7. Linear Inequations (In two variables)

Exercise 7

1. Question

Solve the given inequality $x + y \geq 4$ graphically in two – dimensional plane.

Answer

The graphical representation of $x + y \geq 4$ is given by blue line in the figure below.

This lines divides x-y plane into two parts

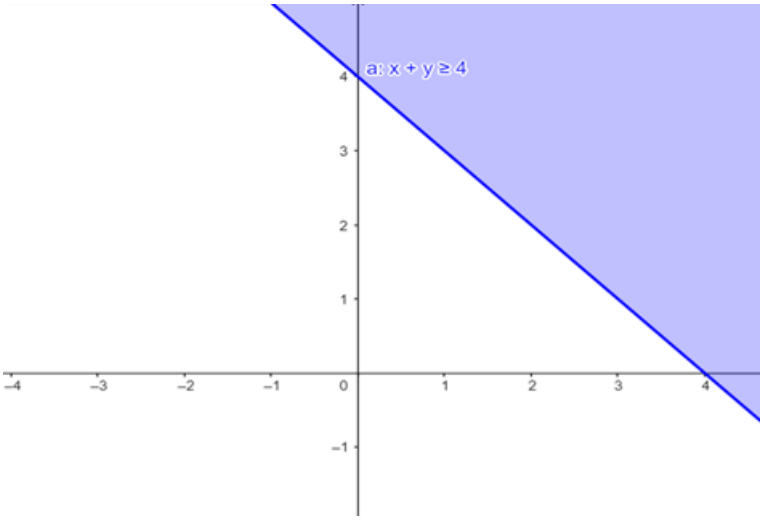
Select a point (not on the line),which lies on one of the two parts, to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 + 0 \geq 4$ *or* $0 \geq 4$ which is false.

Therefore, the solution for the given inequality **including** the points on the line.

This can be represented as follows,



2. Question

Solve the given inequality $x - y \leq 3$ graphically in two – dimensional plane.

Answer

The graphical representation of $x - y \leq 3$ is given by blue line in the figure below.

This lines divides x-y plane into two parts .

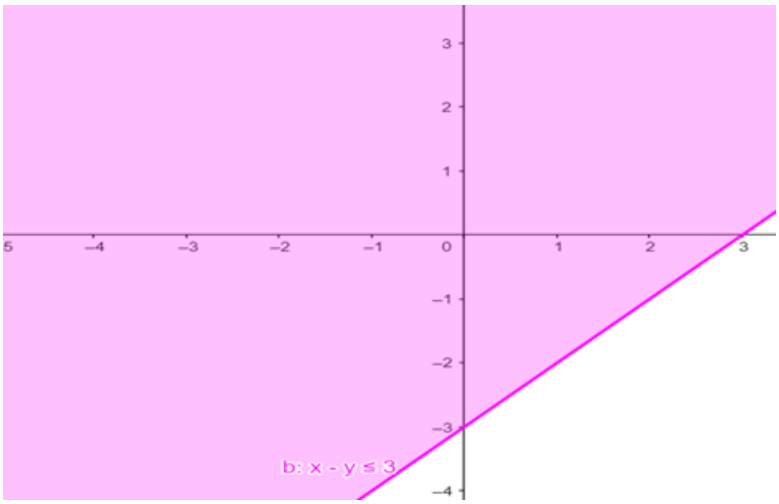
Select a point (not on the line),which lies on one of the two parts, to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 - 0 \leq 3$ *or* $0 \leq 3$ which is true.

Therefore, the solution for the given inequality **including** the points on the line.

This can be represented as follows,



3. Question

Solve the given inequality $y - 2 \leq 3x$ graphically in two – dimensional plane.

Answer

The graphical representation of $y - 2 \leq 3x$ is given by blue line in the figure below.

This lines divides x-y plane into two parts .

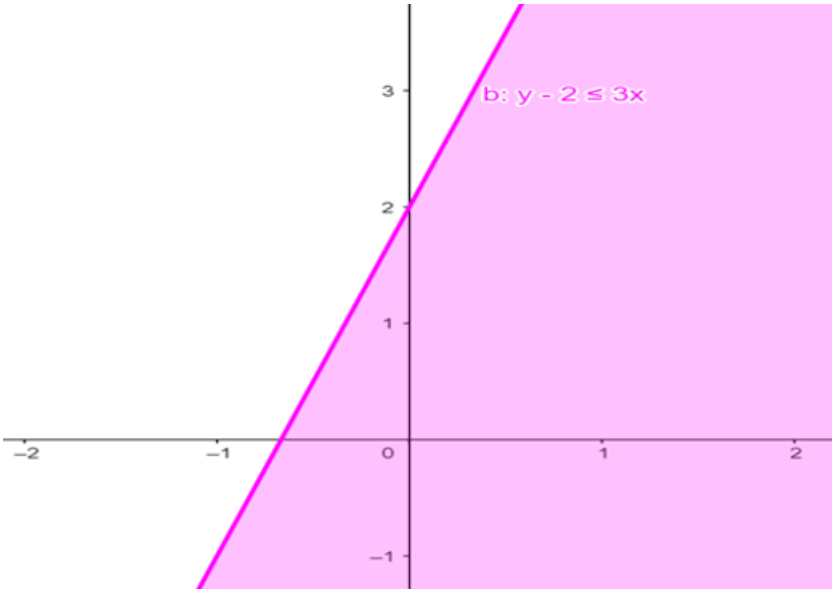
Select a point (not on the line),which lies on one of the two parts,to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 - 2 \leq 3 \times 0 \; or \; -2 \leq 0$ which is true.

Therefore, the solution for the given inequality **including** the points on the line.

This can be represented as follows,



4. Question

Solve the given inequality $x \geq y - 2$ graphically in two – dimensional plane.

Answer

The graphical representation of $x \geq y - 2$ is given by blue dotted line in the figure below.

This lines divides x-y plane into two parts .

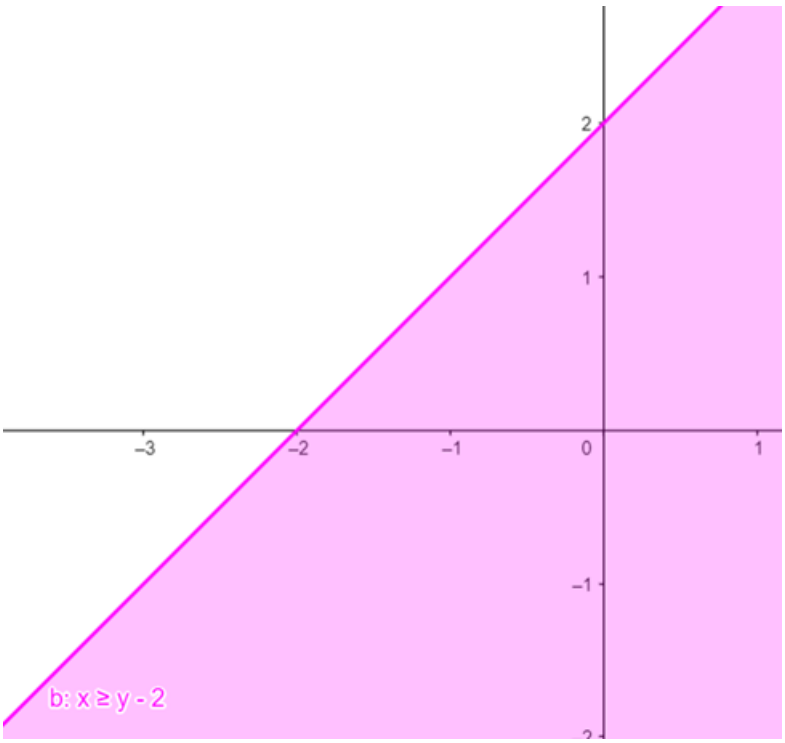
Select a point (not on the line),which lies on one of the two parts,to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 > 0 - 2 \; or \; 0 > -2$ which is false.

Therefore, the solution for the given inequality **excluding** the points on the line.

This can be represented as follows,



5. Question

Solve the given inequality 3x + 2y > 6 graphically in two – dimensional plane.

Answer

The graphical representation of $3x + 2y > 6$ is given by blue dotted line in the figure below.

This lines divides x-y plane into two parts .

