Functions

Exercise 3A

Q. 1. Define a function as a set of ordered pairs.

Answer : Function as a set of ordered pairs: A function is a set of ordered pairs with the property that no two ordered pairs have the same first component and a different second component.

The domain of a function is the set of all first components, x, in the ordered pairs and the range of a function is the set of all second components, y, in the ordered pairs.

For. e.g. {(1,x), (2,y), (3,z)} is a function, since there are no two pairs with the same first component.

Here, Domain is {1, 2, 3} and Range is {x, y, z}

Q. 2. Define a function as a correspondence between two sets.

Answer : Function as a correspondence between two sets: Let A and B be two non – empty sets. Then, a function 'f' from set A to set B is a correspondence (rule) which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B.

Q. 3. What is the fundamental difference between a relation and function? Is every relation a function?

Answer :

Fundamental difference between Relation and Function:

Every function is a relation, but every relation need not be a function.

A relation f from A to B is called a function if

(i) Dom(f) = A

(ii) no two different ordered pairs in f have the same first component.

For. e.g.

Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5}

Some relations f, g and h are defined as follows:

f = {(a, 1), (b, 2), (c, 3), (d, 4)}

g = {(a, 1), (b, 3), (c, 5)}

h = {(a, 1), (b, 2), (b, 3), (c, 4), (d, 5)}

In the relation f,

f = {(a, 1), (b, 2), (c, 3), (d, 4)}

(i) Dom (f) = A

(ii) All first components are different.

So, f is a function.

In the relation g,

(i) Dom (g) ≠ A

So, the condition is not satisfied. Thus, g is not a function.

In the relation h,

h = {(a, 1), (b, 2), (b, 3), (c, 4), (d, 5)}

(i) Dom (h) = A

(i) Two first components are the same, i.e. b has two different images.

So, h is not a function.

No, every relation is not a function.

Q. 4. Let X = {1, 2, 3, 4,}, Y = {1, 5, 9, 11, 15, 16} and F = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}.

Are the following true?

(i) F is a relation from X to Y (ii) F is a function from X to Y. Justify your answer in following true?

Answer : X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16}

and F = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

(i) To show: F is a relation from X to Y

First elements in F = 1, 2, 3, 4

All the first elements are in Set X

So, the first element is from set X

Second elements in F = 5, 9, 1, 11

All the second elements are in Set Y

So, the second element is from set Y

Since the first element is from set X and the second element is from set Y

Hence, F is a relation from X to Y.

(ii) To show: F is a function from X to Y

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

F = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

Here, 2 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

Q. 5. Let X = {-1, 0, 3, 7, 9} and f : X → R : f(x) x^3 + 1. Express the function f as set of ordered pairs.

Answer : Given: f: X → R, f(x) = $x^3 + 1$

Here, X = {-1, 0, 3, 7, 9}

For x = -1

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

For x = 0

$f(0) = (0)^3 + 1 = 0 + 1 = 1$

For x = 3

$f(3) = (3)^3 + 1 = 27 + 1 = 28$

For x = 7

$f(7) = (7)^3 + 1 = 343 + 1 = 344$

For x = 9

$f(9) = (9)^3 + 1 = 729 + 1 = 730$

∴ the ordered pairs are (-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)

Q. 6. Let A = {–1, 0, 1, 2} and B = {2, 3, 4, 5}. Find which of the following are function from A to B. Give reason.

(i) f = {(–1, 2), (-1, 3), (0, 4), 1,5)}

(ii) g = {(0, 2), (1, 3), (2, 4)}

(iii) h = {(-1, 2), (0, 3), (1, 4), (2, 5)}

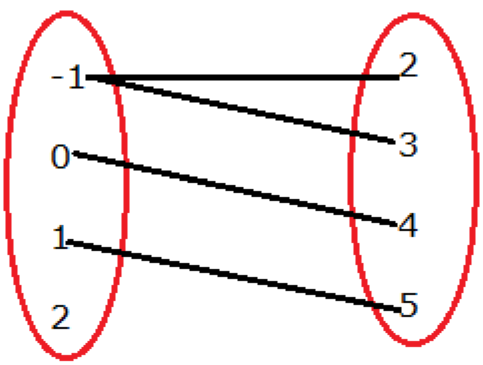
Answer : (i) Given: A = {-1, 0, 1, 2} and B = {2, 3, 4, 5}

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

f = {(–1, 2), (-1, 3), (0, 4), (1, 5)}




f = {(-1, 2), (-1, 3), (0, 4), (1, 5)}

Here, -1 is coming twice.

Hence, it does not have a unique (one) image.

∴ f is not a function

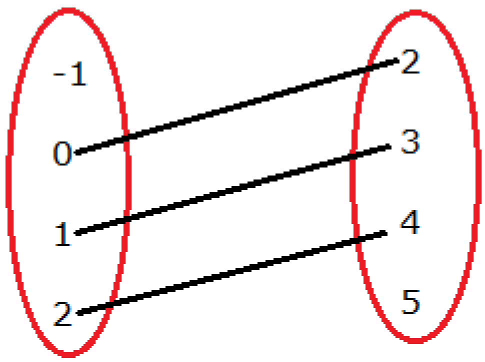
(ii) Given: A = {-1, 0, 1, 2} and B = {2, 3, 4, 5}

Function:

(i) all elements of first set is associated with the elements of second set.

(ii) An element of first set has a unique image in second set.

g = {(0, 2), (1, 3), (2, 4)}



Here, -1 is not associated with any element of set B

Hence, it does not satisfy the condition of the function

∴ g is not a function.

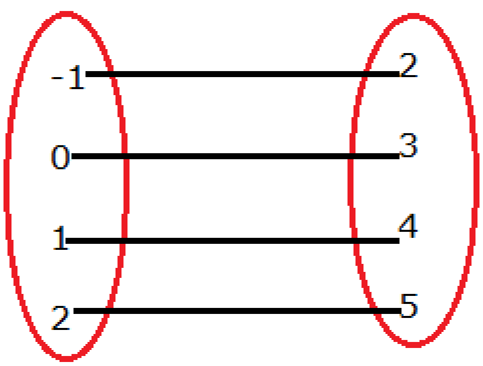
(iii) Given: A = {-1, 0, 1, 2} and B = {2, 3, 4, 5}

Function:

(i) all elements of first set is associated with the elements of second set.

(ii) An element of first set has a unique image in second set.

h = {(-1, 2), (0, 3), (1, 4), (2, 5)}



Here, (i) all elements of set A are associated to element in set B.

(ii) an element of set A is associated to a unique element in set B.

∴ h is a function.

Q. 7. Let A = {1, 2} and B = {2, 4, 6}. Let f = {(x, y) : x ϵ A, y ϵ B and y > 2x + 1}. Write f as a set of ordered pairs. Show that f is a relation but not a function from A to B.

Answer : Given: A = {1, 2} and B = {2, 4, 6}

f = {(x, y): x ∈ A, y ∈ B and y > 2x + 1}

Putting x = 1 in y > 2x + 1, we get

y > 2(1) + 1

⇒ y > 3

and y ∈ B

this means y = 4, 6 if x = 1 because it satisfies the condition y > 3

Putting x = 2 in y > 2x + 1, we get

y > 2(2) + 1

⇒ y > 5

this means y = 6 if x = 2 because it satisfies the condition y > 5.

∴ f = {(1, 4), (1, 6), (2, 6)}

(1, 2), (2, 2), (2, 4) are not the members of 'f' because they do not satisfy the given condition y > 2x + 1

Firstly, we have to show that f is a relation from A to B.

First elements = 1, 2

All the first elements are in Set A

So, the first element is from set A

Second elements in F = 4, 6

All the second elements are in Set B

So, the second element is from set B

Since the first element is from set A and second element is from set B

Hence, F is a relation from A to B.

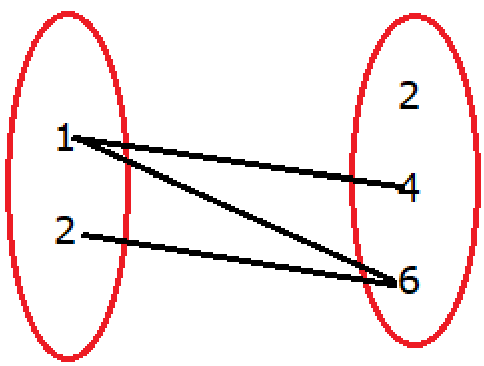
Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

Now, we have to show that f is not a function from A to B

f = {(1, 4), (1, 6), (2, 6)}



f = {(1, 4), (1, 6), (2, 6)}

Here, 1 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

Q. 8. Let A = {0, 1, 2} and B = {3, 5, 7, 9}. Let f = {(x, y) : x ϵ A, y ϵ B and y = 2x + 3}. Write f as a set of ordered pairs. Show that f is function from A to B. Find dom (f) and range (f).

Answer : Given: A = {0, 1, 2} and B = {3, 5, 7, 9}

f = {(x, y): x ∈ A, y ∈ B and y = 2x + 3}

For $x = 0$

$y = 2x + 3$

$y = 2(0) + 3$

$y = 3 \in B$

For $x = 1$

$y = 2x + 3$

$y = 2(1) + 3$

$y = 5 \in B$

For $x = 2$

$y = 2x + 3$

$y = 2(2) + 3$

$y = 7 \in B$

$\therefore f = \{(0, 3), (1, 5), (2, 7)\}$

(0, 5), (0, 7), (0, 9), (1, 3), (1, 7), (1, 9), (2, 3), (2, 5), (2, 9) are not the members of ‘f’ because they are not satisfying the given condition $y = 2x + 3$

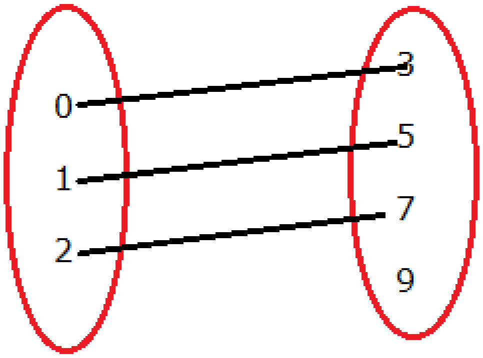
Now, we have to show that f is a function from A to B

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

$f = \{(0, 3), (1, 5), (2, 7)\}$



Here, (i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B.

