}

Q. 12. Let R = {(a, b) : a, b, ϵ N and a < b}.

Show that R is a binary relation on N, which is neither reflexive nor symmetric. Show that R is transitive.

Answer : N is the set of all the natural numbers.

N = {1, 2, 3, 4, 5, 6, 7…..}

R = {(a, b) : a, b, ϵ N and a < b}

R = {(1, 2), (1, 3), (1, 4) …. (2, 3), (2, 4), (2, 5) ……}

For reflexivity,

A relation R on N is said to be reflexive if (a, a) є R for all a є N.

But, here we see that a < b, so the two co-ordinates are never equal. Thus, the relation is not reflexive.

For symmetry,

A relation R on N is said to be symmetrical if (a, b) є R è(b, a) є R

Here, (a, b) є R does not imply (b, a) є R. Thus, it is not symmetric.

For transitivity,

A relation R on A is said to be transitive if (a, b) є R and (b, c) є R è (a, c) є R for all (a, b, c) є N.

Let's take three values a, b and c such that a < b < c. So, (a, b) є R and (b, c) є R è (a, c) є R. Thus, it is transitive.