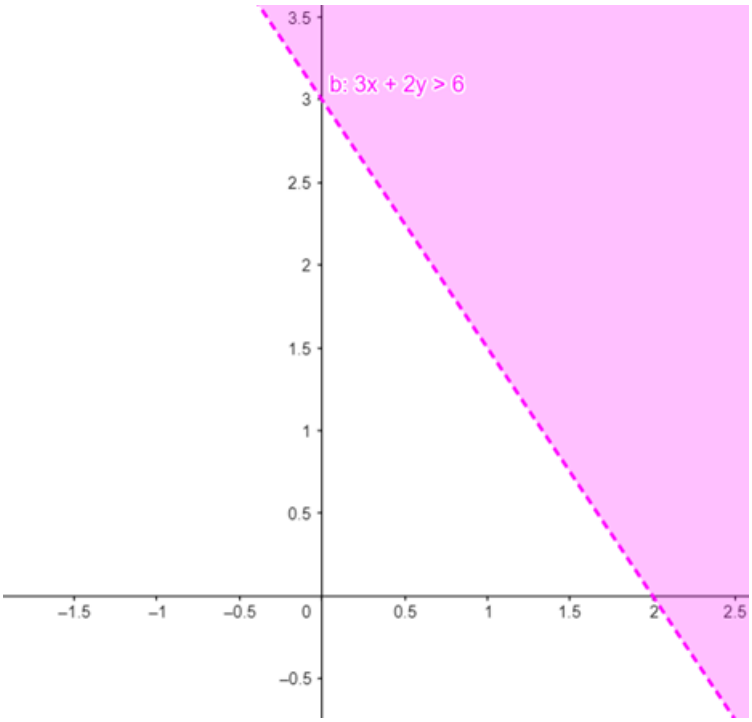
Select a point (not on the line),which lies on one of the two parts,to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 + 0 > 6 \; or \; 0 > 6$ which is false.

Therefore, the solution for the given inequality **excluding** the points on the line.

This can be represented as follows,



6. Question

Solve the given inequality 3x + 5y < 15 graphically in two – dimensional plane.

Answer

The graphical representation of 3x + 5y < 15 is given by blue dotted line in the figure below.

This lines divides x-y plane into two parts .

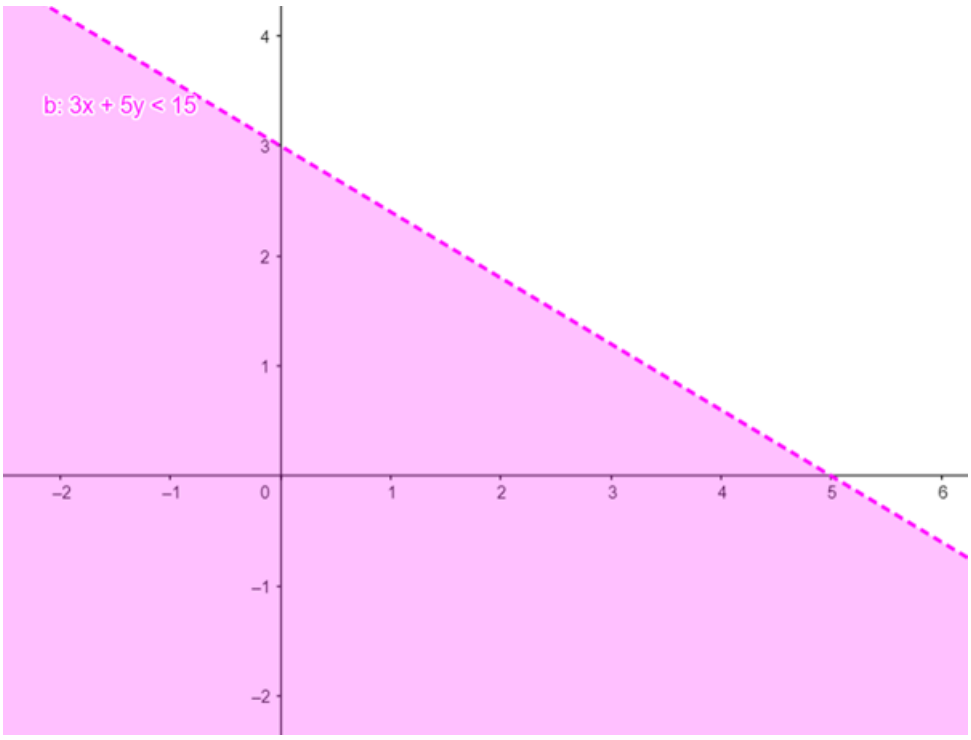
Select a point (not on the line),which lies on one of the two parts,to determine whether the point satisfies the given inequality or not.

We select the point as (0,0)

It is observed that $0 + 0 < 15 \; or \; 0 < 15$ which is true.

Therefore, the solution for the given inequality **excluding** the points on the line.

This can be represented as follows,



7. Question

Solve the given inequalities x ≥ 2y, y ≥ 3 graphically in two – dimensional plane.

Answer

The graphical representation of x ≥ 2y, y ≥ 3 is given by common region in the figure below.

$x \geq 2y$ ..... (1)

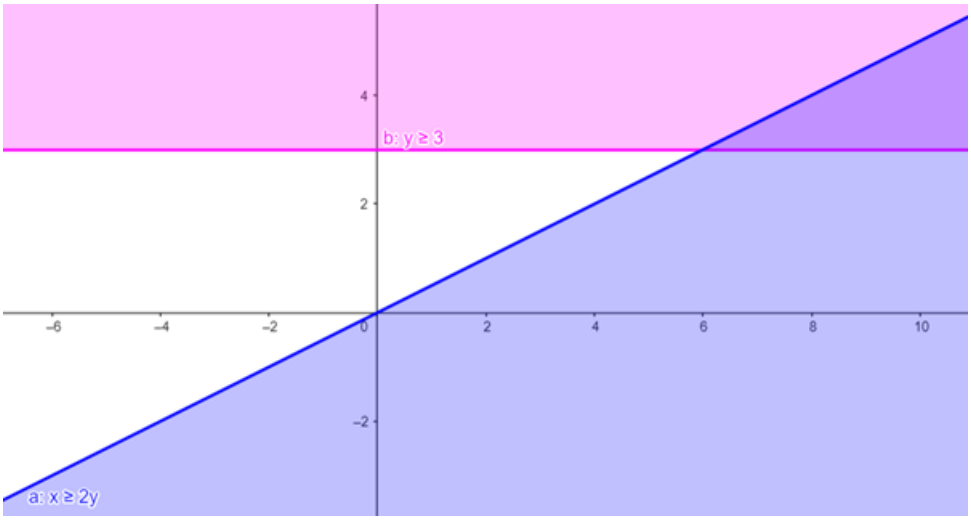
$y \geq 3$ ..... (2)

Inequality (1) represents the region below line x=2y(**including** the line x=2y).

Inequality (2) represents the region above line y=3(**including** the line y=3).

Therefore, every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



8. Question

Solve the given inequalities 3x + 2y ≤ 12, x ≤ 1, y ≥ 2 graphically in two – dimensional plane.

Answer

The graphical representation of $3x + 2y \leq 12, x \leq 1, y \geq 2$ is given by common region in the figure below.

$3x + 2y \leq 12$ ..... (1)

$x \leq 1$ ..... (2)

$y \geq 2$ ..... (3)

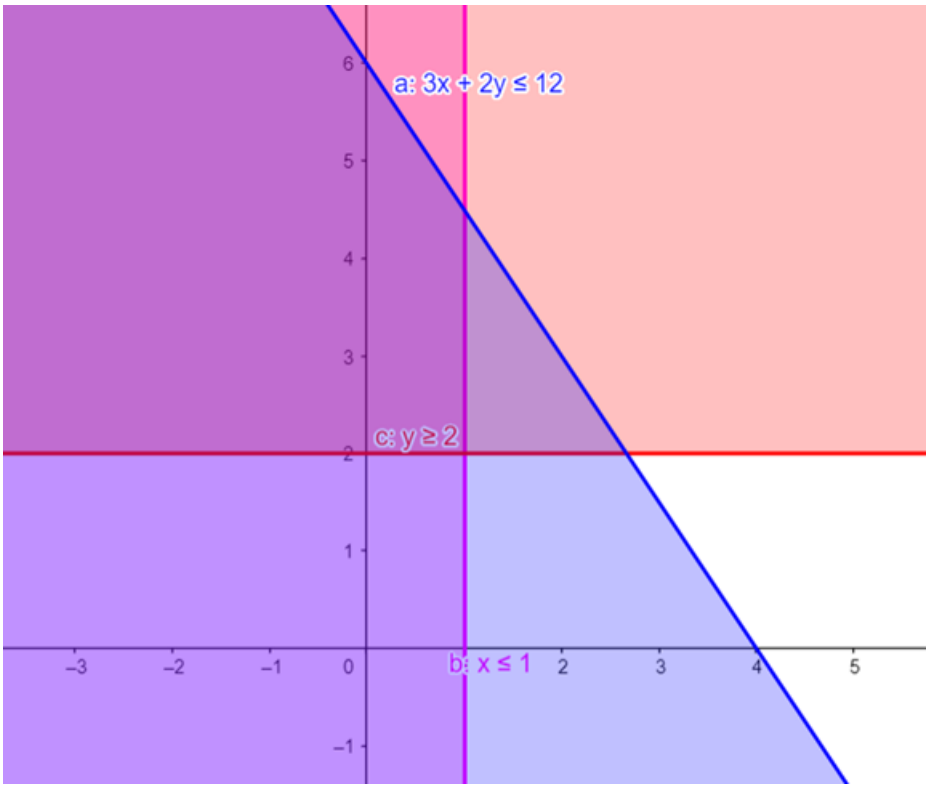
Inequality (1) represents the region below line $3x + 2y = 12$ (**including** the line $3x + 2y = 12$).