∴ f is a function.

Dom (f) = 0, 1, 2

Range (f) = 3, 5, 7

Q. 9. Let A = {2, 3, 5, 7} and B = {3, 5, 9, 13, 15}. Let f = {(x, y) : x ϵ A, y ϵ B and y = 2x – 1}. Write f in roster form. Show that f is a function from A to B. Find the domain and range of f.

Answer : Given: A = {2, 3, 5, 7} and B = {3, 5, 9, 13, 15}

f = {(x, y): x ∈ A, y ∈ B and y = 2x – 1}

For x = 2

y = 2x – 1

y = 2(2) – 1

y = 3 ∈ B

For x = 3

y = 2x – 1

y = 2(3) – 1

y = 5 ∈ B

For x = 5

y = 2x – 1

y = 2(5) – 1

y = 9 ∈ B

For x = 7

y = 2x – 1

y = 2(7) – 1

y = 13 ∈ B

∴ f = {(2, 3), (3, 5), (5, 9), (7, 13)}

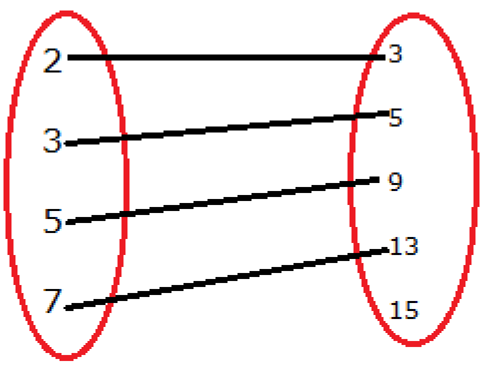
Now, we have to show that f is a function from A to B

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

f = {(2, 3), (3, 5), (5, 9), (7, 13)}



Here, (i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B.

∴ f is a function.

Dom (f) = 2, 3, 5, 7

Range (f) = 3, 5, 9, 13

Q. 10. Let g = {(1, 2), (2, 5), (3, 8), (4, 10), (5, 12), (6, 12)}. Is g a function? If yes, its domain range. If no, give reason.

Answer : Given:

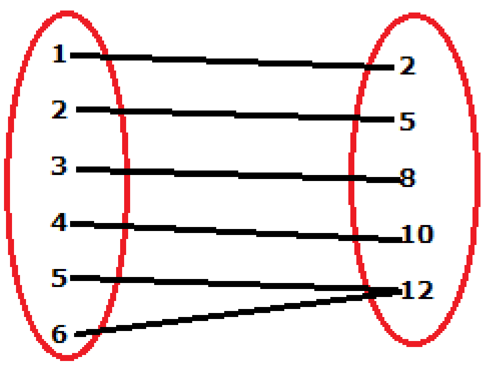
g = {(1, 2), (2, 5), (3, 8), (4, 10), (5, 12), (6, 12)}

We know that,

A function 'f' from set A to set B is a correspondence (rule) which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B



Here, we observe that each element of the given set has appeared as the first component in one and only one ordered pair in 'g.' So, g is a function in the given set.

Dom (g) = 1, 2, 3, 4, 5, 6

Range (g) = 2, 5, 8, 10, 12

Q. 11. Let f = {(0, -5), (1, -2), (3, 4), (4, 7)} be a linear function from Z into Z. Write an expression for f.

Answer : Given that: f = {(0, -5), (1, -2), (3, 4), (4, 7)} be a function from Z to Z defined by linear function.

We know that, linear functions are of the form y = mx + b

Let f(x) = ax + b, for some integers a, b

Here, (0, -5) ∈ f

⇒ f(0) = -5

⇒ a(0) + b = -5

⇒ b = -5 …(i)

Similarly, (1, -2) ∈ f

⇒ f(1) = -2

⇒ a(1) + b = -2

⇒ a + b = -2

⇒ a + (-5) = -2 [from (i)]

⇒ a = -2 + 5

⇒ a = 3

∴ f(x) = ax + b

= 3x + (-5)

f(x) = 3x – 5

Q. 12. If f (x) = x^2, find the value of $\frac{\{(f)(5)-f(1)\}}{(5-1)}$.

Answer : Given: f(x) = x^2

find: $\frac{f(5)-f(1)}{(5-1)}$ ...(i)

Firstly, we find the f(5)

Putting the value of x = 5 in the given eq., we get

f(5) = $(5)^2$

⇒ f(5) = 25

Similarly,

f(1) = $(1)^2$

⇒ f(1) = 1

Putting the value of f(5) and f(1) in eq. (i), we get

$$\frac{f(5)-f(1)}{(5-1)}=\frac{25-1}{5-1}=\frac{24}{4}=6$$

Hence, the value of $\frac{f(5)-f(1)}{(5-1)}=6$

Q. 13. If $f(x) = x^2$, find the value of $\frac{\{(f)(1.1)-f(1)\}}{(1.1)-1}$.

Answer : Given: $f(x) = x^2$

Firstly, we find the f(1.1)

Putting the value of x = 1.1 in the given eq., we get

$f(1.1) = (1.1)^2$

⇒ f(1.1) = 1.21

Similarly,

$f(1) = (1)^2$

⇒ f(1) = 1

Putting the value of f(1.1) and f(1) in eq. (i), we get

$$\frac{f(1.1)-f(1)}{(1.1-1)}=\frac{1.21-1}{1.1-1}=\frac{0.21}{0.1}=2.1$$

Hence, the value of $\frac{f(1.1)-f(1)}{(1.1-1)}=2.1$

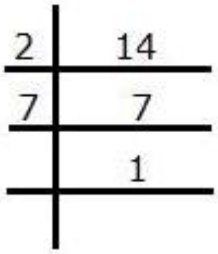
Q. 14. Let X = {12, 13, 14, 15, 16, 17} and f : A → Z : f(x) = highest prime factor of x. Find range (f)

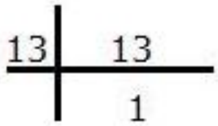
Answer : Given: f(x) = highest prime factor of x

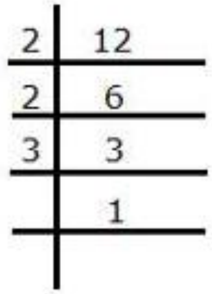
And since x ∈ A, A = {12, 13, 14, 15, 16, 17}

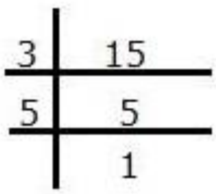
Value of x can only be 12, 13, 14, 15, 16, 17

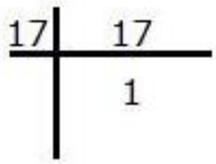
Doing prime factorization of the above, we get











Value of x	Highest Prime Factor of x
12	3
13	13
14	7
15	5
16	2
17	17

Hence, range of f = {2, 3, 5, 7, 13, 17}

Q. 15. Let R^+ be the set of all positive real numbers. Let f : R+→ R : f(x) = $\log_e x$. Find

(i) Range (f)

(ii) {x : x ϵ R^+ and f(x) = -2}.

(iii) Find out whether f(x + y) = f(x). f(y) for all x, y ϵ R.

Answer : Given that f: R+→ R such that f(x) = $\log_e x$

To find: (i) Range of f

Here, f(x) = $\log_e x$

We know that the range of a function is the set of images of elements in the domain.

∴ The image set of the domain of f = R

Hence, the range of f is the set of all real numbers.

To find: (ii) $\{x : x \in R^+$ and $f(x) = -2\}$

We have, f(x) = -2 …(a)

And $f(x) = \log_e x$ …(b)

From eq. (a) and (b), we get

$\log_e x = -2$

Taking exponential both the sides, we get

$\Rightarrow e^{\log_e x} = e^{-2}$

$[\because$ Inverse property .i. e $b^{\log_b x} = x]$

$\Rightarrow x = e^{-2}$

$\therefore \{x : x \in R^+$ and $f(x) = -2\} = \{e^{-2}\}$

To find: (iii) f(xy) = f(x) + f(y) for all x, y ϵ R

We have,

$f(xy) = \log_e(xy)$

$= \log_e(x) + \log e(y)$

[Product Rule for Logarithms]

$= f(x) + f(y)$ $[\because f(x) = \log_e x]$

∴ f(xy) = f(x) + f(y) holds.

Q. 16. Let f : R → R : f(x) =2^x. Find

(i) Range (f)

(ii) {x : f(x) = 1}.

(iii) Find out whether f(x + y) = f(x). f(y) for all x, y ϵ R.

Answer : Given that f: R → R such that $f(x) = 2^x$

To find: (i) Range of x

Here, $f(x) = 2^x$ is a positive real number for every x ∈ R because 2^x is positive for every x ∈ R.

Moreover, for every positive real number x, ∃ $\log 2^x$ ∈ R such that

$$f(\log_2 x) = 2^{\log_2 x}$$

$$= x \left[\because a^{\log_a x} = x\right]$$

Hence, the range of f is the set of all positive real numbers.

To find: (ii) {x : f(x) = 1}

We have, f(x) = 1 …(a)

and $f(x) = 2^x$ …(b)

From eq. (a) and (b), we get

$2^x = 1$

⇒ $2^x = 2^0$ [∵ $2^0 = 1$]

Comparing the powers of 2, we get

⇒ x = 0

∴{x : f(x) = 1} = {0}

To find: (iii) f(x + y) = f(x). f(y) for all x, y ϵ R

We have,

$f(x + y) = 2^{x + y}$

$= 2^x.2^y$

[The exponent "product rule" tells us that, when multiplying two powers that have the same base, you can add the exponents or vice - versa]

= f(x).f(y) [∵$f(x) = 2^x$]

∴ f(x + y) = f(x). f(y) holds for all x, y ϵ R

Q. 17. Let f : R → R : $f(x) = x^2$ and g : C → C: $g(x) = x^2$, where C is the set of all complex numbers. Show that f ≠ g.

Answer : It is given that f : R → R and g : C → C

Thus, Domain (f) = R and Domain (g) = C

We know that, Real numbers ≠ Complex Number

∵, Domain (f) ≠ Domain (g)

∴ f(x) and g(x) are not equal functions

∴ f ≠ g

Q. 18. f, g and h are three functions defined from R to R as following:
(i) f(x) = x^2
(ii) g(x) = x^2 + 1
(iii) h(x) = sin x
That, find the range of each function.

Answer : (i) f: R → R such that f(x) = x^2

Since the value of x is squared, f(x) will always be equal or greater than 0.

∴ the range is [0, ∞)

(ii) g: R → R such that g(x) = x^2 + 1

Since, the value of x is squared and also adding with 1, g(x) will always be equal or greater than 1.

∴ Range of g(x) = [1, ∞)

(iii) h: R → R such that h(x) = sin x

We know that, sin (x) always lies between -1 to 1

∴ Range of h(x) = (-1, 1)

Q. 19. **Let f : R → R : f(x) = x^2 + 1. Find**

(i) f^{-1} {10}
(ii) f^{-1} {–3}.

Answer : Given: f(x) = x^2 + 1

To find: (i) f^{-1}{10}

We know that, if f: X → Y such that y ∈ Y. Then $f^{-1}(y) = \{x \in X: f(x) = y\}$.

In other words, $f^{-1}(y)$ is the set of pre – images of y

Let $f^{-1}\{10\} = x$. Then, f(x) = 10 …(i)

and it is given that $f(x) = x^2 + 1$ …(ii)

So, from (i) and (ii), we get

$x^2 + 1 = 10$

$\Rightarrow x^2 = 10 – 1$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \sqrt{9}$

$\Rightarrow x = \pm 3$

$\therefore f^{-1}\{10\} = \{-3, 3\}$

To find: (ii) $f^{-1}\{-3\}$

Let $f^{-1}\{-3\} = x$. Then, f(x) = -3 …(iii)

and it is given that $f(x) = x^2 + 1$ …(iv)

So, from (iii) and (iv), we get

$x^2 + 1 = -3$

$\Rightarrow x^2 = -3 – 1$

$\Rightarrow x^2 = -4$

Clearly, this equation is not solvable in R

$\therefore f^{-1}\{-3\} = \phi$

Q. 20. The function $F(x)\frac{9x}{5} = +32$ **is the formula to convert x °C to Fahrenheit units. Find**

(i) F(0),

(ii) F(–10),
(iii) The value of x when f(x) = 212.
Interpret the result in each case.

Answer : Given: $F(x) = \frac{9}{5}x + 32$ …(i)

To find: (i) F(0)

Substituting the value of x = 0 in eq. (i), we get

$$F(x) = \frac{9}{5}x + 32$$

$$\Rightarrow F(0) = \frac{9}{5} \times 0 + 32$$

⇒ F(0) = 32

It means 0˚ C = 32˚ F

To find: (ii) F(-10)

Substituting the value of x = -10 in eq. (i), we get

$$F(x) = \frac{9}{5}x + 32$$

$$\Rightarrow F(-10) = \frac{9}{5} \times (-10) + 32$$

⇒ F(-10) = 9 × (-2) + 32

⇒ F(-10) = -18 + 32

⇒ f(-10) = 14

It means -10˚ C = 14˚ F

To find: (iii) the value of x when F(x) = 212

It is given that $F(x) = \frac{9}{5}x + 32$

Substituting the value of F(x) = 212 in the above equation, we get

$$212 = \frac{9}{5}x + 32$$

$$\Rightarrow 212 - 32 = \frac{9}{5}x$$

$$\Rightarrow 180 = \frac{9}{5}x$$

$$\Rightarrow x = 180 \times \frac{5}{9}$$

⇒ x = 20 × 5

⇒ x = 100

It means 212˚F = 100˚C

Exercise 3B

Q. 1. If f(x) = x^2 – 3x + 4 and f(x) = f(2x + 1), find the values of x.

Answer : Given: f(x) = x^2 – 3x + 4 ----- (1)

and f(x) = f(2x + 1)

Need to Find: Value of x

Replacing x by (2x + 1) in equation (1) we get,

f(2x + 1) = $(2x + 1)^2$ – 3(2x + 1) + 4 ----- (2)

According to the given problem, f(x) = f(2x + 1)

Comparing (1) and (2) we get,

$x^2 - 3x + 4 = (2x + 1)^2 - 3(2x + 1) + 4$

$\Rightarrow x^2 - 3x + 4 = 4x^2 + 4x + 1 - 6x - 3 + 4$

$\Rightarrow 4x^2 + 4x + 1 - 6x - 3 + 4 - x^2 + 3x - 4 = 0$

⇒ $3x^2 + x - 2 = 0$

⇒ $3x^2 + 3x - 2x - 2 = 0$

⇒ $3x(x + 1) - 2(x + 1) = 0$

⇒ $(3x - 2)(x + 1) = 0$

So, either $(3x - 2) = 0$ or $(x + 1) = 0$

Therefore, the value of x is either $\frac{2}{3}$ or -1 [Answer]

Q. 2. If $f(x) = \frac{x-1}{x+1}$ **then show that**

(i) $f\left(\frac{1}{x}\right) = -f(x)$

(ii) $f\left(\frac{-1}{x}\right) = \frac{-1}{f(x)} f$

Answer : Given: $f(x) = \frac{x-1}{x+1}$

(i) Need to prove: $f\left(\frac{1}{x}\right) = -f(x)$

Now replacing x by $\frac{1}{x}$ we get,

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1}$$

⇒ $f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$

⇒ $f\left(\frac{1}{x}\right) = \frac{-(x-1)}{(x+1)} = -f(x)$ [Proved]

(ii) Need to prove: $f\left(\frac{-1}{x}\right) = \frac{-1}{f(x)}$

Now replacing x by $-\frac{1}{x}$ we get,

$$f\left(\frac{-1}{x}\right) = \frac{\frac{-1}{x} - 1}{\frac{-1}{x} + 1}$$

$\Rightarrow f\left(\frac{-1}{x}\right) = \frac{-1-x}{-1+x}$

$\Rightarrow f\left(\frac{-1}{x}\right) = \frac{-(x+1)}{x-1}$

$\Rightarrow f\left(\frac{-1}{x}\right) = \frac{-1}{\frac{x-1}{x+1}} = \frac{-1}{f(x)}$ [Proved]

Q. 3. If $f(x) = x^3 - \frac{1}{x^3}$ **then show that** $f(x) + f\left(\frac{1}{x}\right) = 0$

Answer : Given: $f(x) = x^3 - \frac{1}{x^3}$

Need to prove: $f(x) + f\left(\frac{1}{x}\right) = 0$

Replacing x by $\frac{1}{x}$ we get,

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

Now according to the problem,

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = 0$ [Proved]

Q. 4. **If** $f(x)=\frac{x+1}{x-1}$ **then show that f{f(x)} = x.**

Answer : Given: $f(x)=\frac{x+1}{x-1}$

Need to prove: f{f(x)} = x

Now replacing x by f(x) we get,

$$f\{f(x)\}=\frac{f(x)+1}{f(x)-1}$$

$\Rightarrow f\{f(x)\}=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$

$\Rightarrow f\{f(x)\}=\frac{x+1+x-1}{x+1-x+1}$

$\Rightarrow f\{f(x)\}=\frac{2x}{2}$

$\Rightarrow f\{f(x)\}=x$ [Proved]

Q. 5. If f(x) = $\frac{1}{(2x+1)}$ **and** $x\neq\frac{-1}{2}$ **then prove that f{(x)} =** $\frac{2x+1}{2x+3}$**, when it is given that** $x\neq\frac{-3}{2}$**.**

Answer : Given: f(x) =$\frac{1}{(2x+1)}$, where $x\neq\frac{-1}{2}$

Need to prove: f{f(x)} = $\frac{2x+1}{2x+3}$ when $x\neq\frac{-3}{2}$

Now placing f(x) in place of x

$\Rightarrow f\{f(x)\}=\frac{1}{2f(x)+1}$

$\Rightarrow f\{f(x)\}=\frac{1}{2\frac{1}{2x+1}+1}$

$\Rightarrow f\{f(x)\} = \frac{1}{\frac{2+2x+1}{2x+1}} = \frac{2x+1}{2x+3}$, where $x \neq \frac{-3}{2}$ [Proved]

Q. 6. If $f(x) = \frac{1}{(1-x)}$ **then show that f [f{f(x)}] = x.**

Answer : Given: $f(x) = \frac{1}{(1-x)}$

Need to prove: f[f{f(x)}] = x

Replacing x by f(x),

$$f\{f(x)\} = \frac{1}{1-f(x)}$$

$\Rightarrow f\{f(x)\} = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}} = \frac{1-x}{-x}$

Now again replacing x by f(x) we get,

$$f[f\{f(x)\}] = \frac{1-f(x)}{-f(x)}$$

$\Rightarrow f[f\{f(x)\}] = \frac{1-\frac{1}{1-x}}{-\frac{1}{1-x}}$

$\Rightarrow f[f\{f(x)\}] = \frac{\frac{1-x-1}{1-x}}{\frac{-1}{1-x}}$

$\Rightarrow f[f\{f(x)\}] = \frac{-x}{-1} = x$ [Proved]

Q. 7. **If** $f(x) = \frac{2x}{(1+x^2)}$ **then show that f(tanθ) = sin 2θ.**

Answer : Given: $f(x) = \frac{2x}{(1+x^2)}$

Need to prove: f(tanθ) = sin 2θ

$$f(\tan\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$$

$\Rightarrow f(\tan\theta) = \frac{2\tan\theta}{\sec^2\theta}$ [as $1 + \tan^2\theta = \sec^2\theta$]

$\Rightarrow f(\tan\theta) = 2\frac{\sin\theta}{\cos\theta}\cos^2\theta$ [as $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$]

$\Rightarrow f(\tan\theta) = 2\sin\theta\cos\theta = \sin 2\theta$ [Proved]

Q. 8. **If** $y = f(x) = \frac{3x+1}{5x-3}$**, prove that x = f(y).**

Answer : Given: $y = f(x) = \frac{3x+1}{5x-3}$

Need to prove: x = f(y)

Replacing x by y in the function,

$$f(y) = \frac{3y+1}{5y-3}$$

Now, given in the problem that y = f(x)

$$f(y) = \frac{3f(x)+1}{5f(x)-3}$$

$\Rightarrow f(y) = \frac{3\frac{3x+1}{5x-3}+1}{5\frac{3x+1}{5x-3}-3}$

$\Rightarrow f(y) = \frac{9x+3+5x-3}{15x+5-15x+9}$

$\Rightarrow f(y) = \frac{14x}{14} = x$

$\Rightarrow x = f(y)$ [Proved]

Exercise 3C

Q. 1. Find the domain of each of the following real function.

(i) $f(x)=\frac{3x+5}{x^2-9}$

(ii) $f(x)=\frac{2x-3}{x^2+x-2}$

(iii) $f(x)=\frac{x^2-2x+1}{x^2-8x+12}$

(iv) $f(x)=\frac{x^3-8}{x^2-1}$

Answer : (i) Given: $f(x)=\frac{3x+5}{x^2-9}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator to 0.