Inequality (2) represents the region behind line $x = 1$ (**including** the line x=1).

Inequality (3) represents the region above line $y = 2$ (**including** the line y=2).

Therefore, every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



9. Question

Solve the given inequalities x + y ≤ 6, x + y ≥ 4graphically in two – dimensional plane.

Answer

The graphical representation of $x + y \leq 6, x + y \geq 4$ is given by common region in the figure below.

$x + y \leq 6$ ..... (1)

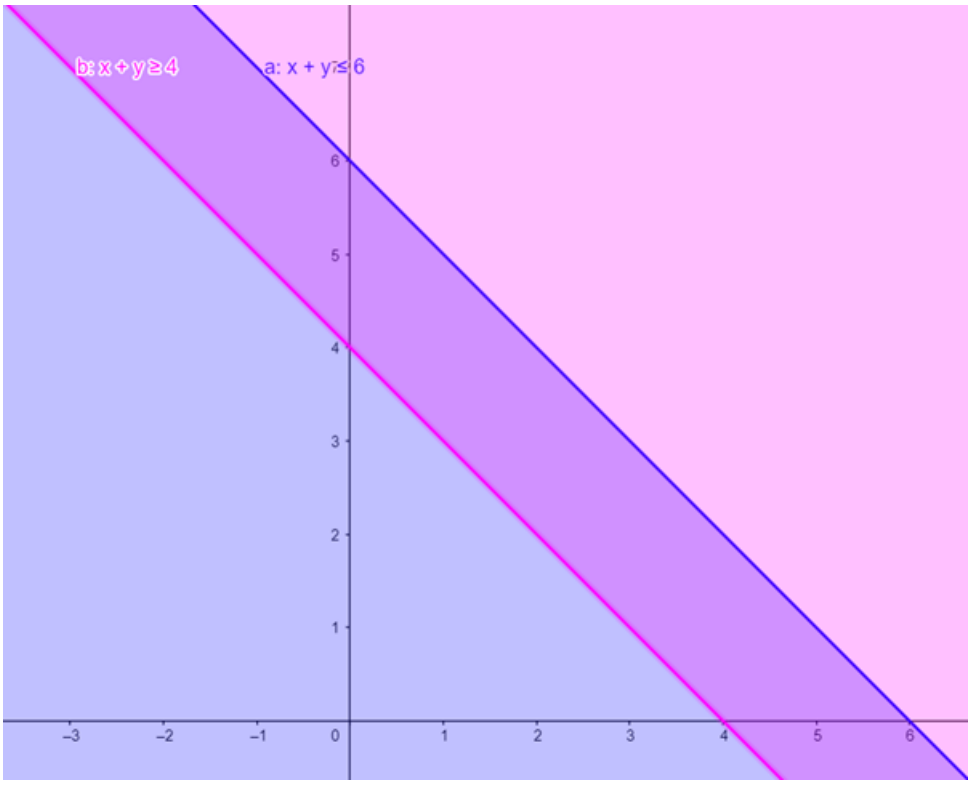
$x + y \geq 4$ ..... (2)

Inequality (1) represents the region below line $x + y = 6$ (**including** the line $x + y = 6$).

Inequality (2) represents the region above line $x + y = 4$ (**including** the line x+y=4).

Therefore, every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



10. Question

Solve the given inequalities 2x + y ≥ 6, 3x + 4y ≤ 12 graphically in two – dimensional plane.

Answer

The graphical representation of $2x + y \geq 6, 3x + 4y \leq 12$ is given by common region in the figure below.

$2x + y \geq 6$ ..... (1)

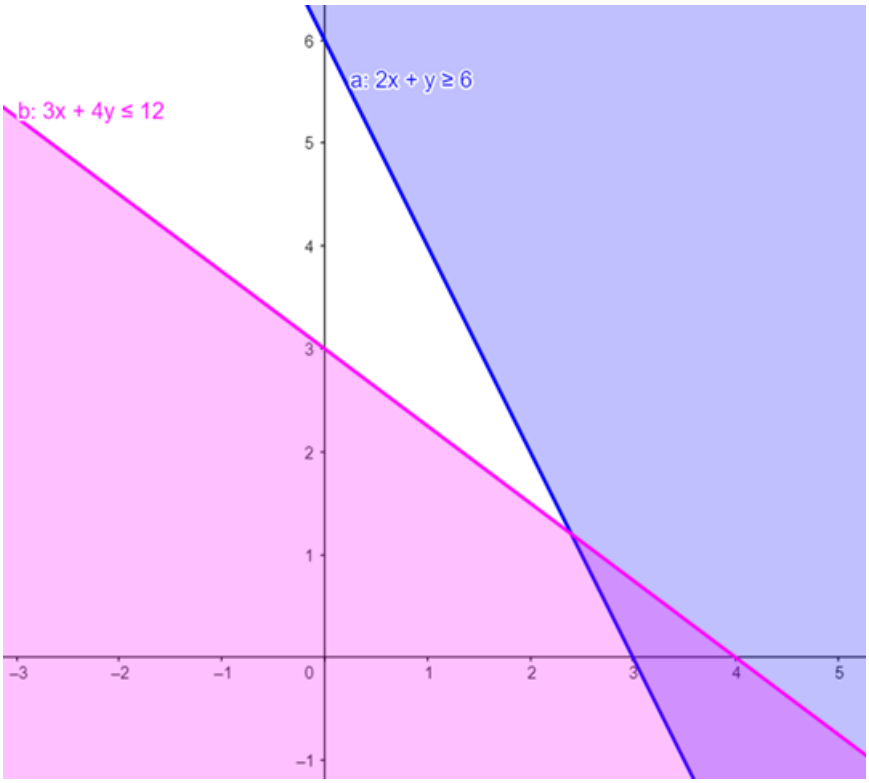
$3x + 4y \leq 12$ ..... (2)

Inequality (1) represents the region above line $2x + y = 6$ (**including** the line $2x + y = 6$).

Inequality (2) represents the region below line $3x + 4y = 12$ (**including** the line 3x+4y=12).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



11. Question

Solve the given inequalities x + y ≤ 9, y < x, x ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $x + y \leq 9, y < x, x \geq 0$ is given by common region in the figure below.

$x + y \leq 9$ ..... (1)

$y < x$ ..... (2)

$x \geq 0$ ..... (3)

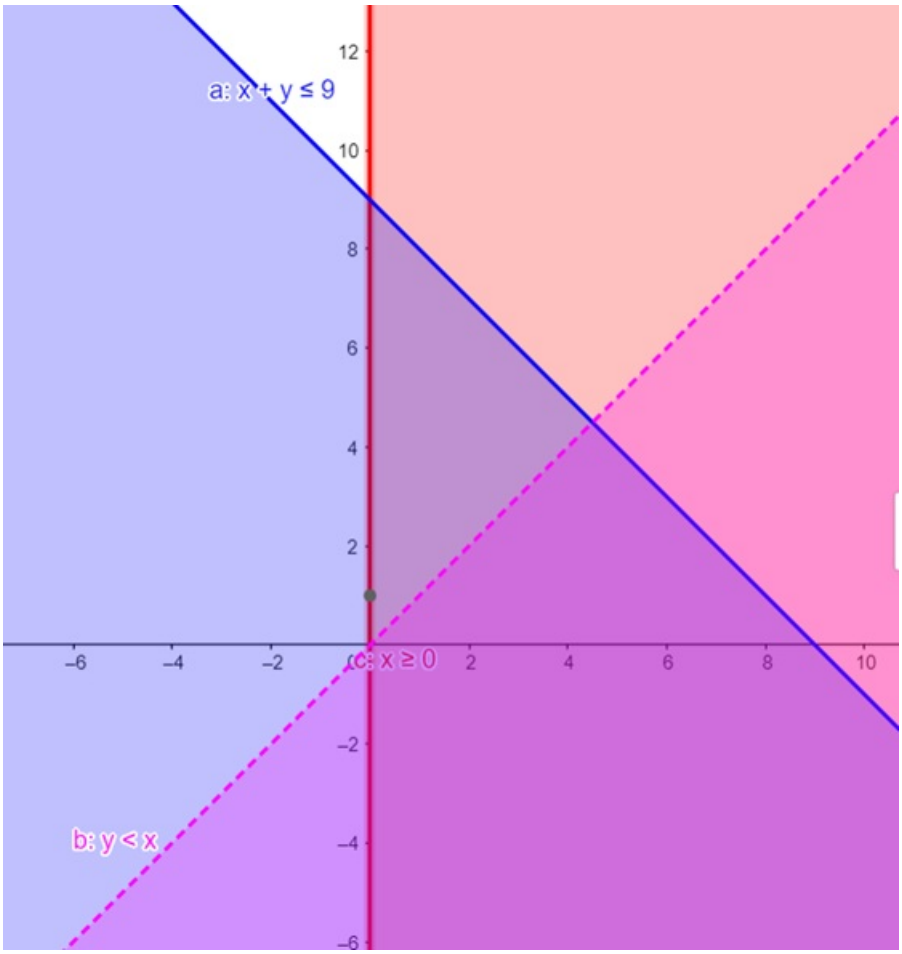
Inequality (1) represents the region below line $x + y = 9$ (**including** the line $x + y = 9$).