Therefore,

$x^2-9=0$

$\Rightarrow x^2=9$

$\Rightarrow x=\pm 3$

It means that the denominator is zero when x = 3 and x = -3

So, the domain of the function is the set of all the real numbers except +3 and -3.

The domain of the function, $D_{f(x)} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

(ii) Given: $f(x)=\frac{2x-3}{x^2+x-2}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator to 0.

Therefore,

$x^2 + x - 2 = 0$

⇒ $x^2 + 2x - x - 2 = 0$

⇒ $x(x + 2) - 1(x + 2) = 0$

⇒ $(x + 2)(x - 1) = 0$

⇒ $x = -2$ & $x = 1$

It means that the denominator is zero when x = 1 and x = -2

So, the domain of the function is the set of all the real numbers except 1 and -2.

The domain of the function, $D_{f(x)} = (-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

(iii) Given: $f(x) = \frac{x^2 - 2x + 1}{x^2 - 8x + 12}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator to 0.

Therefore,

$x^2 - 8x + 12 = 0$

⇒ $x^2 - 2x - 6x + 12 = 0$

⇒ $x(x - 2) - 6(x - 2) = 0$

⇒ $(x - 2)(x - 6) = 0$

⇒ $x = 2$ & $x = 6$

It means that the denominator is zero when x = 2 and x = 6

So, the domain of the function is the set of all the real numbers except 2 and 6.

The domain of the function, $D_{f(x)} = (-\infty, 2) \cup (2, 6) \cup (6, \infty)$.

(iv) Given: $f(x) = \frac{x^3 - 8}{x^2 - 1}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator to 0.

Therefore,

$x^2 - 1 = 0$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

It means that the denominator is zero when x = -1 and x = 1

So, the domain of the function is the set of all the real numbers except -1 and +1.

The domain of the function, $D_{f(x)} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

Q. 2. Find the domain and the range of each of the following real function:

$$f(x) = \frac{1}{x}$$

Answer : Given: $f(x) = \frac{1}{x}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{1}{x} = y$ ---- (1)

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x = 0

It means that the denominator is zero when x = 0

So, the domain of the function is the set of all the real numbers except 0.

The domain of the function, $D_{f(x)} = (-\infty, 0) \cup (0, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{1}{y} = x$$

$$\Rightarrow y = \frac{1}{x} = f(x_1)$$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

x = 0

It means that the denominator is zero when x = 0

So, the range of the function is the set of all the real numbers except 0.

The range of the function, $R_{f(x)} = (-\infty, 0) \cup (0, \infty)$.

Q. 3. Find the domain and the range of each of the following real function: $f(x) = \frac{1}{(x-5)}$

Answer : Given: $f(x) = \frac{1}{(x-5)}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{1}{x-5} = y$ ---- (1)

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x – 5 = 0

⇒ x = 5

It means that the denominator is zero when x = 5

So, the domain of the function is the set of all the real numbers except 5.

The domain of the function, $D_{f(x)}$ = (- ∞, 5) ∪ (5, ∞).

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{1}{y-5} = x$$

⇒ $y - 5 = \frac{1}{x}$

⇒ $y = \frac{1}{x} + 5 = \frac{1+5x}{x} = f(x_1)$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

x = 0

It means that the denominator is zero when x = 0

So, the range of the function is the set of all the real numbers except 0.

The range of the function, $R_{f(x)}$ = (- ∞, 0) ∪ (0, ∞).

Q. 4. **Find the domain and the range of each of the following real function:** $f(x) = \frac{x-3}{2-x}$

Answer : Given: $f(x) = \frac{x-3}{2-x}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{x-3}{2-x} = y$ ---- (1)

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

2 – x = 0

⇒ x = 2

It means that the denominator is zero when x = 2

So, the domain of the function is the set of all the real numbers except 2.

The domain of the function, $D_{f(x)} = (-\infty, 2) \cup (2, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$\frac{y-3}{2-y} = x$

⇒ $y - 3 = 2x - xy$

⇒ $y + xy = 2x + 3$

⇒ $y(1 + x) = 2x + 3$

⇒ $y = \frac{2x+3}{1+x} = f(x_1)$

To find the range of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x + 1 = 0

⇒ x = -1

It means that the denominator is zero when x = -1

So, the range of the function is the set of all the real numbers except -1.

The range of the function, $R_{f(x)} = (-\infty, -1) \cup (-1, \infty)$.

Q. 5. Find the domain and the range of each of the following real function: $f(x) = \frac{3x-2}{x+2}$

Answer : Given: $f(x) = \frac{3x-2}{x+2}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{3x-2}{x+2} = y$ ---- (1)

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x + 2 = 0

⇒ x = -2

It means that the denominator is zero when x = -2

So, the domain of the function is the set of all the real numbers except -2.

The domain of the function, $D_{f(x)} = (-\infty, -2) \cup (-2, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$\frac{3y-2}{2+y} = x$

⇒ $3y - 2 = 2x + xy$

⇒ $3y - xy = 2x + 2$

$\Rightarrow y = \frac{2x+2}{3-x} = f(x_1)$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

3 – x = 0

⇒ x = 3

It means that the denominator is zero when x = 3

So, the range of the function is the set of all the real numbers except 3.

The range of the function, $R_{f(x)} = (-\infty, 3) \cup (3, \infty)$.

Q. 6. Find the domain and the range of each of the following real function: $f(x) = \frac{x^2 - 16}{x - 4}$

Answer : Given: $f(x) = \frac{x^2-16}{x-4}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x – 4 = 0

⇒ x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4.

The domain of the function, $D_{f(x)} = (-\infty, 4) \cup (4, \infty)$.

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 8

So, the range of the function is the set of all the real numbers except 8.

The range of the function, $R_{f(x)} = (-\infty, 8) \cup (8, \infty)$.

Q. 7. Find the domain and the range of each of the following real function: f(x) $= \frac{1}{\sqrt{2x-3}}$

Answer : Given: f(x) = $\frac{1}{\sqrt{2x-3}}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{1}{\sqrt{2x-3}} = y$ ---- (1)

The condition for the function to be defined,

$2x - 3 > 0$

$\Rightarrow x > \frac{3}{2}$

So, the domain of the function is the set of all the real numbers greater than $\frac{3}{2}$.

The domain of the function, $D_{f(x)} = (\frac{3}{2}, \infty)$.

Now putting any value of x within the domain set we get the value of the function always a fraction whose denominator is not equals to 0.

The range of the function, $R_{f(x)} = (0, 1)$.

Q. 8. Find the domain and the range of each of the following real function: f(x) $= \frac{ax-b}{cx-d}$

Answer : Given: f(x) = $\frac{ax-b}{cx-d}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{ax-b}{cx-d} = y$ ---- (1)

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

cx – d = 0

⇒ x = $\frac{d}{c}$

It means that the denominator is zero when x = $\frac{d}{c}$

So, the domain of the function is the set of all the real numbers except d/c.

The domain of the function, $D_{f(x)} = (-\infty, \frac{d}{c}) \cup (\frac{d}{c}, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$\frac{ay-b}{cy-d} = x$

⇒ $ay - b = cxy - dx$

⇒ $ay - cxy = b - dx$

⇒ $y = \frac{b-dx}{a-cx}$

To find the range of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

$a - cx = 0$

⇒ $x = \frac{a}{c}$

It means that the denominator is zero when x = $\frac{a}{c}$

So, the range of the function is the set of all the real numbers except a/c.

The range of the function, $R_{f(x)} = (-\infty, \frac{a}{c}) \cup (\frac{a}{c}, \infty)$.

Q. 9. **Find the domain and the range of each of the following real function:** $f(x) = \sqrt{3x - 5}$

Answer : Given: $f(x) = \sqrt{3x - 5}$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$3x - 5 \geq 0$

$\Rightarrow x \geq \frac{5}{3}$

So, the domain of the function is the set of all the real numbers greater than equals to $\frac{5}{3}$.

The domain of the function, $D_{f(x)} = [\frac{5}{3}, \infty)$.

Putting $\frac{5}{3}$ in the function we get, f(x) = 0

It means the range of the function is defined for all the values greater than equals to 0.

The range of the function, $R_{f(x)} = [0, \infty)$.

Q. 10. Find the domain and the range of each of the following real function: $f(x) = \sqrt{\frac{x - 5}{3 - x}}$

Answer : Given: $f(x) = \sqrt{\frac{x-5}{3-x}}$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$3 - x > 0$

$\Rightarrow x < 3$

So, the domain of the function is the set of all the real numbers lesser than 3.

The domain of the function, $D_{f(x)} = (-\infty, 3)$.

The condition for the range of the function to be defined,

$x - 5 \geq 0$ & $3 - x > 0$

$\Rightarrow x \geq 5$ & $x < 3$

Both the conditions can’t be satisfied simultaneously. That means there is no range for the function f(x).