Inequality (2) represents the region below line $x = y$ (**excluding** the line $x = y$).

Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



12. Question

Solve the given inequalities 2x – y > 1, x – 2y < 1 graphically in two – dimensional plane.

Answer

The graphical representation of $2x - y > 1, x - 2y < 1$ is given by common region in the figure below.

$2x - y > 1$ ..... (1)

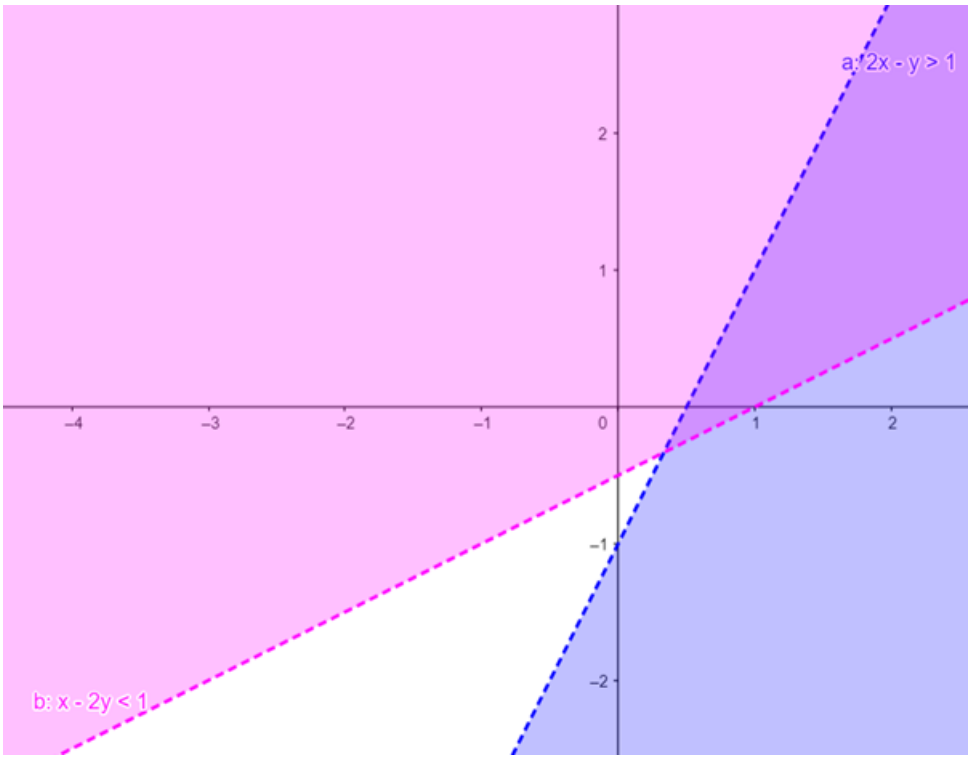
$x - 2y < 1$ ..... (2)

Inequality (1) represents the region below line $2x - y = 1$ (**excluding** the line $2x - y = 1$).

Inequality (2) represents the region above line $x - 2y = 1$ (**excluding** the line $x - 2y = 1$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



13. Question

Solve the given inequalities 5x + 4y ≤ 20, x ≥ 1, y ≥ 2graphically in two – dimensional plane.

Answer

The graphical representation of $5x + 4y \leq 20, x \geq 1, y \geq 2$ is given by common region in the figure below.

$5x + 4y \leq 20$ ..... (1)

$x \geq 1$ ..... (2)

$y \geq 2$ ..... (3)

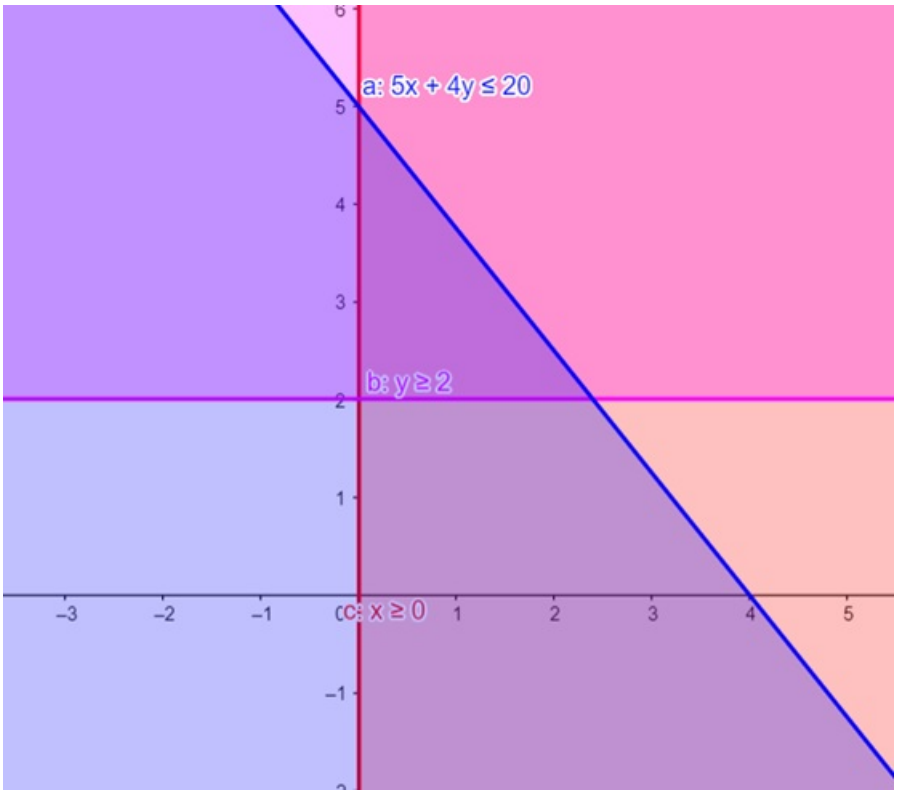
Inequality (1) represents the region below line $5x + 4y = 20$ (**including** the line $5x + 4y = 20$).

Inequality (2) represents the region in front of line $x = 1$ (**including** the line $x = 1$).

Inequality (3) represents the region above line $y = 2$ (**including** the line $y = 2$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



14. Question

Solve the given inequalities 3x + 4y ≤ 60, x + 3y ≤ 30, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$ is given by common region in the figure below.

$3x + 4y \leq 60$ ..... (1)

$x + 3y \leq 30$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

Inequality (1) represents the region below line $3x + 4y = 60$ (**including** the line $3x + 4y = 60$).

Inequality (2) represents the region below line $x + 3y = 30$ (**including** the line $x + 3y = 30$).

Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,

15. Question

Solve the given inequalities 2x + y ≥ 4, x + y ≤ 3, 2x – 3y ≤ 6graphically in two – dimensional plane.

Answer

The graphical representation of $2x + y \geq 4, x + y \leq 3, 2x - 3y \leq 6$ is given by common region in the figure below.

$2x + y \geq 4$ ..... (1)

$x + y \leq 3$ ..... (2)

$2x - 3y \leq 6$ ..... (3)

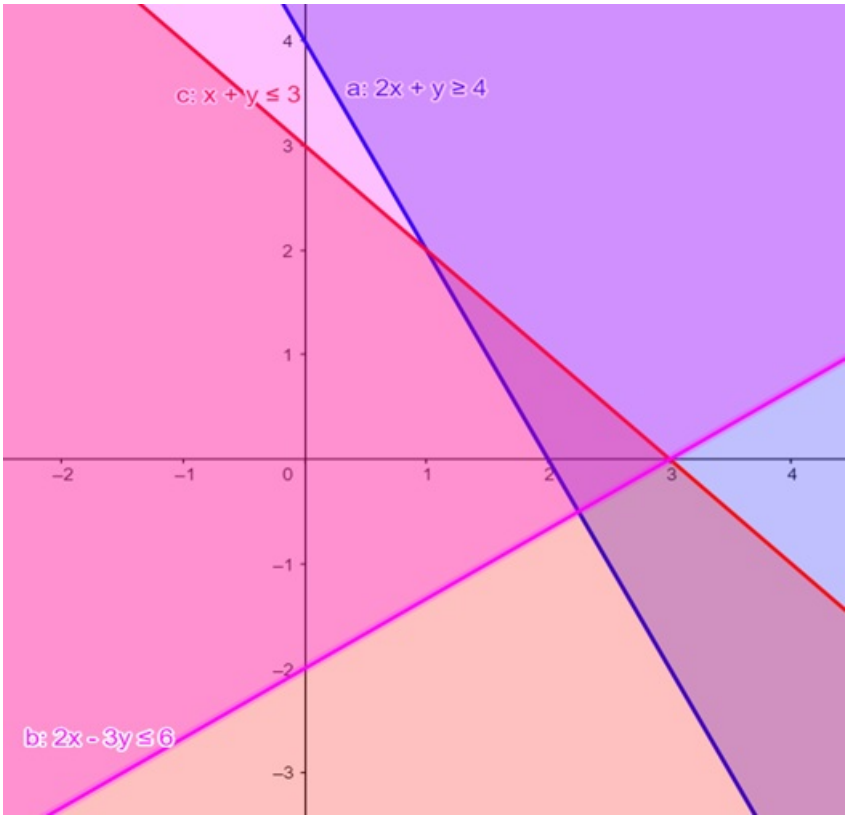
Inequality (1) represents the region above line $2x + y = 4$ (**including** the line $2x + y = 4$).

Inequality (2) represents the region below line $x + y = 3$ (**including** the line $x + y = 3$).

Inequality (3) represents the region above line $2x - 3y = 6$ (**including** the line $2x - 3y = 6$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



16. Question

Solve the given inequalities x + 2y ≤ 10, x + y ≥ 1, x – y ≤ 0, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $x + 2y \leq 10\,, x + y \geq 1\,, y \geq 0$

$x - y \leq 0\,, x \geq 0$ is given by common region in the figure below.

$x + 2y \leq 10$ ..... (1)

$x + y \geq 1$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

$x - y \leq 0$ ..... (5)

Inequality (1) represents the region below line $x + 2y = 10$ (**including** the line $x + 2y = 10$).

Inequality (2) represents the region above line $x + y = 1$ (**including** the line $x + y = 1$).

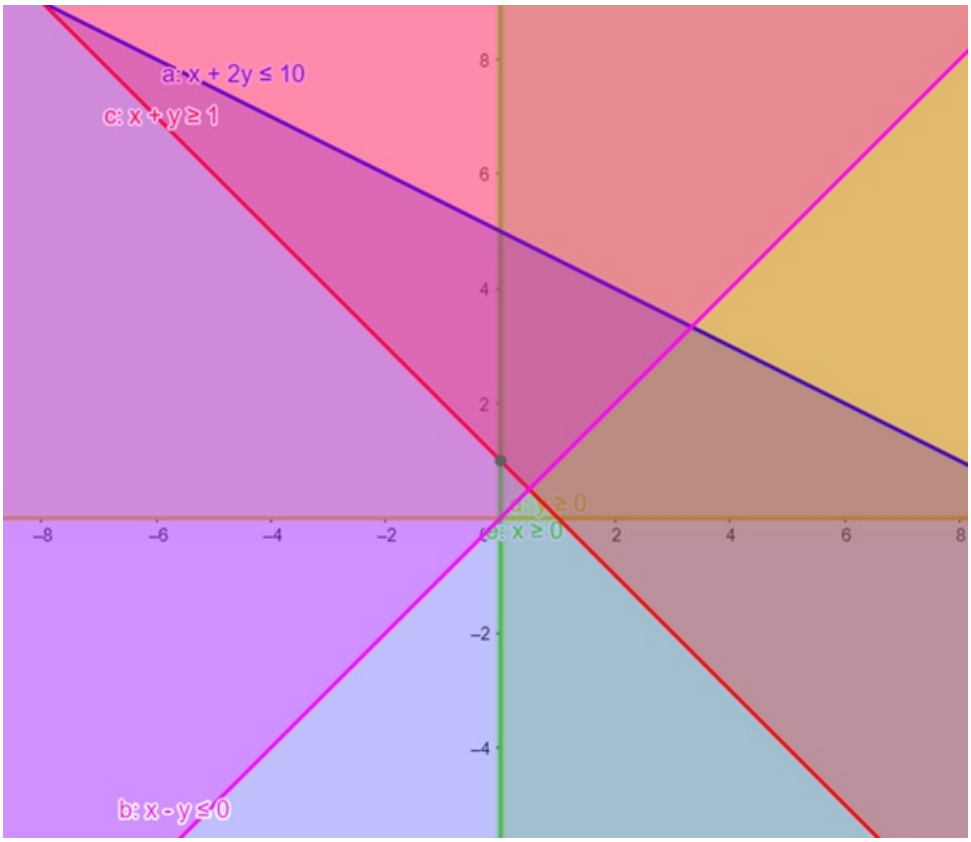
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Inequality (5) represents the region above line $x - y = 0$ (**including** the line $x - y = 0$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



17. Question

Solve the given inequalities 4x + 3y ≤ 60, y ≥ 2x, x ≥ 3, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $4x + 3y \leq 60, y \geq 2x, y \geq 0$

$x \geq 3, x \geq 0$ is given by common region in the figure below.

$4x + 3y \leq 60$ ..... (1)

$y \geq 2x$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

$x \geq 3$ ..... (5)

Inequality (1) represents the region below line $4x + 3y = 60$ (**including** the line $4x + 3y = 60$).

Inequality (2) represents the region above line $y = 2x$ (**including** the line $y = 2x$).

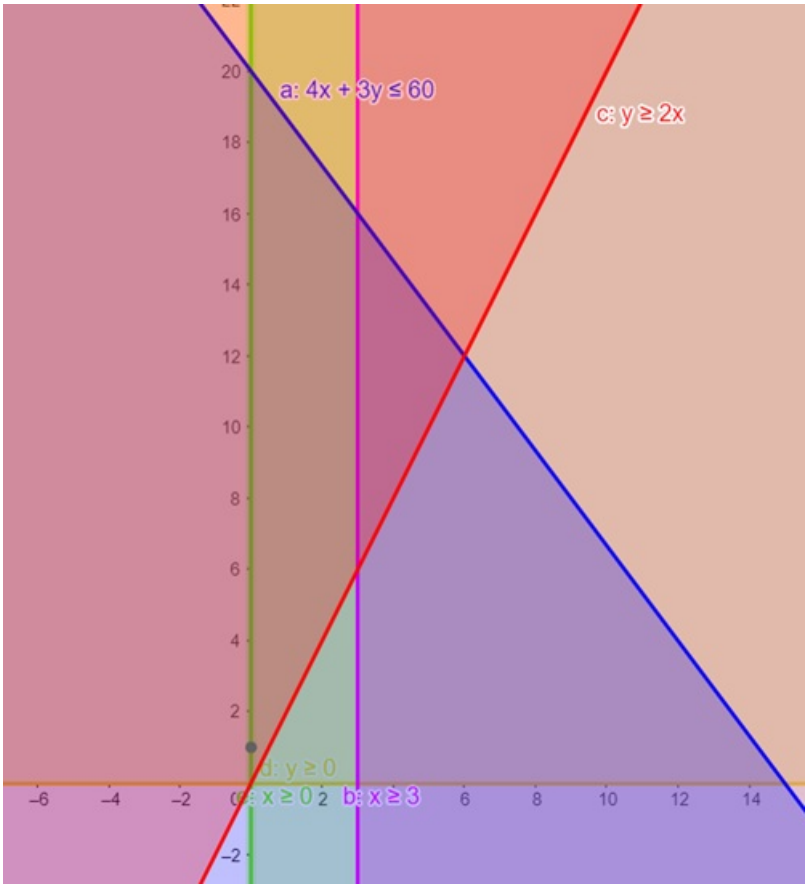
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Inequality (5) represents the region in front of line $x = 3$ (**including** the line $x = 3$)

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



18. Question

Solve the given inequalities x – 2y ≤ 2, x + y ≥ 3, –2x + y ≤ 4, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $x - 2y \leq 2, x + y \geq 3, y \geq 0$

$-2x + y \leq 4, x \geq 0$ is given by common region in the figure below.

$x - 2y \leq 2$ ..... (1)

$x + y \geq 3$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

$-2x + y \leq 4$ ..... (5)

Inequality (1) represents the region above line $x - 2y = 2$ (**including** the line $x - 2y = 2$).

Inequality (2) represents the region above line $x + y = 3$ (**including** the line $x + y = 3$).