Q. 11. Find the domain and the range of each of the following real function: $f(x) = \frac{1}{\sqrt{x^2 - 1}}$

Answer : Given: $f(x) = \frac{1}{\sqrt{x^2-1}}$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$x^2 - 1 > 0$

$\Rightarrow x^2 > 1$

$\Rightarrow x > 1$

So, the domain of the function is the set of all the real numbers greater than 1.

The domain of the function, $D_{f(x)} = (1, \infty)$.

Now putting any value of x within the domain set we get the value of the function always a fraction whose denominator is not equals to 0.

The range of the function, $R_{f(x)} = (0, 1)$

Q. 12. Find the domain and the range of each of the following real function: f(x) = 1 – |x – 2|

Answer : Given: f(x) = 1 – |x – 2|

Need to find: Where the functions are defined.

Since |x – 2| gives real no. for all values of x, the domain set can possess any real numbers.

So, the domain of the function, $D_{f(x)} = (-\infty, \infty)$.

Now the given function is f(x) = 1 – |x – 2|, where | x – 2 | is always positive. So, the maximum value of the function is 1.

Therefore, the range of the function, $R_{f(x)} = (-\infty, 1)$

Q. 13. **Find the domain and the range of each of the following real function:** $f(x) = \frac{|x-4|}{x-4}$

Answer : Given: $f(x) = \frac{|x-4|}{x-4}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

x – 4 = 0

⇒ x = 4

It means that the denominator is zero when x = 4

So, the domain of the function is the set of all the real numbers except 4.

The domain of the function, $D_{f(x)} = (-\infty, 4) \cup (4, \infty)$.

The numerator is an absolute function of the denominator. So, for any value of x from the domain set, we always get either +1 or -1 as the output. So, the range of the function is a set containing -1 and +1

Therefore, the range of the function, $R_{f(x)} = \{ -1 , 1 \}$

Q. 14. Find the domain and the range of each of the following real function: $f(x) = \frac{x^2 - 9}{x - 3}$

Answer : Given: $f(x) = \frac{x^2-9}{x-3}$

Need to find: Where the functions are defined.

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

$x – 3 = 0$

$\Rightarrow x = 3$

It means that the denominator is zero when x = 3

So, the domain of the function is the set of all the real numbers except 3.

The domain of the function, $D_{f(x)} = (-\infty, 3) \cup (3, \infty)$.

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 6

So, the range of the function is the set of all the real numbers except 6.

The range of the function, $R_{f(x)} = (-\infty, 6) \cup (6, \infty)$.

Q. 15. Find the domain and the range of each of the following real function: $f(x) = \frac{1}{2 - \sin 3x}$

Answer : Given: $f(x) = \frac{1}{2-\sin 3x}$

Need to find: Where the functions are defined.

The maximum value of an angle is 2π

So, the maximum value of x = 2π/3.

Whereas, the minimum value of x is 0

Therefore, the domain of the function, $D_{f(x)}$ = (0, 2π/3).

Now, the minimum value of sinθ = 0 and the maximum value of sinθ = 1. So, the minimum value of the denominator is 1, and the maximum value of the denominator is 2.

Therefore, the range of the function, $R_{f(x)}$ = (1/2, 1).

Exercise 3D

Q. 1. Consider the real function f: R → R: f(x) = x + 5 for all

x ϵ R. Find its domain and range. Draw the graph of this function.

Answer : Given: f(x) = x +5 ∀ x ϵ R

To Find: Domain and Range of f(x).

The domain of the given function is all real numbers expect where the expression is undefined. In this case, there is no real number which makes the expression undefined.

As f(x) is a polynomial function, we can have any value of x.

Therefore,

Domain(f) = (-∞, ∞) {x | x ϵ R}

Now,

Let y = f(x)

y = x + 5

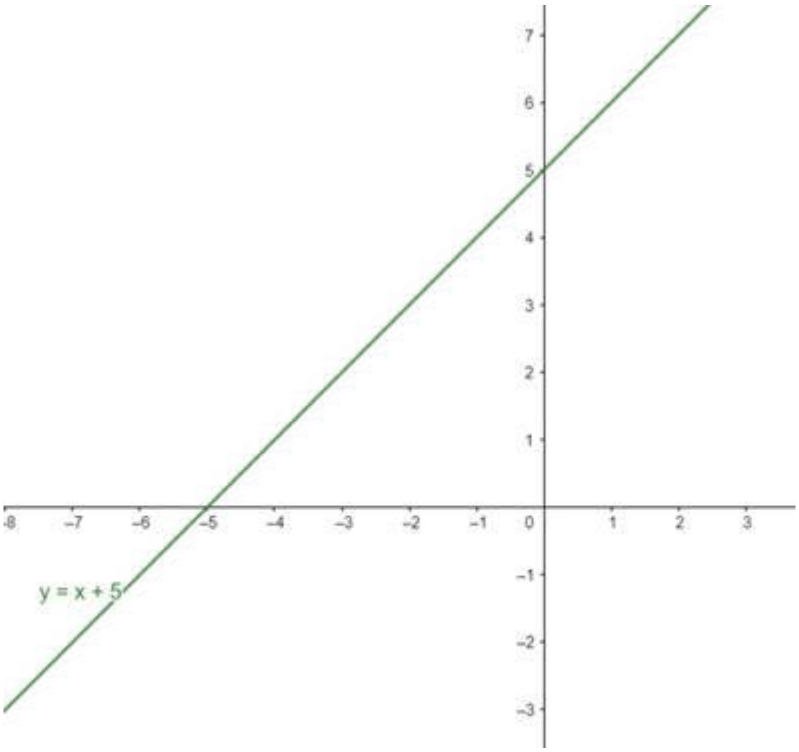
x = y – 5

The range is set of all valid values of y

Therefore,

Range(f) = (- ∞, ∞) {y | y ϵ R}

Graph:



Q. 2. Consider the function f: R → R, defined by

$$f(x) = \begin{cases} 1 - x, \text{when } x < 0 \\ 1x, \text{when } x = 0 \\ x + 1, \text{when } x > 0 \end{cases}$$

Write its domain and range. Also, draw the graph of f(x).

Answer : Given:

$$f(x) = \begin{cases} 1 - x \; when \; x < 0 \\ 1x, when \; x = 0 \\ x + 1, wwhen \; x > 0 \end{cases}$$

To Find:

Domain and Range of f(x)

When f(x) = 1 – x | x < 0

In this case there is no value of x (x < 0) which makes the above expression undefined.

Therefore,

Domain(f) = (-∞, 0) …(1)

When f(x) = x | x = 0

In this case there is no value other than 0 which makes the above expression undefined.

Therefore,

Domain(f) = 0 …(2)

When f(x) = x + 1 | x > 0

In this case there is no value of x (x > 0) which makes the above expression undefined.

Therefore,

Domain(f) = (0, ∞) …(3)

From equations (1),(2) & (3) We can say that the domain of f(x) as a whole :

Domain(f) = (-∞, ∞)

Now when, f(x) = 1 -x

x = 1 – f(x)

As x ranges from -∞ to 0, then f(x) ranges from 1 to ∞

Therefore,

Range(f) = (1, ∞) …(4)

Now when, f(x) =x

As x = 0

Therefore,

Range(f) = 0 …(5)

Now when, f(x) = x +1

x = f(x) - 1

As x ranges from 0 to ∞, then f(x) ranges from 1 to ∞

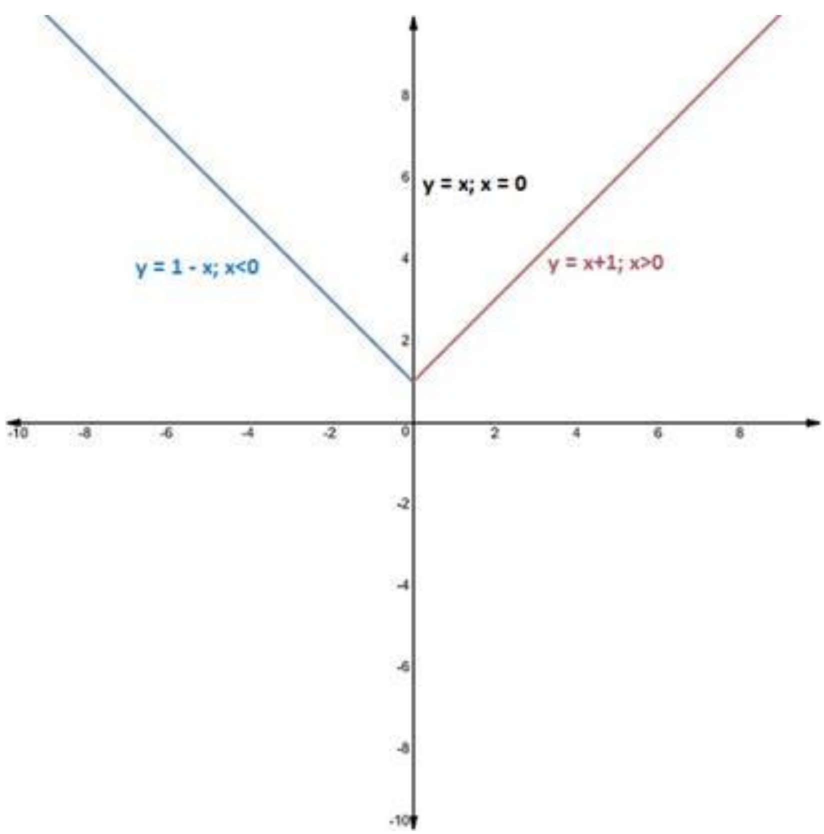
Therefore,

Range(f) = (1, ∞) …(6)

From (4), (5) & (6) the range of f(x) as whole:

Range(f) = 0 ∪ (1, ∞)

Graph:



Q. 3. Find the domain and the range of the square root function,

f: R^+ U {0} → R f(x) = $\sqrt{x}$ for all non-negative real numbers.
Also, draw its graph.

Answer : Given:

f(x) $= \sqrt{x}$

To Find: Domain and Range of f(x).