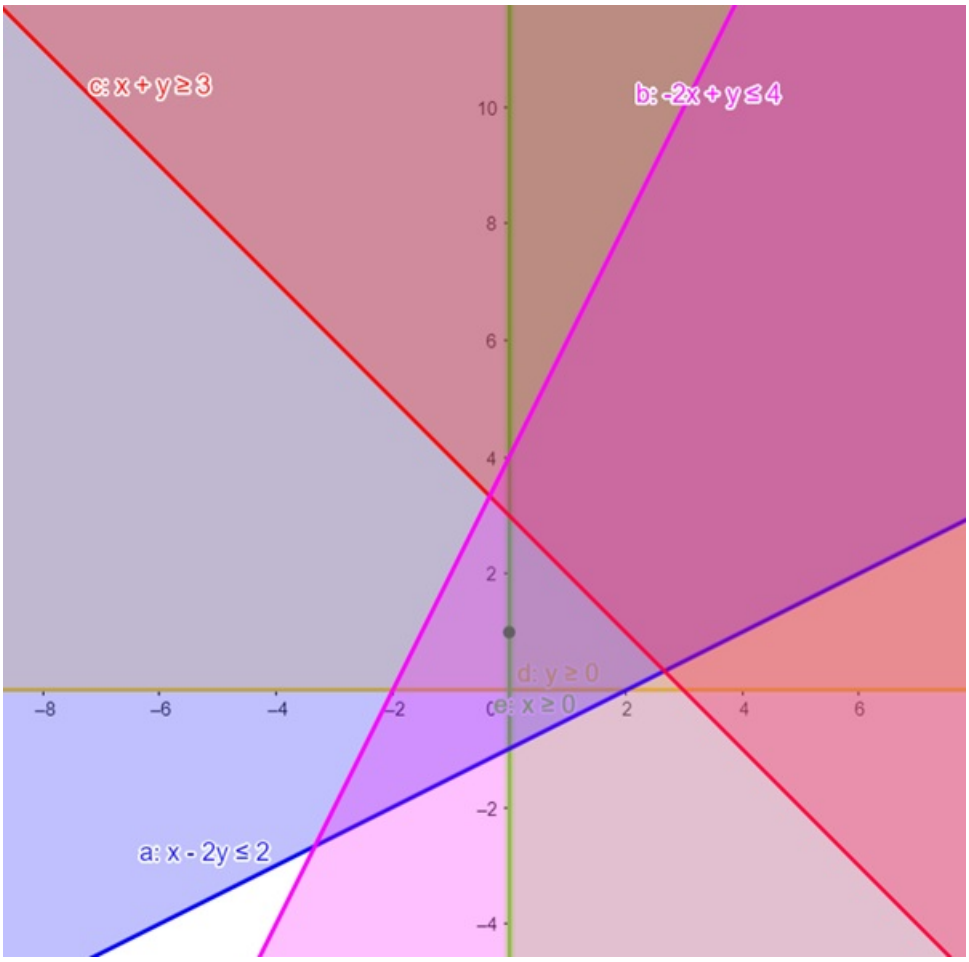
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Inequality (5) represents the region below line $-2x + y = 4$ (**including** the line $-2x + y = 4$).

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



19. Question

Solve the given inequalities x + 2y ≤ 100, 2x + y ≤ 120, x + y ≤ 70, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $x + 2y \leq 100\,, 2x + y \leq 120$

$x + y \leq 70, y \geq 0\,, x \geq 0$ is given by common region in the figure below.

$x + 2y \leq 100$ ..... (1)

$2x + y \leq 120$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

$x + y \leq 70$ ..... (5)

Inequality (1) represents the region below line $x + 2y = 100$ (**including** the line $x + 2y = 100$).

Inequality (2) represents the region below line $2x + y = 120$ (**including** the line $2x + y = 120$).

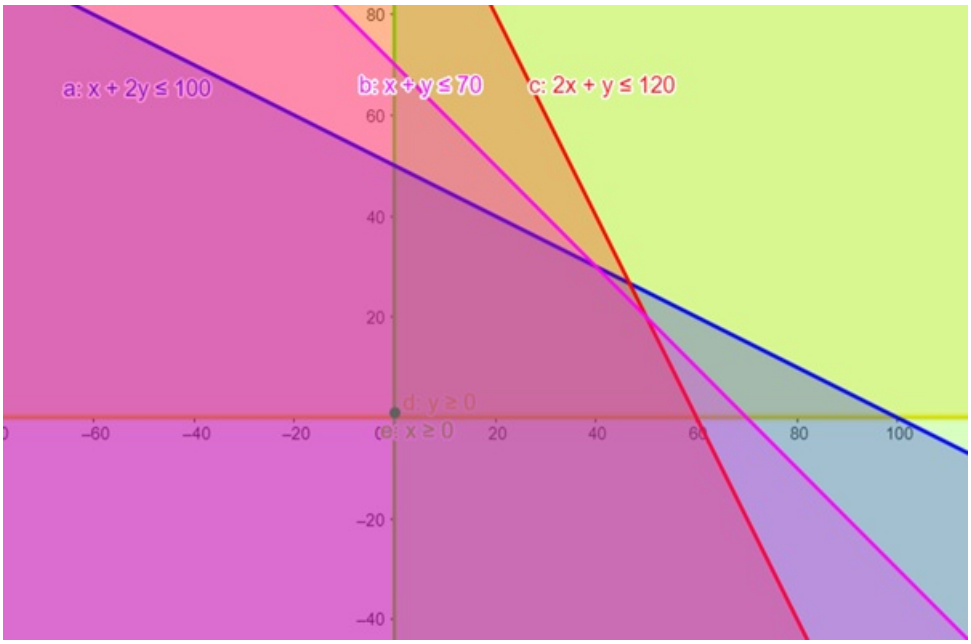
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Inequality (5) represents the region below line $x + y = 70$ (**including** the line $x + y = 70$)

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



20. Question

Solve the given inequalities x + 2y ≤ 2000, x + y ≤ 1500, y ≤ 600, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $x + 2y \leq 2000\,, x + y \leq 1500$

$y \leq 600, y \geq 0\,, x \geq 0$ is given by common region in the figure below.

$x + 2y \leq 2000$ ..... (1)

$x + y \leq 1500$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

$y \leq 600$ ..... (5)

Inequality (1) represents the region below line $x + 2y = 2000$ (**including** the line $x + 2y = 2000$).

Inequality (2) represents the region below line $x + y = 1500$ (**including** the line $x + y = 1500$).

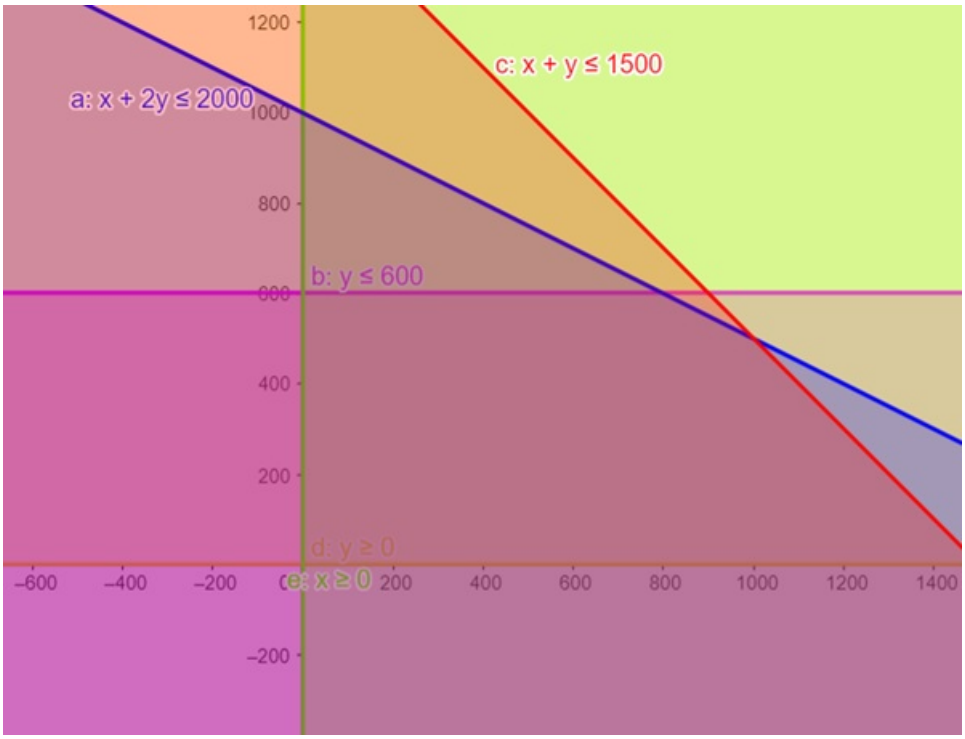
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Inequality (5) represents the region below line $y = 600$ (**including** the line $y = 600$)

Therefore,every point in the common shaded region including the points on the respective lines represents the solution for the given inequalities.

This can be represented as follows,



21 A. Question

Solve the given inequalities 3x + 2y ≥ 24, 3x + y ≤ 15, x ≥ 4,graphically in two – dimensional plane.

Answer

The graphical representation of $3x + 2y \geq 24, 3x + y \leq 15$

$x \geq 4$ is given by common region in the figure below.

$3x + 2y \geq 24$ ..... (1)

$3x + y \leq 15$ ..... (2)

$x \geq 4$ ..... (3)

Inequality (1) represents the region above line $3x + 2y = 24$ (**including** the line $3x + 2y = 24$).

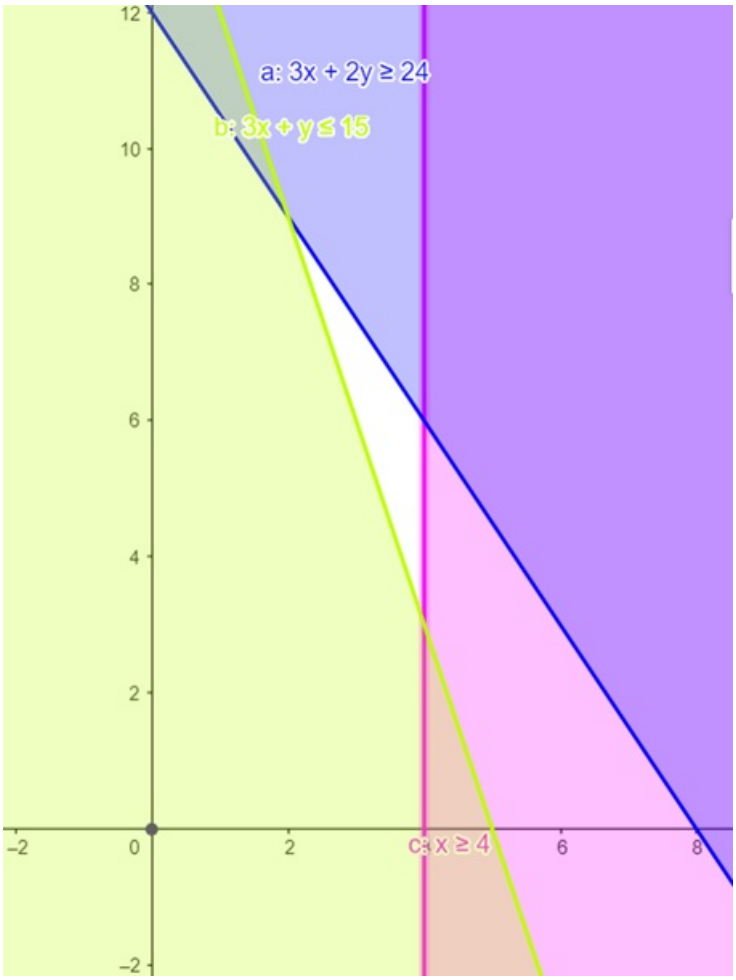
Inequality (2) represents the region below line $3x + y = 15$ (**including** the line $3x + y = 15$).

Inequality (3) represents the region in front of line $x = 4$ (**including** the line $x = 4$).

Therefore, we can see in the figure that there is no common shaded region.

So there linear inequalities in equations has no solution.

This can be represented as follows,



21 B. Question

Solve the given inequalities 2x – y ≤ –2, x – 2y ≥ 0, x ≥ 0, y ≥ 0graphically in two – dimensional plane.

Answer

The graphical representation of $2x - y \leq -2, x - 2y \geq 0$

$x \geq 0, y \geq 0$ is given by common region in the figure below.

$2x - y \leq -2$ ..... (1)

$x - 2y \geq 0$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

Inequality (1) represents the region above line $2x - y = -2$ (**including** the line $2x - y = -2$).

Inequality (2) represents the region below line $x - 2y = 0$ (**including** the line $x - 2y = 0$).

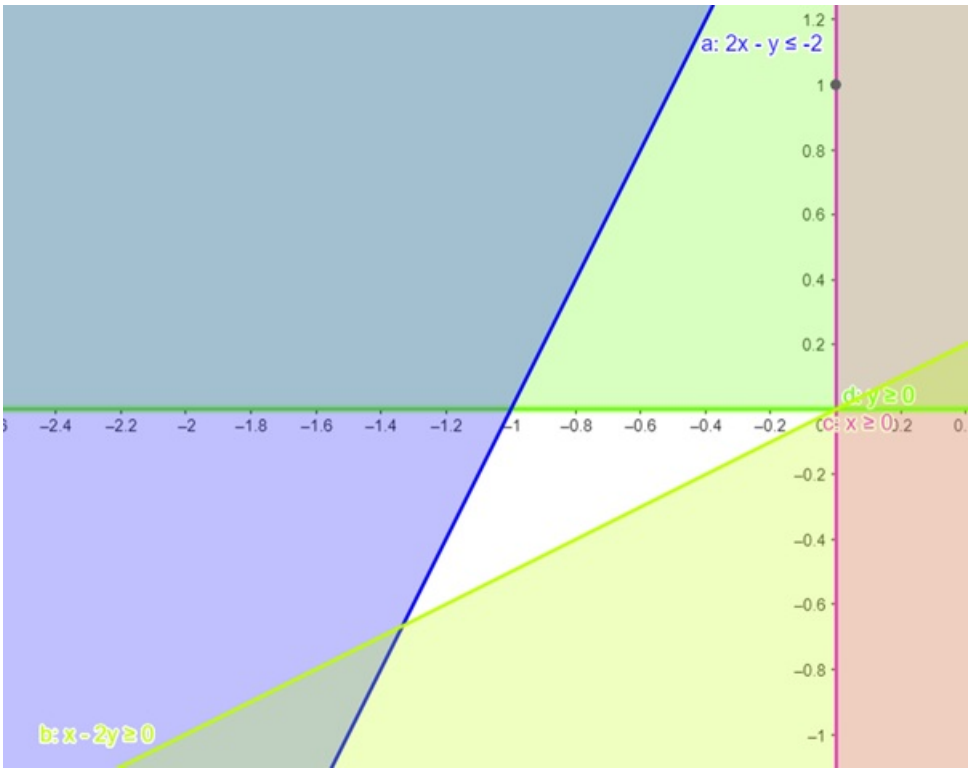
Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$).

Therefore, we can see in the figure that there is no common shaded region.

So there linear inequalities in equations has no solution.

This can be represented as follows,



22. Question

Solve the given inequalities 3x + y ≥ 12, x + y ≥ 9, x ≥ 0, y ≥ 0.graphically in two – dimensional plane.

Answer

The graphical representation of $3x + y \geq 12$, $x + y \geq 9$

$x \geq 0$, $y \geq 0$ is given by common region in the figure below.

$3x + y \geq 12$ ..... (1)

$x + y \geq 9$ ..... (2)

$x \geq 0$ ..... (3)

$y \geq 0$ ..... (4)

Inequality (1) represents the region above line $3x + y = 12$ (**including** the line $3x + y = 12$).

Inequality (2) represents the region above line $x + y = 9$ (**including** the line $x + y = 9$).

Inequality (3) represents the region in front of line $x = 0$ (**including** the line $x = 0$).

Inequality (4) represents the region above line $y = 0$ (**including** the line $y = 0$)

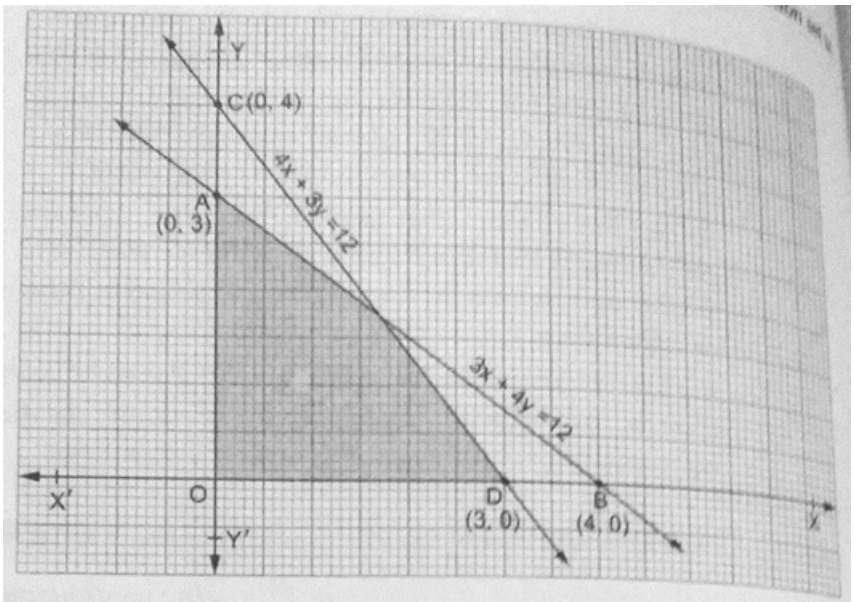
It is clear from the graph , that the region is unbounded.

Therefore , the following system of inequation is an unbounded set.

This can be represented as follows,

23. Question

Find the linear inequalities for which the shaded area is the solution set in the figure given below.



Answer

We have seen that the shaded region and origin are on the same side of the line $3x + 4y = 12$

For (0,0) we have $0 + 0 - 12 < 0$. So the shaded region satisfies the inequality $\mathbf{3x + 4y \leq 12}$.

We have seen that the shaded region and origin are on the same side of the line $4x + 3y = 12$

For (0,0) we have $0 + 0 - 12 < 0$. So the shaded region satisfies the inequality $\mathbf{4x + 3y \leq 12}$.