The domain of the given function is set of all positive real

Numbers including 0. In this case, if the value of x is a negative

The number then it makes the expression undefined.

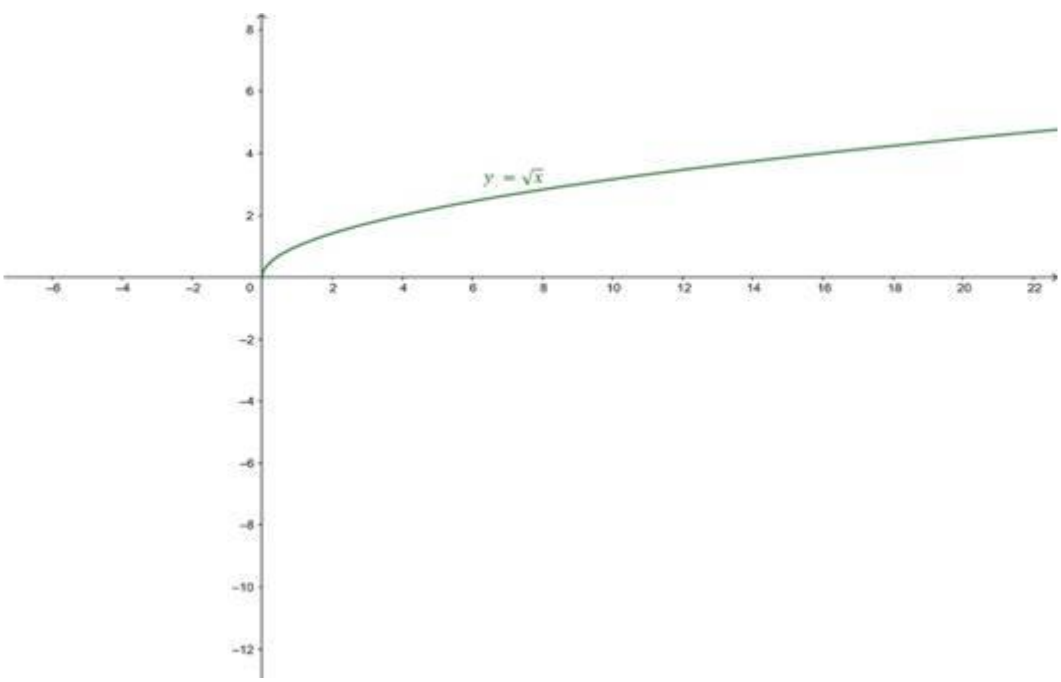
Therefore,

Domain(f) = [0, ∞) ∀ x є R^+ υ {0}

As the value of x varies from 0 to ∞, value of $\sqrt{x}$ varies from $\sqrt{0}\ to\ \sqrt{\infty}$, Hence,

Range(f) = [0, ∞) ∀ x є R^+ υ {0}

Graph:



Q. 4. **Find the domain and the range of the cube root function,**

f: R → R: $f(x) = x^{1/3}$ for all x ϵ R. Also, draw its graph.

Answer : Given:

$f(x) = x^{1/3} \forall x \in R$

To Find: Domain and range of the given function.

Here, $f(x) = x^{1/3}$

The domain of the above function would be,

Domain(f) = (-∞, ∞) {x | x ϵ R}

Because all real numbers have a cube root. There is no value of x which makes the function undefined.

Now, to find the range

Consider f(x) = y

Then, $y = x^{1/3}$

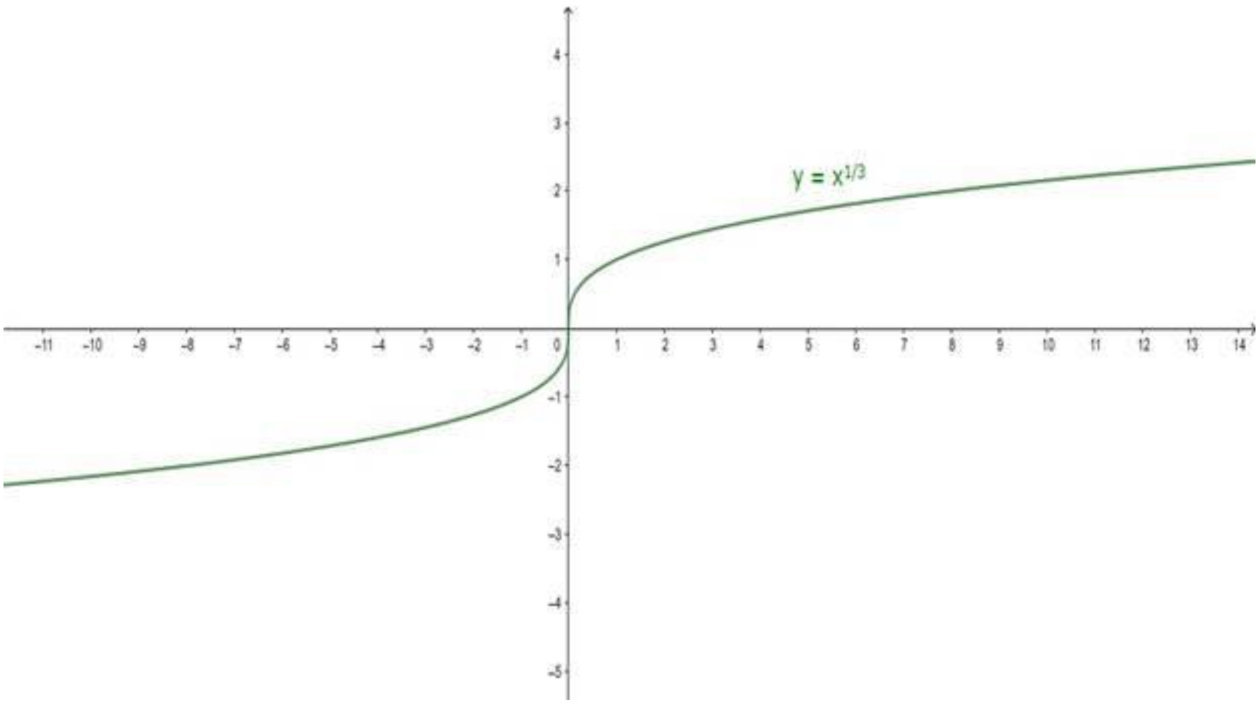
$y^3 = x$

Since f(x) is continuous, it follows that

Range(f) = (-∞, ∞) {y | y ϵ R}

Because for every value of y there would be a cube of that value.

Graph:



Exercise 3E

Q. 1. Let f : R → R : f(x) = x + 1 and g : R → R : g(x) = 2x – 3.Find

(i) (f + g) (x)
(ii) (f – g) (x)
(iii) (fg) (x)
(iv)(f/g) (x)

Answer : (i) Given:

f(x) = x + 1 and g(x) = 2x -3

(i) To find: (f + g) (x)

(f + g) (x) = f(x) + g(x)

= (x + 1) + (2x -3)

= x + 1 + 2x – 3

= 3x – 2

Therefore,

(f + g) (x) = 3x -2

(ii) To find: (f - g) (x)

(f - g) (x) = f(x) - g(x)

= (x + 1) - (2x -3)

= x + 1 - 2x + 3

= 4 - x

Therefore,

(f - g) (x) = 4 - x

(iii) To find: (fg)(x)

(fg)(x) = f(x). g(x)

= (x+1) (2x-3)

= x(2x) -3(x) +1(2x) -1(3)

= $2x^2 - 3x + 2x - 3$

= $2x^2 - x - 3$

Therefore,

(fg)(x) = $2x^2 - x - 3$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

Sol. $\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$

$= \left(\frac{x+1}{2x-3}\right)$

Therefore,

$$\left(\frac{f}{g}\right)(x) = \left(\frac{x+1}{2x-3}\right)$$

Q. 2. Let f : R → R : f(x) = 2x + 5 and g : R → R : g(x) = x^2 + x.
Find
(i) (f + g) (x)
(ii) (f – g) (x)
(iii) (fg) (x)
(iv) (f/g)(x)

Answer : (i) Given:

f(x) = 2x + 5 and g(x) = $x^2 + x$

(i) To find: (f + g) (x)

(f + g) (x) = f(x) + g(x)

= $(2x + 5) + (x^2 + x)$

= $2x + 5 + x^2 + x$

= $x^2 + 3x + 5$

Therefore,

(f + g) (x) = $x^2 + 3x + 5$

(ii) To find: (f - g) (x)

(f - g) (x) = f(x) - g(x)

= $(2x + 5) - (x^2 + x)$

= $2x + 5 - x^2 - x$

= $-x^2 + x + 5$

Therefore,

(f + g) (x) = $-x^2 + x + 5$

(iii) To find: (fg)(x)

(fg)(x) = f(x).g(x)

= $(2x + 5).(x^2 + x)$

= $2x(x^2) + 2x(x) + 5(x^2) + 5x$

$= 2x^3 + 2x^2 + 5x^2 + 5x$

$= 2x^3 + 7x^2 + 5x$

Therefore,

$(fg)(x) = 2x^3 + 7x^2 + 5x$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{2x+5}{x^2+x}\right)$$

Therefore,

$$\left(\frac{f}{g}\right)(x) = \left(\frac{2x+5}{x^2+x}\right)$$

Q. 3. . Let f: R → R: f(x) = x^3 + 1 and g: R → R: g(x) = (x + 1). Find:

(i) (f + g) (x)
(ii) (f – g) (x)
(iii) (1/f) (x)
(iv) (f/g) (x)

Answer : (i) Given:

$f(x) = x^3 + 1$ and $g(x) = x + 1$

(i) To find: (f + g) (x)