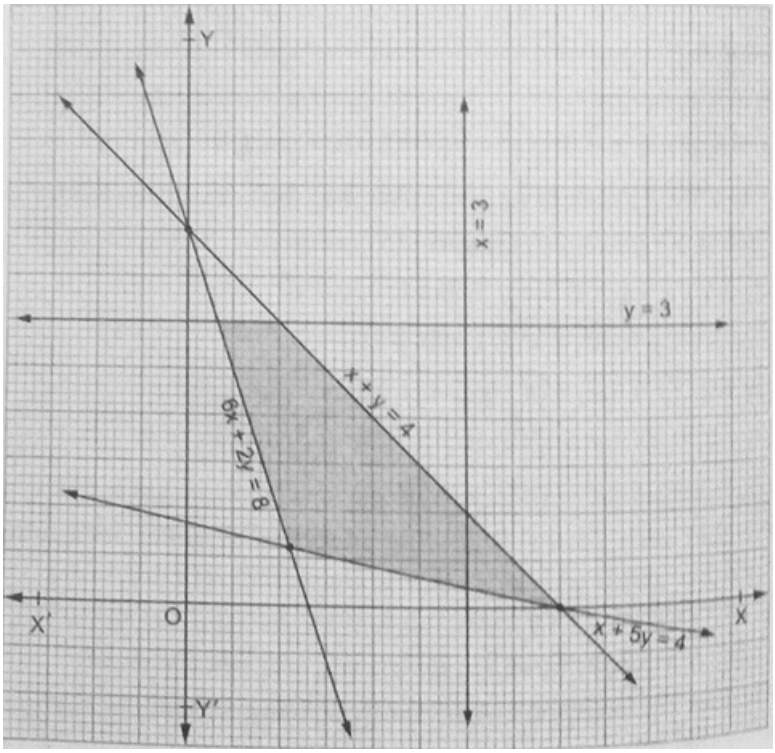
Also , the region lies in the first quadrent . Therefore $\boldsymbol{x \geq 0}$

and $\boldsymbol{y \geq 0}$

Thus the linear inequation comprising the given solution set are $\mathbf{+4y \leq 12}$, $\mathbf{4x + 3y \leq 12}$, $\boldsymbol{x \geq 0}$, $\boldsymbol{y \geq 0}$

24. Question

Find the linear inequalities for which the shaded area is the solution set in the figure given below.



Answer

We have seen that the shaded region and origin are on the opposite side of the line $6x + 2y = 8$

For (0,0) we have $0 + 0 - 8 < 0$. So the shaded region satisfies the inequality $\boldsymbol{6x + 2y \geq 8}$.

We have seen that the shaded region and origin are on the opposite side of the line $x + 5y = 4$

For (0,0) we have $0 + 0 - 4 < 0$. So the shaded region satisfies the inequality $\boldsymbol{x + 5y \geq 4}$.

We have seen that the shaded region and origin are on the same side of the line $x + y = 4$

For (0,0) we have $0 + 0 - 4 < 0$. So the shaded region satisfies the inequality $\boldsymbol{x + y \leq 4}$.

We have seen that the shaded region and origin are on the same side of the line $y = 3$

For (0,0) we have $0 - 3 < 0$. So the shaded region satisfies the inequality $\boldsymbol{y \leq 3}$.

Thus the linear inequation comprising the given solution set are $\boldsymbol{+2y \geq 8}$, $\boldsymbol{x + 5y \geq 4}$, $\boldsymbol{x + y \leq 4}$, $\boldsymbol{y \leq 3}$

25. Question

A furniture dealer deals in only two items : tables and chairs. He has 30000 to invest and a space to store at most 60 pieces. A table costs him 1500 and a chair 300. Formulate the data in the form of inequations and draw a graph representing the solution of these inequation.

Answer

Let the number of tables and chairs be x and y respectively.

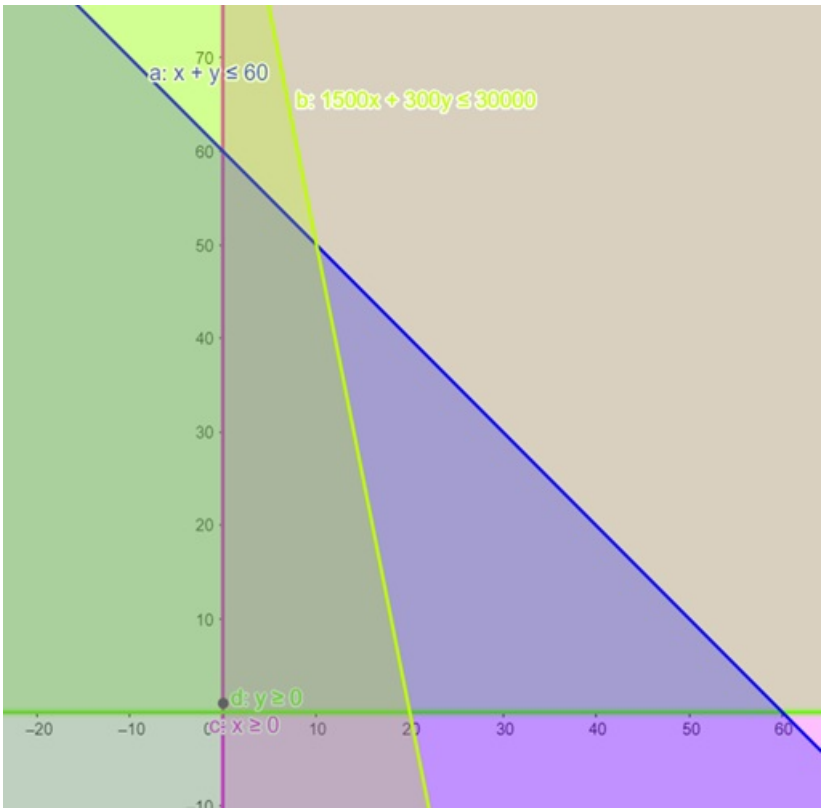
Therefore $x \geq 0 , y \geq 0$

Now the maximum number of pieces he can store = 60.

Therefore , $\boldsymbol{x + y \leq 60}$ .....(1)

Also it is given that maximum amount he can invest = 30000

Therefore , $\boldsymbol{1500x + 300y \leq 30000}$ ..... (2)



Therefore , the shaded protion (i.e. A) together with its boundary represents the solution set of the given inequation.

No. of tables = x = 10

No. of chair = y = 50

26. Question

If a young man rides his motorcycle at 40 km per hour, he has to spend 6 per km on petrol and if he rides it at 50 km hour, the petrol cost rises to 10 per km. He has 500 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Formulate the data in the form of inequation and draw a graph representing the solution of these inequations.

Answer

Let the distance covered with speed 40 km/hr = x km

And the distance covered with speed 50 km/hr = y km

We know that,

$$Time = \frac{Distance}{Speed}$$

Therefore , maximum speed covered within one hour is

$$\frac{x}{40} + \frac{y}{50} \leq 1$$

Thus , according to equation ,

Maximum speed covered , $Z_{max} = x + y$

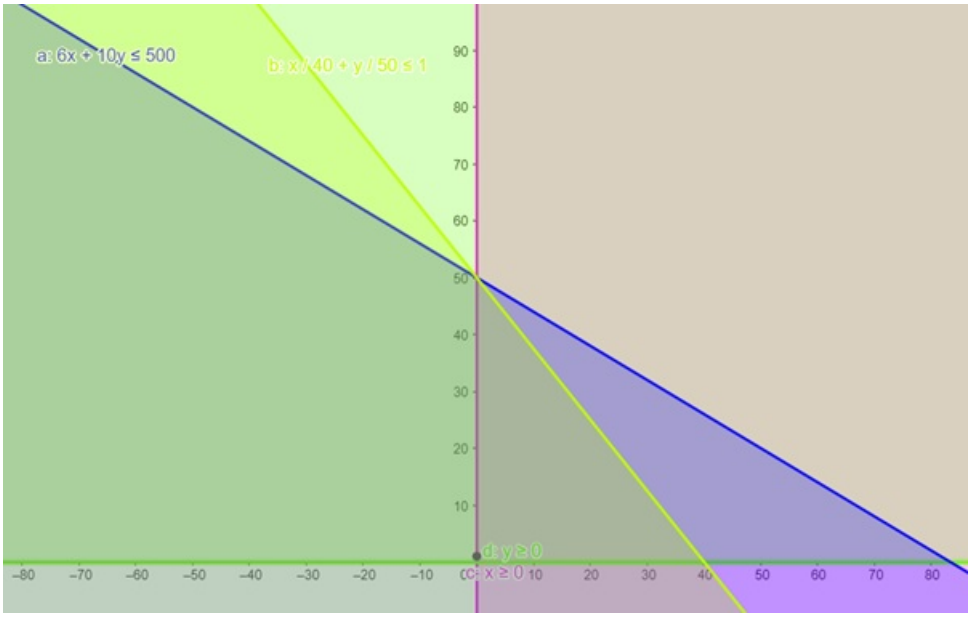
Subject to the constraint ,

$$6x + 10y \leq 500$$

$$\frac{x}{40} + \frac{y}{50} \leq 1$$

$$x, y \geq 0$$

Now plotting both the line on graph paper , we have ,



Distance covered with speed 40 km/hr = x = 0

Distance covered with speed 50 km/hr = y = 50

Therefore , **maximum distance covered = 0 + 50 = 50 km**