$(f + g)(x) = f(x) + g(x)$

$= (x^3 + 1) + (x + 1)$

$= x^3 + 1 + x + 1$

$= x^3 + x + 2$

Therefore,

$(f + g)(x) = x^3 + x + 2$

(ii) To find: $(f - g)(x)$

$(f - g)(x) = f(x) - g(x)$

$= (x^3 + 1) – (x + 1)$

$= x^3 + 1 – x - 1$

$= x^3 - x$

Therefore,

$(f - g)(x) = x^3 - x$

(iii) To find: $\left(\frac{1}{f}\right)(x)$

$$\left(\frac{1}{f}\right)(x) = \left(\frac{1}{f(x)}\right)$$

$$= \left(\frac{1}{x^3 + 1}\right)$$

Therefore,

$$\left(\frac{1}{f}\right)(x) = \left(\frac{1}{x^3 + 1}\right)$$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{x^3 + 1}{x + 1}\right)$$

$$= \left(\frac{x^3 + 1^3}{x + 1}\right)$$

$= \left(\frac{(x+1)(x^2-x+1)}{x+1}\right)$ (Because $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$)

Therefore,

$\left(\frac{f}{g}\right)(x) = x^2 - x + 1$

Q. 4. . Let f: R → R; f(x)= $\frac{x}{c}$,where c is a constant. Find
(i) (cf) (x)
(ii) (c^2 f) (x)
(iii) $\left(\frac{1}{c}f\right)(x)$

Answer : Given:

$f(x) = \frac{x}{c}$

(i) To find:(cf) (x)

(cf)(x) = c.f(x)

$= c.\left(\frac{x}{c}\right)$

= x

Therefore,

(cf)(x) = x

(ii) To find: (c^2f) (x)

(c^2f) (x) = c^2. f(x)

$= c.\left(\frac{x}{c}\right)$

= cx

Therefore,

(c^2f) (x) = cx

(iii) To find: $\left(\frac{1}{c}f\right)(x)$

$\left(\frac{1}{c}f\right) = \frac{1}{c}$ .f(x)

$= \frac{1}{c}\left(\frac{x}{c}\right)$

Therefore,

$$\left(\frac{1}{c}f\right)(x) = \frac{x}{c^2}$$

Q. 5. Let f:(2, ∞) → R: f(x) = $\sqrt{x-2}$, and g: (2, ∞) → R: g(x) = $\sqrt{x+2}$ Find:
(i) (f + g) (x)
(ii) (f - g) (x)
(iii) (fg) (x)

Answer : Given:

f(x)=$\sqrt{x-2}$: x > 2 and g(x)=$\sqrt{x+2}$:x > 2

(i) To find: (f + g) (x)

Domain(f) = (2, ∞)

Range(f) = (0, ∞)

Domain(g) = (2, ∞)

Range(g) = (2, ∞)

(f + g) (x) = f(x) + g(x)

$= \sqrt{x-2} + \sqrt{x+2}$

Therefore,

(f + g) (x) = $\sqrt{x-2} + \sqrt{x+2}$

(ii) To find:(f - g)(x)

Range(g) $\subseteq$ Domain(f)

Therefore,

(f - g)(x) exists.

(f - g)(x) = f(x) – g(x)

= $\sqrt{x-2}+\sqrt{x+2}$

Therefore,

(f - g) (x) = $\sqrt{x-2}-\sqrt{x+2}$

(iii) To find:(fg)(x)

(fg)(x) = f(x).g(x)

= $\left(\sqrt{x-2}\right).\left(\sqrt{x+2}\right)$

= $\sqrt{(x-2)(x+2)}$

= $\sqrt{x^2-\ 2^2}\ \left(\because\ a^2-\ b^2\ =(a-b)(a+b)\right)$

= $\sqrt{x^2-4}$

Therefore,

(fg)(x) = $\sqrt{x^2-4}$

Exercise 3F

Q. 1. Find the set of values for which the function f(x) = 1 – 3x and g(x) = 2x^2 – 1 are equal.

Answer : f(x) = 1 – 3x, g(x) = 2x^2 – 1

To find:- Set of values of x for which f(x) = g(x)

Consider,

f(x) = g(x)

$1 - 3x = 2^{x^2} - 1$

$2^{x^2} + 3x - 2 = 0$

$2^{x^2} + 4x - x - 2 = 0$

2x (x + 2) –(x + 2) = 0

(x+2)(2x-1) = 0

x = -2 or x = �

The set values for which f(x) and g(x) have same value is { -2 , � }.

Q. 2. Find the set of values for which the function f(x) = x + 3 and g(x) = $3x^2 - 1$ are equal.

Answer : f(x) = x + 3, g(x) = $3x^2 - 1$

To find:- Set of values of x for which f(x) = g(x)

Consider,

f(x) = g(x)

$x+3 = 3^{x^2} - 1$

$3^{x^2} - x - 4 = 0$

$3^{x^2} - 4x + 3x - 4 = 0$

x(3x-4) +(3x-4) = 0

(3x - 4)(x + 1) = 0

x = 4/3 or x= -1

The set values for which f(x) and g(x) have same value is { 4/3 , -1}.

Q. 3. Let X = {–1, 0, 2, 5} and f : X → R Z: f(x) = $x^3 + 1$. Then, write f as a set of ordered pairs.

Answer : Given, X = {–1, 0, 2, 5}

f : X → R Z: f(x) = $x^3 + 1$

Finding f(x) for each value of x,

(1) $f(-1) = (-1)^3 + 1 = -1 + 1 = 0$

(2) $f(0) = (0)^3 + 1 = 0 + 1 = 1$

(3) $f(2) = (2)^3 + 1 = 8 + 1 = 9$

(4) $f(5) = (5)^3 + 1 = 125 + 1 = 126$

f in ordered pair is represented as

f = {(-1,0),(0,1),(2,9),(5,126)}

Q. 4. Let A = {–2, –1, 0, 2} and f : A → Z: f(x) = x^2 – 2x – 3. Find f(A).

Answer : Given :

A = {–2, –1, 0, 2}

f : A → Z: f(x) = $x^2 – 2x – 3$

Finding f(x) for each value of x,

$f(-2) = (-2)^2 – 2(-2) – 3 = 4 + 4 -3 = 5$

$f(-1) = (-1)^2 – 2(-1) – 3 = 1 + 2 -3 = 0$

$f(0) = (0)^2 – 2(0) – 3 = 0 + 0 -3 = -3$

$f(2) = (2)^2 – 2(2) – 3 = 4 - 4 -3 = -3$

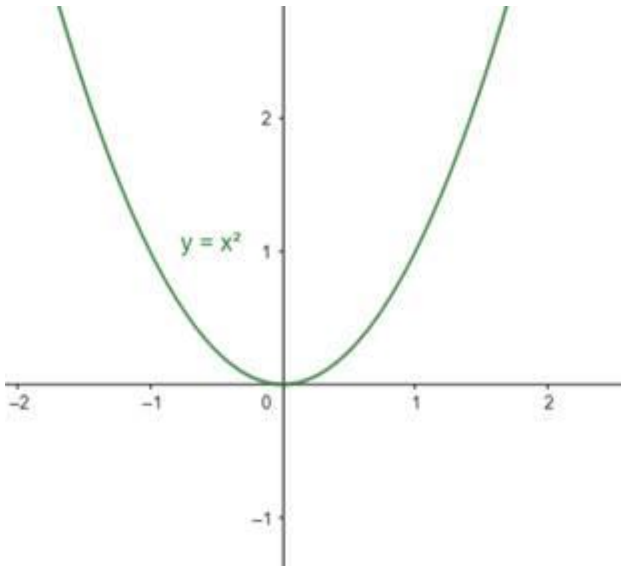
f in ordered pair is represented as

f = {(-2,5),(-1,0),(0,-3),(2,-3)}

Q. 5. Let f : R → R : f(x) = x^2.
Determine (i) range (f) (ii) {x : f(x) = 4}

Answer : Given: f(x) = x^2

The graph for the given function is



(i) Range(f):

For finding the range of the given function, let y = f(x)

Therefore,

$y = x^2$

$x = \sqrt{y}$

The value of $y \geq 0$.

Hence, Range(f) is $[0, \infty)$.

(ii) Let y = f(x) = x^2

Given y = 4.

Therefore, x^2=4

x = 2 or x = -2

The set of values for which y = 4 is x = {2,-2}.

Q. 6. Let f : R → R : f(x) = x^2 + 1. Find f^{-1} {10}.

Answer : Given:

f : R → R : f(x) = x^2 + 1

To find inverse of f(x)

Let y = f(x)

$y = x^2 + 1$

$y - 1 = x^2$

$x = \sqrt{y - 1}$

$f^{-1}(x) = \sqrt{x - 1}$

Substituting x = 10,

$f^{-1}(10) = \sqrt{10 - 1} = \sqrt{9} = 3$

Q. 7. Let f : $R^+ \rightarrow R$: f(x) = $\log_e x$. Find {x : f(x) = –2}.

Answer : Given, f : $R^+ \rightarrow R$: f(x) = $\log_e x$

f(x) = -2

$\log_e x = -2$

Taking antilog on both sides

$x = e^{-2}$

Hence, the value of x for which f(x) = -2 is e^{-2}.

Q. 8. Let A = {6, 10, 11, 15, 12} and let f : A → N : f(n) is the highest prime factor of n. Find range (f).

Answer : Given, A = {6, 10, 11, 15, 12}

f : A → N : f(n) is the highest prime factor of n

(1) When n = 6, the highest prime factor of 6 is 3.

Hence, f(6) = 3.

(2) When n = 10, the highest prime factor of 10 is 5.

Hence, f(10) = 5.

(3) When n = 11, the highest prime factor of 11 is 11 as 11 itself is a prime number.
Hence, f(11) = 11.

(4) When n = 15, the highest prime factor of 15 is 5.

Hence, f(15) = 5.

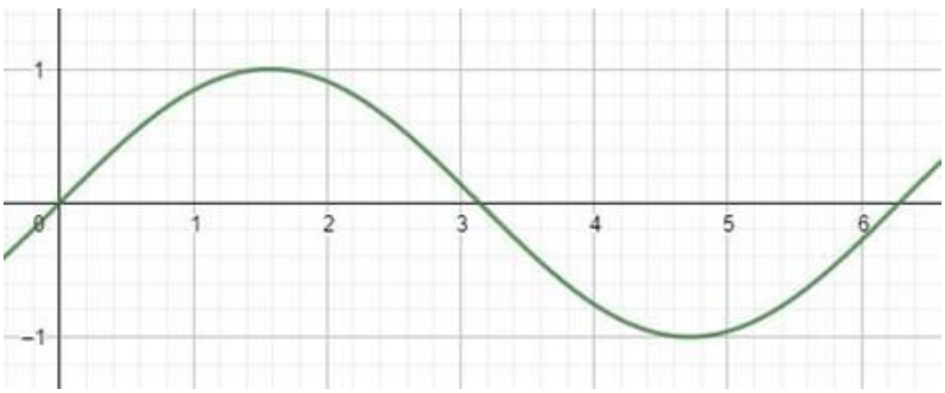
(5) When n = 12, the highest prime factor of 12 is 3.

Hence, f(12) = 3.

Hence range of f is { 3,5,11}.

Q. 9. Find the range of the function f(x) = sin x.

Answer : The graph of sin(x) is



Sin(x) is a periodic function whose values always lies between -1 to +1. The maximum value is attained at $n\frac{\pi}{2}$ where n is odd and minimum when n is even.
Hence, Range is [-1, + 1].

Q. 10. Find the range of the function f(x) = |x|.

Answer : |x| is defined as

|x|= x; x>=0

-x; x<0

The value of |x| is never a negative value.

Hence range of |x| is [0, ∞).

Q. 11. Write the domain and the range of the function, $f(x)=\sqrt{x-[x]}$.

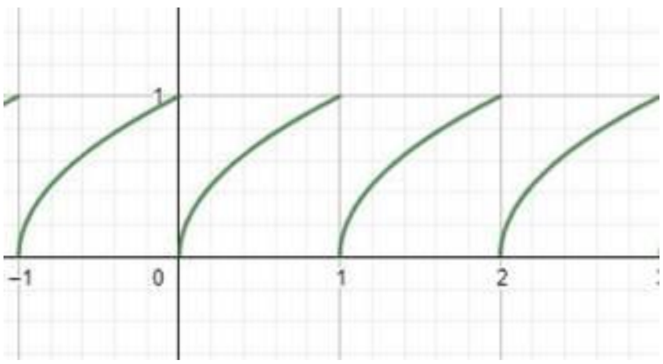
Answer : Given, f(x) = $\sqrt{x-[x]}$

Where [x] is the Greatest Integer Function of x.

f(x) = $\sqrt{\{x\}}$

Where {x} is fractional part of x.

The graph of f(x) is



(i) dom(f)

Domain of{x} is R.

The value of the fractional part of x is always either positive or zero.

Hence domain of f(x) is R.

(ii) range(f)

Range of {x} is [0, 1).

As the root value [0, 1) between interval lies between [0,1).

Hence range of f(x) is [0, 1).

Q. 12. If $f(x) = \frac{x-5}{5-x}$ **then find dom (f) and range (f).**

Answer : Given, $f(x) = \frac{x-5}{5-x}$

(i) dom(f)

Here f(x) is a polynomial function whose domain is R except for points at which denominator becomes zero.

Hence x ≠ 5

The domain is (−∞,∞) – {5}

(ii) range(f)

Let $y = \frac{x-5}{5-x}$

For the specified domain

y = - 1

Range is {-1}.

Q. 13. Let f = {(1, 6), (2, 5), (4, 3), (5, 2), (8, –1), (10, –3)} and g = {(2, 0), (3, 2), (5, 6), (7, 10), (8, 12), (10, 16)}.

Find (i) dom (f + g) (ii) dom $\left(\frac{f}{g}\right)$**.**

Answer : Given, f = {(1, 6), (2, 5), (4, 3), (5, 2), (8, –1), (10, –3)}

g = {(2, 0), (3, 2), (5, 6), (7, 10), (8, 12), (10, 16)}

(1) Domain of f ={1,2,4,5,8,10}

Domain of g ={2,3,5,7,8,10}

Domain of (f + g) = {x : x ∈D f ∩ Dg }

Where Df = Domain of function f, Dg = Domain of function g

Domain of (f + g) = {2,5,8,10}.

(2) Domain of quotient function f/ g = {x : x ∈D f ∩ Dg and g (x) ≠ 0}

Domain of (f/g) = {2,5,8,10}.

Q. 14. If $f(x)=\frac{x-1}{x}$ **, find the value of** $\left\{f\frac{1}{x}\right\}$ **.**

Answer : Given, $f(x)=\frac{x-1}{x}$

F(x) = 1 – 1/x

To find f(1/x) replacing x by 1/x

F(1/x) = 1 – 1/(1/x)

F(1/x) = 1 – x

Q. 15. If $f(x)=\frac{kx}{x+1}$**, where x ≠ –1 and f{f(x)} = x for x ≠ –1 then find the value of k.**

Answer : Given. $f(x)=\frac{kx}{x+1}$, x ≠ –1

F(f(x)) = f($\frac{kx}{x+1}$)

= $\frac{k\frac{kx}{x+1}}{\frac{kx}{x+1}+1}$

= $\frac{k^2 x}{kx+x+1}$

Given that f(f(x)) = x

x = $\frac{k^2 x}{kx+x+1}$

Dividing both sides by x

1 = $\frac{k^2}{kx+x+1}$

kx + x + 1 = k^2

1 k^2 – kx – (x+1) = 0

k = $\frac{-(-x)+\sqrt{(-x)^2-4(1)(-(x+1))}}{2(1)}$ or k = $\frac{-(-x)-\sqrt{(-x)^2-4(1)(-(x+1))}}{2(1)}$

k = $\frac{x+\sqrt{x^2+4x+4}}{2}$ or k = $\frac{x-\sqrt{x^2+4x+4}}{2}$

k = $\frac{x+x+2}{2}$ or k = $\frac{x-x-2}{2}$

k = x + 1 or k = -1

As value of x is variable we take k = -1.

Therefore, k= -1

Q. 16. Find the range of the function, $f(x) = \frac{x}{|x|}$**.**

Answer : |x| is defined as

|x|= x; x>=0

-x; x<0

$\frac{1}{|x|}$ = $\frac{1}{x}$; x>0

= $\frac{-1}{x}$; x<0

$\frac{x}{|x|}$ = 1; x >0

= -1; x<0

Hence f(x) gives output values 1 and -1 only.

Range is {1,-1}.

Q. 17. Find the domain of the function, f(x) = log |x|.

Answer : log x function has domain R^+ .

When x is replaced by |x|, the function f shows value as

f(x) = log(x); x>0

= log(-x); x<0

Hence in the function x cannot be zero as log function is not defined for x=0.

Domain of f(x) is R – {0}

Q. 18. If $f\left(x+\frac{1}{x}\right)$ = $\left(x^2+\frac{1}{x^2}\right)$ for all x ϵ R – {0} then write an expression for f(x).

Answer : Given, $f\left(x+\frac{1}{x}\right)$ = $\left(x^2+\frac{1}{x^2}\right)$

Let y = x + $\frac{1}{x}$

xy = x^2 + 1

x^2 -xy + 1 =0

X= $\frac{-(-y)\pm\sqrt{(-y)^2-4(1)(1)}}{2}$

X= $\frac{y\pm\sqrt{y^2-4}}{2}$

f(y) = ($(\frac{y\pm\sqrt{y^2-4}}{2})^2+\frac{1}{(\frac{y\pm\sqrt{y^2-4}}{2})^2}$)

f(y) = (($\frac{y^2+y^2-4\pm2y\sqrt{y^2-4}}{4}$) + ($\frac{4}{y^2+y^2-4\pm2y\sqrt{y^2-4}}$))

f(y) = y^2 – 2.

Q. 19. Write the domain and the range of the function, $f(x) = \frac{ax+b}{bx-a}$ **.**

Answer : (i) domain

$$f(x) = \frac{ax+b}{bx-a}$$

As f(x) is a polynomial function whose domain is R except for the points where the denominator becomes 0.

Hence $x \neq \frac{a}{b}$

Domain is R-$\{\frac{a}{b}\}$

(ii) Range

Let $y = \frac{ax+b}{bx-a}$

Y(bx-a) = ax +b

byx -ay = ax + b

byx -ax= ay +b

x(by -a) = ay + b

x = $\frac{ay+b}{by-a}$

x is not defined when denominator is zero.

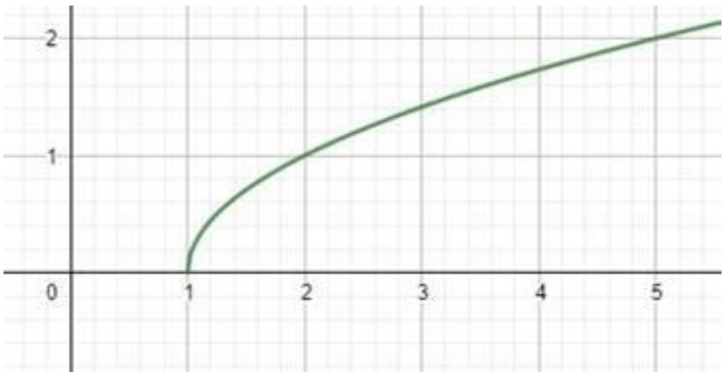
by – a ≠ 0

y≠a/b

Range is R-{a/b}.

Q. 20. Write the domain and the range of the function, f(x) = $\sqrt{x-1}$ **.**

Answer : The graph of f(x) is



(i) Domain

Domain for $\sqrt{x}$ is [0, ∞).

Hence, domain for $\sqrt{x-1}$ is [1, ∞).

(ii) Range

As the range of function f(x) = √x is given by the interval [0, +∞).

The graph of the given function f(x) = √x - 1 is the graph of √ x shifted 1 unit to the right. A shift to the right does not affect the range.

Hence the range of f(x) = √x - 1 is also given by the interval: [0 , +∞).

Q. 21. Write the domain and the range of the function, f(x) = –|x|.

Answer : (i) Domain

|x | is defined for all real values.

Hence -|x| is also defined for all real values.

The domain is R.

(ii) Range

Range for |x| is [0, ∞)

Therefore, range for - |x| is (-∞,0].